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ERRATUM

Erratum to: Sharp upper bound for the first eigenvalue

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In our paper "Sharp upper bounds for the first eigenvalue" [1], we proved the following theorem.

Theorem 1 *Let $(\overline{M}, ds^2)$ be a non compact rank-1 symmetric space and M be a closed hypersurface in $\overline{M}$ which encloses the bounded region Ω. Then*

$$\lambda_1(M) \leq \lambda_1(S(R)) \left(\frac{Vol(M)}{Vol(S(R))} \right) + \frac{1}{\sinh^2 R \, Vol(S(R))} \int_M \| \nabla^M \sinh r \|^2$$

where $R > 0$ is such that $Vol(\Omega) = Vol(B(R))$; here $B(R)$ and $S(R)$ are the geodesic ball and geodesic sphere respectively of radius R.

Further, the equality holds if and only if M is a geodesic sphere of radius R.

The proof of this theorem uses the following inequality of [1]: For a closed hypersurface M in the noncompact rank-1 symmetric space $\overline{M}$,

$$\lambda_1(M) \int_M f^2 dm \leq \int_M \| \nabla^M f \|^2 \, dm + \int_M f^2 \left(\lambda_1(S(r)) - \sum_{i=1}^{kn} \left(\frac{\partial f_i}{\partial \eta} \right)^2 \right) dm$$

where $f = \sinh r$.

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Substituting the value of $\lambda_1(S(r))$ and applying Lemmas 1 and 2 of [1], we get

$$\lambda_1(M)Vol(S(R))\sinh^2 R \le (kn-1)Vol(M) - (k-1)\tanh^2 R\, Vol(S(R)) + \int_M \| \nabla^M \sinh r \|^2\, dm \tag{0.1}$$

when $k = 1$, that is for $\overline{M} = \mathbb{H}^n$, we get the required inequality stated in Theorem 1.

In a later inspection, we observed that when $k > 1$, we can not assert the validity of the inequality

$$\lambda_1(M)Vol(S(R))\sinh^2 R \le \big((kn-1) - (k-1)\tanh^2 R\big)\, Vol(M) + \int_M \| \nabla^M \sinh r \|^2\, dm$$

from Eq. (0.1). This was used to complete the proof Theorem 1.

As the above inequality does not hold in general, the Theorem 1 stated as above needs correction. The correct statement and proof of the theorem are as follows.

Theorem 2 *Let $(\overline{M}, ds^2)$ be a non-compact rank-1 symmetric space with $\dim \overline{M} = kn$ where $k = \dim_{\mathbb{R}}\mathbb{K}$; $\mathbb{K} = \mathbb{R}, \mathbb{C}, \mathbb{H}$ or $\mathbb{C}a$. Let M be a closed hypersurface in $\overline{M}$ which encloses the bounded region Ω. Then for $k = 1$, we have*

$$\frac{\lambda_1(M)}{\lambda_1(S(R))} \le \frac{Vol(M)}{Vol(S(R))} + \frac{1}{(n-1)Vol(S(R))}\int_M \| \nabla^M \sinh r \|^2$$

and for $k > 1$, we have

$$\lambda_1(M) \le \lambda_1(S(R))\left(\frac{Vol(M)}{Vol(S(R))}\right) + \frac{k-1}{\cosh^2 R}\left(\frac{Vol(M)}{Vol(S(R))}\right) + \frac{1}{\sinh^2 R\, Vol(S(R))}\int_M \| \nabla^M \sinh r \|^2$$

where $R > 0$ is such that $Vol(\Omega) = Vol(B(R))$; here, $B(R)$ and $S(R)$ are the geodesic ball and geodesic sphere, respectively, of radius R. Further, the equality holds in above two inequalities if and only if M is a geodesic sphere of radius R.

Proof When $k = 1$, the inequality (0.1) reduces to

$$\lambda_1(M)Vol(S(R))\sinh^2 R \le (n-1)Vol(M) + \int_M \| \nabla^M \sinh r \|^2\, dm.$$

Using the fact that $\lambda_1(S(r)) = \frac{n-1}{\sinh^2 r}$ for all $r > 0$, we get the required result

$$\frac{\lambda_1(M)}{\lambda_1(S(R))} \le \frac{Vol(M)}{Vol(S(R))} + \frac{1}{(n-1)Vol(S(R))}\int_M \| \nabla^M \sinh r \|^2 \tag{0.2}$$

for hypersurfaces in $\mathbb{H}^n$.

When $k > 1$, we get

$$\begin{aligned}\lambda_1(M) \leq & \left(\frac{kn-1}{\sinh^2 R} - \frac{k-1}{\cosh^2 R}\right)\frac{Vol(M)}{Vol(S(R))} \\ & + \frac{1}{Vol(S(R))}\left(\frac{k-1}{\cosh^2 R}Vol(M) + \frac{1}{\sinh^2 R}\int_M \parallel \nabla^M \sinh r \parallel^2\right) \\ & = \lambda_1(S(R))\left(\frac{Vol(M)}{Vol(S(R))}\right) + \frac{k-1}{\cosh^2 R}\left(\frac{Vol(M)}{Vol(S(R))}\right) \\ & + \frac{1}{\sinh^2 R\, Vol(S(R))}\int_M \parallel \nabla^M \sinh r \parallel^2 .\end{aligned} \tag{0.3}$$

The equality in (0.2) and in (0.3) follows from the equality criterion in Lemmas 1 and 2 and $\frac{\partial f_i}{\partial \eta}(q) = 0$ for all $i = 1, \ldots, kn$ for all points $q \in M$. This happens if and only if M is a geodesic sphere. □

References

1. Binoy, R., Santhanam, G.: Sharp upper bound for the first eigenvalue. Geometriae Dedicata **169**(1), 397–410 (2014)