J Geod (2015) 89:189–194
DOI 10.1007/s00190-014-0765-1

COMMENT

A comment on the article "A collinearity diagnosis of the GNSS geocenter determination" by P. Rebischung, Z. Altamimi, and T. Springer

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Received: 10 June 2014 / Accepted: 24 September 2014 / Published online: 7 October 2014

Abstract Meindl et al. (Adv Space Res 51(7):1047–1064, 2013) showed that the geocenter z-component estimated from observations of global navigation satellite systems (GNSS) is strongly correlated to a particular parameter of the solar radiation pressure (SRP) model developed by Beutler et al. (Manuscr Geod 19:367–386, 1994). They analyzed the forces caused by SRP and the impact on the satellites' orbits. The authors achieved their results using perturbation theory and celestial mechanics. Rebischung et al. (J Geod doi:10.1016/j.asr.2012.10.026, 2013) also deal with the geocenter determination with GNSS. The authors carried out a collinearity diagnosis of the associated parameter estimation problem. They conclude "without much exaggerating that current GNSS are insensitive to any component of geocenter motion". They explain this inability by the high degree of collinearity of the geocenter coordinates mainly with satellite clock corrections. Based on these results and additional experiments, they state that the conclusions drawn by Meindl et al. (Adv Space Res 51(7):1047–1064, 2013) are questionable. We do not agree with these conclusions and present our arguments in this article. In the first part, we review and highlight the main characteristics of the studies performed by Meindl et al. (Adv Space Res 51(7):1047–1064, 2013) to show that the experiments are quite different from those performed by Rebischung et al. (J Geod doi:10.1016/j.asr.2012.10.026,2013) . In the second part, we show that normal equation (NEQ) systems are regular when estimating geocenter coordinates, implying that the covariance matrices associated with the NEQ systems may be used to assess the sensitivity to geocenter coordinates in a standard way. The sensitivity of GNSS to the components of the geocenter is discussed. Finally, we comment on the arguments raised by Rebischung et al. (J Geod doi:10.1016/j.asr.2012.10.026, 2013) against the results of Meindl et al. (Adv Space Res 51(7):1047–1064, 2013).

Keywords GNSS · Geocenter · Collinearity · Solar radiation pressure · Celestial mechanics

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1 A review of the article (Meindl et al. 2013)

Data basis and motivation Time series of geocenter coordinates were determined, separately from GPS and GLONASS observations, for the years 2008–2011. The four years of data were recorded by 92 globally distributed and permanently observing GPS/GLONASS receivers.

Whereas the x- and y-components of the geocenter were found to agree quite well for the two systems, large periodic excursions were visible in the GLONASS z-component. The variations show an eye-catching correlation with the maximum and minimum values of the angle β_s, the Sun's elevation above/below the orbital planes, suggesting a correlation of the z-component with one of the estimated orbit parameters (or with a linear combination thereof).

Mathematical/physical approach Meindl et al. (2013) apply the methods of first-order perturbation theory and celestial mechanics to study the correlation mechanism. Each orbit is modeled by six initial osculating elements and five SRP parameters (Meindl et al. 2013). In a first step, they decompose the constituents of the SRP model into the components R, S, and W of a system co-rotating with a GNSS satellite. The first axis e_R of this system points from the geocenter to the satellite; the second axis e_S is orthogonal to e_R and lies in the orbital plane; the third axis e_W is normal to the orbital plane. They show that an acceleration associated with the direct SRP parameter D_0 always (except for $\beta_s = 0°$) causes an acceleration $W_D = D_0 \sin\beta_s$, which is orthogonal to the orbital plane.

In a next step, they explicitly solve the Gaussian perturbation equations (Beutler 2005, Vol I, Chapter 6.3) of a GNSS-like satellite for a constant acceleration W in W-direction. They find that the resulting perturbed orbit is shifted in parallel w. r. t. a mean orbit by the distance of

$$\delta w = \frac{W}{n^2} \quad \text{(in units of meters)},$$

where n is the mean motion of the satellite. Consequently, the constant SRP parameter D_0 is always associated with a parallel shift of the orbit.

Relation of geocenter z-coordinate and SRP Based on three different (theoretical) GNSS configurations, the authors explain the mechanism of how a parallel shift of an orbital plane may be (partly) compensated by a z-shift of the geocenter. They raise the question whether an estimated geocenter shift may be described by the differences of two sets of D_0 parameters, where one D_0-set is estimated together with geocenter coordinates and the other without. Based on the results from perturbation theory, they derive the equation

$$\delta z = \frac{\sum_{\ell=1}^{p} \Delta D_\ell \sin \beta_{s\ell}}{n^2 \cos i}, \qquad (1)$$

describing the geocenter z-shift δz as a function of the mean plane-specific D_0-differences ΔD_ℓ, $\ell = 1, 2, \ldots, p$, of all p orbital planes. The angle $\beta_{s\ell}$ is the elevation angle of the Sun w. r. t. the orbital plane ℓ and i is the inclination of the planes (assumed to be the same for all satellites).

Meindl et al. (2013) applied Eq. (1) to the four-year GLONASS and GPS time series and were able to represent the estimated z-shifts of the geocenter to a remarkable degree solely based on the D_0-differences. Not only the size but also the complicated signature is represented very accurately for both systems, GLONASS as well as GPS.

The results were illustrated by Figs. 7 and 10 in (Meindl et al. 2013), showing the estimated geocenter z-components, the corresponding values calculated with Eq. (1), and the differences of both. As the two figures show the key findings of the study, they are reproduced here for convenience as Figs. 1 and 2.

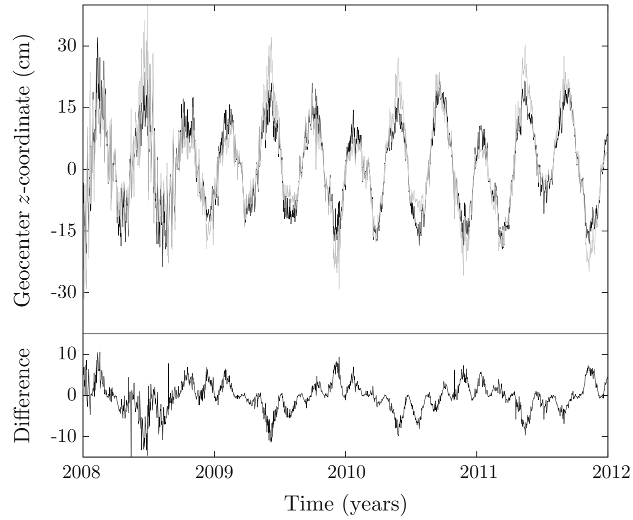

Fig. 1 Estimated (*black*) and calculated (*gray*) z-coordinates of the geocenter and their differences (*bottom*) for GLONASS

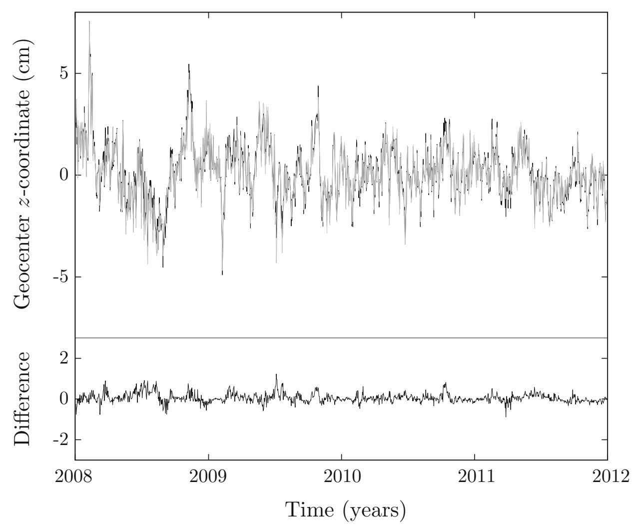

Fig. 2 Estimated (*black*) and calculated (*gray*) z-coordinates of the geocenter and their differences (*bottom*) for GPS

The two figures directly illustrate the quality of the approximations by Eq. (1): It is better than about 30 % for GLONASS and 15 % for GPS. This approximation is sufficient for the analysis presented by Meindl et al. (2013). Refinements of formula (1), including the impact of other SRP parameters, would be possible but were not considered by Meindl et al. (2013).

2 Estimating geocenter coordinates with GNSS

Definition Meindl et al. (2013) understand geocenter estimation as the determination of the components of the offset vector $\boldsymbol{G}$ of the entire network of n tracking stations w. r. t. the Earth's instantaneous center of mass. The geocentric position vector of a single station $i = 1, \ldots, n$ is given by $\boldsymbol{R}_i - \boldsymbol{G}$, where $\boldsymbol{R}_i$ is the station position w. r. t. the origin of the terrestrial reference frame.

Normal equation systems Geocenter coordinates, i. e., the components of $\boldsymbol{G}$, must be estimated together with all other parameters occurring in a data analysis of global networks. Following the least-squares principle, the normal equation (NEQ) system

$$\boldsymbol{A}^T\boldsymbol{P}\boldsymbol{A}\,\boldsymbol{x} - \boldsymbol{A}^T\boldsymbol{P}\,\boldsymbol{l} = \boldsymbol{N}\boldsymbol{x} - \boldsymbol{b} = \boldsymbol{0}, \quad (2)$$

can be established, where $\boldsymbol{A}$ is the first design matrix, $\boldsymbol{P}$ is the weight matrix associated with the GNSS observations, and $\boldsymbol{l}$ is the difference between the observations and the mathematical model evaluated at the a priori values. The definitions for the NEQ matrix $\boldsymbol{N}$ and vector $\boldsymbol{b}$ are obvious from Eq. (2). The NEQ system can be solved for $\boldsymbol{x}$, the unknown parameter increments w. r. t. the a priori values.

NEQ regularization Matrix $\boldsymbol{N}$ may or may not be regular. In global GNSS analyses, it is usually not, e. g., due to correlations of polar motion and UT1-UTC parameters with station network rotations and with the right ascension of the ascending nodes of the satellites.

It is in particular not possible to solve for geocenter motion $\boldsymbol{G}$ and station coordinates $\boldsymbol{R}_i$ simultaneously without providing additional information, because $\boldsymbol{G}$ represents a linear combination (weighted mean value) of all individual station coordinate changes. This singularity may be removed by introducing a no-net-translation (NNT) condition for the estimated coordinate corrections w. r. t. the corresponding a priori values for each of the three components x, y, and z.

The regularization conditions may be written as observations of special linear combinations of the estimated parameters

$$\boldsymbol{H}\boldsymbol{x} = \boldsymbol{0}, \quad (3)$$

where $\boldsymbol{H}$ is the coefficient matrix. To distinguish these observations from the ordinary GNSS observations, we call them pseudo-observations. Consult (Dach et al. 2007, Sect 9.3.9) for details concerning the coefficient matrix $\boldsymbol{H}$ of NNT conditions.

Applying the NNT conditions through the pseudo-observation equations (3) is a necessity if the geocenter is estimated together with the station coordinates. Other correlations and resulting singularities are not of our interest here and we just assume that the necessary regularizations are applied correctly. The normal equation contribution associated with Eq. (3) is

$$\boldsymbol{H}^T\boldsymbol{P}_c\,\boldsymbol{H}\,\boldsymbol{x} = \boldsymbol{0}\,. \quad (4)$$

The dimension of the weight matrix $\boldsymbol{P}_c$ equals the sum of all scalar conditions to be imposed, e. g., three for the NNT conditions. The weight matrix is usually assumed as diagonal and the i-th diagonal element is defined as $P_{c,i} = \sigma_0^2/\sigma_{c,i}^2$ (σ_0 is the a priori error of unit weight and $\sigma_{c,i}$ is the admissible RMS error of the network translation values, respectively).

The resulting complete NEQ system now reads as

$$\left(\boldsymbol{N} + \boldsymbol{H}^T\boldsymbol{P}_c\,\boldsymbol{H}\right)\boldsymbol{x} = \tilde{\boldsymbol{N}}\boldsymbol{x} = \boldsymbol{b}. \quad (5)$$

Alternatively to the regularization based on NNT conditions, one may constrain the station coordinates on their a priori values (by eliminating them from the NEQ system) and leave only the common offset $\boldsymbol{G}$ in the system. The latter avenue was actually chosen by Meindl et al. (2013), where the a priori coordinates were defined according to Meindl (2011, p. 98 "Multi-year solution").

Sensitivity of GNSS to geocenter motion The normal equation matrix $\tilde{\boldsymbol{N}}$ may be decomposed into a part $\tilde{\boldsymbol{N}}_{11}$ related to all parameters except the geocenter coordinates, and a part $\tilde{\boldsymbol{N}}_{22}$ related to the three geocenter coordinates:

$$\tilde{\boldsymbol{N}} = \begin{pmatrix} \tilde{\boldsymbol{N}}_{11} & \tilde{\boldsymbol{N}}_{12} \\ \tilde{\boldsymbol{N}}_{12}^T & \tilde{\boldsymbol{N}}_{22} \end{pmatrix}. \quad (6)$$

$\tilde{\boldsymbol{N}}_{11}$ is the normal equation matrix corresponding to a global analysis without geocenter estimation. By pre-eliminating all parameters of $\tilde{\boldsymbol{N}}_{11}$, we obtain the following (3×3)-NEQ matrix for the three geocenter coordinates:

$$\tilde{\boldsymbol{N}}_{\mathrm{GCC}} = \tilde{\boldsymbol{N}}_{22} - \tilde{\boldsymbol{N}}_{12}^T\,\tilde{\boldsymbol{N}}_{11}^{-1}\tilde{\boldsymbol{N}}_{12}\,. \quad (7)$$

Equation (7) assumes the regularity of $\tilde{\boldsymbol{N}}_{11}$.

The reduced NEQ matrix $\tilde{\boldsymbol{N}}_{\mathrm{GCC}}$ is regular: A singular value decomposition (for the arbitrarily chosen day 60 of year 2011) results in the 2-norm condition numbers 2.9 and 6.5 for the GPS-only and GLONASS-only matrices, respectively.

The covariance matrix $\sigma_p^2 \cdot \tilde{\boldsymbol{N}}_{\mathrm{GCC}}^{-1}$ (where σ_p is the a posteriori standard deviation of unit weight) therefore tells how well the geocenter may be determined. Table 1 contains the RMS errors of the estimated geocenter coordinates when set up as unknown parameters.

To illustrate the sensitivity of the GNSS NEQ systems w. r. t. the z-component of the geocenter furthermore, we generated global solutions without solving for the geocenter, but keeping it fixed at values shifted w. r. t. the suspected true value.

If the NEQ systems were not sensitive to the geocenter, the a posteriori standard deviations of unit weight would not change as a function of the fixed geocenter positions. Figure 3 shows that the a posteriori standard deviation of the

Table 1 Average RMS (in mm) of daily geocenter estimates from GPS and GLONASS for 2011; the a posteriori standard deviations of unit weight are around 1.3 mm for both systems

Component	GPS	GLONASS
x	0.9	1.6
y	0.9	1.6
z	1.3	4.4

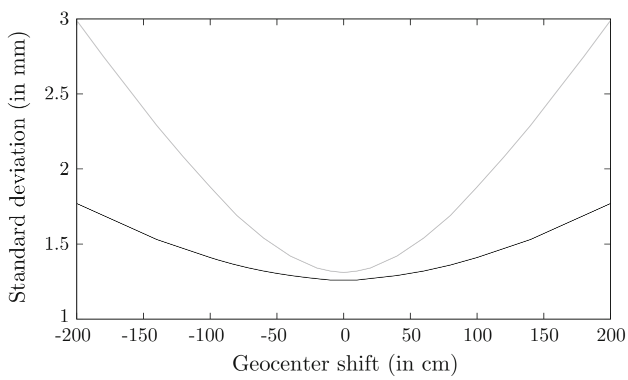

Fig. 3 A posteriori standard deviation of unit weight as a function of the geocenter shift for GPS (*gray*) and GLONASS (*black*)

observations is a function of the introduced z-shifts of the geocenter. Similar figures may be generated for the x- and y-components of the geocenter showing even more pronounced sensitivities. Let us point out that Fig. 3 merely is an illustration. Table 1 tells how well the geocenter may be estimated from GPS and GLONASS, respectively, in a statistically perfect environment.

Based on Table 1 and Fig. 3 we conclude that properly constrained GNSS NEQ systems are sensitive to the geocenter coordinates.

3 Remarks concerning section *"8 Discussion"* in (Rebischung et al. 2013)

Rebischung et al. (2013) dedicate their section *"8 Discussion"* almost completely to commenting the results achieved by Meindl et al. (2013). Subsequently, we will address the most important aspects raised in sections 8.1 and 8.2 of this article.

"8.1 Comments to Meindl et al. 2013 Rebischung et al. (2013)" state that the main argument of Meindl et al. (2013) relies on experiments that are similar in essence to their simulations. However, as summarized in Sect. 1, the main topic of Meindl et al. (2013) is the analysis of the correlation of the geocenter z-component with the SRP parameters, especially the constant D_0 parameter. This analysis is purely based on perturbation theory and celestial mechanics and is as such independent of any experimental setup. The GPS and GLONASS time series, each based on four years of observation data, were then used to successfully confirm the findings.

Rebischung et al. (2013) introduce an artificial geocenter shift $\delta z = 1$ cm in their simulations and in real solutions (provided by ESA) to test Eq. (1). They find that they were unable to reproduce the results from Meindl et al. (2013) and implicitly conclude that Meindl et al. (2013) must be wrong.

As Rebischung et al. (2013) were not able to reproduce our results, we performed their experiment using, however, the data sets and general analysis procedures outlined previously. The two solutions required for Eq. (1) were computed with the geocenter offset $\boldsymbol{G}$ fixed to zero and to (0, 0, 1) cm, respectively. The mean value and standard deviation of the daily shifts resulting from Eq. (1) are 9.4 and 1.4 mm for GPS and 10.5 and 3.2 mm for GLONASS. Both mean values are very close to the introduced shift $\delta z = 1$ cm. The day-to-day variability of the z-shifts (expressed in percent) is comparable to the differences in the original experiment as shown in Figs. 1 and 2.

In Sect. 2, it was mentioned that the regularization of the NEQ system may either be realized by a NNT condition or alternatively by tightly constraining the station coordinates to properly defined a priori values. All results presented by Meindl et al. (2013) were based on the latter option. To rule out the datum definition as a potential reason for the discrepancies between Rebischung et al. (2013) and Meindl et al. (2013), the "1 cm experiment" was repeated for GPS and GLONASS following an alternative processing scheme.

The first set of D_0 parameters was computed from a solution where only a no-net-rotation (NNR) condition was imposed; the geocenter was implicitly realized by a translation of the entire station network. Subsequently, all estimated station coordinates have been shifted by $\delta z = 1$ cm, re-introduced as new a priori values, and constrained by an additional NNT condition. The resulting second set of D_0 parameters thus refers to a solution where the (implicit) geocenter is shifted by 1 cm in z-direction and kept fixed. This approach closely follows the procedure used by Rebischung et al. (2013, email communication).

The resulting time series of z-shifts computed with Eq. (1) exhibits a mean value of 7.8 mm and a standard deviation of 1.2 mm for GPS and 9.5 and 2.9 mm for GLONASS, respectively. The results are not the same as those achieved with fixed coordinates and explicit geocenter parameters, but they are still close to the expected shift of 1 cm. The definition of the geodetic datum (fixed vs. NNT condition) can thus be dismissed as a cause why Eq. (1) seems to be not valid for the experiments in Rebischung et al. (2013).

"8.2 Lower quality of GLONASS-derived geocenter time series" Rebischung et al. (2013) state that the geocenter time series derived from GLONASS observations are of much lower quality than GPS-derived geocenter time series. This

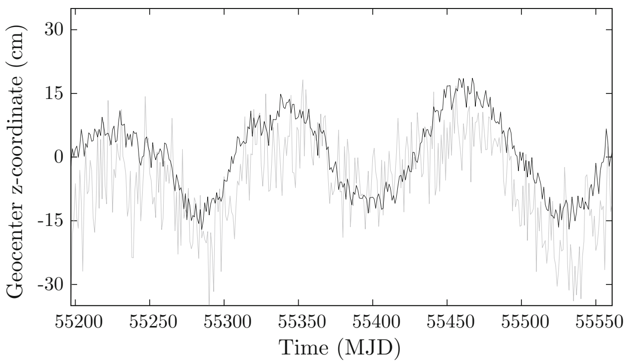

Fig. 4 GLONASS-derived geocenter z-coordinates for year 2010 with (*black*) and without (*gray*) ambiguity resolution

is a somewhat pessimistic point of view. The quality of the GLONASS time series (at least in the years 2009–2011) is, although not as good as, at least comparable to that of GPS. Table 1 shows that the RMS ratio of the GLONASS geocenter time series w.r.t. GPS is of the order of 2–3. Moreover, the GPS- and GLONASS-specific time series of the geocenter x- and y-components agree very well as shown in Meindl et al. (2013, Fig. 2). It is, however, true that the z-component shows large periodic variations for GLONASS, whereas GPS does not exhibit such artifacts.

Rebischung et al. (2013) propose that these excursions may be caused by the large number of unresolved ambiguities: fewer than 50 % of the ambiguities are resolved for GLONASS as compared to about 90 % for GPS (Meindl et al. 2013). They support their assumption by the findings of Springer (2000), who inspected GPS geocenter time series from the CODE analysis center for the years 1993–1999. Significant geocenter z-variations of about 20 cm were visible before ambiguity resolution was activated in 1994.

The CODE processing protocols revealed, however, that the average number of resolved ambiguities was only around 20 % during that time period. With this number in mind, it seems not plausible that (the comparatively high) ambiguity resolution rate of about 50 % for GLONASS is responsible for the large periodic variations. To further clarify this issue, we have recomputed the GLONASS geocenter time series for the year 2010, once with ambiguity resolution, once without resolving any ambiguities. Figure 4 shows that although the noise is clearly higher for the ambiguity-float solution, the signature is comparable in both, size and period, for the two solutions.

4 Summary and conclusion

The estimation of geocenter coordinates from GNSS observations was recently discussed in two articles.

Meindl et al. (2013) derived the correlation between the SRP parameter D_0 and the z-component of the geocenter using the methods of perturbation theory and celestial mechanics. They validated the results with geocenter time series derived from four years of GPS and GLONASS observations.

Rebischung et al. (2013), on the other hand, studied the collinearity of the geocenter coordinates with other GNSS-typical parameters. They base their study on a simulated set of observations and find a very high collinearity of the geocenter coordinates with the satellite clock corrections, whereas the SRP parameters do not play a significant role. They conclude that: (a) GNSS are in general insensitive to geocenter variations and (b) the results from Meindl et al. (2013) are questionable.

In Sect. 2, we have shown that the NEQ matrices associated with geocenter determination are regular and that the geocenter coordinates can be estimated with an accuracy derived from the covariance matrix (at least in the absence of systematic errors). The high collinearity of a parameter with a linear combination of other parameters is not an indicator for the determinability of almost collinear parameters. A well-known example are troposphere parameters, station height, and clock corrections (Rothacher and Beutler 1998). Although highly correlated, the individual parameters still can be determined quite well.

In Sect. 3, we have demonstrated that the experiments and argumentation of Rebischung et al. (2013) do not allow it to reject the results from Meindl et al. (2013). We have in particular reproduced the "1 cm experiment" of Rebischung et al. (2013) and Eq. (1) works very well in our realization of this experiment; the z-shift computed from the D_0-differences is quite close to the expected 1 cm for both, GPS and GLONASS. Additional experiments showed that the geodetic datum definition (fixed coordinates vs. NNT condition) is not responsible for Eq. (1) being valid or not. Finally, ambiguity resolution has been ruled out as a reason for the periodic variations of the GLONASS-derived geocenter z-coordinate.

From our perspective, the results of the two articles (Rebischung et al. 2013; Meindl et al. 2013) are not in conflict as they address different aspects: the statistical properties (collinearity) of geocenter coordinates as parameters in a GNSS analysis on one hand; the mathematical correlation of the geocenter z-coordinate with the direct SRP parameter D_0 on the other hand. Both approaches give interesting insights in the mechanism of geocenter estimation with GNSS and are, as such, valuable scientific contributions.

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