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Experimental investigation of axial dispersion in a horizontal rotating cylinder

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Abstract This work reports experimental measurements of the dispersion of particles during rotation in a horizontal cylinder. The axial dispersion of a pulse of approximately monodisperse black glass ballotini into a bed of clear glass ballotini of the same size is analysed. This is done using a sectioning technique, where the concentration is determined throughout the cylinder for a given rotation time and speed. The concentration profile is fitted to an appropriate solution of Fick's second law to determine the dispersion coefficient. The dispersion coefficient is compared for various drum rotation rates and glass ballotini sizes. The cylinder was filled to 35 % by volume and rotated at a range of speeds between 5 and 20 rpm. The particle sizes vary from 1.14 to 3.15 mm. The dispersion coefficient was found to be dependent on both particle size and rotation speed. As the rotation speed, ω, was increased the dispersion coefficient increased proportionally to $\omega^{0.8}$. As the particle diameter, d_p, was increased the dispersion coefficient increased proportionally to $d_p^{1.84}$. These results are compared with previous experimental and simulation data, in particular the simulations of Third et al. (Powder Technol 203:510, 2010). Strong agreement was found between the simulations of Third et al. and the experimental results.

Keywords Axial dispersion · Rotating cylinders · Kiln · Diffusion

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1 Introduction

Many materials from manufacturing fields including mineral, ceramic, metallurgical, chemical, pharmaceutical, waste and food are processed using rotating drums. As simple as this plant appears, there remains much that is unknown about the behaviour of grains within rotating cylinders [1–4]. One particular aspect of these systems which has received considerable attention is axial dispersion [5–9]. An understanding of dispersion within rotating cylinders is important because it controls the residence time distribution (RTD) in industrial kilns and is a critical parameter in models of axial segregation [10–12]. However, precise experimental measurements of axial dispersion are difficult to obtain, with the result that there have been conflicting reports regarding the dependence of the rate of axial dispersion on operating parameters.This paper seeks to explore the reported unexplained discrepancies of the dependence of rotation rate and particle diameter [5,6].

It is now generally accepted that dispersion of homogeneous material in horizontally rotating cylinders follows Fick's second law [13–15], Eq. (1)

$$\frac{\partial C(z,t)}{\partial t} = D_{\mathrm{ax}} \frac{\partial^2 C(z,t)}{\partial z^2} \qquad (1)$$

where z is axial position, D_{ax} is a constant dispersion coefficient, t is time and C is the concentration of 'marked' particles. In a typical experiment a pulse of marked particles ($C = 1$) is initially located axially in the centre of the cylinder. The remainder of the cylinder is filled with non-marked particles ($C = 0$). The length of the cylinder is $2L$ and the length of the pulse is $2l$. This initial configuration is shown schematically in Fig. 1.

Taberlet and Richard [16] simulated the diffusion of a granular pulse in a rotating drum using the discrete ele-

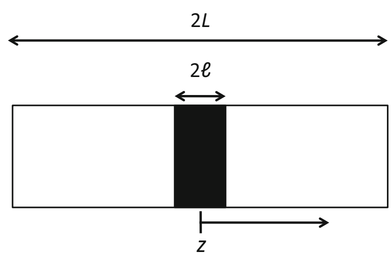


Fig. 1 Pulse initial condition used in measurements of axial dispersion

ment method (DEM). They modelled the particles as perfect spheres and considered two particle sizes, the smaller grains having a diameter of $d_s = 5\,\text{mm}$ and the larger a diameter of $2d_s$. The drum length was varied from $2L = 60d_s$ to $2L = 420d_s$ with the initial pulse length set to $2l = 25d_s$. The cylinder was filled to 37 % and rotated at 30 rpm. It was reported that the axial dispersion coefficient was independent of grain size.

Kahn and Morris experimentally observed the diffusion of a pulse in a rotating drum, varying the grain types and drum rotation rates [6]. The experimental set up consisted of a Pyrex tube 600 mm long with an inner diameter of 28.5 mm, filled to 28 % and rotated at either 0.31 or 0.62 rev/s. They used two different particle sizes. The larger grains were cubic white table salt or transparent glass spheres with a size range of 300–420 μm. The smaller grains were irregularly shaped black sand or bronze spheres ranging in size from 177–212 μm. The pulse was 1.5 mm wide of the smaller particles. Using a bulk imaging technique by placing a light source behind, they created a two dimensional projection of the radial core. Kahn and Morris observed that the pulse of small grains does not mix into the large grains but rather sinks below the surface of the larger grains and spreads axially in a radial core. They calculated the small grains disperse axially as t^α where $\alpha \sim 1/3 < 1/2$, independent of drum rotation rate and grain type. Thus the process is subdiffusive.

Parker et al. [17] used positron emission particle tracking (PEPT) to track the motion of a radioactive tracer particle within a partially filled drum. The experiments were conducted using 1.5 mm glass spheres in a 136 mm diameter drum or 3 mm glass spheres in either a 100 or a 144 mm diameter drum. The drums were rotated at speeds ranging from 10 to 65 rpm. The cylinder was filled to approximately 1/3 by volume. The results showed that the axial dispersion coefficient was independent of drum diameter but strongly dependent of particle size. This is in contradiction of Taberlet and Richard [16], who found that dispersion was independent of grain size. Parker et al. [17] observed that the dispersion was not proportional to drum speed. They showed a model of proportionality between the dispersion coefficient and the circulation frequency, defined as the frequency at which a particle travels its circular path, rotating with the cylinder within the particle bed followed by sliding down the free surface of the bed.

Third et al. [18] used the discrete element method to calculate axial dispersion coefficients for monosized particles in a rotating cylinder. They observed the effect of particle size, rotation speed, cylinder diameter, and gravity on the dispersion. The particle diameter was varied from 1 to 3.5 mm. Third et al. [18] found that the axial dispersion is proportional to $\omega^{0.74}$ for all particle sizes tested. Here ω is the rotation speed of the cylinder. As long as the drum size is sufficiently large, the size of the drum did not affect the dispersion coefficient. Third et al. [18] suggested there exists a critical value for the ratio drum diameter to particle diameter, D/d_p, equal to 25. Above this value a change in drum diameter did not affect the dispersion coefficient. The influence of drum diameter on axial dispersion was explored for a wider range of D by Third and Müller [19]. Third et al. [18] found the dispersion coefficient to be proportional to the particle diameter to the power of 1.9, $D_{\text{ax}} \propto d_p^{1.9}$. For the range of parameters they simulated they found the relation, $D_{\text{ax}} \propto (\omega d^2)(\frac{g}{\omega^2 d})^\lambda$ with $\lambda \approx 0.1$.

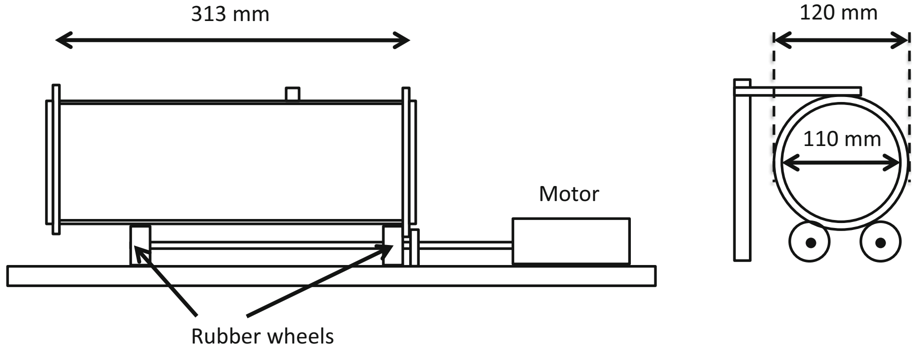
Sherritt et al. [20] reviewed literature on rotary kilns for both continuous and batch mixing, then proposed new design equations for the axial dispersion coefficient in terms of rotational speed, fill level, drum diameter, and particle diameter. They concluded that mixing in the axial direction is purely diffusive, caused by random collisions of particles in the active region. The axial dispersion coefficient was found to range from 10^{-7} to $10^{-4}\text{m}^2/\text{s}$. In contrast to Third et al. [18] they found that the dispersion coefficient is always proportional to the drum diameter. They suggested that the dispersion coefficient is proportional to the square root of the particle diameter and, in the rolling and cascading regimes, to the square root of the rotation speed.

This work reports experimental measurements of the axial dispersion of glass ballotini within horizontal rotating cylinders. Measurements are made using a destructive sectioning technique and the effects of particle size and rotation speed are examined.

2 Experimental method

The experimental set up consisted of an acrylic cylinder filled with glass ballotini and rotated about its axis by a motor. A shaft connected to the motor rotated rubber wheels, which in turn rotated the cylinder. The cylinder had a constant fill level (35 % by volume) and the rotation speed was varied from 5 to 20 rpm. Three different sizes of glass ballotini were used and these had size ranges of 1–1.3, 2–2.4 and 2.85–3.45 mm. Before placing the beads in the cylinder, both the beads and the interior of the cylinder were sprayed with anti-static spray to avoid the build up of static electricity. Hereafter each par-

Fig. 2 Experimental setup



ticle size will be described using the mean particle diameter, i.e. 1.14, 2.14 and 3.15 mm. For each of these particle sizes runs were performed at three different rotation speeds, 5, 10 and 20 rpm and for three different rotation time periods. A schematic of the experimental setup is shown in Fig. 2. The motor used was a Maxon EPOS 2 24/5 and was operated in 'velocity mode'. A high speed camera (Nikon, 496RC2) placed along the axis of the cylinder recorded the rotation of the cylinder to control for slip between the cylinder and the rubber wheels turning the cylinder. The same camera was also used in the process to count the beads (please see below).

The whole system was located on a Thor UltraLight Series I Breadboard with adjustable supports and a bubble level was used to ensure the system was flat. A perspex cylinder 313 mm long with an inner diameter of 110 mm and an outer diameter of 120 mm was used. The inside of the cylinder was sanded to avoid slip between the surface of the cylinder and the particles. Flanges were glued on to each end. The flanges have two functions: they allow the end plates to be attached to the cylinder and serve as a guide for the rubber wheels, which prevents movement of the cylinder in the axial direction. On one end the end plate was glued to the flange. The flange on the other side has holes which allow the end plate to be affixed with bolts. This design enables the end plate to be removed so that the cylinder could be loaded with particles, or the bed of particles to be sectioned using the splitter. The cover on this end is two half circles which are independently removable. This allowed one half to be removed and the splitter inserted without the particles falling out.

Experiments were performed using a 'pulse' initial condition, Fig. 1. An $l = 10.2$ mm pulse of black ballotini was loaded into a U shaped channel and slid into the centre of the cylinder, then the colourless ballotini were placed on either side. To ensure the same fill level for each run the ballontini were weighed before being placed into the cylinder. Once the particles had been loaded, the U shaped channel was removed, the end plate was fitted to the cylinder and it was rotated for the desired amount of time at a constant speed.

The axially-resolved concentration of black beads in the cylinder was calculated by dividing the axial length of the cylinder into bins. This was done using a splitter that creates 30 bins each 10.4 mm wide except for the two ends which are 10.9 mm wide. The splitter is constructed of 29 steel semi-circles with a radius of 110 mm and a thickness of 0.4 mm. The face-to-face spacing of the semi-circles was 10 mm and they were arranged on three 3 mm diameter steel rods for stability. To ensure the 10 mm spacing, 10 mm steel spacers are placed between each of the steel semi-circles. An acrylic semi-circular cover at the end of the splitter is used as a cover when turning the cylinder with the splitter inserted. Once the rotation had been completed, the splitter was slid into the top half of the cylinder. The cylinder was then rotated half a rotation such that the beads would fall into the splitter. After the beads had been rotated the half turn into the splitter, the splitter was pulled out one bin at a time to collect the beads.

The total number of ballotini in each bin was calculated by weighing the contents of the bin and dividing by the average mass of a bead. The number of black ballotini was then calculated by spreading the beads out into a mono-layer and photographing them. The resulting image was processed using the program ImageJ to count the black beads. The concentration of black ballotini was then calculated as the ratio of the number of black beads to the total number of beads in the bin.

Once the concentration of each bin was determined, the dispersion coefficient was calculated by fitting the data to Eq. (2)

$$C(Z,T) = \frac{1}{2}\sum_{n=-\infty}^{+\infty}\left[\mathrm{erf}\left(\frac{Z+\ell-2n}{\sqrt{4T}}\right)-\mathrm{erf}\left(\frac{Z-\ell-2n}{\sqrt{4T}}\right)\right] \tag{2}$$

with the dimensionless variables

$$Z = (z-s)/L, \quad T = t/(L^2/D_{\mathrm{ax}})$$

Equation 2 is the solution to Eq. (1) for the initial condition shown in Fig. 1. The variable s was added to account for the possibility that the pulse was not perfectly centred in the experiment.

The curve fitting tool in Matlab was used to find the optimum values for the variables D_{ax} and s within 95 %

confidence bounds. The average R-square value was 0.997 and the values obtained for s were small, having a maximum value of 2.4 mm. When the curve was fitted, only three terms were used to reduce the computational load: $n = -1$, 0 and 1. Due to the low values of T of all the experiments reported here, this simplification does not influence the accuracy of the results.

3 Results

Figure 3 shows the concentration profiles obtained for 1.14 mm glass ballotini. Data are shown for rotation speeds of 5, 10 and 20 rpm and for each rotation speed profiles are shown for three different values of t. For each profile Eq. (2) is plotted as a solid line using the best-fit values of D_{ax} and s. The experimental data is well described by Eq. (2).

Table 1 summarises the axial dispersion coefficients obtained for all the particle diameters, rotation speeds and times considered here. In most cases the values of D_{ax} obtained for different values of t are very similar, which supports the assumption that axial dispersion is governed by Fick's second law with a constant dispersion coefficient.

To determine the reproducibility of the results the 2.14 mm particles were rotated two separate times at 20 rpm for 360 s. The dispersion coefficients calculated were $1.84 \times 10^{-6}\ \mathrm{m^2/s}$ and $1.85 \times 10^{-6}\ \mathrm{m^2/s}$. This shows a strong repeatability of dispersion values.

Additionally the root mean square error was calculated for the fits of the concentration profile. It was found to be an average of 0.005 and no more than 0.0120.

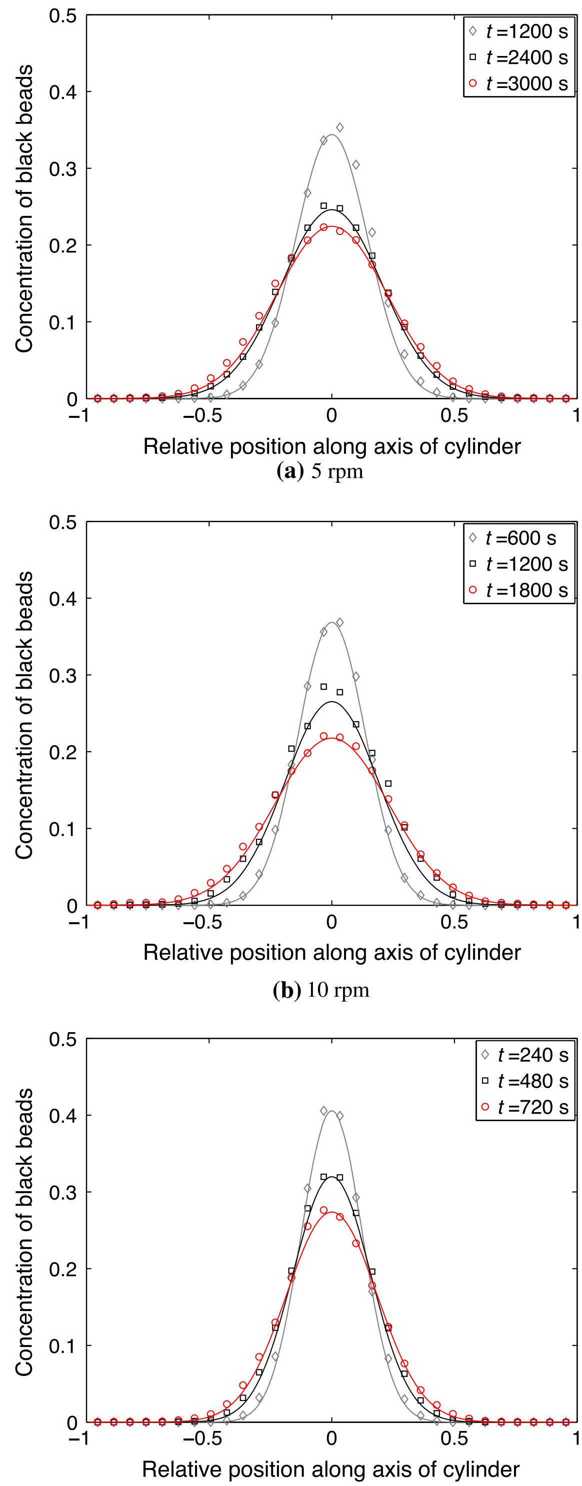
Figure 4 shows a log–log plot of the axial dispersion coefficient as a function of particle size for the three rotation speeds considered here. The linear relationship seen in Fig. 4 demonstrates the relation $D_{ax} \propto d_p^q$. The exact value of q shows a small dependence on the rotation speed, Table 2. Using the average value of q for all the rotation speeds considered results in the relation $D_{ax} \propto d_p^q$ with $q = 1.84 \pm 0.06$.

The effect of rotation speed on the axial dispersion coefficient is shown in Fig. 5. These data indicate that the dispersion coefficient is proportional to ω^k, where $k < 1$ and shows a small dependence on the particle size. For the sizes considered in this work, k ranges from 0.74 to 0.85 with an average of 0.80 ± 0.05, Table 3.

4 Discussion

The nature of dispersion and the effects of operating parameters on the dispersion in horizontal rotating cylinders is widely debated [15].

The data presented in Table 1 indicate that D_{ax} is independent of the rotation time of the cylinder for most of the



(c) 20 rpm

Fig. 3 Concentration as a function of the relative position in the cylinder for three time periods at rotation speeds of 5, 10 and 20 rpm. Plotted is the calculated concentration of each bin for the rotation period. The *solid lines* show Eq. (2), using the fitted values of D_{ax} and s. The particles are 1.14 mm glass ballotini and are rotated in a horizontal cylinder with an internal diameter of 110 mm and a length of 313 mm filled to 35 % by volume. The initial central pulse of black glass ballotini is 20.4 mm long

Table 1 Axial dispersion coefficients

d_p (mm)	ω (rpm)	t (s)	D_{ax} (m^2/s)	Average D_{ax} (m^2/s)
1.14	5	1,200	2.1×10^{-7}	2.1×10^{-7}
		2,400	2.1×10^{-7}	
		3,000	2.1×10^{-7}	
1.14	10	600	3.6×10^{-7}	3.6×10^{-7}
		1,200	3.6×10^{-7}	
		2,400	3.6×10^{-7}	
1.14	20	240	7.3×10^{-7}	6.4×10^{-7}
		480	6.1×10^{-7}	
		720	5.7×10^{-7}	
2.14	5	600	6.8×10^{-7}	6.8×10^{-7}
		900	7.0×10^{-7}	
		1,200	6.6×10^{-7}	
2.14	10	360	1.1×10^{-6}	1.1×10^{-6}
		600	1.1×10^{-6}	
		720	1.2×10^{-6}	
2.14	20	180	1.9×10^{-6}	1.9×10^{-6}
		270	1.9×10^{-6}	
		360	1.8×10^{-6}	
3.15	5	180	1.3×10^{-6}	1.3×10^{-6}
		360	1.4×10^{-6}	
		540	1.3×10^{-6}	
3.15	10	120	2.4×10^{-6}	2.4×10^{-6}
		240	2.3×10^{-6}	
		360	2.4×10^{-6}	
3.15	20	120	4.6×10^{-6}	4.4×10^{-6}
		180	5.0×10^{-6}	
		240	3.4×10^{-6}	

cases studied here. Furthermore, Fig. 3 shows agreement between the experimentally determined concentration profiles and Eq. (2). These findings are consistent with the theory that axial dispersion within rotating cylinders is governed by Fick's second law with a constant dispersion coefficient. This result is in agreement with the numerical results presented by Taberlet and Richard [16] and Third et al. [18,19] and with experimental measurements made by Hogg et al. [13], Ingram et al. [7] and Parkeret al. [17].

There have been conflicting reports regarding the effect of particle diameter on the rate of axial dispersion within rotating cylinders. Parker et al. [17] reported that the dispersion coefficient of 3 mm ballotini is six times larger than that of 1.5 mm ballotini. The DEM simulations of Third et al. [18] also indicated that axial dispersion is strongly influenced by particle diameter, although the dependence was not found to be as strong as reported by Parker et al. [17]. Third et al. [18] proposed the relation $D_{ax} \propto d_p^{1.9}$. In contrast, DEM simulations performed by Taberlet and Richard [16] showed that D_{ax} was independent of d_p. Sherritt et al. [20] suggested that D_{ax} is proportional to the square root of the particle diameter. From the measurements reported here the relation $D_{ax} \propto d_p^q$, with $q = 1.84 \pm 0.06$ was determined. This finding agrees well with the correlation proposed by Third et al. [18]. A possible explanation for the discrepancy between the results of Parker et al. [17] and those reported here and by Third et al. [18] is slip between the cylinder wall and the outermost layer of particles. Parker et al. [17] showed that there was significant slip between the cylinder wall and the bed of particles in their experiments. In the current work the cylinder wall was sanded to avoid this phenomenon and Third et al. [18] simulated 'wall rougheners' to prevent slip between the particles and the cylinder.

Third et al. [18] showed that the dispersion coefficient was proportional to $\omega^{0.74}$ for all particle sizes simulated. Parker et al. [17] also found the rate of dispersion to increase more slowly than linearly with the drum rotation. Further, Sherritt et al. [20] reported that $D_{ax} \propto \omega^{0.5}$. In the current work the dispersion coefficient was found to be proportional to ω^k, with k approximately 0.8. The value of k changes slightly with particle size, though all values are within 8 % of 0.8 and no systematic variation with particle size was observed.

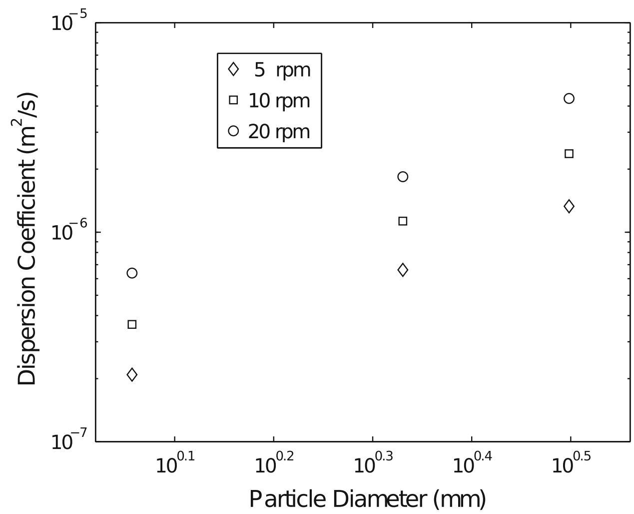


Fig. 4 Log–log plot of the average dispersion coefficient as a function of particle diameter in a horizontal rotating cylinder. The cylinder has an inner diameter of 110 mm, a length of 313 mm and is filled to 35 % by volume

Table 2 Calculated values of q for each rotation speed

Rotation speed (rpm)	q	SD
5	1.87	0.042
10	1.84	0.014
20	1.82	0.064

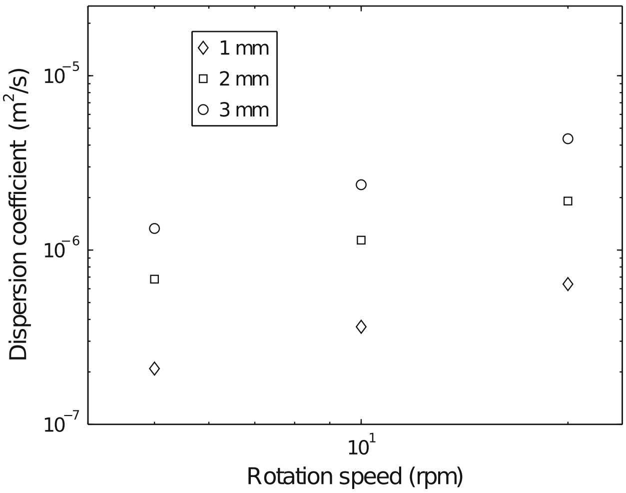


Fig. 5 Log–log plot of the average dispersion coefficient as a function of the rotation speed in a horizontal rotating cylinder. The cylinder has an inner diameter of 110 mm, a length of 313 mm and is filled to 35 % by volume

Table 3 Calculated values of k for each particle diameter

Diameter of ballotini (mm)	k	SD
1.14	0.81	0.026
2.14	0.74	0.012
3.15	0.85	0.109

Thus, the dependence $D_{ax} \propto \omega^{0.8}$ determined here agrees well with the simulations performed by Third et al. [18], i.e. $D_{ax} \propto \omega^{0.74}$.

5 Conclusions

The axial dispersion of approximately monodisperse spheres in a horizontal rotating drum has been investigated using a sectioning method. Particle sizes between 1.14 and 3.15 mm and rotation speeds between 5 to 20 rpm were employed. For each combination of particle size and rotation speed concentration profiles were measured for three different rotation periods. The resulting concentration profiles were found to be well described by Fick's second law with a constant dispersion coefficient.

Within the range of parameters tested, the dispersion coefficient was shown to be proportional to the rotation speed to the power of 0.8, $D_{ax} \propto \omega^{0.8}$. The particle diameter was found to have the following relationship with the dispersion coefficient: $D_{ax} \propto d_p^{1.84}$. Both of these results are very similar to the simulations reported by Third et al. [18] who found $D_{ax} \propto \omega^{0.74}$ and $D_{ax} \propto d_p^{1.9}$. The strong agreement between the experimental data reported here and the simulations reported by Third et al. [18] helps to validate the DEM as a method to predicted accurately the behaviour of real granular media.

Conflict of interest The authors have no relevant financial relationships to disclose. The data presented here have not been published previously.

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