Optical response for a discrete stripe

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The optical response of a charge stripe in the presence of pinning impurities is investigated. We address the issue of a discrete description of the stripe, and discuss its quantitative relevance with respect to a continuum one in the light of recent optical-conductivity measurements in cuprate compounds.

The role of charge inhomogeneities in doped two-dimensional antiferromagnets has been recently the subject of intense experimental and theoretical investigations [1]. In particular, the physics of high-T_c cuprates stimulated the interest on theoretical models for charge segregation in one-dimensional stripes acting as domain walls for the surrounding antiferromagnetism. In the present paper we study the optical response of a string of holes (stripe) in the presence of pinning impurities.

Let us consider a single vertical line of holes enumerated by n on a square lattice with lattice constant a. The holes can move only in the transversal (x) direction. The phenomenological Hamiltonian describing the system is

$$H = \sum_n \left[-t\cos(p_n a) + \frac{J}{2a^2}(u_{n+1} - u_n)^2 \right], \qquad (1)$$

where t is the hopping parameter, a is the lattice constant, u_n is the displacement of the n-th hole from the equilibrium position, p_n is its conjugate transversal momentum, and J is the stripe stiffness, determined by the surrounding antiferromagnetism. We put $\hbar = k_B = 1$. As it has been shown in Ref. [2], at $t/J > 4/\pi^2$ the stripe depins from the lattice. In this regime we can expand the cos-term as $\cos(p_n a) \sim \text{const.} + (p_n a)^2/2$, so

that the corresponding action for the dimensionless field $\psi_n = u_n/a$ reads

$$S_0 = \frac{1}{2at} \sum_n \int d\tau \left[\left(\frac{\partial \psi}{\partial \tau} \right)^2 + (Jt)(\psi_{n+1} - \psi_n)^2 \right]. \qquad (2)$$

The (bare) Green function for the ψ field reads:

$$\mathscr{D}_0(q, \omega_m) = (at)[\omega_m^2 + \omega_J^2 \sin^2(qa/2)]^{-1}, \qquad (3)$$

with $\omega_J = 2\sqrt{Jt}$. Thus, in the long-wavelength limit ($q \to 0$) the ψ field displays a sound-like behavior $\omega = vq$ with the velocity $v = a\omega_J/2$. When the electric field is applied perpendicular to the stripe, a current arises $J = e\sum_n(\partial\psi_n/\partial t)$, and the transversal wandering of the holes shows perfect Fröhlich's conductivity:

$$\Re\sigma(\omega) = -e^2\omega\Im\mathscr{D}_0(q = 0, i\omega_m \to \omega - i\varepsilon), \qquad (4)$$

where the analytical continuation of the Green function in the lower half-plane has been performed. In Ref. [3], by assuming from the beginning a continuum ($a \to 0$) description for the field $\psi_n \to \psi(y)$ and $\psi_{n+1} - \psi_n \to \partial\psi/\partial y$, it was shown that the interaction with the impurities destroys the perfect conductivity $\Re\sigma(\omega) = e^2\pi at\delta(\omega)$ of the clean case, and $\sigma(\omega)$ displays a peak at finite frequency. The value of this frequency was evaluated by means of a diagrammatic approach for the disorder, both in the weak- and strong-pinning regimes. Here we investigate how these results are modified within a discrete description for the stripe.

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We represent the pinning by the impurities as a parabolic confining potential, provided by the scattering centers located at coordinates i along the stripes, with strength V_0, i.e. $S = S_0 + (V_0/2) \int d\tau \sum_i (\psi_i - \beta_i)^2$. We describe the full, time-dependent solution $\psi_n(t) = \phi_n(t) + \psi_n^0$ as a fluctuation around the equilibrium configuration ψ_n^0 which the hole would assume in the absence of dynamical fluctuations. As a consequence, Eq. (3) will refer now to the dressed Green function $\mathscr{D}$ of the fluctuating field ϕ_n, evaluated according to the Dyson equation as $\mathscr{D}^{-1}(q,\omega_m) = \mathscr{D}_0^{-1}(q,\omega_m) - \Sigma = (\omega_m^2 + v^2 q^2 - \Gamma t)/(at)$, where we have performed a rescaling $\Sigma = \Gamma/a$ such that Γ has dimensions of energy. In Ref. [3] it has been suggested that the weak-pinning regime is the relevant one to describe stripe pinning in cuprates. In this regime, Γ can be evaluated in the Born approximation as $\Gamma(\omega_m) = -(V_0/2) n_i a + (V_0/2)^2 n_i a L^{-1} \sum_q D_0(q,\omega_m)$, where $\mathscr{D}_0$ was used for calculating the second-order correction and the density of impurities n_i arises after averaging over the impurity positions. On defining the frequency $\omega_0 = n_i v = n_i a \sqrt{Jt}$, the dimensionless quantities $\tilde{\omega} = \omega/\omega_0$ and $G = \Gamma t/\omega_0^2$, the parameter $\alpha = V_0/(2n_i a J)$, and performing the analytical continuation, the equation for Γ takes the form

$$G(\omega) = -\alpha - i \frac{\alpha^2}{2\tilde{\omega}} \frac{\omega_J}{\sqrt{\omega_J^2 - \omega^2}}, \quad \omega < \omega_J \tag{5}$$

$$G(\omega) = -\alpha - \frac{\alpha^2}{2\tilde{\omega}} \frac{\omega_J}{\sqrt{\omega^2 - \omega_J^2}}, \quad \omega > \omega_J. \tag{6}$$

The interaction with impurities generates both a real and an imaginary part in the self-energy $G = G' + iG''$. The real part G' determines the energy at which the zero-energy delta-like peak of the optical conductivity shifts, and G' controls the spreading of the peak around this value. Indeed, according to Eq. (4), the real part of the optical conductivity is $\Re\sigma(\omega) = -\sigma_0 \tilde{\omega} G''/[(\tilde{\omega}^2 + G')^2 + G''^2]$, where $\sigma_0 = e^2 at/\omega_0$. From Eqs. (5) and (6) one sees that at $\omega > \omega_J$ G'' vanishes and consequently also $\sigma(\omega)$. This result is a consequence of assuming a finite lower cut-off a for the lengths: this reflects in turn in an upper bound for the momenta and consequently for the frequencies. On the other hand, the relevant physical processes occur at an energy scale lower than ω_J, as we will discuss below. In particular, when $\omega \ll \omega_J$ Eq. (5) reduces to

$$G = -\alpha - \frac{i\alpha^2}{2\tilde{\omega}}. \tag{7}$$

which is exactly the result obtained in Ref. [3]. In such a case, the resulting conductivity reads

$$\Re\sigma(\omega) = \sigma_0 \frac{2(\omega/v)^2}{4(\omega/v)^2 [(\omega/v)^2 - 1]^2 + \alpha^4}, \tag{8}$$

with $v = \sqrt{\alpha}\omega_0$. We note that $\Re\sigma(\omega) \to 0$ both as $\omega \to 0$ and as $\omega \to \infty$, and has a peak at $\omega \simeq v$. The optical conductivity $\sigma(\omega)$ evaluated with the self-energy (5) and

(6) shows remarkable differences from Eq. (8) only when $v \sim \omega_J$, but as we shall discuss below this case is not physically relevant because in any case $v < \omega_J$. Dimensional estimates of Eq. (2) indicate that the effect of pinning can be understood in terms of trapping the sound mode on a finite length scale λ, so that $v = v/\lambda$. In the weak-pinning case ($\alpha \ll 1$) considered here λ is the Larkin–Ovchinnikov length. Because λ cannot be smaller than the lattice spacing a, $v < \omega_J$ always.

Let us discuss now the physical values of the parameters in comparison with recent measurements of the infra-red response of La-based cuprates [4]. The optical conductivity of these compounds displays a huge peak in the far infra-red region at a frequency around 5–25 meV (depending on doping), which we attributed to the pinning of transversal stripe wandering [3]. For cuprates, physical values of the parameters are $t \sim J \sim 0.1$ eV. By considering the dopants themselves as scattering centers, we can also estimate on average $n_i a = 0.5$ [3]. This means that $\omega_0 \sim 50$ meV, and because $n_i a \sim 1$ corresponds to weak pinning, we expect that the peak frequency $v = \sqrt{\alpha}\omega_0$ is further reduced with respect to this value, in agreement with experimental data. Note that for these physical values of t,J: (i) $t/J \sim 1$ so that the stripe is depinned from the lattice; (ii) $\omega_J = 200$ meV, thus v is always found in the range where $v \ll \omega_J$ and as a consequence Eq. (7) is a good approximation for the self-energy (5).

In summary, we evaluated the optical response of a discrete hole stripe in the presence of impurities. The impurity centers pin the sound-like mode associated to the transversal motion of the holes. By retaining the lattice parameter a as a lower cut-off for the length we find an upper limit for the optical response of the system. However, for parameter values relevant for cuprates the pinning process always takes place at energies $v \ll \omega_J$, where the details of the discrete lattice description can be neglected and a continuum ($a \to 0$) description for the elastic stripe can be adopted.

References

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