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ORIGINAL PAPER

Slope Stability Analysis Based on Autocorrelated Shear Strength Parameters

La stabilité d'un versant basé sur l'autocorrélation de la résistance de cisaillement

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Abstract The stability of a slope is governed by the spatial average of the shear strength over the extent of the failure surface. In Eurocode 7 the average soil properties are taken into account by defining the characteristic soil parameter as being "a cautious estimate of the value affecting the occurrence of the limit state" and further stating that this value should be based on, among other factors, "the extent of the zone of ground governing the behavior of the geotechnical structure at the limit state being considered". To completely quantify the characteristic shear strength along a failure surface, three statistical values are required: the arithmetic mean, the variance and the spatial correlation. The mean soil properties and to a lesser degree the variance (or equivalently the standard deviation or the coefficient of variation) are known and used by most geotechnical engineers for the selection of characteristic soil properties. The scale of fluctuation, however, is not generally used. The scale of fluctuation is a measure of the soil spatial variability and can be understood as the range within which soil properties are correlated and beyond which they are statistically uncorrelated. This paper investigates the influence of the variability of shear strength on the reliability of slopes based on simulated autocorrelated random fields created by the turning bands method. In particular, the influence of the length of the failure surface on the characteristic value is investigated. Numerical Monte Carlo analyses verify the validity of a simplified practical approach presented to determine the characteristic soil properties according to Eurocode 7.

Résumé *La stabilité d'un versant est régnée par la moyenne spatiale de la résistance de cisaillement sur l'extension de la surface coulissante. Dans l'Eurocode 7 une moyenne des propriétés du sol est tenue compte, par définir le paramètre caractéristique du sol comme "une estimation prudent de la valeur, qui concerne l'apparition de l'état limité" et puis, que cette valeur devrait, entre autres facteurs, être basée sur "l'implication de l'extension de la zone du sol régné du comportement de la structure géotechnique dans l'état limité". Pour pouvoir quantifier complètement les caractéristiques de la résistance de cisaillement le long de la surface coulissante, il y en a besoin de trois valeurs: la valeur moyenne arithmétique, la variance et la corrélation spatiale. La valeur moyenne des propriétés du sol, et dans une moindre mesure la*

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variance (ou bien aussi l'écart normal ou le coefficient de variation) sont connus et utilisés d'une majorité des ingénieurs en géotechnique pour sélectionner des propriétés caractéristiques du sol. La corrélation spatiale n'est pas beaucoup utilisée en général. C'est une mesure pour l'échelle de fluctuation, par exemple la marge entre les propriété du sol qui sont en corrélation et au-delà quelles ne sont statistiquement pas en corrélation. Cette mémoire recherche l'influence de la variabilité de la résistance de cisaillement sur la fiabilité d'un versant, basé sur des prélèvements simulés en autocorrélation avec la turning bands method. L'influence de la longueur de la surface coulissant sur la valeur caractéristique est en particulier recherchée. L'analyse numérique de Monte Carlo vérifie la validité d'une approche plus simple et plus pratique, présenté pour déterminer les caractéristiques des propriétés du sol selon l'Eurocode 7.

Keywords Slope stability · Characteristic value · Eurocode 7 · Spatial variability · Monte Carlo analysis · Random fields

1 Introduction

The assessment of slope stability involves the essential task of selecting geotechnical parameters represented by their characteristic values according to Eurocode 7 (EN 1997-1). No guidance is offered in Eurocode 7 to quantitatively determine characteristic values for practical applications. Therefore considerable difficulties exist in selecting characteristic values that take into account both data and model uncertainty.

Parameter uncertainty relates to the basic availability of data, transformation uncertainty, measurement errors and statistical errors derived from the measured values, the statistical interdependence of soil parameters (correlation) and the spatial variability (e.g. Christian et al. 1994; Phoon and Kulhawy 1999). Model uncertainty relates to the assumed conditions used in the different calculation methods and is neglected here.

Characteristic values according to Eurocode 7 are dependent, among other factors, on the extent of the failure surface, which in turn influences the variance of soil properties.

In this paper the influence of the spatial variability of soil when estimating the characteristic value and its implications on slope stability is investigated. Numerical Monte Carlo analyses are employed to verify the validity of a simplified practical approach presented to determine the characteristic soil properties according to Eurocode 7.

2 Materials and Methods

2.1 Description of Cases

In order to assess the different methods for estimating the characteristic values of soil properties, two typical slope stability examples are introduced. The method of slices (Fellenius 1936) is employed with a constant unit weight γ as well as a constant slope angle and dimensions.

The first example represents a saturated clay slope with a mean undrained shear strength of $c_u = 40$ kPa (friction angle $\phi = 0°$). The slope angle is 26.57° and the distance of the failure surface perpendicular to the slope surface is limited to 20 m as shown in Fig. 1.

The second example (see Fig. 2) represents a slope with an angle of 36.87°, consisting of a clayey sand with a mean friction angle of $\phi = 30°$ and a mean cohesion of $c = 8$ kPa.

The statistical parameters describing the inherent soil variability, which are: the mean μ, the coefficient of variation CV ($=\sigma/\mu$), the vertical correlation length ρ_v, the horizontal scale of fluctuation δ_h (see Sect. 2.3 for ρ_v *and* δ_h), and the anisotropy ratio (i.e. ρ_h/ρ_v) for the two examples, are shown in Table 1 (cohesion c) and Table 2 (friction angle ϕ) and are based on (Phoon and Kulhawy 1999).

2.2 Calculation of the Global Factor of Safety

The computation of the global factor of safety F_S of a slope is based on:

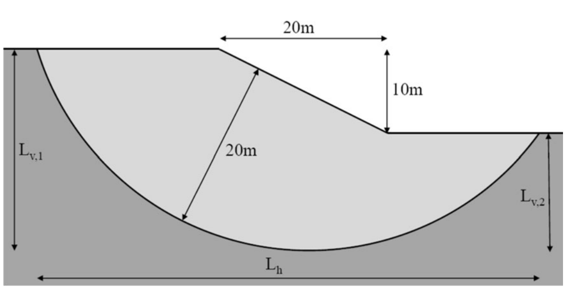

Fig. 1 Example 1, undrained clay slope ($F_s = 1.171$)

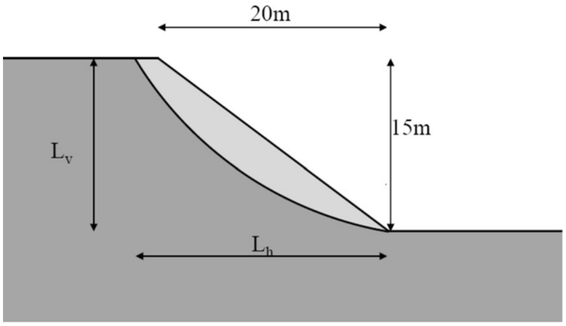

Fig. 2 Example 2, slope in clayey sand ($F_s = 1.155$)

Table 1 Statistical parameters for cohesion

Cohesion	Example 1	Example 2
μ	40 kPa	8 kPa
CV	0–0.45	0–0.45
ρ_v	0.5–∞ m	0.5–∞ m
δ_v	1–∞ m	1–∞ m
$\rho_h/\rho_v = \delta_h/\delta_v$	10	10

Typically, CV_c is 0.3 and $\rho_{v,c}$ is 0.5–1 m and $\delta_{v,c}$ is 1–2 m, respectively

Table 2 Statistical parameters of the friction angle

Friction angle (°)	Example 1	Example 2
μ	0	30°
CV	–	0–0.2
ρ_v	–	0.5–∞ m
δ_v	–	1–∞ m
$\rho_h/\rho_v = \delta_h/\delta_v$	–	10

Typically, CV_ϕ is 0.1 and $\rho_{v,\phi}$ is 0.5–1 m and $\delta_{v,c}$ is 1–2 m, respectively

- Estimating of the *characteristic values* x_k of cohesion and friction angle as the 5 % confidence limit for estimating the average of n measurements of a normally distributed population (t is the 5 % percentile of Student's t-distribution with n degrees of freedom) (e.g. Schneider 1997; Orr and Breysse 2008; Bond and Harris 2008):

$$x_k = \mu_x - \frac{t}{\sqrt{n}}\sigma_x = \mu_x\left(1 - \frac{t}{\sqrt{n}}CV_w\right) \quad (1)$$

- Calculating the *global factor of safety* F_S using:

$$F_S = \frac{cL + \tan(\varphi)\gamma\int H\cos(\alpha)dx}{\gamma\int H\sin(\alpha)dx} \quad (2)$$

In Eq. (2) L is the length of the failure plane, H the vertical depth of the failure plane from the surface and α the angle of the failure plane relative to the horizontal.

- Find the minimum global factor of safety among all possible slip surfaces

$$F_{S,\min} = \min\{F_S\} \quad (3)$$

A simplified equation for determining the characteristic value according to Eq. (1), which gives good values in practical terms (implicitly valid for about L/δ equal to about 8–10) was found by (Schneider 1997). Thereby, the characteristic value could be calculated as:

$$x_k = \mu_x - 0.5\sigma_x = \mu_x(1 - 0.5CV) \quad (4)$$

2.3 Spatial Variability

Spatial variability describes the variation of geotechnical parameters in one, two, or three spatial dimensions. It is assumed that measurements are correlated if the distance between two locations is small, and are uncorrelated or statistically independent, respectively, if the distance between two locations is large. A mathematical function that describes the (auto) correlation $r(h)$ is given by:

$$r(h) = \exp(-h/\rho) \quad (5)$$

This exponential autocorrelation function is a function of the distance between two locations, h, and the correlation length, ρ. In geotechnical applications the *scale of fluctuation* (Vanmarcke 1983) is mostly used which is defined as $\delta = 2\rho$ for this particular auto-correlation function (i.e. the exponential model) and leads to

$$r(h) = \exp(-2h/\delta) \quad (6)$$

The scale of fluctuation of geotechnical parameters shows a dependency on direction, namely on the vertical and the horizontal distances, due to a soil's deposition and loading history. It is implicitly assumed that the autocorrelation function is separable (i.e. it is expressed as a product of the autocorrelation functions in each direction), as usually done.

This produces two exponential autocorrelation functions; one with the horizontal and one with the vertical correlation lengths. The ratio of the horizontal and the vertical correlation lengths ρ_h/ρ_v is called the anisotropy ratio.

2.4 Simplified formula regarding spatial variability

A major difficulty in assessing the characteristic value according to EN 1997-1 is to account for "the zone of ground affecting the limit state".

In a zone larger than the scale of fluctuation the spatially variable properties tend to "average out", whereas within a distance smaller than the scale of fluctuation the spatial average varies considerably, in an extreme case by as much as the variance of the samples (Vanmarcke 1977).The averaging out occurs because of an increasing probability that high property values are balanced by low property values at other points (NRC 1995), when the correlation (i.e. the scale of fluctuation) decreases and/or the size (length, area, volume) of the failure mechanism increases. This effect is known as variance reduction due to spatial averaging.

Thus, the *effect* of the scale of fluctuation depends on the size of the investigated area, or in this context on the length of the failure surface *L*.

Taking into account the spatial variability in a simplified way, the characteristic value as defined in Eurocode 7 can be estimated by Eqs. (7a, 7b)

$$x_k = \mu_x\left(1 - 1.645 \cdot CV \cdot \sqrt{\frac{\delta}{L}}\right) \quad (7a)$$

$$x_k = \mu_x(1 - 1.645 \cdot CV \cdot \Gamma) \quad (7b)$$

where δ is the scale of fluctuation, L is the length of the governing failure mechanism, $\Gamma^2 = \delta/L$ is the variance reduction factor, and 1.645 is the 5 % fractile of the normal probability distribution. Equation (7a) is valid for $L \geq \delta$. For the case of $L \leq \delta, \frac{\delta}{L} = 1$ (Vanmarcke 1983). Note that Eqs. (7a, 7b) only accounts for the inherent variability, whereas Eq. (1) accounts for the statistical uncertainty. The two uncertainties can be linked in the following simplified equation:

$$x_k = \mu_x\left(1 - 1.645 \cdot CV \cdot \sqrt{\frac{\delta}{L} + \frac{1}{n}}\right) \quad (8)$$

In Eq. (8) it is assumed, that a large population of data or expert knowledge is available, so that the Student t distribution converges to the standard normal distribution. More details on the derivation of the equation are given in (Schneider 2010).

With the focus of this paper on the quantification of the effect of spatial variability on slope stability, no statistical measurement error is introduced in Example 1 and 2. The consequence of this aussumption is that Eq. (8) converges to Eqs. (7a, 7b), because 1/n in Eq. (1) approaches zero for the above assumption. The Eqs. (7a, 7b) and (8) however still account for the inherent variability, whereas Eq. (1), which accounts only for statistical error, reduces to $x_k = \mu_k$, irrespective of the coefficient of variation.

In a situation where the lognormal distribution is appropriate (e.g. when the coefficient of variation *CV* is larger than about 0.3 or when available information indicates that the lognormal distribution is more adequate) Eq. (9) can be used to estimate the characteristic value:

$$x_k = \mu_x \frac{0.193^{\sqrt{\ln\left(1+\frac{\delta}{L}CV^2\right)}}}{\sqrt{1+\frac{\delta}{L}CV^2}} \quad (9)$$

With this characteristic value a first-order approximation of the 5 % percentile of the global factor of safety can be obtained.

2.5 Conditions of Spatial Averaging

Spatial averaging is only possible if the governing failure mechanism is capable of redistributing forces or stresses along failure surfaces.

True cohesion is often not ductile, i.e. brittle (is lost after small strains) and the amount mobilized on the failure surface depends on stress history and stress level. Cohesion can therefore not generally be averaged because the strains acting on the failure surface generally vary along the failure surface. Many practitioners are well aware of this fact and will consequently neglect cohesion in most slope stability calculations. Despite this fact, in the calculations here it is assumed for generality that the cohesion is redistributed on the failure surface.

For the frictional resistance, the same general remarks as for cohesion apply. For more details refer to (Schneider and Fitze 2011).

3 Methodology and Results

3.1 Anisotropic Spatially Variable Slip Surfaces

The variance reduction factor Γ^2 along the slip surface of an anisotropic soil can be described as:

$$\Gamma^2 = \frac{\delta_v}{L_v} \quad for \; \frac{\delta_v}{L_v} < \frac{\delta_h}{L_h}, \quad otherwise \quad \Gamma^2 = \frac{\delta_h}{L_h} \tag{10}$$

where $L_v = L_{v1} + L_{v2}$. See Figs. 1 and 2 for the definitions of L.

3.2 Calculation methodology

Equations (1), (4), (7a, 7b) and (9) are used to estimate the characteristic values in order to determine the 5 % fractile for the global factor of safety. Additionally, Monte Carlo (MC) analyses with the soil described by autocorrelated random fields (Matheron 1973; Tietje and Richter 1992; Mantoglou 1987) are performed to compute the 5 %-fractile for the global factor of safety as well. The autocorrelated random fields are generated with the turning bands method. A 2-dimensional standard normal distribution is produced for a spatially dependent random variable Z. Each realization of Z is then transformed to obtain a lognormal distribution of the generated parameter field. Thus the logarithm of the generated parameter field is spatially correlated with correlation function of Eq. (5) and the correlation length or the scale of fluctuation, respectively (Fig. 3).

Figures 4, 5, 6 and 7 show the results of the following three different MC-methods, Methods 1, 2 and 3, to obtain the 5 %-fractile of the factor of safety (F_s) as well as the results obtained using the three simplified Eqs. (1), (4), and (9) for comparison. The main features of the three methods and the three equations are:

- *Method 1* MC analysis with a spatially homogeneous soil, but accounts for spatial variability because the reduced variance ($CV \cdot \sqrt{\delta/L}$) is used as input in the MC analysis. For Examples 1 and 2 a fixed midpoint and radius of the failure surface was determined with average values of c and ϕ.
- *Method 2* MC analysis with (auto)correlated random fields. Each MC run creates a spatially variable random soil with a correlation length ρ. Method 2 uses the fixed midpoint and radius of the failure surface determined in Method 1, although the spatial variability could imply a different failure surface in each MC run.
- *Method 3* MC analysis with correlated random fields and a search for the critical failure surface according to Eq. (3) in each MC run. Method 3 selects as the failure surface the circle for which the generated soil shows the most unfavourable strength.
- *Equation* (1) neglects spatial variability as n (degrees of freedom, resp. number of measurements) is assumed to be infinite so that $x_k = \mu_k$. n approaches infinite, because of the assumption of a negligible statistical error. Note: Eq. (1) is only able to account for the statistical uncertainty (estimates the 5 %-fractile of the *average* of a population). It does not account for the inherent variability inside a population.
- *Equation* (4)—as a rule of thumb—accounts for both statistical error and the spatial variability. However the uncertainty is only roughly accounted for by just using the *CV* and neglecting the correlation structure (δ) and the length (L) of the governing failure mechanism.
- *Equation* (9), and also Eqs. (7a, 7b), account explicitly for the spatial variability and the size of the governing failure mechanism. They calculate the variance reduction simply by dividing the scale of the fluctuation (δ) by the size (L) of the governing failure mechanism. In these comparisons, Eq. (9) was used.

Because all other methods or equations can be derived from Method 3 by means of simplification, this method is used as the reference method for the comparison.

3.3 Comparative Results

For both examples a sensitivity analysis is presented. Figure 4 (Example 1) and Fig. 6 (Example 2) show the sensitivity of the methods for calculatingthe 5 %-fractile of the *global* F_s value to the coefficient of variation, when the vertical scale of fluctuation is fixed ($\delta_v = 2$ m). Figure 5 (Example 1) and Fig. 7 (Example 2) show the sensitivity of the methods for calculating the 5 %-fractile of the *global* F_s value to the vertical scale of fluctuation, when the coefficient of variation is fixed.

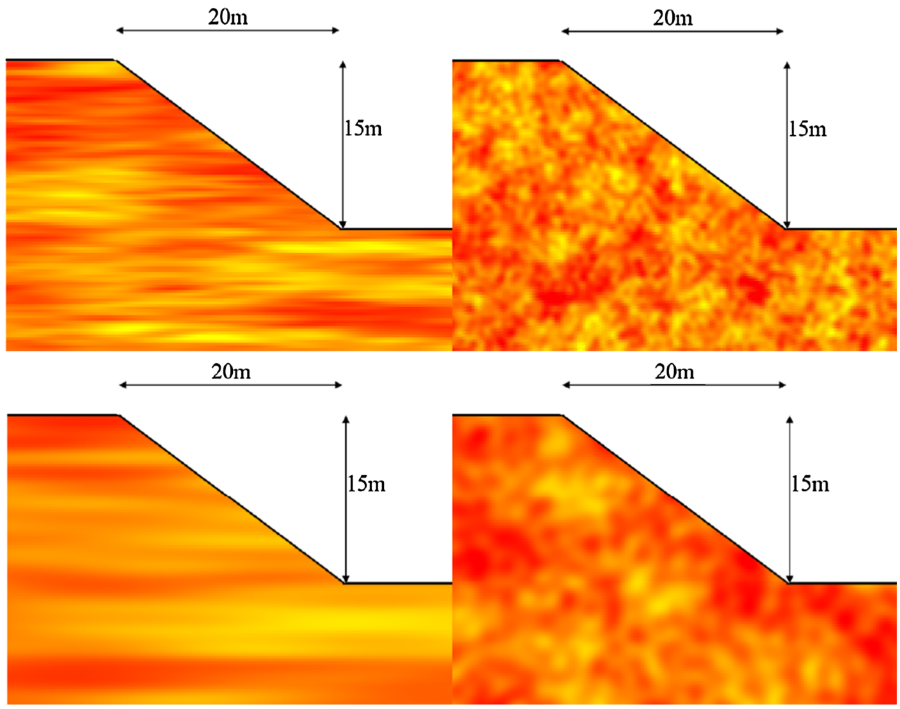

Fig. 3 Random fields in Example 2: dark zones with weak cohesion, light zones with strong cohesion. *Upper part* with correlation length 1 m, *lower part* with correlation length 3 m, *right slopes* isotropic, *left slopes* with anisotropy ratio 10

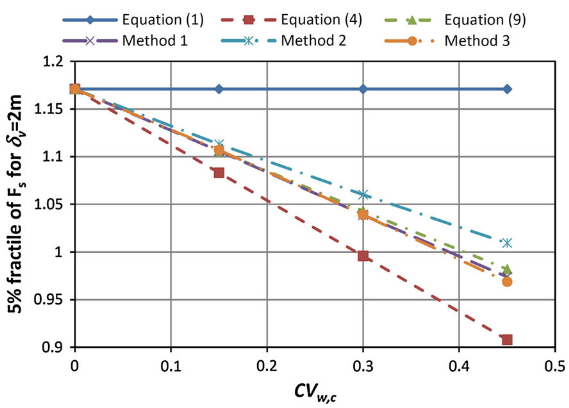

Fig. 4 Example 1: 5 %-fractile of *global* F_s as a function of CV_c ($\delta_v = 2$ m)

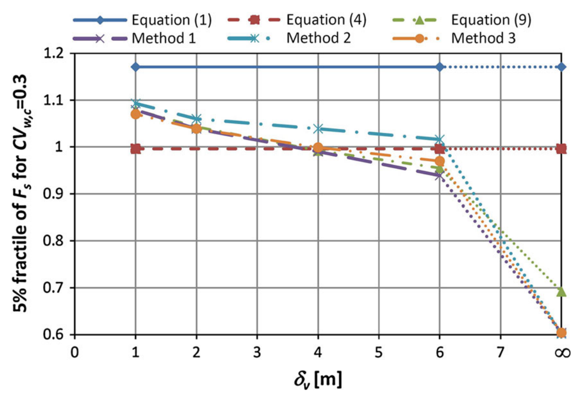

Fig. 5 Example 1: 5 %-fractile of *global* F_s as a function of δ_v ($CV = 0.3$)

Using Eq. (1), with the assumtion of $1/n = 0$, the global factor of safety F_s calculated by Eq. (2) is overestimated, especially if *CV* is large and/or the scale of fluctuation is large (see Figs. 4 and 5). It should be noted here that the global factors of safety F_s presented in this paper are meant to be used for a relative comparison of the different methods. By no means do the absolute values of F_s imply the slope to be safe or unsafe.

Using Eq. (4) in Eq. (2) results in an *global* F_s value that is slightly lower than the reference (Method 3) and thus is slightly conservative for a typical scale of fluctuation (2 m) and for the governing failure mechanism assumed here. However for a large correlation length and/or a small governing failure mechanism, Eq. (4) might overestimate the *global* F_s. (Figs. 5 or 7).

Using Eqs. (7a, 7b) and (9) in Eq. (2) yields very good results. The simple variance reduction used for the circular slip surface (Eq. (10)) is slightly conservative. The results show that there is a little difference in the F_s values obtained using Eqs. (7a, 7b) or (9) and the reference Method 3.

The results of Example 1 and 2 are very similar. In Example 2 both, ϕ and c, are spatially variable and are

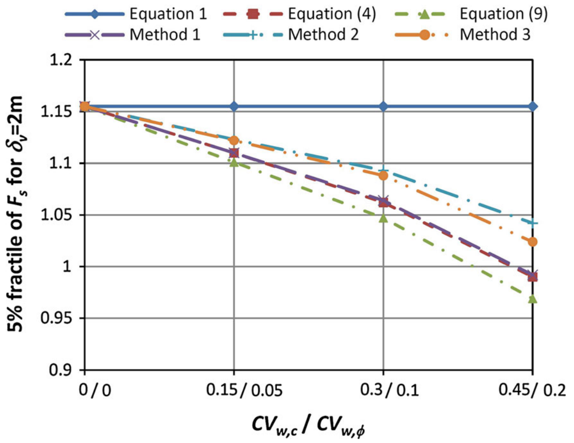

Fig. 6 Example 2: 5 %-fractile of *the global* F_s as a function of the *CV* ($\delta_v = 2$ m)

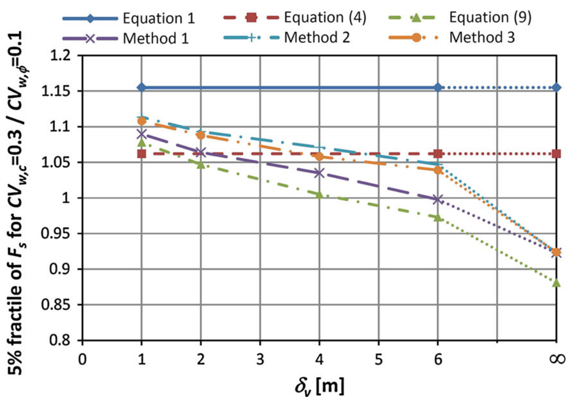

Fig. 7 Example 2: 5 %-fractile of *the global* F_s as a function of δ_v ($CV_c = 0.3$ and $CV_\phi = 0.1$)

uncorrelated. Thus the critical slip surfaces determined in each MC run in Method 3 are not much different from the one critical slip surface used in Method 2.

4 Conclusions

The numerical investigations show that the risk of slope failure depends not only the variability of the soil properties, but also on the scale of fluctuation of the soil properties and the extent of the failure mechanism. The cross-correlation between the strength parameters (i.e. angle of internal friction and cohesion) has been neglected in this study.

The variance of the cohesion and friction angle is reduced due to spatial averaging, if δ/L decreases. A higher variance reduction due to spatial averaging leads to higher characteristic values and larger factor of safety respectively and vice versa. Equations (7a, 7b) and (9) explicitly account for spatial variability through the scale of fluctuation (i.e. the autocorrelation) and the size of the governing failure mechanism. The validity of Eqs. (7a, 7b) and (9) has been proven by independent MC-analyses.

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