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Relative timing jitter measurements with an indirect phase comparison method

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ABSTRACT We propose and demonstrate experimentally a method for the sensitive measurement of the relative timing jitter of two mode-locked lasers, which can be either free-running or timing-synchronized to a common reference oscillator. The method is based on the indirect comparison of the phases of two photodetector outputs, using a microwave oscillator, the noise of which does not affect the results, electronic mixers, and a sampling oscilloscope. We carefully analyze and experimentally demonstrate the potential of this method. Compared to phase detector methods, it has a broader scope of applications and a lower sensitivity to intensity noise. We also obtained data on the coupling of intensity to timing noise in photodetectors.

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1 Introduction

The timing jitter of actively or passively mode-locked lasers is important for many applications. For example, it can adversely affect optical data transmission, optical sampling measurements, and various kinds of experiments with synchronized lasers. Therefore, techniques for the accurate measurement of timing jitter are of high interest. Although various techniques have been demonstrated, they all have their shortcomings, usually in the form of limited performance, limited applicability, and sometimes also in terms of cost of the equipment.

The measurement of absolute timing jitter of a laser requires a timing reference with lower timing noise than the laser itself. As the timing jitter of mode-locked solid-state lasers can be very small, the demands on the reference oscillator are typically rather high. For some lasers, only the most expensive electronic oscillators can meet such demands. Note that in the case of a commonly used technique, based on the spectral analysis of the signal from a fast photodiode [1], the timing reference is given by the tunable local oscillator of the spectrum analyzer, the phase noise of which can severely limit the sensitivity of such timing jitter measurements (as will be discussed in Sect. 3.4).

The need for an ultrastable reference oscillator can be eliminated by using two lasers of the same kind and measuring the relative timing jitter, which also allows one to estimate the timing jitter of a single laser. A typical technique is based on the phase detector method [2], where the relative phase of the photodiode signals from the two lasers is kept near $\pi/2$, and an electronic mixer generates a signal which is proportional to the deviation of the relative phase from $\pi/2$, at least for small deviations. With further sophistication, including the combination of two phase detectors operating on different harmonics of the photodiode signals [3], timing stabilization and measurement have become possible with relative rms jitter values in the order of only 1 fs. However, this approach requires quite refined electronics to minimize the coupling of intensity noise to timing noise (AM-PM conversion [2]). Subfemtosecond relative jitter has also been measured with an optical phase detector based on cross-correlation in a nonlinear crystal [4]. Also note that an electronic phase detector is only applicable to cases where the relative timing deviation is well below one pulse period; for an optical phase detector, the limit is even in the order of the pulse duration. This means that electronic and optical phase detectors can usually only be applied to lasers which are timing-synchronized in some way, e.g., by active mode locking with a common reference clock.

The new technique proposed and demonstrated in this article can be used either with timing-stabilized or with free-running passively mode-locked lasers, where the drift of the relative timing within the measurement period may be much larger than the pulse period. Furthermore, it allows to simultaneously measure timing and intensity noise while keeping AM-PM conversion at low levels. It requires an electronic oscillator, but the timing noise of this oscillator does not affect the results, and the achieved noise floor is rather low at all noise frequencies.

This article is structured as follows. The proposed measurement technique is described in Sect. 2. Its potential is theoretically evaluated and compared to the potential of alternative techniques in Sect. 3. In Sect. 4, we present experimental results, confirming the usefulness of the technique. Finally, conclusions are discussed in Sect. 5.

2 Description of the measurement technique

Figure 1 schematically shows the proposed measurement setup. It contains two mode-locked lasers, the timing of which is to be compared. These lasers might be either timing-synchronized (e.g., by active mode locking with

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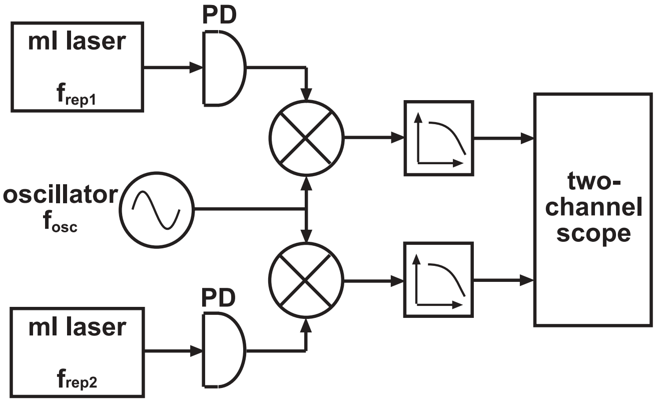

FIGURE 1 Experimental setup for our jitter measurements. ml = mode-locked, PD = fast photodiode. The photodiode signals are downconverted to low frequencies, low-pass filtered and digitally recorded

a common timing reference) or free-running; the averaged pulse repetition rates should be similar but do not have to be equal.

Two fast photodetectors are used to monitor the laser outputs. Ideally, these photodetectors should be based on photodiodes which can handle high photocurrents, as this allows one to minimize the detection noise (see Sect. 3.1). Both photodiode outputs are fed into mixers (using preamplifiers before the mixers, if required), the local oscillator ports of which receive a sinusoidal signal from an electronic oscillator. This oscillator should be tuned to a frequency which differs from the pulse repetition rates by e.g., 100 kHz. Each mixer output then contains the difference frequency between the laser repetition rate and the oscillator frequency, among mixing products with higher frequencies which are subsequently removed with a low-pass filter. The filtered signals are simultaneously recorded with a two-channel digital oscilloscope (or a PC equipped with a sampling card). A personal computer is used to retrieve and process the resulting data.

A possible modification of the setup would use two frequency-to-voltage converters before the digital oscilloscope. This would reduce the required sampling rate and memory, but we do not discuss this further, as the benefit does not seem to justify the additional effort to use frequency-to-voltage converters and to characterize their frequency response and noise properties.

Note that the two recorded signals are affected by phase noise of the electronic oscillator, which can be stronger than the timing phase noise of the lasers. However, this oscillator noise affects both channels equally, and the difference of the phase values of the two channels, as obtained with the last mixer, is not affected. The same holds for timing noise of the digital oscilloscope, as both channels are recorded simultaneously. If two timing-stabilized lasers are used, which are both timing-locked to the same reference, then the timing noise of this reference has an influence on the measured results if the lasers do not equally react to the control signals.

In the following we describe in detail the numerical processing required to extract the relative timing noise as well as the intensity noise of the two lasers.

One noise measurement will typically involve the simultaneous recording of N samples in two channels, where N is a power of two to allow the use of a simple fast Fourier transform (FFT) algorithm. These values are first multiplied with a slowly varying window function which is zero at the ends of the recording interval; we use e.g. a $\sin^2$ function. Such a window function basically removes the discontinuity of the signals at beginning and end of the recorded trace, which would otherwise lead to strong artifacts in the spectrum. For later convenience, we also normalize the data to a fixed mean amplitude. The Fourier-transformed data of both channels each contain a peak at the difference between laser repetition rate and oscillator frequency. The sidebands around these peaks contain information on intensity and phase fluctuations. The positions of these peaks are evaluated (as a kind of "center of gravity"), and subsequently the spectra are shifted so that the peaks are now located at $f = 0$. At the same time, the frequency range is reduced to a fraction of the original range, depending on the required range of noise frequencies. Inverse FFT then generates two time-dependent complex amplitudes (phasors). The modulus of these reflect the fluctuations of the laser powers (multiplied with the mentioned window function); these values are also affected by intensity noise of the electronic oscillator, which however is usually much weaker. The phase of each complex amplitude represents the difference between the timing phase of the laser and the phase of the oscillator, minus the linear part of the evolution of the relative phase (due to the above mentioned shift of the spectra). Although both phase values are affected by phase noise of the oscillator, this influence cancels out in the difference of both phase values, which finally reflects the difference of the timing phases of the two lasers, apart from a phase term proportional to the time, resulting from the difference of the shifts of both spectra, and some noise from detection and digital sampling.

The time-dependent intensity and relative phase values can again be Fourier-transformed to obtain noise spectra. However, note that the time-dependent values contain more information than noise spectra, the latter showing only the squared modulus of the noise amplitudes. For example, we have seen situations where due to a technical problem the repetition rate of a laser exhibited oscillations with a frequency which varied over the measurement period. The resulting broad bump in the spectra revealed much less information than directly displaying the time-dependent phase.

We also emphasize that even in the absence of intensity noise, the obtained relative phase noise spectrum is not identical with the sidebands of a beat signal which could be generated from the photodiode outputs, unless the phase excursions are limited to small values ($\ll 1$ rad). Therefore, our Fourier method serves not only to separate phase and intensity noise, but also to retrieve the true phase noise under conditions of large phase excursions, as typically occur for free-running lasers.

Another important issue, in particular for the timing phase noise of free-running passively mode-locked lasers, is the use of an appropriate window function before performing the FFT for obtaining noise spectra. Severe errors can arise from the so-called leakage phenomenon [5] in situations where the power density diverges at zero frequency [6]. Apart from using an appropriate window function, processing of frequency fluctuations instead of phase fluctuations can be a solution [6].

The choice of oscillator frequency and sampling parameters is also of crucial importance. This issue is discussed in detail in Sect. 3.4.

3 Discussion of the potential of the method

In the following we discuss the limitations of the proposed measurement technique, showing that it has a very good potential for accurate and versatile timing noise characterization. Basic limitations arise from noise in the photodetectors (Sect. 3.1) and from digital sampling (Sect. 3.2), but these limitations can be strongly affected by proper choice of components and sampling parameters. In Sect. 3.3 we show that the coupling of intensity to timing noise can be rather weak. The results then allow a comparison with other techniques in Sect. 3.4.

3.1 *Noise from detection electronics*

Obviously, the photodetection noise leads to a basic limitation of the sensitivity of the proposed measurement technique. Basically we are dealing with two different kinds of noise: electronic noise, which arises partly from thermal noise in electronic components, and shot noise, which is a quantum effect. For all calculations we use the notation of [6], with two-sided power densities. (Note that the engineering disciplines usually use one-sided power densities, which are two times larger.)

A typical kind of fast photodetector consists simply of a reverse-biased photodiode and a 50-Ω resistor to convert the photocurrent into a voltage. If the output is connected to other electronics (e.g., a preamplifier) with a 50-Ω input impedance, we effectively have an impedance of 25 Ω. If this is purely resistive, it contributes thermal noise with a two-sided power density

$$S_i(f) = \frac{2\,k_\mathrm{B}T}{R} \tag{1}$$

of the current. Here, we have $R = 25\ \Omega$ (the effective resistive impedance), and T is the temperature (typically room temperature) at which the electronics are operated. The resulting equivalent power density of the detected optical power is

$$S_P(f) = \left(\frac{h\nu}{\eta e}\right)^2 \frac{2\,k_\mathrm{B}T}{R}\,, \tag{2}$$

where η is the quantum efficiency of the detector and $h\nu$ is the photon energy. The corresponding relative intensity noise is characterized by

$$S_I(f) = \left(\frac{h\nu}{\eta e}\right)^2 \frac{1}{\bar{P}^2}\frac{2\,k_\mathrm{B}T}{R} = \frac{2\,k_\mathrm{B}T}{\bar{i}^2 R}\,, \tag{3}$$

where $\bar{P}$ is the average power of the detected beam and $\bar{i}$ the average photocurrent.

If an electronic preamplifier is used to boost the power of the resulting signal, the relative intensity noise in the amplified signal is increased by the noise figure of the amplifier, which is typically a few dB.

Apart from intensity noise, thermal noise also leads to an apparent timing phase noise

$$S_\varphi(f) = \frac{2\,k_\mathrm{B}T}{\bar{i}^2 R}\,, \tag{4}$$

with the timing phase $\varphi = 2\pi f_\mathrm{rep}\Delta t$, where Δt is the timing error and f_rep the pulse repetition rate.

We can also have the influence of shot noise, leading to intensity noise with a relative power density

$$S_I(f) = \frac{h\nu}{\eta\bar{P}} = \frac{e}{\bar{i}}\,. \tag{5}$$

This occurs together with a timing phase noise which has the same power density. As this type of noise decreases less rapidly with increasing laser power, it can dominate over thermal noise for high detected powers, i.e., for

$$\bar{P} > \frac{2h\nu\,k_\mathrm{B}T}{\eta e^2 R} \quad \text{or} \quad \bar{i} > \frac{2\,k_\mathrm{B}T}{eR}\,. \tag{6}$$

For example, for a 1535-nm laser, $\eta = 0.8$, room temperature, $R = 25\ \Omega$, and 1 mW optical average power leads to a noise floor of −155 dBc/Hz, if no preamplifier is required, or otherwise a few dB more. (We always specify noise power densities with dBc/Hz, calculated as 10 times the logarithm of the two-sided power density.) Shot noise would dominate only above ≈ 2 mW, if a preamplifier is not required, or otherwise at higher powers. In any case, it is difficult to measure noise on this low level.

Note that pulse trains with low repetition rates and short pulses have a higher ratio of peak to average power, so that lower detected average powers are required to avoid saturation. In that case, the thermal noise floor can easily become much higher, e.g., −135 dBc/Hz for 0.1 mW.

3.2 *Noise from digital sampling*

Our method involves digital sampling of data, which inevitably leads to sampling errors. However, a suitable choice of the sampling parameters allows to achieve a rather low noise level.

A mathematical complication results from the fact that the sampling errors of different samples are partially correlated. For a first estimate of the noise floor, we neglect such correlations, and also assume that the sampling is not affected by any additional noise. When the sampling is done with n bits and the sampled voltage can vary between 0 and U_max, the sampling resolution is $\delta U \approx 2^{-n} U_\mathrm{max}$, and the maximum sampling error is $\delta U/2$. The root mean squared (rms) error is then $\sigma_U = \frac{\delta U}{2\sqrt{3}}$. For a sinusoidal signal with the maximum amplitude of $\approx U_\mathrm{max}/2$, sampled with a time resolution δt, this leads to a noise floor with power density

$$S_U(f) = \frac{(\delta U)^2}{12}\frac{8}{U_\mathrm{max}^2}\delta t = \frac{2}{3}2^{-2n}\delta t \tag{7}$$

half of which is intensity noise, and the other half phase noise. This shows that the bit resolution is rather important: each additional bit reduces the noise level by 6 dB; the same could be achieved only with a fourfold increase of the sampling rate.

Note that the noise floor is higher when the maximum range of input voltages is not used by the signal. In practice, this will often lead to the loss of ≈ 1 effective bit.

We can reduce this noise floor by increasing the sampling rate $f_s = 1/\delta t$. On the other hand, the lowest measurable noise frequency is $1/(N\delta t) = f_s/N$, so that a high sampling rate increases the required number N of samples for a given lower noise frequency. A more effective way to lower the noise floor is to increase n, the number of bits.

As a numerical example, assume a sampling resolution of 12 bits and a moderate sampling rate of 1 MHz. This leads to an estimated noise floor of ≈ 131 dBc/Hz. Estimating the quantum-limited timing noise of a miniature 10-GHz Er:Yb:glass laser similar to the one described in [7], using calculations of [8], we find that such a low noise level can be expected only at noise frequencies well above 10 kHz. Solid-state lasers with lower repetition rates (longer cavities) could have significantly lower noise levels at high frequencies; the limit will then be given by the photodiodes and/or the electronics, while the sampling noise could be further reduced with a higher sampling rate. For a lower sampling resolution of e.g., 8 bits, significantly higher sampling rates would be required, so that the demand on sampling memory would be rather high for measurement times of several seconds.

In reality, the assumption of uncorrelated sampling errors is not exactly fulfilled: there is some correlation between the errors of nearby samples. We did numerical simulations to explore this regime, which is difficult to treat with analytical means. Here, we assumed e.g. a noiseless sinusoidal signal. When its frequency is chosen so that a period corresponds to an integer number of samples, the sampling errors are perfectly periodic, and we do not get a constant noise floor but rather a sequence of peaks at harmonics of the signal frequency. However, a more realistic situation for our measurements is a signal frequency which is not simply related to the sampling rate. In typical situations, one obtains basically a flat noise floor, which can be somewhat lower than estimated from (7), with superimposed peaks of moderate height. By addition of white input noise with an rms value somewhat below δU, we can reduce these peaks without strongly raising the noise floor. Typically, (7) was found to give a reasonable estimate. Note that actual noise on the input signal further randomizes the sampling errors, making the estimate of (7) better.

3.3 *Coupling of intensity noise to timing noise*

Several other techniques for timing jitter measurements are severely plagued by the coupling of intensity noise to timing noise within the measurement apparatus. Our method, however, can be expected to be quite immune to this problem. In the following, we discuss various ways in which AM-PM conversion may occur.

For our method, AM-PM conversion requires some kind of nonlinearity. For operation at low enough optical power levels, the photodetectors are operating in the linear regime, avoiding any AM-PM coupling. However, there is a need to maximize the photocurrent in order to minimize the effects of thermal noise and/or shot noise. One might, therefore, have to find the maximum photocurrent level where AM-PM coupling due to detector saturation is still acceptable. At the end of Sect. 4, we present experimental results, showing that there is some AM-PM coupling, but at a level which is far from affecting our timing jitter measurements – even for the maximum allowed photocurrents.

The mixers are inherently nonlinear devices. However, for low enough input signals (and given power levels applied to the local oscillator ports), the outputs are still linearly dependent on the inputs. Thus, we also do not have to expect significant AM-PM coupling here. Note that mixer offsets, which are known to lead to AM-PM conversion in electronic phase detectors, do not have this effect for our method.

Significant nonlinearities are also not to be expected in the A/D converter of the oscilloscope. Even with some nonlinearity at this location, AM-PM conversion should not occur at this stage, since a nonlinearity should primarily distort the shape of the recorded oscillating signal (i.e., add harmonics to its spectrum) but not affect its phase.

In conclusion, the proposed method appears to be quite immune to AM-PM coupling, provided that excessive power levels or electrical amplitudes are avoided, and the experimental results of section 4 confirm this.

3.4 *Choice of oscillator frequency and sampling frequency*

In the following we discuss several conditions which have to be met by a proper choice of the oscillator frequency f_{osc} and the sampling frequency $f_s = 1/\delta t$, when the goal is to obtain a measurement for noise frequencies in the range f_{min} to f_{max} with sufficiently low noise floor S_{fl}. The (average) repetition rates of the lasers are denoted by f_{rep1} and f_{rep2}, and we assume that sufficient memory for N samples is available.

In order to reach a maximum noise frequency f_{max}, we must meet the condition

$$\min\left(\left|f_{rep1} - f_{osc}\right|, \left|f_{rep2} - f_{osc}\right|\right) > f_{max} \tag{8}$$

so that the noise sidebands around the peaks at the repetition frequency are just translated to low frequencies but not folded.

The next condition arises from the Nyquist theorem, which says that the sampling frequency has to be at least twice the frequency of the sampled signal:

$$f_s > 2\left[\max\left(\left|f_{rep1} - f_{osc}\right|, \left|f_{rep2} - f_{osc}\right|\right) + f_{max}\right] \tag{9}$$

(Note that we need to properly sample the full noise sidebands.) This will at least require that $f_s > 4 f_{max}$, but the limit can be higher if the laser repetition rates are significantly different.

Furthermore, the sampling frequency must be high enough to obtain a low enough noise floor; using (7), we obtain the condition

$$f_s > \frac{1}{3} 2^{-2n} / S_{fl} \,. \tag{10}$$

Of course, the detection noise (see Sect. 3.1) must also be low enough to achieve this noise floor.

Finally, to obtain information on noise down to a frequency f_{min}, we need a sufficiently long measurement time

$T = N\delta t = N/f_s$. This leads to the condition $T > 1/f_{min}$, but in many cases we are loosing at least the lowest frequency sample due to the leakage problem (see Sect. 2). Thus we should at least have $T > 2/f_{min}$ or

$$N > 2\frac{f_s}{f_{min}} . \tag{11}$$

If the required sampling rate is rather high due to a high value of f_{max}, different laser repetition rates, or the requirement of a low noise level, even moderate choices of f_{min} may lead to rather large values of N. Even if the sampling card is able to handle millions of samples, the processing time is then increased. This can be easily avoided by doing two (or even more) measurements for different frequency regions: the high-frequency region is treated with a high sampling rate but moderate value of N, and the low-frequency region is measured separately with a lower sampling rate, accepting a higher noise floor for this part. This higher noise floor may not matter as the noise of mode-locked lasers tends to be much higher at low noise frequencies.

3.5 *Comparison with other techniques*

3.5.1 Spectral analysis of photodiode signal. The probably most popular technique for timing jitter measurements is based on the direct Fourier analysis of a photodiode current (reflecting the output power of a mode-locked laser) with an electronic spectrum analyzer. It has been shown [1] that at least in simple cases there are two contributions to the noise sidebands of different harmonics of the repetition rate in the obtained spectrum: a contribution from intensity noise, which is the same for all sidebands, and a contribution from timing noise, the power of which scales with the square of the sideband order. For high enough sideband orders, timing noise may dominate, so that the timing noise spectrum can be obtained. Although this technique is quite simple, it has a number of fundamental and practical limitations, which are shortly summarized in the following.

The analysis of [1] is based on a linear expansion of timing phase errors, which is usually justified for actively mode-locked lasers, but is problematic for passively mode-locked lasers (without a timing stabilization) where the timing errors can grow without bound. In the latter case, the analysis stays applicable for limited measurement times, effectively leading to a lower limit for the accessible noise frequencies.

Another important aspect arises from possible correlations between pulse parameters like energy, temporal position and center frequency. Such correlations can lead to additional contributions to the noise sidebands [9, 10], so that the timing jitter can no more easily be obtained. Unfortunately, there is no simple method to check whether such correlations affect the results.

Besides these fundamental limitations, a number of practical problems can occur. To begin with, for lasers with multi-GHz repetition rates one can often record only very few noise sidebands (or even only one), so that it becomes difficult to distinguish between intensity and timing noise. An often overlooked problem is phase noise of the local oscillator in the spectrum analyzer; we show in Sect. 4 that this can easily mask the noise of the laser under test. Effectively we are comparing the timing of the laser with the timing of a tunable electronic oscillator, which can not easily achieve a better timing precision than a good laser. AM-PM conversion can occur in the photodetector, but this problem will probably be far less severe than local oscillator noise in most cases. Finally, noise measurements with electronic spectrum analyzers are easily affected by a variety of errors if they are not done with great care; for example, it is often overlooked that the correct detector mode (sample mode) has to be chosen, and that corrections have to be applied to the results, taking into account aspects like the averaging of logarithmic values and the difference between 3-dB bandwidth and effective noise bandwidth. In fact, a lot of published noise specifications may be too low due to the fact that such corrections have not been applied.

Apart from AM-PM conversion in the photodetector, the proposed new measurement scheme does not suffer from any of these limitations. The method works even for very large timing deviations between the lasers; only the repetition rates should not differ too much. Intensity and timing phase noise are clearly distinguished. Correlations between the fluctuations of different pulse parameters should not affect the results. High repetition rates are no problem as long as fast enough detectors and mixers are available, apart from the reference oscillator. Finally, noise of the electronic oscillator does not affect the timing noise measurement.

The only apparent disadvantages of the proposed method are that two lasers are required – either two lasers of the same kind or one laser to be tested and a more stable reference laser – and that an additional (preferably tunable) electronic oscillator is required, even though its noise properties are not critical.

3.5.2 Electronic phase detector. A frequently used method for the measurement of phase deviations of electronic signals involves an electronic mixer, which is operated as a phase detector [2]. Here, the phase of one of the two oscillating signals is adjusted so that on average the relative phase difference between the signals is $\pi/2$. For small deviations from this value, the mixer output is proportional to the phase deviation. This output voltage can then be recorded.

Obviously, this method is applicable only when the relative timing deviation of the two lasers stays well below the pulse period for all times. This is usually the case e.g., for two actively mode-locked lasers using the same modelocker signal, or when a passively mode-locked laser is timing-locked to another laser using some feedback control. Two free-running passively mode-locked lasers, however, can not be investigated with this method.

A serious technical problem is AM-PM conversion in the mixer. An ideal mixer would not exhibit this effect, as e.g., for a constant phase offset of $\pi/2$ there is no output signal, regardless of intensity noise. However, mixer offsets introduce AM-PM conversion. Although such offsets can in principle be electronically compensated, offset drifts make this difficult or impractical. Advanced techniques have been applied to mitigate such problems [2] at the cost of increased complexity of the electronics design.

Compared to the electronic phase detector method, the proposed new measurement scheme has a wider range of applications (e.g., to free-running passively mode-locked lasers) and a lower susceptibility to AM-PM conversion, while there appears to be no significant disadvantage.

3.5.3 Optical phase detector. Another option is to do a phase comparison in the optical domain, using e.g., an intensity cross-correlation by sum frequency generation in a nonlinear crystal. As the cross-correlation itself is fully subject to intensity noise, the difference of the signals from two cross correlators with a slight timing offset have to be used [4]. This significantly reduces the sensitivity for intensity noise, although this has not yet been checked in detail.

The optical phase detector technique basically eliminates all problems with photodetector noise, and in particular for short pulse durations it can deliver a very sensitive phase error signal. Such a signal has been shown to be suitable for a timing stabilization with sub-femtosecond relative timing jitter [4].

Of course, the phase error signal is available only for timing offsets which are approximately smaller than the pulse duration. The restriction to the allowed timing offset is thus even significantly more severe than for electronic phase detectors.

Compared with the novel jitter measurement method introduced here, the optical phase detector technique has the advantage of a potentially much lower noise floor by basically eliminating the influence of photodetection noise. However, it is applicable only for small timing deviations and requires a more complicated optical setup.

4 Experimental demonstration

In the following we describe the experimental setup with which we have tested the new measurement technique.

The two lasers are passively mode-locked Er:Yb:glass lasers (ERGO PGLs from GigaTera) operating with a 10-GHz pulse repetition rate, similar to the laser described in [7] apart from an improved mechanical setup. They deliver output powers of ≈ 15 mW, and their pulse durations are around 1.5 ps. The cavities have been constructed to be mechanically quite stable, with all optical elements attached to a single metal frame, which itself is fixed in a bigger metallic housing for shielding against vibrations and air currents.

To minimize the impact of detection noise (see Sect. 3.1), we have chosen to use a special kind of traveling-wave photodiodes (XPDV 2020R from u2t photonics, Germany) which can handle high peak photocurrents of up to 30 mA and have a 3-dB bandwidth of ≈ 50 GHz. The obtained RF powers in the 10-GHz peaks are ≈ 12 dBm for average photocurrents of ≈ 2.3 mA, sufficient to directly drive the mixers (Miteq DM0416LW2) without using preamplifiers, which would introduce additional noise. The oscillator is a HP 83650A, tunable from 10 MHz to 50 GHz. It was set to deliver a power of 10 dBm to the local oscillator port of each mixer.

The mixer outputs are sufficiently powerful to be directly recorded with the two inputs of a sampling card (National Instruments NI PCI-5122) in a personal computer. The card has a 14-bit resolution and allows one to record up to ≈ 16 million samples per channel with a sampling rate of up to 100 MHz. The computer is subsequently used to read out the recorded traces and to process these data according to the algorithm described in Sect. 2.

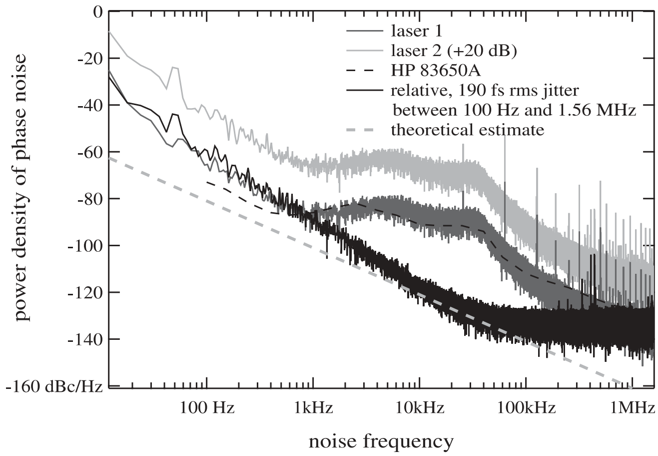

FIGURE 2 Power densities of the timing phase noise of two free-running lasers (*gray curves*) and of their phase difference (*black curve*). *Upper dashed curve*: noise specs of the electronic oscillator. *Lower* (*and straight*) *dashed curve*: theoretical estimate for quantum-limited timing noise of the lasers

Figure 2 shows timing noise spectra for the two free-running lasers and for the difference signal, obtained for a sampling rate of 50 MHz with 10 million samples per channel. Each spectrum has been averaged over four measurements. The average photocurrents were 2.3 mA. The spectra for the two lasers are hardly distinguishable, while the power density for the difference signal is significantly lower for ≈ 1–400 kHz, indicating that noise from the electronic oscillator is dominating here but cancels out in the difference signal. Indeed, the upper dashed curve shows that this noise is close to the specifications of the electronic oscillator. For frequencies higher than ≈ 50 kHz, there is a noise floor for the difference signal which is caused by sampling noise (see below). For frequencies below ≈ 50 kHz, the difference curve shows the relative timing noise of the lasers. This can be compared with the dashed line, which is a theoretical estimate based on theoretical results for quantum-limited timing noise from [8] and on estimated parameters of the lasers, with 3 dB added for the relative noise between two independent lasers. We see that for 10–100 kHz the laser noise is close to the limit given by the quantum noise influence in the gain medium (assuming a perfectly stable laser cavity), while at lower noise frequencies there is an increasing discrepancy which might be due to drifts of the cavity length. Narrow noise peaks due to mechanical vibrations are not observed, while there is a weak peak at 50 Hz which is apparently picked up through the driver electronics of the lasers.

Figure 3 shows similar data for the situation where the two lasers are timing-stabilized to the same electronic reference, using feedback to the cavity length via a piezo actuator behind the SESAM. The timing stabilization effectively suppresses the low-frequency noise, as is seen in the spectrum of the relative timing phase noise. We expect that further optimization of the mechanical cavity setup and the feedback electronics should result in a higher feedback bandwidth and a lower low-frequency noise floor. Note that the low-frequency noise is already well below the mentioned quantum limit, which is not

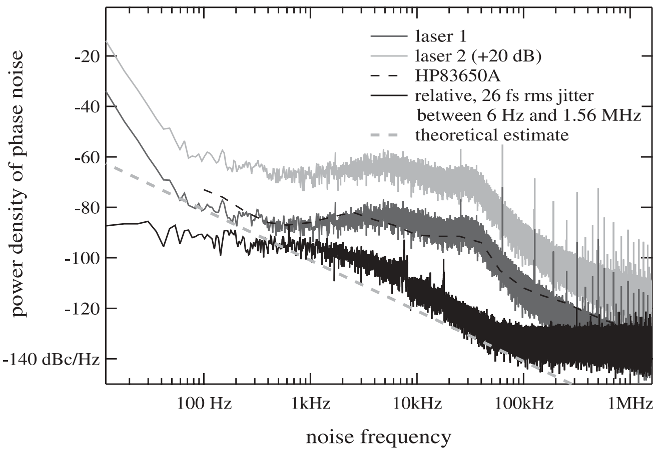

FIGURE 3 Power densities of the timing phase noise of two timing-stabilized lasers (*gray curves*) and of their phase difference (*black curve*). *Upper dashed curve*: noise specs of the electronic oscillator. *Lower dashed curve*: theoretical estimate for quantum-limited timing noise

surprising since this is only the limit for a laser subject only to quantum noise, but not the absolute quantum limit which is far lower [8].

The lower noise trace in Fig. 4 shows the difference signal for the new method when both photodiodes detect parts of the output of a single laser, so that any relative timing noise in the optical signal is removed, apart from weak peaks at 60, 180, and 300 Hz and negligible contributions from mechanical instabilities of the relative path lengths to the detectors. As expected, this shows a flat noise floor, since the sampling errors are essentially uncorrelated. The higher black curve is the same as in Fig. 3 for two timing-stabilized lasers. For comparison, the figure also shows a noise trace obtained with the frequently used method based on the spectral analysis of the signal from a fast photodiode [1] (see Sect. 3.5.1), using a single laser, and with 3 dB added for comparison with the relative jitter measurement. In most spectral regions, the latter curve is higher and is close to the curve indicating the noise specs of the HP 8565EC spectrum analyzer used for this measurement. This local oscillator noise basically limits the sensitivity of the method. Particularly at low noise frequencies, the noise floor of our new method is far lower, even though the noise of our electronic oscillator is much stronger than that of the HP 8565EC spectrum analyzer.

For a quantitative test, we compare the noise floor of our method with theoretical expectations. The calculated noise levels are −162 dBc/Hz for shot noise, −163 dBc/Hz for the thermal detector noise, which is increased to −154 dBc/Hz by mixer noise, and a theoretical limit of −166 dBc/Hz for sampling noise (with 50 MSamples/s). The observed noise floor is $\approx$ 136 dBc/Hz (with 50 MSamples/s), which we attribute to excess noise from the sampling card, because the observed noise level depends on the sampling rate. A dedicated high-quality digital oscilloscope as a replacement for the (cheaper) sampling card may offer a significantly improved noise performance which would directly translate into an even lower noise floor of the jitter measurements. With a significantly better oscilloscope, the next limitation would be mixer noise, the impact of which could be reduced by using low-noise preamplifiers.

Figure 5 shows the timing phase errors versus time for a short temporal slice of the traces recorded with two free-running lasers. The recorded timing phase errors for both lasers are very similar, as the noise of these signals is actually dominated by common noise from the electronic oscillator. The noise of the difference signal is much weaker. With timing-stabilized lasers, the results are very similar, except that the vertical offset between the curves for both lasers is strongly reduced. Note that the timing stabilization can suppress timing noise only on time scales which are longer than the shown span, because the feedback bandwidth is limited.

Figure 6 shows intensity noise spectra for both free-running lasers. Switching on the timing stabilization does not significantly change the result. One can observe broad (but not very high) peaks at the relaxation oscillation frequencies of

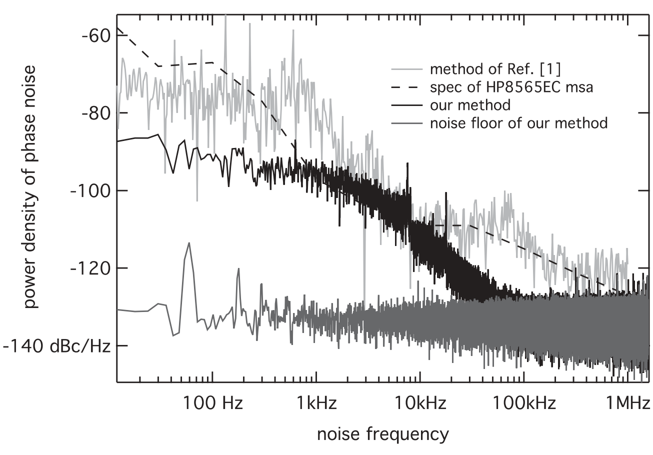

FIGURE 4 Noise floor of phase noise measurements (*dark gray curve*) obtained by measuring the phase noise between two parts of the output from a single laser. *Black*: relative phase noise of two timing-stabilized lasers. *Light gray*: phase noise of timing-stabilized laser measured with the method of [1]. The *dashed curve* shows the noise specs of the spectrum analyzer used in this measurement

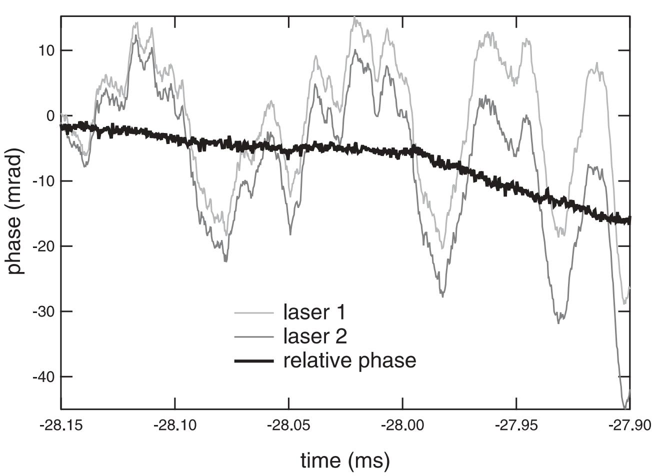

FIGURE 5 Timing phase errors versus time recorded for two free-running lasers (*dark* and *light gray curves*). The phase errors are dominated by noise from the microwave oscillator, which cancels out in the phase difference (*black*)

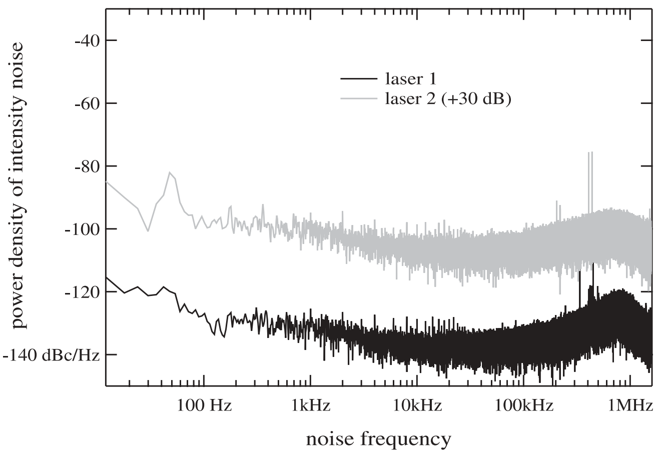

FIGURE 6 Relative intensity noise power spectra of both free-running lasers

≈ 750 kHz, apart from a 50-Hz peak which is probably caused by the laser drivers. For ≈ 2–100 kHz, the noise is close to the noise floor set by sampling noise, which is ≈ 20 dB above the shot noise level.

Finally, we investigated AM-PM conversion in photodetection. For this purpose, we exposed both detectors to the output of a single laser, where the intensity of one beam was modulated with a mechanical chopper wheel by typically a few percent peak to peak. The intensity noise spectrum for the modulated detector as well as the relative phase noise spectrum exhibits a peak at the chopper frequency. By comparing the peak heights, we can quantify the AM-PM conversion. For example, for a moderate average photocurrent of 2.3 mA, the peak in the relative phase noise spectrum was ≈ 16 dB lower than in the amplitude noise spectrum, indicating a moderate degree of AM-PM conversion. This difference increased to 48 dB for a lower photocurrent of 0.2 mA, showing that AM-PM conversion becomes significant only for the largest allowed photocurrents. In our case, the larger photocurrent of 2.3 mA corresponds to a peak current in the order of the maximum allowed value of 30 mA. As the pulse duration is well below the response time of the detector, the optical peak power is already well above the value which would be allowed for longer pulses. In any case, given the low intensity noise of our lasers, AM-PM conversion is found to be totally insignificant for our timing jitter measurements. If required in other cases, a strong further reduction of AM-PM conversion could be achieved with lower photocurrents at the expense of a higher noise floor.

5 Conclusions

We have proposed and demonstrated a novel technique which allows the precise measurement of the relative jitter for free-running or timing-stabilized mode-locked lasers. The potential of this technique has been carefully analyzed and experimentally demonstrated. Its noise floor is very low, particularly if suitable high-current photodetectors and sampling electronics with high bit resolution are used, and is flat, rather than strongly rising at low frequencies, as is the case for some other techniques. In our case, the sampling noise has been found to be the limiting factor and to be higher than theoretically possible. AM-PM conversion has been found to be very low, totally insignificant for our timing jitter measurements. For two passively mode-locked miniature Er:Yb:glass lasers in a free-running and a timing-stabilized configuration, we have been able to precisely measure the timing noise, which is not far from the quantum limit in a wide frequency range.

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