Randomness enhances cooperation: A resonance-type phenomenon in evolutionary games

Jie Ren,[1] Wen-Xu Wang,[2] and Feng Qi[3,*]

[1]*Department of Physics, University of Fribourg, CH-1700, Fribourg, Switzerland*
[2]*Department of Electronic Engineering, City University of Hong Kong, Hong Kong SAR, China*
[3]*Biotechnology and Bioengineering Center and Department of Physiology, Medical College of Wisconsin, Milwaukee,*
Wisconsin 53226, USA

We investigate the effect of randomness in both relationships and decisions on the evolution of cooperation. Simulation results show, in such randomness' presence, the system evolves more frequently to a cooperative state than in its absence. Specifically, there is an optimal amount of randomness, which can induce the highest level of cooperation. The mechanism of randomness promoting cooperation resembles a resonancelike fashion, which could be of particular interest in evolutionary game dynamics in economic, biological, and social systems.

Cooperation is ubiquitous in the real world, ranging from biological systems to economic and social systems. However, the unselfish, altruistic actions apparently contradict Darwinian selection. Thus understanding the conditions for the emergence and maintenance of cooperative behavior among selfish individuals becomes a central issue [1]. In the last decades, several natural mechanisms of enforcing cooperation have already been explored such as kin selection [2], retaliating behavior [3], reciprocity [4], voluntary participation [5], development of reputation [6], or spatial extensions [7].

Since the pioneering work on iterated games by Axelrod [3], the evolutionary Prisoner's Dilemma game (PDG) as a general metaphor for studying cooperative behavior has drawn much attention from scientific communities. Szabó presented a stochastic evolutionary rule to capture the bounded rationality of individuals for better characterizing the dynamics of games in real systems [8]. The individuals can follow only two simple strategies: C (cooperate) or D (defect), described by

$$s = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ or } \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{1}$$

respectively. Each individual plays the PDG with its "neighbors" defined by their spatial relationships. The total income of the player at the site x can be expressed as

$$M_x = \sum_{y \in \Omega_x} s_x^T P s_y, \tag{2}$$

where s_x and s_y denote the strategy of node x and y. The sum runs over all the neighboring sites of x (this set is indicated by Ω_x) and the payoff matrix has a rescaled form suggested by Nowak and May [9]:

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*Electronic address: fqi@mcw.edu

$$P = \begin{pmatrix} 1 & 0 \\ b & 0 \end{pmatrix}, \tag{3}$$

where $1 < b < 2$. Then, the individual x randomly selects a neighbor y for possible updating of its strategy. The probability that x follows the strategy of the selected node y is determined by the total payoff difference between them:

$$W_{s_x \leftarrow s_y} = \frac{1}{1 + \exp[(M_x - M_y)/T]}, \tag{4}$$

where T characterizes the stochastic uncertainties, including errors in decision, individual trials, etc., $T=0$ denotes the complete rationality, where the individual always adopts the best strategy determinately. While $T>0$, it introduces some dynamical randomness that there is a small probability to select the worse one. $T \rightarrow \infty$ denotes the complete randomness of the decision. This choice of W takes into account the fact of bounded rationality of individuals in sociology and also reflects natural selection based on the relative fitness in terms of evolutionism. Szabó *et al.* studied the effect of dynamical randomness T on the stationary concentration of cooperators in Ref. [10].

In a recent paper, Perc studied the evolutionary PDG by introducing the random disorder in the payoff matrix [11]. The reported results indicated a resonancelike behavior that the frequency of the cooperators reaches its maximum at an intermediate disorder. Using a different approach, Traulsen *et al.* also found that the additive noise on the classical replicator dynamics can enhance the average payoff of the system in a resonancelike manner [12]. Vainstein and Arenzon also reported that the disorder in the underlying site diluted lattices can enhance the fraction of cooperators [13].

It is well-known that intrinsically noisy and disordered processes can generate surprising phenomena, such as stochastic resonance [14], coherence resonance [15], ordering spatiotemporal chaos by disorder [16], disorder-enhanced synchronization [17], ordering chaos by randomness [18], etc. In evolutionary games, the enhancement of the frequency of cooperators at intermediate noise intensities re-
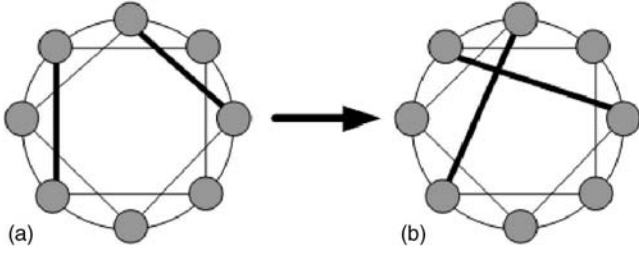
FIG. 1. (a) Illustration of a regular ring graph with connectivity $z=4$. Two edges are chosen and marked by thick lines. (b) Swap the ends of the two chosen edges. The swapped edges are marked by thick lines.

sembles the response of nonlinear systems to purely noisy excitations.

Presently, much interest has been given to evolutionary games on complex graphs or in structured population [5,7,19,20] by considering the fact that who-meets-whom is determined by spatial relationships or underlying networks. Complex networks provides a natural framework to describe the population structure. It has been shown that, in many real-life cases, relationships among networked individuals are neither completely random nor completely regular, but somewhere in between [21,22]. In other words, real networks have some degree of topological randomness. It is well-accepted that the topology of a network often plays crucial roles in determining the dynamics [23]. Therefore it is natural to ask whether this new type of randomness will play some constructive roles for the dynamics of the evolutionary games, i.e., is of benefit to the cooperation, such as stochastic uncertainties T [10], disordered payoff matrix [11], noise of replicator dynamics [12], or disordered environments [13].

In this Rapid Communication, we study the effects of both the topological randomness in individual relationships and the dynamical randomness in decision makings on the evolution of cooperation. We found that there exists an optimal amount of randomness, inducing the highest level of cooperation. The mechanism of randomness promoting cooperations resembles a resonancelike fashion, wherein the randomness-induced prevalence of the "good" strategy, i.e., cooperations, evokes the positive effect of the topological and dynamical randomness on the system.

To explore the topological randomness, we consider a homogeneous small-world network (HSWN) [24]. Starting from an undirected regular graph with fixed connectivity z and size N, a two-step circular procedure is introduced: (i) choose two different edges randomly, which have not been used yet in step (ii) and (ii) swap the ends of the two edges. Here, duplicate connections and disconnected graphs are avoided. The annealed randomness is characterized by the parameter p, which denotes the fraction of swapped edges in the network. (An illustration of the swap process is shown in Fig. 1.) In contrast to the Watts-Strogatz (WS) model [21], this network has small-world effect together with keeping the degree of each individual unchanged, so that the pure topological randomness can be investigated by avoiding any associated heterogeneity of degree distribution [24,25].

In all cases below, simulations start from a population of $N=1000$ individuals located on the vertices of a regular ring
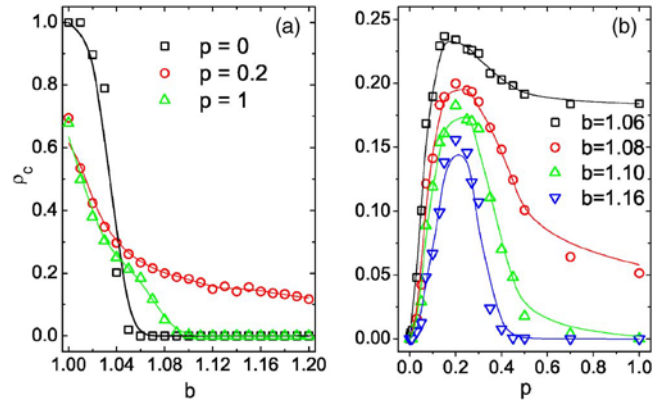

FIG. 2. (Color online) (a) The frequencies of cooperators ρ_c vs the temptation to defect b for $p=0$, 0.2, and 1, respectively, with $T=0.08$. (b) ρ_c as a function of the topological randomness p with various values of the temptation to defect b for $T=0.08$. The lines are used to guide eyes.

graph of $z=6$ with periodic boundary conditions. Initially, an equal percentage of strategies (cooperators or defectors) was randomly distributed among the population. Equilibrium frequencies of cooperators (ρ_c) were obtained by averaging over 5000 generations after a transient time of 10 000 generations. Each data is obtained by averaging over ten different network realizations with ten runs for each realization. Here, we adopted a synchronous updating rule.

Figure 2(a) shows the frequencies of cooperators ρ_c on the HSWN as a function of b for different values of the topological randomness P with $T=0.08$. In the equilibrium state, ρ_c is independent of the initial state and decreases monotonically as b increases. One can find that when $b<1.04$, cooperators dominate defectors significantly on the regular ring graph ($p=0$) and the more randomness of the network, the worse the cooperation is. While for $b>1.04$, the cooperator is nearly extinct in the cases of $p=0$ and $p=1$, which correspond to the complete regular network and the complete random network, respectively. However, the cooperator can survive around $p=0.2$, i.e., intermediate topological randomness.

The dependence of ρ_c on the topological randomness p is presented in Fig. 2(b). It illustrates that there is a clear maximum ρ_c around $p=0.2$, where cooperation can be revitalized and maintained for substantially large values of b. This phenomenon reveals that there exists somewhat resonant behaviors reflected by the optimal cooperation level at intermediate topological randomness, similar to the effects of noise and disorder in nonlinear systems. However, it is worth mentioning that the dynamics leading to these equilibriums is the same, and the resonant dependence of cooperation on p results from the changes of the equilibrium states. Moreover, in Fig. 2(b), one can find that as b increases, the positive effect of topological randomness on cooperation is restricted by the favored defection action, which is demonstrated by the reduction of the maximum value of ρ_c.

In the case of regular ring graph, the local spatial relationship constrains the spreading of cooperators. However, when $p \neq 0$, the shortcut generated by the edge-swapping reduces the average distance of the relationship network and pro-

motes the strategy spreading efficiency, which induces the survival and enhancement of cooperation. At this point, the individuals in the system can keep clustering locally, meanwhile they can communicate to each other more effectively due to the random shortcuts. As $p \rightarrow 1$, where the edges are exchanged sufficiently, the spatial relationships of the individuals are completely random and the system satisfies the mean-field approximation. Based on the classical mean-field theory, the average payoff for the C and D strategies are $M_C = z\rho_c$ and $M_D = z\rho_c b$, where $M_D > M_C$ always since $b > 1$. According to the dynamical rule (4), we can write down the following equation for the motion of the frequency of cooperators:

$$\frac{\partial \rho_c}{\partial t} = \rho_c(1 - \rho_c)[W_{D \leftarrow C} - W_{C \leftarrow D}]$$

$$= -\rho_c(1 - \rho_c)\tanh\left(\frac{M_D - M_C}{2T}\right). \tag{5}$$

It indicates that ρ_c tends to zero for the arbitrary value of T as $M_D > M_C$. On the other hand, in the absence of topological randomness ($p = 0$), the regular relationship graph can be considered as a one-dimensional system in which cooperators also die out [26]. While in the optimal region of p, the underlying network has the "small-world" property: the short average distance promotes the spreading of cooperators; the common cluster structure induces the clustering of cooperators, leading to the surviving and enhancement of cooperation [19]. Thus the optimal topological randomness p emerges.

To quantify the ability of topological randomness p to facilitate and maintain cooperation for various b more precisely, we study ρ_c depending on b and p together, as shown in Fig. 3(a). One can find that when $b < 1.04$, ρ_c is a monotonically decreasing function of p. We call it the harmful region [denoted by II in Fig. 3(a)] because the topological randomness p always decreases ρ_c. While for $b > 1.04$ [the region is denoted by I in Fig. 3(a)], there exists an optimal level of p around 0.2, resulting in the maximum value of ρ_c. The positive effect of the appropriate topological randomness p on the dynamics indicates the existence of an interesting resonancelike manner in the evolutionary game. Thus we call I the resonant region. In fact, there exists a region III beyond the shown range of b in Fig. 3(a), where the cooperators vanish and there is no p that can persist or enhance the cooperation. We call it the absorbing region.

Besides the topological randomness p, we have studied the effect of the dynamical randomness T. Figure 3(b) illustrates the phase diagram of the three regions of ρ_c in the parameter space (b, T). It shows clearly that as the dynamical randomness T increases, the resonant region reduces, i.e., the area of the range of b where the optimal p can promote the cooperation decreases, indicating the constructive effect of the optimal topological randomness is restricted by the higher dynamical randomness.

To investigate the combined effect of both the topological randomness and the dynamical randomness on the evolutionary dynamics, we fix $b = 1.08$, and calculate ρ_c in dependence on various p and T, as shown in Fig. 4. It is found that there
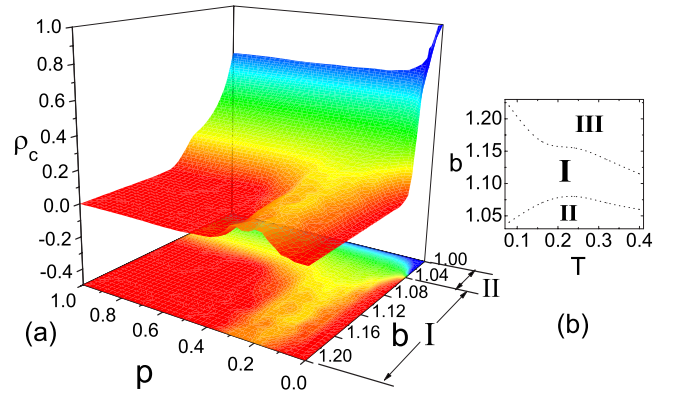

FIG. 3. (Color online) (a) The frequencies of cooperators ρ_c vs the parameter space (b, p) for $T = 0.08$. This figure illustrates two regions for b: I is the resonant region where there is an optimal amount of topological randomness p enhancing ρ_c; II is the harmful region where the topological randomness p decreases the level of the cooperation ρ_c. In fact, there exists a region III beyond the shown range of b in this figure, where the cooperators vanish and there is no p that can persist or enhance the cooperation. We call it the absorbing region. (b) The phase diagram of the three regions of ρ_c in the parameter space (b, T). It illustrates that the resonant region decreases as the dynamical randomness T increases.

exists a clear "optimal island" in the parameter space (T, p) where ρ_c reaches the highest value, indicating that the cooperation can be promoted by both the appropriate topological and the dynamical randomness. In other words, the resonance induced by the dynamical randomness can be enhanced by the topological randomness, just as the noise-induced temporal and spatiotemporal order can be greatly enhanced by an appropriately pronounced small-world connectivity of coupled elements [27].

In addition, we study the PDG on the WS network [21] to give a comparison with the cases on HSWNs [28]. In Fig. 5, we calculate the dependence of ρ_c on the rewiring probability p_r of the WS model. Contrary to the results in Fig. 2(b), there is not any optimal amount of the topological randomness. Instead, ρ_c rapidly approaches a plateau at $p_r = 0.2$, which is the optimal value in the case of HSWNs. Since the only
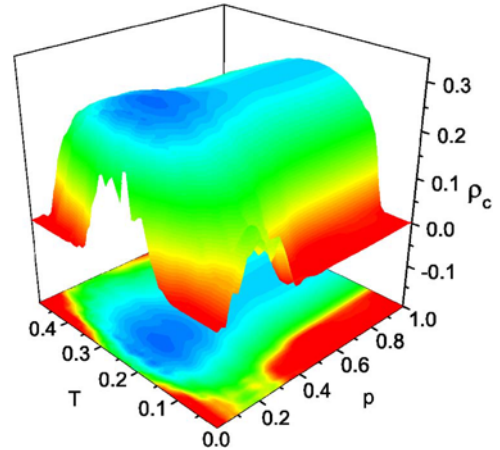

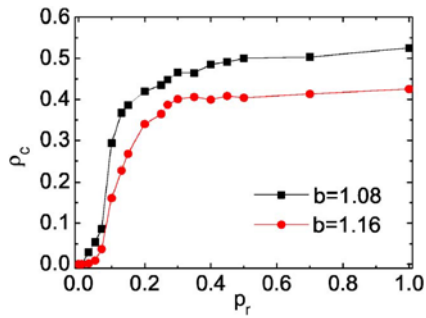FIG. 4. (Color online) The frequencies of cooperators ρ_c vs the parameter space (T, p) for fixed $b = 1.08$.

FIG. 5. (Color online) The frequencies of cooperators ρ_c as a function of the rewired probability p_r with $b=1.08$ and 1.16 for $T=0.08$. The lines are used to guide eyes.

difference between the HSWN and the WS model is that the variance of the degrees in the latter is nonzero [24], we conclude that when p_r is over 0.2, ρ_c can be enhanced considerably as a result of the redundant heterogeneity on relationships among individuals. Moreover, due to the additive heterogeneity, the plateau in Fig. 5 is clearly higher than the corresponding maximal values in Fig. 2(b). Thus the plateau of ρ_c is the combined effect of the topological randomness and the heterogeneous spatial relationships.

To summarize, we have studied the effects of both the topological randomness and the dynamical randomness on the evolutionary Prisoner's Dilemma game and found that there exists an optimal amount of randomness, leading to the highest level of cooperation. The mechanism of randomness promoting cooperation resembles an interesting resonance-like phenomenon, wherein the randomness-induced prevalence of the cooperation evokes the positive role of the topological and dynamical randomness in the system. Moreover, we find that the heterogeneity in the underlying relationship net also enhances the cooperation. Although our work is exclusively based on the evolutionary Prisoner's Dilemma game, the "resonant" behavior may play a significant role in other styles of evolutionary dynamics.

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