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RESEARCH ARTICLE

# Dark energy and dark gravity: theory overview

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**Abstract** Observations provide increasingly strong evidence that the universe is accelerating. This revolutionary advance in cosmological observations confronts theoretical cosmology with a tremendous challenge, which it has so far failed to meet. Explanations of cosmic acceleration within the framework of general relativity are plagued by difficulties. General relativistic models are nearly all based on a dark energy field with fine-tuned, unnatural properties. There is a great variety of models, but all share one feature in common—an inability to account for the gravitational properties of the vacuum energy. Speculative ideas from string theory may hold some promise, but it is fair to say that no convincing model has yet been proposed. An alternative to dark energy is that gravity itself may behave differently from general relativity on the largest scales, in such a way as to produce acceleration. The alternative approach of modified gravity (or dark gravity) provides a new angle on the problem, but also faces serious difficulties, including in all known cases severe fine-tuning and the problem of explaining why the vacuum energy does not gravitate. The lack of an adequate theoretical framework for the late-time acceleration of the universe represents a deep crisis for theory—but also an exciting challenge for theorists. It seems likely that an entirely new paradigm is required to resolve this crisis.

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## 1 Introduction

The current "standard model" of cosmology is the inflationary cold dark matter model with cosmological constant, usually called LCDM, which is based on general relativity and particle physics (i.e. the Standard Model and its minimal supersymmetric extensions). This model provides an excellent fit to the wealth of high-precision observational data, on the basis of a remarkably small number of cosmological parameters [1,2]. In particular, independent data sets from cosmic microwave background (CMB) anisotropies, galaxy surveys and supernova luminosities, lead to a consistent set of best-fit model parameters—which represents a triumph for LCDM.

The standard model is remarkably successful, but we know that its theoretical foundation, general relativity, breaks down at high enough energies, usually taken to be at the Planck scale,

$$E \gtrsim M_p \sim 10^{16}\,\mathrm{TeV}. \tag{1}$$

The LCDM model can only provide limited insight into the very early universe. Indeed, the crucial role played by inflation belies the fact that inflation remains an effective theory without yet a basis in fundamental theory. A quantum gravity theory will be able to probe higher energies and earlier times, and should provide a consistent basis for inflation, or an alternative that replaces inflation within the standard cosmological model (for recent work in different directions, see, e.g. Refs. [3–8]).

An even bigger theoretical problem than inflation is that of the late-time acceleration in the expansion of the universe [9–17]. In terms of the fundamental energy density parameters, the data indicates that the present cosmic energy budget is given by

$$\Omega_\Lambda \equiv \frac{\Lambda}{3H_0^2} \approx 0.75, \quad \Omega_m \equiv \frac{8\pi G\rho_{m0}}{3H_0^2} \approx 0.25, \quad \Omega_K \equiv \frac{-K}{H_0^2} \approx 0, \tag{2}$$

$$\Omega_r \equiv \frac{8\pi G\rho_{r0}}{3H_0^2} \approx 8 \times 10^{-5}. \tag{3}$$

Here $H_0$ is the present value of the Hubble parameter, $\Lambda$ is the cosmological constant, $K$ is spatial curvature, $\rho_{m0}$ is the present matter density and $\rho_{r0}$ is the present radiation density. $G$ is Newton's constant. The Friedman equation is

$$\begin{aligned}\left(\frac{\dot a}{a}\right)^2 &\equiv H^2 = \frac{8\pi G}{3}(\rho_m + \rho_r) + \frac{\Lambda}{3} - \frac{K}{a^2}\\ &= H_0^2\left[\Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_\Lambda + \Omega_K(1+z)^2\right],\end{aligned} \tag{4}$$

where $a$ denotes the scale factor which is related to the cosmological redshift by $z = a^{-1} - 1$. We normalize the present value of the scale factor to $a_0 = 1$. Together with the energy conservation equation this implies

$$\frac{\ddot a}{a} = -\frac{4\pi G}{3}(\rho_m + 2\rho_r) + \frac{\Lambda}{3}. \tag{5}$$

The observations, which together with Eq. (4) lead to the values given in Eq. (2), produce via Eq. (5) the dramatic conclusion that the universe is currently accelerating,

$$\ddot{a}_0 > 0. \tag{6}$$

This conclusion holds only if the universe is (nearly) homogeneous and isotropic, i.e. a Friedmann–Lemaître model. In this case the distance to a given redshift $z$, and the time elapsed since that redshift, are tightly related via the only free function of this geometry, $a(t)$. If the universe instead is isotropic around us but not homogeneous, i.e. if it resembles a Tolman–Bondi–Lemaître solution with our galaxy cluster at the centre, then this tight relation between distance and time for a given redshift would be lost and present data would not necessarily imply acceleration. This point is discussed in detail in the contribution by Enqvist [18]. Of course isotropy without homogeneity violates the Copernican principle as it puts us in the centre of the Universe. However, it has to be stressed that up to now observations of homogeneity are very limited, unlike isotropy, which is firmly established. Homogeneity is usually inferred from isotropy together with the Copernican principle. With future data, it will in principle be possible to distinguish observationally an isotropic but inhomogeneous universe from an isotropic and homogeneous universe (see, e.g. [19]). In the following, we disregard this possibility and assume that the Copernican principle applies.

The data also indicate that the universe is currently (nearly) spatially flat,

$$|\Omega_K| \ll 1. \tag{7}$$

It is common to assume that this implies $K = 0$ and to use inflation as a motivation. However, inflation does *not* imply $K = 0$, but only $\Omega_K \to 0$. In the late universe, the distinction may be negligible. But in the very early universe, a non-zero curvature can have significant effects (see, e.g. [20]). In fact, if curvature is small but non-vanishing, neglecting it in the analysis of Supernova data can sometimes induce surprisingly large errors, as discussed in the contribution by Hlozek et al. [21].

These results are illustrated in Fig. 1 (taken from [22,23]). A detailed discussion of the experimental aspects of the late-time acceleration is given in the contributions by Leibundgut [24], Nichol [25] and Sarkar [26].

The simplest option is probably a cosmological constant, i.e. the LCDM model. Even though the cosmological constant can be considered as simply an additional gravitational constant (in addition to Newton's constant), a cosmological constant enters the Einstein equations in exactly the same way as a contribution from the vacuum energy, i.e. via a Lorentz-invariant energy–momentum tensor $T_{\mu\nu}^{\rm vac} = -(\Lambda/8\pi G)g_{\mu\nu}$. The only observable signature of both a cosmological constant and vacuum energy is their effect on spacetime—and so a vacuum energy and a classical cosmological constant cannot be distinguished by observation. Therefore the "classical" notion of the cosmological constant is effectively physically indistinguishable from a quantum vacuum energy.

Even though the absolute value of vacuum energy cannot be calculated within quantum field theory, *changes* in the vacuum energy (e.g. during a phase transition) can be calculated, and they do have a physical effect—for example, on the energy

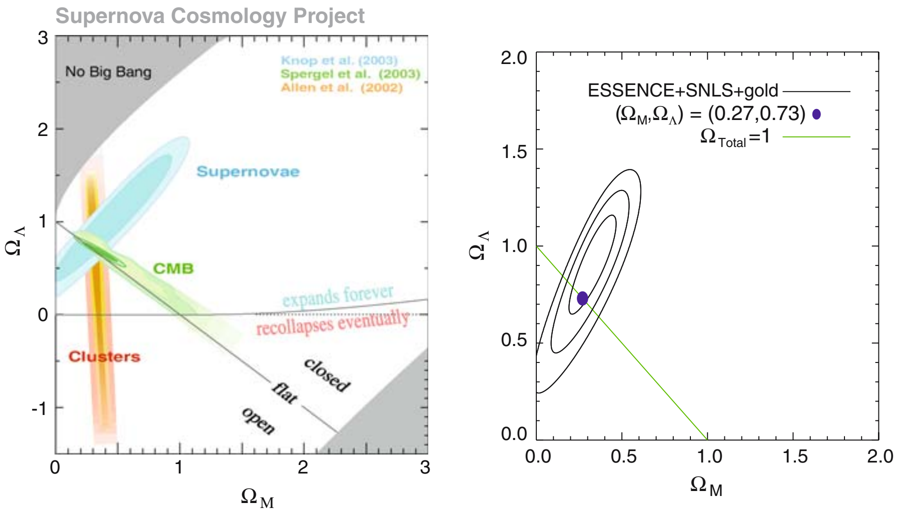


**Fig. 1** Observational constraints in the $(\Omega_m, \Omega_\Lambda)$ plane: joint constraints (*left*) (from [22]); recent compilation of supernova constraints (*right*) (from [23]). The *line* $\Omega_K = \Omega_m + \Omega_\Lambda - 1 = 0$ is also indicated

levels of atoms (Lamb shift), which is well known and well measured. Furthermore, differences of vacuum energy in different locations, e.g. between or on one side of two large metallic plates, have been calculated and their effect, the Casimir force, is well measured [27,28]. Hence, there is no doubt about the reality of vacuum energy. For a field theory with cutoff energy scale $E$, the vacuum energy density scales with the cutoff as $\rho_{\rm vac} \sim E^4$, corresponding to a cosmological constant $\Lambda_{\rm vac} = \rho_{\rm vac}/(8\pi G)$. If $E = M_p$, this yields a renormalization of the "cosmological constant" of about $\Lambda_{\rm vac} \sim 10^{38}\,{\rm GeV}^2$, whereas the measured effective cosmological constant is the sum of the "bare" cosmological constant and the contribution from renormalization,

$$\Lambda_{\rm eff} = \Lambda_{\rm vac} + \Lambda \simeq 10^{-83}\,{\rm GeV}^2. \tag{8}$$

Hence a cancellation of about 120 orders of magnitude is required. This is called the *fine tuning* or *size* problem of dark energy: a cancellation is needed to arrive at a result which is many orders of magnitude smaller than each of the terms.[1] It is possible that the quantum vacuum energy is much smaller than the Planck scale. But even if we set it to the lowest possible SUSY scale, $E_{\rm susy} \sim 1$TeV, arguing that at higher energies vacuum energy exactly cancels due to supersymmetry, the required cancellation is still about 60 orders of magnitude. These issues are discussed in the contributions by Padmanabhan [29] and Bousso [30].

A reasonable attitude towards this open problem is the hope that quantum gravity will explain this cancellation. But then it is much more likely that we shall obtain

[1] In quantum field theory we actually have to add to the cut-off term $\Lambda_{\rm vac} \simeq E_c^4/M_{pl}^2$ the unmeasureable "bare" cosmological constant. In this sense, the cosmological constant problem is a fine tuning between the unobservable "bare" cosmological constant and the term coming from the cut-off scale.

directly $\Lambda_{\text{vac}} + \Lambda = 0$ and not $\Lambda_{\text{vac}} + \Lambda \simeq 3\rho_m(t_0)/(8\pi G)$. This unexpected observational result leads to a second problem, *the coincidence problem*: given that

$$\rho_\Lambda = \frac{\Lambda_{\text{eff}}}{8\pi G} = \text{constant}, \quad \text{while} \;\; \rho_m \propto (1+z)^3, \tag{9}$$

why is $\rho_\Lambda$ of the order of the *present* matter density $\rho_m(t_0)$? It was completely negligible in most of the past and will entirely dominate in the future.

Instead of a cosmological constant, one may also introduce a scalar field or some other contribution to the energy–momentum tensor which has an equation of state $w < -1/3$. Such a component is called "dark energy". So far, no consistent model of dark energy has been proposed which can yield a convincing or natural explanation of either of these problems. A variety of such models is discussed in the contribution by Linder [31].

Alternatively, it is possible that there is no dark energy field, but instead the late-time acceleration is a signal of a *gravitational* effect. Within the framework of general relativity, this requires that the impact of inhomogeneities somehow acts to produce acceleration, or the appearance of acceleration (within a Friedman–Lemaître interpretation). One possibility is the Tolman–Bondi–Lemaître model discussed in this volume [18]. Another possibility is that the "backreaction" of inhomogeneities on the background, treated via non-linear averaging, produces effective acceleration. This is discussed in the contribution by Buchert [32].

A more radical version is the "dark gravity" approach, the idea that gravity itself is weakened on large-scales, i.e. that there is an "infrared" modification to general relativity that accounts for the late-time acceleration. Specific classes of models which modify gravity are discussed in the contributions by Capozziello and Francaviglia [33] and by Koyama [34]. Schematically, we are modifying the geometric side of the field equations,

$$G_{\mu\nu} + G_{\mu\nu}^{\text{dark}} = 8\pi G T_{\mu\nu}, \tag{10}$$

rather than the matter side,

$$G_{\mu\nu} = 8\pi G \left( T_{\mu\nu} + T_{\mu\nu}^{\text{dark}} \right), \tag{11}$$

as in the general relativity approach. Modified gravity represents an intriguing possibility for resolving the theoretical crisis posed by late-time acceleration. However, it turns out to be extremely difficult to modify general relativity at low energies in cosmology, without violating the low-energy solar system constraints, or without introducing ghosts and other instabilities into the theory. Up to now, there is no convincing alternative to the general relativity dark energy models—which themselves are not convincing.

The plan of the remainder of this paper is as follows. In Sect. 2 we discuss constraints which one may formulate for a dark energy or modified gravity (dark gravity) theory from basic theoretical requirements. In Sect. 3 we discuss models that address the dark energy problem within general relativity. In Sect. 4 we present modified gravity models. The ideas outlined in Sects. 3 and 4 are discussed in more detail in the specific contributions of this issue which are devoted to them. In Sect. 5 we conclude.

## 2 Constraining effective theories

The theories of both dark matter and dark energy often have very unusual Lagrangians that cannot be quantized in the usual way, e.g. because they have non-standard kinetic terms. We then simply call them "effective low energy theories" of some unspecified high energy theory which we do not elaborate. In this section, we want to point out a few properties which we nevertheless can require of low energy effective theories. We first enumerate the properties which we can require from a good basic physical theory at the classical and at the quantum level. We then discuss which of these requirements are inherited by low energy effective descriptions.

### 2.1 Fundamental physical theories

Here we give a minimal list of properties which we require from a fundamental physical theory. Of course, all the points enumerated below are open for discussion, but at least we should be aware of what we lose when we let go of them.

In our list we start with very basic requirements which become more strict as we go on. Even though some theorists would be able to live without one or several of the criteria discussed here, we think they are all very well founded. Furthermore, all known current physical theories, including string- and M-theory, do respect them.

1. **A physical theory allows a mathematical description**
   This is the basic idea of theoretical physics. It may well be wrong at some stage, but it has been a working hypothesis for all of what we call theoretical physics. If it has limitations these may well be called the limitations of theoretical physics itself.
2. **A physical theory allows a Lagrangian formulation**
   Fundamental physical theories have a Lagrangian formulation. This requirement is of course much stronger than the previous one. But it has been extremely successful in the past and was the guiding principle for the entire development of quantum field theory and string theory in the twentieth century. If we drop it, anything goes. We can then just say the evolution of the scale factor of the universe obeys $a(t) = At^{1/2} + Bt^{2/3} + C\exp(t/t_0)$, call this our physical theory and fit the four parameters $A$, $B$, $C$ and $t_0$ from cosmological data. Of course something like this does not deserve the name "theory"; it is simply a fit to the data.
   Nevertheless sometimes fits of this kind are taken more seriously then they should be. Some "varying speed of light theories" without Lagrangian formulation leave us more or less free to specify the evolution of the speed of light during the expansion history of the universe. However, if we introduce a Lagrangian formulation, we realize that most of these theories are simply some variant of scalar tensor theories of gravity, which are of course well defined and have been studied in great detail.

If we want to keep deep physical insights like Nöther's theorem, which relates symmetries to conservation laws, we need to require a Lagrangian formulation for a physical theory. A basic ingredient of a Lagrangian physical theory is that every physical degree of freedom has a kinetic term which consists (usually) of first order time derivatives and may also have a "potential term" which does not involve derivatives. In the Lagrangian formulation of a fundamental physical theory, we do not allow for external, arbitrarily given functions. Every function has to be a degree of freedom of the theory so that its evolution is determined self-consistently via the Lagrangian equations of motion, which are of first or second order. It is possible that the Lagrangian contains also higher than first order derivatives, but such theories are strongly constrained by the problem of ghosts which we mention below, and by the fact that the corresponding equations of motion are usually described by an unbounded Hamiltonian, i.e. the system is unstable (Ostrogradski's theorem [35,36]). For example, for the gravitational Lagrangian in four dimensions, this means that we may only allow for a function depending on $R$ and its derivatives, where $R$ is the Riemann curvature scalar.

3. **Lorentz invariance**
   We also want to require that the theory be Lorentz invariant. Note that this requirement is much stronger than demanding simply "covariance". It requires that there be no "absolute element" in the theory apart from true constants. Lorentz covariance can always be achieved by rewriting the equations. As an example, let us consider a Lagrangian given in flat space by $(\partial_t\phi)^2 - (\partial_x\phi)^2$. This is clearly not Lorentz invariant. However, we can trivially write this term in the covariant form $\alpha^{\mu\nu}\partial_\nu\partial_\mu\phi$, by setting $(\alpha^{\mu\nu}) = \mathrm{diag}(1, -1, 0, 0)$. Something like this should of course not be allowed in a fundamental theory. A term of the form $\alpha^{\mu\nu}\partial_\nu\partial_\mu\phi$ is only allowed if $\alpha^{\mu\nu}$ is itself a dynamical field of the theory. This is what we mean by requiring that the theory is not allowed to contain "absolute elements", i.e. it is Lorentz invariant and not simply covariant.
4. **Ghosts**
   Ghosts are fields whose kinetic term has the wrong sign. Such a field, instead of slowing down when it climbs up a potential, is speeding up. This unstable situation leads to severe problems when we want to quantize it, and it is generally accepted that one cannot make sense of such a theory, at least at the quantum level. This is not surprising, since quantization usually is understood as defining excitations above some ground state, and a theory with a ghost has no well defined ground state. Its kinetic energy has the wrong sign and the larger $\dot{\phi}^2$ is, the lower is the energy.
5. **Tachyons**
   These are degrees of freedom that have a negative mass squared, $m^2 < 0$. Using again the simple scalar field example, this means that the second derivative of the potential about the "vacuum value" ($\phi = 0$ with $\partial_\phi V(0) = 0$) is negative, $\partial_\phi^2 V(0) < 0$. In general, this need not mean that the theory makes no sense, but rather that $\phi = 0$ is a bad choice for expanding around, since it is a maximum rather than a minimum of the potential and therefore an unstable equilibrium.
   This means also that the theory cannot be quantized around the classical solution $\phi = 0$, but it may become a good quantum theory by a simple shift, $\phi \to \phi - \phi_0$,

where $\phi_0$ is the minimum of the potential. If the potential of a fundamental scalar field has no minimum but only a maximum, the situation is more severe. Then the theory is truly unstable.

The last two problems, together with the Ostrogradski instability that appears in theories with higher derivatives, can be summarized in the requirement that a meaningful theory needs to have an energy functional which is bounded from below.

6. **Superluminal motion and causality**

A fundamental physical theory which does respect Lorentz invariance must not allow for superluminal motions. If this condition is not satisfied, we can construct closed curves along which a signal can propagate, in the following way.

Consider modes of a field $\phi$ which can propagate faster than the speed of light, with velocities $v_1 > 1$ and $v_2 > 1$. Consider a reference frame $R$ and a frame $R'$ that is boosted with respect to $R$ by a velocity $v$ in the direction $x$, and which coincide at the origin, $q_0$ (see Fig. 2). We choose $v$ such that $1/v_1 < v < 1$. An observer in $R$ now sends a signal from $q_0$, whose coordinates are $(t, x) = (0, 0) = (t', x')$, with signal speed $v_1$ in the direction $x$. At time $t_1$ this signal arrives at event $q_1$, with coordinates $(t_1, x_1)$ in the $R$-frame, where $x_1 = v_1 t_1$. There it is received by an observer who is at rest with respect to $R'$, and who returns the signal with speed $v_2'$ in the direction $-x$ to event $q_2 = (t_2, 0)$ (see Fig. 2). We want to show that for an appropriate choice of $v_2'$, the time $t_2$ becomes negative.

We denote positions and times in the boosted frame $R'$ with a prime. We then have $x_2' - x_1' = v_2'(t_2' - t_1')$. Applying the usual formulas for Lorentz transformations, we find that $0 = x_2 = \gamma(x_2' + vt_2')$ and $t_2 = \gamma^{-1} t_2'$. On the other hand, we have $x_1' = \gamma(x_1 - vt_1) = \gamma(v_1 - v)t_1$ and $t_1' = \gamma(t_1 - vx_1) = \gamma(1 - vv_1)t_1$. Note that, since we require $vv_1 > 1$, it follows that $t_1'$ is negative. A signal which is travelling at a speed greater than $1/v$ in the frame $R$ is moving towards the past in the frame $R'$. With respect to this frame, the event $(t_1', x_1')$ at which the signal has reached $x_1'$, is earlier than the event $(0, 0)$ when it left the position 0. With respect to the frame $R$ the situation is opposite: the signal left 0 before it reached $x_1$, $t_1 > 0$. The same happens if we now send back a signal to 0 in $R'$ with a velocity $|v_2'| > 1/v$. This signal will travel backwards in time $t$ with respect to $R$, and will arrive before the time $t_1$ when it was emitted. To achieve $\Delta t' = t_2' - t_1' = \gamma(\Delta t - v\Delta x) = \gamma(1 - vv_2)\Delta t > 0$, and at the same time $\Delta t < 0$, we need $vv_2 > 1$, hence $v_2 > 1/v$. As is evident from Fig. 2, $v_2$, which is the inverse of the slope of the straight line connecting $q_2$ and $q_1$, must be smaller than $v_1$, which is the inverse of the slope from $q_0$ to $q_1$. Hence we need $1/v_1 < 1/v_2 < v < 1$. For more details see [37].

The loop generated in this way is not "causal" since both the trajectory from $q_0$ to $q_1$ and the one from $q_1$ to $q_2$ are spacelike. So we cannot speak of the formation of closed causal loops, but it is nevertheless a closed loop along which a signal can propagate and which therefore enables the construction of a time machine, leading to the usual problems with causality and entropy.

It is well known that in relativity events with spacelike separation, like $q_0$ and $q_1$ or $q_1$ and $q_2$, have no well defined chronology. Depending on the reference frame, one of them is earlier than the other. Therefore superluminal motion leads to the

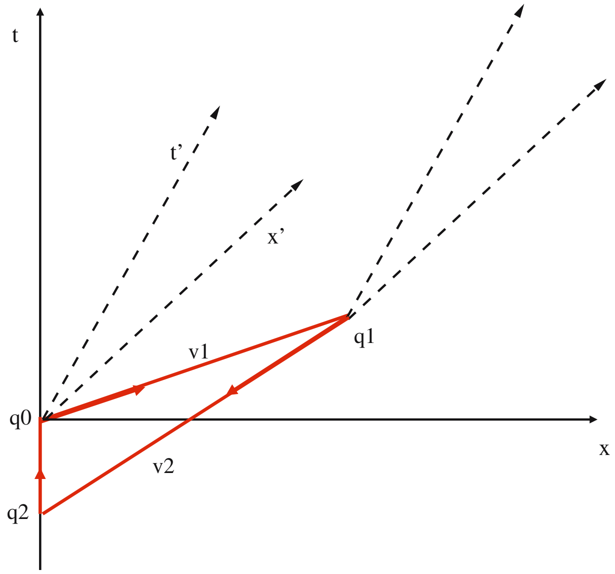


**Fig. 2** The frame $R'$ with coordinates $(t', x')$ moves with speed $v$ in the $x$-direction. A signal is sent with velocity $v_1$ from $q_0$ to $q_1$ in the frame R. Since $v_1 > 1/v$, this signal travels backward in time with respect to frame $R'$. Then a signal is sent with speed $v_2$ from $q_1$ to $q_2$. Since $|v_2| > 1/v$, this signal, which is sent forward in time in frame $R'$, travels backward in time with respect to $R$ and can arrive at an event $q_2$ with $t_2 < 0$

possibility of time machines. Hence superluminal motion is not compatible with the equivalence of all inertial frames. Once we allow for superluminal motion, but still require that signals can only be sent forward in time, an event, like $q_2$ lying in the past of $q_1$ in frame $R$ can be reached with a signal emitted in frame $R'$, but it cannot be reached if the signal is emitted from a source in $R$.

In a reference frame which moves with $v = 1/v_1$ with respect to $R$, a mode which propagates with velocity $v_1$ in R, has infinite velocity. This means that the propagation equation for this mode is no longer hyperbolic, but is elliptic, i.e. it has become a constraint equation. In this frame the evolution of the mode in the forward light cone of a small patch can no longer be determined by knowing the field values (and their first derivatives) in the small patch; the mode equation is non-local. In all reference frames moving with a velocity $v > 1/v_1$ with respect to $R$, there exist two directions in which modes with propagation velocity $v_1$ obey elliptic equations of motion. Hence the Cauchy problem is not well posed in these frames. This nullifies the equivalence of all reference frames.

At first sight one might think that a Lorentz invariant Lagrangian will automatically forbid superluminal motions. But the situation is not so simple. Already in the 1960s Velo and Zwanziger [38,39] discovered that generic Lorentz invariant higher spin theories, $s \geq 1$, lead to superluminal motion. While the equations are manifestly Lorentz invariant, their characteristics in general do not coincide with the light cone and can very well be spacelike. There are exceptions to this rule, among which are Yang Mills theories for spin 1 and the linearized Einstein equations for spin 2.

One may object to this restriction, on the grounds that general relativity, which is certainly a theory that is acceptable (at least at the classical level), can lead to closed causal curves, even though it does not admit superluminal motion. Several solutions of general relativity with closed causal curves have been constructed in the past, see, e.g. Refs. [40–42]. But these constructions usually need infinite energy, as in Ref. [41], with two infinitely long straight cosmic strings, or they have to violate the dominant energy condition, as in Ref. [40], where wormholes are used, or the closed causal curve is hidden behind an event horizon, as in Ref. [42], where an ordinary closed circle in space is converted into a causal curve by moving it behind a horizon which is such that the corresponding angular coordinate becomes timelike. Nevertheless, the possibility of closed causal curves in general relativity under certain conditions does remain a worry, see, e.g. [43].

The situation is somewhat different if superluminal motion is only possible in a background which breaks Lorentz-invariance. Then one has in principle a preferred frame and one can specify that perturbations should always propagate with the Green's function that corresponds to the retarded Green's function in this frame [44]. Nevertheless, one has to accept that there will be boosted frames relative to which the Cauchy problem for the superluminal modes is not well defined. The physics experienced by an observer in such a frame is most unusual (to say the least).

Causality of a theory is intimately related to the analyticity properties of the $S$-matrix of scattering, without which perturbative quantum theory does not make sense. Furthermore, we require the $S$ matrix to be unitary. Important consequences of these basic requirements are the Kramers Kronig dispersion relations, which are a result of the analyticity properties and hence of causality, and the optical theorem, which is a result of unitarity. The analyticity properties have many further important consequences, such as the Froissart bound, which implies that the total cross section converges at high energy [45,46].

## 2.2 Low energy effective theories

The concept of low energy effective theories is extremely useful in physics. As one of the most prominent examples, consider superconductivity. It would be impossible to describe this phenomenon by using full quantum electrodynamics with a typical energy scale of MeV, where the energy scale of superconductivity is milli-eV and less. However, many aspects of superconductivity can be successfully described with the Ginzburg–Landau theory of a complex scalar field. Microscopically, this scalar field is to be identified with a Cooper pair of two electrons, but this is irrelevant for many aspects of superconductivity.

Another example is weak interaction and four-Fermi theory. The latter is a good approximation to weak interactions at energy scales far below the $Z$-boson mass. Most physicists also regard the standard model of particle physics as a low energy effective theory which is valid below some high energy scale beyond which new degrees of freedom become relevant, be this supersymmetry, GUT or string theory.

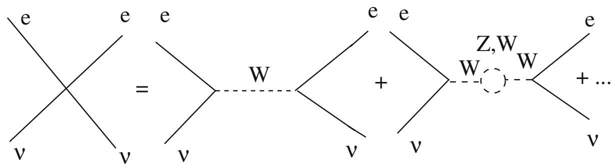


**Fig. 3** The four Fermi interaction is the sum over all "virtual" W and $Z$ boson exchanges. At low energy mainly the "tree" graph contributes

We now want to investigate which of the properties in the previous section may be lost if we "integrate out" high energy excitations and consider only processes which take place at energies below some cutoff scale $E_c$. We cannot completely ignore all particles with masses above $E_c$, since in the low energy quantum theory they can still be produced "virtually", i.e. for a time shorter than $1/E_c$. This is not relevant for the initial and final states of a scattering process, but plays a role in the interaction. As an example we consider four-Fermi theory. The vertex in the four fermion interaction is obtained by integration over $W$ and $Z$ exchanges, shown in Fig. 3. Even though the final states of this theory contain only electrons and neutrinos, the virtual presence of massive $W$'s and $Z$'s is vital for the interaction between them.

Coming back to our list in the previous section, we certainly want to keep the first point—a mathematical description. But the Lagrangian formulation will also survive if we proceed in a consistent way by simply integrating out the high energy degrees of freedom.

What about higher order derivatives in the Lagrangian? To address this question let us briefly repeat the basic argument of Ostrogradski's theorem simply for a (1-dimensional) point particle with time dependent position $q(t)$. If the Lagrangian depends only on $q$ and $\dot{q}$, the requirement $\delta S = 0$ results in the ordinary Euler Lagrange equation,

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = 0. \tag{12}$$

We can now introduce the canonical coordinates $q$ and $p \equiv \frac{\partial L}{\partial \dot{q}}$. The Hamiltonian is then given by the Legendre transform of $L$ in the variable $\dot{q}$,

$$H(q, p) = p\dot{q} - L, \tag{13}$$

and the Euler–Lagrange equation implies the canonical equations

$$\dot{q} = \frac{\partial H}{\partial p}, \quad \dot{p} = -\frac{\partial H}{\partial q}. \tag{14}$$

This procedure is well defined if the Lagrangian is non-degenerate, i.e. if the equation $p \equiv \frac{\partial L}{\partial \dot{q}}$ can be solved for $\dot{q}(q, p)$. Locally, this is equivalent to $\frac{\partial^2 L}{\partial \dot{q}^2} \neq 0$. We assume the system to be autonomous (no external time dependence). Then $H = E$ is an integral of motion, the energy of a solution, and the system is called stable if $H$ is bounded from below. If $H$ is not bounded from below, interactions of the system with, e.g. radiation will lead to an enormous production of radiation (massless particles) by driving the system to lower and lower energy.

If $L$ depends also on $\ddot{q}$, i.e. $L(q, \dot{q}, \ddot{q})$, the variational principle yields

$$\frac{d^2}{dt^2}\frac{\partial L}{\partial \ddot{q}} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} + \frac{\partial L}{\partial q} = 0. \tag{15}$$

This is a fourth order differential equation and its solutions depend on four initial data, $q(0)$, $\dot{q}(0)$, $\ddot{q}(0)$ and $\dddot{q}(0)$. A Hamiltonian formulation will now require four canonical variables, which can be chosen as

$$q_1 \equiv q, \quad q_2 \equiv \dot{q}, \quad \text{and} \quad p_1 \equiv \frac{\partial L}{\partial \dot{q}} - \frac{d}{dt}\frac{\partial L}{\partial \ddot{q}}, \quad p_2 \equiv \frac{\partial L}{\partial \ddot{q}}. \tag{16}$$

The Hamiltonian obtained by Legendre transforming the Lagrangian with respect to the coordinates $\dot{q} \equiv q_2$ and $\ddot{q}$ yields

$$H(q_1, q_2, p_1, p_2) = p_1 q_2 + p_2 \ddot{q}(q_1, q_2, p_2) - L\left(q_1, q_2, \ddot{q}(q_1, q_2, p_2)\right). \tag{17}$$

This procedure is well defined if the Lagrangian is non-degenerate in the sense that $p_2 \equiv \partial L/\partial \ddot{q}$ can be inverted to determine $\ddot{q}$. Locally this requires $\partial^2 L/\partial \ddot{q}^2 \neq 0$.

It is easy to check that the canonical equations are satisfied and $H$ is an integral of motion. But since the Lagrangian is only a function of three and not four variables, $p_1$ is not needed to express $\ddot{q}$ in terms of the canonical variables. It appears only linearly in the term $p_1 q_2$ and therefore $H$ cannot be bounded from below; i.e. the system is unstable. Of course it is possible to find well behaved solutions of this system, since for a given solution energy is conserved. But as soon as the system is interacting, e.g. with a harmonic oscillator, it will lower its energy and produce more and more oscillating modes.

This is especially serious when one quantizes the system. The vacuum is exponentially unstable to simultaneous production of modes of positive and negative energy. Of course one cannot simply "cut away" the negative energy solutions without violating unitarity. And even if the theory under consideration is only a low energy effective theory, it should at least be "unitary at low energy".

It is clear that introducing even higher derivatives only worsens the situation, since the degree of the Euler–Lagrange equation is enhanced by 2 with each new degree of freedom. Hence if the Lagrangian has degree $2+n$, there are $n+1$ pairs of canonical variables needed to describe the Hamiltonian, and of these only $n+2$ are needed to invert the Lagrangian. Hence $n$ momenta appear only linearly in the terms $p_j \dot{q}_j(q_1, \ldots, q_{n+1}, p_{n+1})$, and the Hamiltonian has $n$ unstable directions.

In this argument, it does not at all matter whether the degrees of freedom we are discussing are fundamental or only low energy effective degrees of freedom. Even if we modify the Hamiltonian at high energies, the instability, which is a *low energy* *problem*, will not disappear. There are only two ways out of the Ostrogradski instability: Firstly, if the necessary condition that $L$ be non-degenerate is not satisfied. The second possibility is via constraints. In a system with $m$ constraints, one can in principle eliminate $m$ variables. Hence if a $2 + n$ order system has $n$ constraints one might be able to eliminate all the unstable directions. In practice, this has to be studied on a case by case basis.

An important example for the dark energy problem is modified gravity Lagrangians of the form

$$L = \sqrt{-g} f(R, R_{\mu\nu}R^{\mu\nu}, C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta}). \tag{18}$$

Here $R_{\mu\nu}$ is the Ricci tensor, $C_{\mu\nu\alpha\beta}$ is the Weyl tensor and $f(x_1, x_2, x_3)$ is an arbitrary (at least three times differentiable) function. Since the curvature tensors contain second derivatives of the metric, the resulting equations of motion will in general be fourth order and Ostrogradski's theorem applies. The usual Hamiltonian formulation of general relativity leads to six independent metric components $g_{ij}$ which all acquire higher derivative terms. There is actually only one way out, which is the case $\partial_2 f = \partial_3 f = 0$, i.e. $f$ may only depend on $R$.[2] The reason is that in the Riemann scalar $R$, only a single component of the metric contains second derivatives. In this case, the consequent new degree of freedom can be fixed completely by the $g_{00}$ constraint, so that the only instability in $f(R)$ theories is the usual one associated with gravitational collapse (see [36]).

Therefore, the only acceptable generalizations of the Einstein–Hilbert action of general relativity are $f(R)$ theories, reviewed in the contribution [33].

If the Ostradgradski theorem does not apply, we have still no guarantee that the theory has no ghosts or that the potential energy is bounded from below (no "serious" tachyon). The limitation from the Ostragradski theorem, but also the ghost and tachyon problem, can be cast in the requirement that the theory needs to have an energy functional which is bounded from below. This condition can certainly not disappear in a consistent low energy version of a fundamental theory which satisfies it.

Like ghosts, the Ostrogradski instability can in principle be cured by adding a term $\propto (\Phi/m)^2(\nabla\varphi)^2$ to an unstable mode $\varphi$, where $\Phi$ is a very heavy particle with mass $M \gg m$, which has been neglected in the low energy approximation of the theory. However, this means that the full low energy theory actually must contain a term $(M/m)^2(\nabla\varphi)^2$. Consequences worked out within the low energy theory neglecting this term can in general not be trusted. Only a detailed case by case analysis can then reveal which low energy results still apply and which ones are modified by the coupling to the massive field $\Phi$.

Furthermore, the high energy cut-off will be given by some mass scale, i.e. some Lorentz invariant energy scale of the theory, and therefore the effective low energy

[2] Another (trivial) possibility is the addition of a Gauss Bonnet term, $L_{GB} = \sqrt{-g}\left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho}\right)$, which in four dimensions contributes only a surface term and does not enter the equations of motion. However, such a term becomes interesting in scalar–tensor theories of gravity where one may consider a contribution of the form $\phi L_{GB}$ to the Lagrangian.

theory should also admit a Lorentz invariant Lagrangian. Lorentz invariance is not a high energy phenomenon which can simply be lost at low energies.

What about superluminal motion and causality? We do not want to require certain properties of the $S$ matrix of the low energy theory, since the latter may not have a meaningful perturbative quantum theory; like the four-Fermi theory, it may not be renormalizable. Furthermore, one can argue that in cosmology we do have a preferred frame, the cosmological frame, hence Lorentz-invariance is broken and we can simply demand that all superluminal modes of a field propagate forward in cosmic time. Then no closed signal curves are possible.

But this last argument is very dangerous. Clearly, most solutions of a Lagrangian theory do break several or most of the symmetries of the Lagrangian spontaneously. But when applying a Lorentz transformation to a solution, we produce a new solution that, from the point of view of the Lagrangian, has the same right of existence. If some modes of a field propagate with superluminal speed, this means that their characteristics are spacelike. The condition that the mode has to travel forward in time with respect to a certain frame implies that one has to use the retarded Green's function in this frame. Since spacelike distances have no frame-independent chronology, for spacelike characteristics this is a frame-dependent statement. Depending on the frame of reference, a given mode can represent a normal propagating degree of freedom, or it can satisfy an elliptic equation, a constraint.

Furthermore, to make sure that the mode propagates forward with respect to one fixed reference frame, one would have to use sometimes the retarded, sometimes the advanced and sometimes a mixture of both functions, depending on the frame of reference. In a cosmological setting this can be done in a consistent way, but it is far from clear that such a prescription can be unambiguously implemented for generic low energy solutions. Indeed in Ref. [47] a solution is sketched that would not allow this, so that closed signal curves are again possible.

Therefore, we feel that Lorentz invariant low energy effective Lagrangians which allow for superluminal propagation of certain modes, have to be rejected. Nevertheless, this case is not as clear-cut and there are opposing opinions in the literature, e.g. [44].

With the advent of the "landscape" [48,49], physicists have begun to consider anthropic arguments to justify their theory, whenever it fits the data. Even though the existence of life on earth is an experimental fact, we consider this argument weak, nearly tantamount to giving up physics: "Things are like they are since otherwise we would not be here". We nevertheless find it important to inquire also from a purely theoretical point of view, whether really "anything goes" for effective theories. In the following sections we shall come back to the basic requirements which we have outlined in this section.

## 3 General relativistic approaches

The "standard" general relativistic interpretation of dark energy is based on the cosmological constant as vacuum energy:

$$G_{\mu\nu} = 8\pi G \left[T_{\mu\nu} + T_{\mu\nu}^{\rm vac}\right], \quad T_{\mu\nu}^{\rm vac} = -\frac{\Lambda_{\rm eff}}{8\pi G}\, g_{\mu\nu}, \tag{19}$$

where the vacuum energy–momentum tensor is Lorentz invariant. This approach faces the problem of accounting for the incredibly small and highly fine-tuned value of the vacuum energy, as summarized in Eq. (8).

String theory provides a tantalizing possibility in the form of the "landscape" of vacua [48,49]. There appears to be a vast number of vacua admitted by string theory, with a broad range of vacuum energies above and below zero. This is discussed in the contribution by Bousso [30]. The idea is that our observable region of the universe corresponds to a particular small positive vacuum energy, whereas other regions with greatly different vacuum energies will look entirely different. This multitude of regions forms in some sense a "multiverse". This is an interesting idea, but it is highly speculative, and it is not clear how much of it will survive the further development of string theory and cosmology.

An alternative view of LCDM is the interpretation of $\Lambda$ as a classical geometric constant (see, e.g. Ref. [50]), on a par with Newton's constant $G$. Thus the field equations are interpreted in the geometrical way,

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}. \tag{20}$$

In this approach, the small and fine-tuned value of $\Lambda$ is no more of a mystery than the host of other fine-tunings in the constants of nature. For example, more than a 2% change in the strength of the strong interaction means that no atoms beyond hydrogen can form, so that stars and galaxies would not emerge. However, this classical approach to $\Lambda$ does not evade the vacuum energy problem—it simply shifts that problem to "why does the vacuum not gravitate?" The idea is that particle physics and quantum gravity will somehow discover a cancellation or symmetry mechanism to explain why $\rho_{\text{vac}} = 0$. This would be a simpler solution than that indicated by the string landscape approach, and would evade the disturbing anthropic aspects of that approach. Nevertheless, it is not evident, whether this distinction between $\Lambda$ and $\rho_{\text{vac}}$ is really a physical statement, or a purely theoretical statement that cannot be tested by any experiments.

Within general relativity, various alternatives to LCDM have been investigated.

## 3.1 Dynamical dark energy: quintessence

Here we replace the constant $\Lambda/8\pi G$ by the energy density of a scalar field $\varphi$, with Lagrangian

$$L_\varphi = \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + V(\varphi), \tag{21}$$

so that in a cosmological setting,

$$\rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + V(\varphi), \quad p_\varphi = \frac{1}{2}\dot{\varphi}^2 - V(\varphi), \tag{22}$$

$$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = 0, \tag{23}$$

$$H^2 + \frac{K}{a^2} = \frac{8\pi G}{3}\left(\rho_r + \rho_m + \rho_\varphi\right). \tag{24}$$

The field rolls down its potential and the dark energy density varies through the history of the universe. "Tracker" potentials have been found for which the field energy density follows that of the dominant matter component. This offers the possibility of solving or alleviating the fine tuning problem of the resulting cosmological constant. Although these models are insensitive to initial conditions, they do require a strong fine-tuning of the *parameters of the Lagrangian* to secure recent dominance of the field, and hence do not evade the coincidence problem. More generally, the quintessence potential, somewhat like the inflaton potential, remains arbitrary, until and unless fundamental physics selects a potential. There is currently no natural choice of potential.

In conclusion, there is no compelling reason as yet to choose quintessence above the LCDM model of dark energy. Quintessence models do not seem more natural, better motivated or less contrived than LCDM. Nevertheless, they are a viable possibility and computations are straightforward. Therefore, they remain an interesting target for observations to shoot at. More details and references can be found in the contribution [31].

### 3.2 Dynamical dark energy: more general models

It is possible to couple quintessence to cold dark matter, so that the energy conservation equations become

$$\dot{\varphi}\left[\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi)\right] = J, \tag{25}$$

$$\dot{\rho}_m + 3H\rho_m = -J, \tag{26}$$

where $J$ is the energy exchange [51,52].

Another possibility is a scalar field with non-standard kinetic term in the Lagrangian, for example,

$$L_\varphi = F(\varphi, X) + V(\varphi), \quad \text{where } X \equiv \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi. \tag{27}$$

The standard Lagrangian has $F(\varphi, X) = X$. Some of the non-standard $F$ models may be ruled out on theoretical grounds. An example is provided by "phantom" fields, with negative kinetic energy density (ghosts), $F(\varphi, X) = -X$. They have $w < -1$, so that their energy density *grows* with expansion. This bizarre behaviour is reflected in the instability of the quantum vacuum for phantom fields.

Another example is "k-essence" fields [53], which have $F(\varphi, X) = \varphi^{-2} f(X)$. These theories have no ghosts, and they can produce late-time acceleration. The sound speed of the field fluctuations for the Lagrangian in Eq. (27) is

$$c_s^2 = \frac{F_{,X}}{F_{,X} + 2XF_{,XX}}. \tag{28}$$

For a standard Lagrangian, $c_s^2 = 1$. But for the class of $F$ that produce accelerating k-essence models, it turns out that there is always an epoch during which $c_s^2 > 1$, so that these models may be ruled out according to our causality requirement. They violate standard causality [54,55].

For models not ruled out on theoretical grounds, there is the same general problem as with quintessence, i.e. that no model is better motivated than LCDM, none is selected by fundamental physics and any choice of model is more or less arbitrary. Quintessence then appears to at least have the advantage of simplicity—although LCDM has the same advantage over quintessence.

When investigating generic dark energy models we always have to keep in mind that since both dark energy and dark matter are only detected gravitationally, we can only measure the total energy momentum tensor of the dark component,

$$T_{\mu\nu}^{\text{dark}} = T_{\mu\nu}^{\text{de}} + T_{\mu\nu}^{\text{dm}}. \tag{29}$$

Hence, if we have no information on the equation of state of dark energy, there is a degeneracy between the dark energy equation of state $w(t)$ and $\Omega_{\text{dm}}$. Without additional assumptions, we cannot measure either of them [56]. This degeneracy becomes even worse if we allow for interactions between dark matter and dark energy.

### 3.3 Dark energy as a non-linear effect from structure

As structure forms and the matter density perturbation becomes non-linear, there are two questions that are posed: (1) what is the back-reaction effect of this non-linear process on the background cosmology?; (2) how do we perform a covariant and gauge-invariant averaging over the inhomogeneous universe to arrive at the correct FRW background? The simplistic answers to these questions are: (1) the effect is negligible since it occurs on scales too small to be cosmologically relevant; (2) in light of this, the background is independent of structure formation, i.e. it is the same as in the linear regime. A quantitative analysis is needed to fully resolve both issues. However, this is very complicated because it involves the non-linear features of general relativity in an essential way.

There have been claims that these simplistic answers are wrong, and that, on the contrary, the effects are large enough mimic an accelerating universe. This would indeed be a dramatic and satisfying resolution of the coincidence problem, without the need for any dark energy field. Of course, the problem of why the vacuum does not gravitate would remain. This issue is discussed in the contribution [32].

However, these claims have been disputed, and it is fair to say that there is as yet no convincing demonstration that acceleration could emerge naturally from non-linear effects of structure formation; see Refs. [57–72] for some claims and counter-claims. We should however note the possibility that backreaction/averaging effects could be significant, even if they do not lead to acceleration.

It might also be possible that the universe around us resembles more a spherically symmetric but inhomogeneous solution of Einstein's equation, a Tolman–Bondi–Lemaître universe, than a Friedmann–Lemaître universe. In this case, what appears as cosmic acceleration to us can be explained within simple matter models which only contain dust. However, this would imply that we are situated very close to the centre of a huge (nearly) spherical structure. Apart from violating the Copernican principle, this poses another fine tuning problem. This idea is discussed in the contribution [18].

## 4 The modified gravity approach: dark gravity

Late-time acceleration from non-linear effects of structure formation is an attempt, within general relativity, to solve the coincidence problem without a dark energy field. The modified gravity approach shares the assumption that there is no dark energy field, but generates the acceleration via "dark gravity", i.e. a weakening of gravity on the largest scales, due to a modification of general relativity itself.

Could the late-time acceleration of the universe be a gravitational effect? (Note that this would also not remove the problem of explaining why the vacuum energy does not gravitate.) A historical precedent is provided by attempts to explain the anomalous precession of Mercury's perihelion by a "dark planet", named Vulcan. In the end, it was discovered that a modification to Newtonian gravity was needed.

As we have argued in Sect. 2, a consistent modification of general relativity requires a covariant formulation of the field equations in the general case, i.e. including inhomogeneities and anisotropies. It is not sufficient to propose ad hoc modifications of the Friedman equation, of the form

$$f(H^2) = \frac{8\pi G}{3}\rho \quad \text{or} \quad H^2 = \frac{8\pi G}{3} g(\rho), \tag{30}$$

for some functions $f$ or $g$. Apart from the fundamental problems outlined in Sect. 2, such a relation allows us to compute the supernova distance/redshift relation using this equation—but we *cannot* compute the density perturbations without knowing the covariant parent theory that leads to such a modified Friedman equation. And we also cannot compute the solar system predictions.

It is very difficult to produce infrared corrections to general relativity that meet all the minimum requirements:

- Theoretical consistency in the sense discussed in Sect. 2.
- Late-time acceleration consistent with supernova data.
- A matter-dominated era with an evolution of the scale factor $a$ that is consistent with the requirements of structure formation.
- Density perturbations that are consistent with the observed matter power spectrum, CMB anisotropies and weak lensing power spectrum.
- Stable static spherical solutions for stars and vacuum, and consistency with terrestrial and solar system observational constraints.
- Consistency with binary pulsar period data.

### 4.1 Scalar–tensor theories

General relativity has a unique status as a theory where gravity is mediated by a massless spin-2 particle, and the field equations are second order. If we introduce modifications to the Einstein–Hilbert action of the general form

$$\int d^4x\,\sqrt{-g}\,R\;\rightarrow\;\int d^4x\,\sqrt{-g}\,f(R, R_{\mu\nu}R^{\mu\nu}, C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta}), \tag{31}$$

then the field equations become fourth-order, and gravity is carried also by massless spin-0 and spin-1 fields. In order to avoid the Ostrogradski instability discussed in Sect. 2, we impose $f = f(R)$, and we assume $f''(R) \neq 0$. However, it turns out to be extremely difficult for this simplified class of modified theories to pass the observational and theoretical tests. An example is [73]

$$f(R) = R - \frac{\mu}{R}. \tag{32}$$

For $|\mu| \sim H_0^4$, this model achieves late-time acceleration as the $\mu/R$ term starts to dominate. But the model suffers from non-linear matter instabilities and violation of solar system constraints [74–76].

Variations of $f(R)$ theories have been introduced to evade these problems [77–79]. These are based on a "chameleon" mechanism to alter the modification of general relativity across the boundary between a massive body and its vacuum exterior. Although such mechanisms may be successful, the models look increasingly unnatural and contrived—and suffer from very strong fine-tuning.

All $f(R)$ theories lead to just one fourth order equation [36]. The corresponding additional degree of freedom can be interpreted as a scalar field and in this sense, $f(R)$ theories are mathematically equivalent to scalar–tensor theories via

$$\psi \equiv f'(R), \quad U(\psi) \equiv \psi - f(R(\psi)), \tag{33}$$

$$L = \frac{1}{16\pi G}\sqrt{-g}\,[\psi R + U(\psi)]\,. \tag{34}$$

This Lagrangian can be conformally transformed into ordinary gravity with a scalar field, i.e. a quintessence model, via the transformation

$$\tilde{g}_{\mu\nu} = \psi g_{\mu\nu}, \quad \varphi = \sqrt{\frac{3}{4\pi G}}\ln\psi. \tag{35}$$

In terms of $\tilde{g}_{\mu\nu}$ and $\varphi$ the Lagrangian then becomes a standard scalar field Lagrangian,

$$\sqrt{-g}f(R) = \sqrt{\tilde{g}}\left[\tilde{R} + \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + V(\varphi)\right], \tag{36}$$

where

$$V(\varphi) = \frac{1}{16\pi G}\,\frac{U(\psi(\varphi))}{\psi(\varphi)^2}. \tag{37}$$

This example shows that modifying gravity (dark gravity) or modifying the energy momentum tensor (dark energy) can be seen as a different description of the same physics. Only the coupling of the scalar field $\varphi$ to ordinary matter, shows that this theory originates from a scalar–tensor theory of gravity—and this non-standard coupling reflects the fact that gravity is also mediated by a spin-0 degree of freedom, in contrast to general relativity with a standard scalar field.

More general scalar–tensor theories [80–83], which may be motivated via low-energy string theory, have an action of the form

$$\int d^4x \sqrt{-g}\left[F(\psi)R + \frac{1}{2}g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi + U(\psi)\right], \tag{38}$$

where $\psi$ is the spin-0 field supplementing the spin-2 graviton. In the context of late-time acceleration, these models are also known as "extended quintessence". Scalar–tensor theories contain two functions, $F$ and $U$. This additional freedom allows for greater flexibility in meeting the observational and theoretical constraints. However, the price we pay is additional complexity—and arbitrariness. The $f(R)$ theories have one arbitrary function, and here there are two, $F(\psi)$ and $U(\psi)$. There is no preferred choice of these functions from fundamental theory.

In summary, modifications of the Einstein–Hilbert action, which lead to fourth-order field equations, either fail to meet the minimum requirements in the simplest cases, or contain more complexity and arbitrary choices than quintessence models in general relativity. Therefore, none of these models appears to be a serious competitor to quintessence in general relativity.

## 4.2 Brane-world models

We turn now to a class of brane-world models whose background is no more complicated than that of LCDM, offering the promise of a serious dark gravity contender. However, there are hidden complexities and problems, as we will explain below.

An infra-red modification to general relativity can emerge within the framework of quantum gravity, in addition to the ultraviolet modification that must arise at high energies in the very early universe. The leading candidate for a quantum gravity theory, string theory, is able to remove the infinities of quantum field theory and unify the fundamental interactions, including gravity. But there is a price—the theory is only consistent in nine space dimensions. Branes are extended objects of higher dimension than strings, and play a fundamental role in the theory, especially D-branes, on which open strings can end. Roughly speaking, the endpoints of open strings, which describe the standard model particles like fermions and gauge bosons, are attached to branes, while the closed strings of the gravitational sector can move freely in the higher-dimensional "bulk" spacetime. Classically, this is realized via the localization of matter and radiation fields on the brane, with gravity propagating in the bulk (see Fig. 4).

The implementation of string theory in cosmology is extremely difficult, given the complexity of the theory. This motivates the development of phenomenology, as an intermediary between observations and fundamental theory. (Indeed, the development

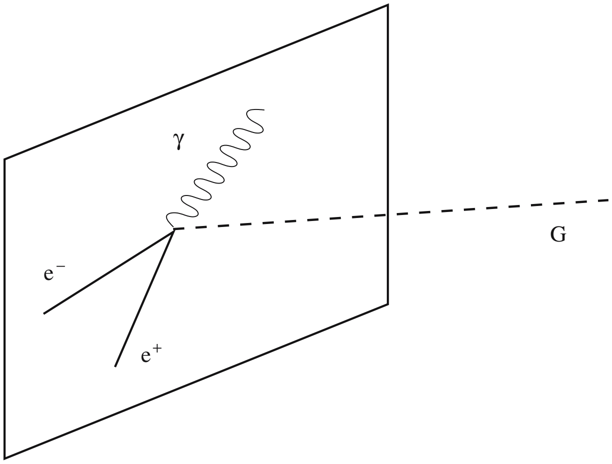


**Fig. 4** The confinement of matter to the brane, while gravity propagates in the bulk (from [84])

of inflationary cosmology has been a very valuable exercise in phenomenology.) Brane-world cosmological models inherit key aspects of string theory, but do not attempt to impose the full machinery of the theory. Instead, simplifications are introduced in order to be able to construct cosmological models that can be used to compute observational predictions (see [85–91] for reviews in this spirit). Cosmological data can then be used to constrain the brane-world models, and hopefully provide constraints on string theory, as well as pointers for the further development of string theory.

It turns out that even the simplest brane-world models are remarkably rich—and the computation of their cosmological perturbations is complicated, and still incomplete. A key reason for this is that the higher-dimensional graviton produces a tower of 4-dimensional massive spin-2 modes on the brane, in addition to the standard massless spin-2 mode on the brane (or in some cases, instead of the massless mode). In the case of some brane models, there are in addition a massless gravi-scalar and gravi-vector which modify the dynamics.

Most brane-world models modify general relativity at high energies. The main examples are those of Randall-Sundrum (RS) type [94,95], where a FRW brane is embedded in an anti de Sitter bulk, with curvature radius $\ell$. At low energies $H\ell \ll 1$, the zero-mode of the graviton dominates on the brane, and general relativity is recovered to a good approximation. At high energies, small scales, $H\ell \gg 1$, the massive modes of the graviton dominate over the zero mode, and gravity on the brane behaves increasingly 5-dimensional. On the brane, the standard conservation equation holds,

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{39}$$

but the Friedmann equation is modified by an ultraviolet correction:

$$H^2 = \frac{8\pi G}{3}\rho\left(1 + \frac{2\pi G\ell^2}{3}\rho\right) + \frac{\Lambda}{3}. \tag{40}$$

The $\rho^2$ term is the ultraviolet correction. At low energies, this term is negligible, and we recover $H^2 \propto \rho + \Lambda/8\pi G$. At high energies, gravity "leaks" off the brane and $H^2 \propto \rho^2$. This 5D behaviour means that a given energy density produces a greater rate of expansion than it would in general relativity. As a consequence, inflation in the early universe is modified in interesting ways [85–91].

By contrast, the brane-world model of Dvali–Gabadadze–Porrati [92] (DGP), which was generalized to cosmology by Deffayet [93], modifies general relativity at late times. This model produces "self-acceleration" of the low-energy universe due to a weakening of gravity. Like the RS model, the DGP model is a 5-dimensional model with infinite extra dimension. (We effectively assume that five of the extra dimensions in the "parent" string theory may be ignored at low energies.)

The action is given by

$$\frac{1}{16\pi G}\left[\frac{1}{r_c}\int_{\text{bulk}} d^5x\,\sqrt{-g^{(5)}}\,R^{(5)} + \int_{\text{brane}} d^4x\,\sqrt{-g}\,R\right]. \tag{41}$$

The bulk is assumed to be 5D Minkowski spacetime. Unlike the AdS bulk of the RS model, the Minkowski bulk has infinite volume. Consequently, there is no normalizable zero-mode of the (bulk) graviton in the DGP brane-world. Gravity leaks off the 4D brane into the bulk at large scales, $\lambda > r_c$, where the first term in the sum (41) dominates. On small scales, gravity is effectively bound to the brane and 4D dynamics is recovered to a good approximation, as the second term dominates. The transition from 4- to 5-D behaviour is governed by the crossover scale $r_c$; the weak-field gravitational potential behaves as

$$\Psi \propto \begin{cases} r^{-1} & \text{for } r \ll r_c, \\ r^{-2} & \text{for } r \gg r_c. \end{cases} \tag{42}$$

Gravity leakage at late times initiates acceleration—not due to any negative pressure field, but due to the weakening of gravity on the brane. 4D gravity is recovered at high energy via the lightest massive modes of the 5D graviton, effectively via an ultra-light metastable graviton.

The energy conservation equation remains the same as in general relativity, but the Friedman equation is modified:

$$\dot\rho + 3H(\rho + p) = 0, \tag{43}$$

$$H^2 - \frac{H}{r_c} = \frac{8\pi G}{3}\rho. \tag{44}$$

This shows that at early times, i.e. $Hr_c \gg 1$, the general relativistic Friedman equation is recovered. By contrast, at late times in a CDM universe, with $\rho \propto a^{-3} \to 0$, we have

$$H \to H_\infty = \frac{1}{r_c}, \tag{45}$$

so that expansion accelerates and is asymptotically de Sitter. Since $H_0 > H_\infty$, in order to achieve self-acceleration at late times, we require

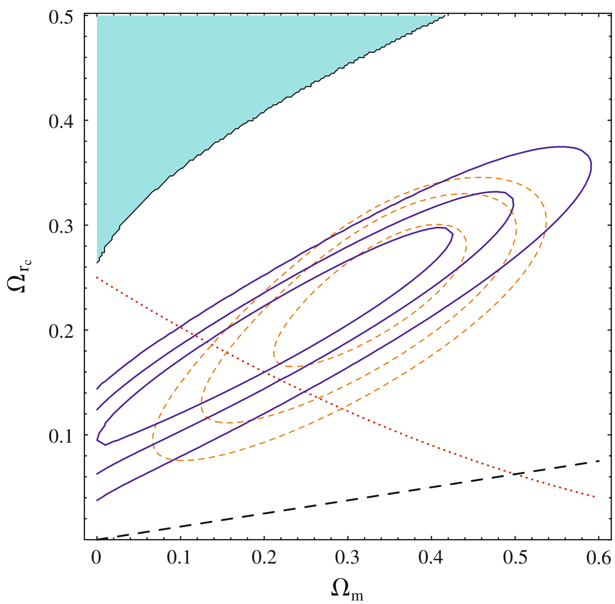


**Fig. 5** The confidence contours for supernova data in the DGP density parameter plane. The *blue* (*solid*) contours are for SNLS data, and the *brown* (*dashed*) contours are for the Gold data. The *red* (*dotted*) curve defines the flat models, the *black* (*dot-dashed*) curve defines zero acceleration today, and the *shaded region* contains models without a big bang. (From [96])

$$r_c \gtrsim H_0^{-1}, \tag{46}$$

and this is confirmed by fitting supernova observations, as shown in Fig. 5. The dimensionless cross-over parameter is

$$\Omega_{r_c} = \frac{1}{4(H_0 r_c)^2}, \tag{47}$$

and the LCDM relation,

$$\Omega_m + \Omega_\Lambda + \Omega_K = 1, \tag{48}$$

is modified to

$$\Omega_m + 2\sqrt{\Omega_{r_c}}\sqrt{1 - \Omega_K} + \Omega_K = 1. \tag{49}$$

LCDM and DGP can both account for the supernova observations, with the fine-tuned values $\Lambda \sim H_0^2$ and $r_c \sim H_0^{-1}$ respectively. The degeneracy may be broken by observations based on structure formation, since the two models suppress the growth of density perturbations in different ways [97,98]. The distance-based observations draw only upon the background 4D Friedman equation (44) in DGP models—and therefore there are quintessence models in general relativity that can produce precisely the same supernova distances as DGP [99]. By contrast, structure formation observations

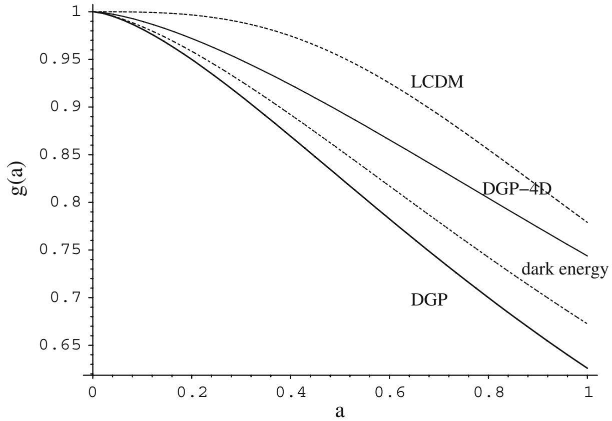


**Fig. 6** The growth factor $g(a) = \Delta(a)/a$ for LCDM (*long dashed*) and DGP (*solid, thick*), as well as for a dark energy model with the same expansion history as DGP (*solid, thick*). DGP-4D (*solid, thin*) shows the incorrect result in which the 5D effects are set to zero. (From [100])

require the 5D perturbations in DGP, and one cannot find equivalent quintessence models [100]. (However, 4D general relativity models allowing for anisotropic stresses can in principle mimick DGP [102].)

For LCDM, the analysis of density perturbations is well understood. For DGP it is much more subtle and complicated. This is discussed in the contribution [34]. Although matter is confined to the 4D brane, gravity is fundamentally 5D, and the bulk gravitational field responds to and back-reacts on density perturbations. The evolution of density perturbations requires an analysis based on the 5D nature of gravity. In particular, the 5D gravitational field produces an anisotropic stress on the 4D universe. If one neglects this stress and all 5D effects, and simply treats the perturbations as 4D perturbations with a modified background Hubble rate—then as a consequence, the 4D Bianchi identity on the brane is violated, i.e. $\nabla^\nu G_{\mu\nu} \neq 0$, and the results are inconsistent.

When the 5D effects are incorporated [100, 101], the 4D Bianchi identity is satisfied. The consistent modified evolution equation for density perturbations on sub-Hubble scales is

$$\ddot{\Delta} + 2H\dot{\Delta} = 4\pi G \left\{ 1 - \frac{(2Hr_c - 1)}{3[2(Hr_c)^2 - 2Hr_c + 1]} \right\} \rho\Delta, \tag{50}$$

where the term in braces encodes the 5D correction. The linear growth factor, $g(a) = \Delta(a)/a$ (i.e. normalized to the flat CDM case, $\Delta \propto a$), is shown in Fig. 6.

In addition to the complexity of the cosmological perturbations, a deeper problem is posed by the fact that the late-time asymptotic de Sitter solution in DGP cosmological models has a ghost [103]. This ghost in the gravitational sector is more serious than the ghost in a phantom scalar field. It is actually this ghost degree of freedom which is responsible for acceleration in the DGP model. Nevertheless, it may still be useful to study DGP as a toy model for dark gravity.

## 5 Conclusion

The evidence for a late-time acceleration of the universe continues to mount, as the number of experiments and the quality of data grow—dark energy or dark gravity appear to be an unavoidable reality of the cosmos. This revolutionary discovery by observational cosmology, confronts theoretical cosmology with a major crisis—how to explain the origin of the acceleration. The core of this problem may be "handed over" to particle physics, since we require at the most fundamental level, an explanation for why the vacuum energy either has an incredibly small and fine-tuned value, or is exactly zero. Both options violently disagree with naive estimates of the vacuum energy.

If one accepts that the vacuum energy is indeed non-zero, then the dark energy is described by $\Lambda$, and the LCDM model is the best current model. The cosmological model requires completion via developments in particle physics that will explain the value of the vacuum energy. In many ways, this is the best that we can do currently, since the alternatives to LCDM, within and beyond general relativity, do not resolve the vacuum energy crisis, and furthermore have no convincing theoretical motivation. None of the contenders appears any better than LCDM.

Presently, perhaps the simplest and most appealing contender is the DGP braneworld model. However, the simplicity of its Friedman equation is deceptive, and the complexity of its cosmological perturbations includes the problem of its ghost.

In view of all this, it is fair to say that at the theoretical level, there is as yet no serious challenger to LCDM. It remains worthwhile to continue investigating alternative dark energy and dark gravity models, in order better to understand the space of possibilities, the variety of cosmological properties, and the observational strategies needed to distinguish them.

At the same time, it is in principle possible that cosmological observations, having discovered dark energy/dark gravity, could rule out LCDM, by showing, to some acceptable level of statistical confidence, that $w \neq -1$.

Finally, the theoretical crisis does not have only negative implications: dark energy/dark gravity in the cosmos provides exciting challenges for theory and observations.

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