

Jozsó's Legacy: Chemical and kinetic freeze-out in heavy-ion collisions

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Abstract. We review J. Zimányi's key contributions to the theoretical understanding of dynamical freeze-out in nuclear collisions and their subsequent applications to ultra-relativistic heavy-ion collisions, leading to the discovery of a freeze-out hierarchy where chemical freeze-out of hadron yields precedes the thermal decoupling of their momentum spectra. Following Zimányi's lines of reasoning we show that kinetic freeze-out necessarily leads to a dependence of the corresponding freeze-out temperature on collision centrality. This centrality dependence can be predicted within hydrodynamic models, and for Au+Au collisions at RHIC this prediction is shown to reproduce the experimentally observed centrality dependence of the thermal decoupling temperature, extracted from hadron momentum spectra. The fact that no such centrality dependence is observed for the chemical decoupling temperature, extracted from the hadron yields measured in these collisions, excludes a similar kinetic interpretation of the chemical decoupling process. We argue that the chemical decoupling data from Au+Au collisions at RHIC can only be consistently understood if the chemical freeze-out process is driven by a phase transition, and that the measured chemical decoupling temperature therefore measures the critical temperature of the quark-hadron phase transition. We propose additional experiments to further test this interpretation.

1 Jozsó's pioneering work and early encounters

Due to political complications, József Zimányi began his long and successful scientific career as an experimental physicist. However, in 1977 Jozsó jump-started his reputation as an outstanding nuclear theorist by writing, together with J.P. Bondorf and S.I.A. Garpman, a very influential paper with the title “A simple analytical model for expanding fireballs” [1]. In this paper, Jozsó and his friends discovered a class of scaling solutions of the non-relativistic Euler equations for the hydrodynamic evolution of spherically symmetric fireballs, with power-law radial density and velocity profiles, which, due to its simplicity and elegance, has continued until today to spawn follow-up papers (in particular by his students and colleagues T. Biró and T. Csörgő) generalizing it to systems with less symmetry and undergoing relativistic expansion.

More important for the present work is, however, Section 3 of that paper [1], with the title “Geometric concept of the break-up”. This section heading is slightly misleading since what is being developed is really a *dynamic* freeze-out concept (even if it is derived with the help of a geometric sketch of the expanding fireball). It introduces the ideas of the competition

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between (i) the “separation velocity” of neighboring volume elements (in modern language: flow velocity gradients) and the thermal velocity, and (ii) the expansion rate (again controlled by velocity gradients) and the local scattering rate, as drivers of the freeze-out process. A key point following from these ideas, although not emphasized in [1], is that the thermal velocity and scattering rate both depend on the particle species, leading to the concept of *differential freeze-out* where different particles (or processes, see below) decouple at different times.

The space-time trajectories of Zimányi and one of us (UH) first joined for extended periods in the summers of 1985/86 when UH, employed by Brookhaven National Laboratory and preparing for the “imminent” start of the Relativistic Heavy-Ion Collider (RHIC eventually became operational in 2000!), worked together with Mark Rhoades-Brown at the University of Stony Brook and our common student Kang Seok Lee on the problem of hadronization of a quark-gluon plasma (QGP), while Jozsó (supported by an NSF-Hungarian Academy of Sciences exchange program) visited his friend Nándor Balázs at Stony Brook to work on the same problem. We discussed a number of thermodynamic issues related to this hadronization process, having to do with subtleties of the Maxwell construction between a QGP and a thermally and chemically equilibrated hadron resonance gas (HG) when two different types of quantum numbers (baryon number and strangeness) needed to be conserved, without really solving some of the questions that flummoxed us before our paths separated again. In fact, our group first got it wrong in a paper that we wrote early in 1986 [2] and whose puzzling (and, as it turned out, incorrect) results spurred Jozsó to look more closely into the problem. In early 1987 three papers appeared in short sequence [3–5] which independently (although clearly triggered by discussions between members of the different author groups – Horst Stöcker visited Stony Brook and BNL in 1986, too) solved the problem correctly (Jozsó [3] beat us all to it by a few months). These papers also pointed out the possibility of strangeness separation in a first-order QGP-HG phase transition. For those who experienced Jozsó’s knowledge and love of good wines and spirits it should not be too surprising that he was the first to mention the analogy of this process with that of distilling alcohol, and to coin the phrase “strangeness distillation” [6].

Mark, Kang Seok and UH tried to find a way to experimentally measure this “distillation” effect, picking up an idea of Shoji Nagamiya [7] who exploited the different mean free paths of K^+ mesons and protons and pions in nuclear matter to explain the observed hierarchy $T_{K^+} > T_p > T_\pi$ of the slopes of their energy spectra measured in heavy-ion collisions at the BEVALAC. We wanted to apply it to K^+ and K^- mesons which carry opposite strangeness. While trying to understand the influence of collective expansion of the fireball and its interplay with the mean free path in the freeze-out process, we ran into Jozsó Zimányi who once again visited Stony Brook. That’s when he pointed us to his 1978 paper with Bondorf and Garpman [1]. The result of this interaction was a paper on “ K^+ and K^- slope parameters as a signature for deconfinement at finite baryon density” [8] which discusses (we believe) for the first time the interplay of differential freeze-out and radial flow, using Jozsó’s hydrodynamic model from 1978 [1] to calculate the expansion rate in the dynamical freeze-out criterion $\tau_{\text{exp}} = \frac{1}{\partial \cdot u} < \tau_{\text{scatt}} = \frac{1}{\rho(\sigma v)}$, motivated by that same 1978 paper. Since the cross section σ and hence the scattering rate τ_{scatt} depends on the particle species, and the density of scatterers ρ depends on the temperature and baryon chemical potential, this freeze-out criterion leads to different freeze-out points (T_f, μ_f) for K^+ and K^- , with K^+ predicted to freeze out earlier, and thus with less radial flow, than K^- . Due to the strangeness distillation effect and the reheating of the matter during a first-order phase transition at finite net baryon density, we predicted a change of the ordering between the K^+ and K^- slopes with and without a quark-hadron phase transition [8]. This works only in fireballs with large net baryon density, however, since in baryon-free matter the difference between the mean free paths of K^+ and K^- disappears.

2 Dynamic freeze-out and the chemical-thermal decoupling hierarchy

2.1 Dynamic freeze-out at approximately constant decoupling temperature

In 1987 UH moved to the University of Regensburg. One of his first three students there, Ekkard Schnedermann, further developed the expanding fireball description of hadron spectra

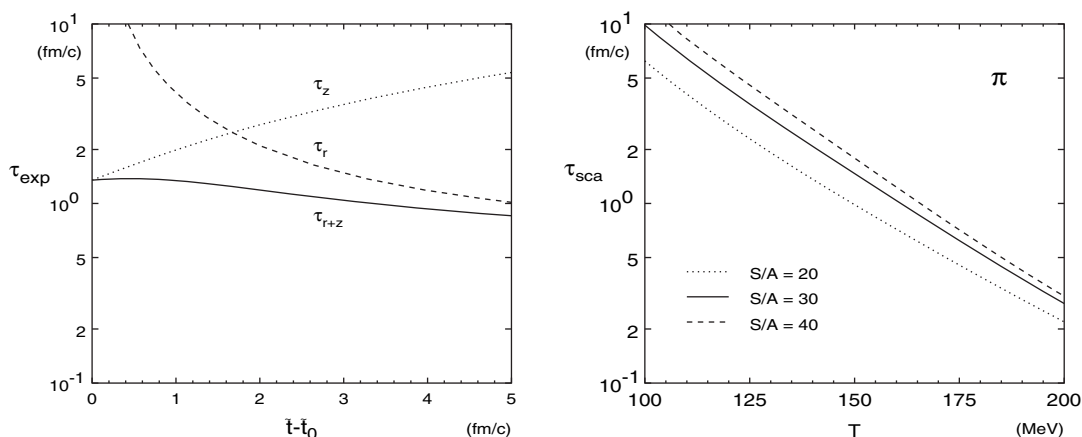


Fig. 1. Expansion (left panel) and scattering time scales (right panel, for pions in hot nuclear matter with different values of entropy per baryon S/A as indicated) for azimuthally expanding fireballs formed in S+S collisions at the SPS. For the expansion time scale, contributions from longitudinal (τ_z) and transverse expansion (τ_r) are shown separately. Figures taken from the second paper in [9].

from heavy-ion collisions in his diploma thesis (1989, unpublished) and generalized it to a genuinely dynamical “global hydrodynamics” model (with azimuthal instead of spherical symmetry, assuming longitudinal boost-invariance) in his PhD thesis (1992) [9]. By applying Jozsó’s kinetic freeze-out criterion as written above, and also comparing it with a geometric picture where freeze-out happens once the mean free path exceeds the fireball radius, he was able to demonstrate two important facts: (i) In relativistic heavy-ion collisions, freeze-out is driven by the collective expansion. In particular due to the accelerating transverse flow (which compensates for the slowing boost-invariant longitudinal expansion, see left panel in Fig. 1), the “Hubble radius” $\sim \tau_{\text{exp}}$ of the exploding fireball in the “Little Bang” is always much smaller ($\sim 1\text{--}2\text{ fm/c}$) than its geometric radius ($\sim 4\text{ fm}$ in S+S), so freeze-out is controlled by dynamics, not by geometry, just as in the Big Bang. (ii) Due to its dependence on the density of scatterers ρ , the scattering time scale τ_{scatt} has a very steep (exponential) temperature dependence (right panel in Fig. 1). Freeze-out (i.e. equality of the scattering and expansion time scales) thus happens approximately at constant temperature $T_{\text{dec}} \approx \text{const.}$

2.2 Differential chemical and thermal freeze-out

At this time we also began to realize that there is a conceptual difference between chemical and thermal freeze-out in heavy-ion collisions. **Chemical freeze-out** describes the point where *inelastic processes* that convert one kind of hadronic species into a different one cease and the hadron abundances stop changing. **Thermal freeze-out** defines the point where the momenta of the particles stop changing, i.e. where *all types of momentum-changing collisions, elastic and inelastic* cease. Zimányi’s kinetic freeze-out criterion predicts that in a dynamically evolving fireball these two freeze-out points do not coincide. To see how this comes about let us rewrite it once again for a medium consisting of a mixture of different particle species which interact with each other both elastically and inelastically:

$$\tau_{\text{exp}}(x) \equiv \frac{1}{\partial \cdot u(x)} = \xi \tau_{\text{scatt}}^{(i)}(x) \equiv \xi \frac{1}{\sum_j \langle \sigma_{ij} v_{ij} \rangle \rho_j(x)}, \quad (1)$$

where ξ is an (unknown) parameter of order 1. Here we exhibit not only the already mentioned dependence of the criterion on the particle species i , but also the fact that it is a *local* criterion, depending on the local fluid expansion rate $\partial \cdot u(x)$ and on the local scattering rate $\sum_j \langle \sigma_{ij} v_{ij} \rangle \rho_j(x)$ (where σ_{ij} is the scattering cross section between particle species i and j and v_{ij} their relative velocity in the pair center of mass frame). Thus, different parts of the expanding fireball will, in general, decouple at different times, depending on the flow velocity and

density profiles $u(x)$ and $\rho_j(x)$. The ensemble of points $(\mathbf{x}, \tau_f(\mathbf{x}))$ satisfying Eq. (1) defines the **freeze-out hypersurface**. It is a 3-dimensional surface imbedded in 4-dimensional space-time.

Since chemical (particle number changing) reactions involve different types of cross sections than thermal (momentum changing) reactions, equality of the two sides of the equation will be satisfied on different freeze-out surfaces: Chemical reactions exploit only a small fraction $\sigma_{ij}^{\text{inel}}$ of the total transport cross section σ_{ij}^{tot} corresponding to all possible momentum changing processes. Since $\sigma_{ij}^{\text{inel}} < \sigma_{ij}^{\text{tot}}$, the mean free time τ_{scatt} between two inelastic scattering processes is longer than that between two arbitrary momentum changing processes. Consequently, chemical processes decouple before the elastic scattering processes, and the hadron abundances freeze out earlier than the momentum spectra [10]:

$$T_{\text{chem}}^{(i)} > T_{\text{therm}}^{(i)}. \quad (2)$$

As it turns out once again, this key relation had been anticipated by Jozsó years earlier: In 1978 he and Montvay wrote another seminal paper on “Hadron chemistry in heavy ion collisions” [11] where they calculated the chemical reaction rates among a few of the key hadronic species created in such collisions and applied them to a schematic model for the fireball expansion. Most of the work in that paper relates to the calculation of the averaged inelastic scattering cross section $\langle \sigma_{ij} v_{ij} \rangle$ that appears in the scattering rate in Eq. (1). Montvay and Zimányi worked it out assuming thermal momentum distributions for the hadrons. This implies the assumption that, where chemical equilibrium breaks and individual reaction rates become important, thermal equilibrium is still valid, just as dictated by the inequality (2).

Since the scattering rate is particle specific, different hadrons should still freeze out at different temperatures, both chemically and thermally. To implement differential *thermal* freeze-out into a hydrodynamic model for the fireball expansion is, however, difficult since it would require the introduction of sophisticated loss terms describing the decoupling of the particles from the fluid. Fortunately, usually there is one species that dominates the scattering cross section (nucleons at low collision energies, pions at high collision energies) whose freeze-out triggers all others.

3 Chemical and thermal freeze-out and the QCD phase transition

3.1 The experimental situation anno 2000

In 2000, after 15 years of fixed-target collision experiments with ultra-relativistic heavy-ion beams at the Brookhaven AGS and CERN SPS, the Relativistic Heavy Ion Collider RHIC began colliding countercirculating beams of Au ions at BNL. Following the theoretical concepts developed during this period and outlined above, a large body of data had been collected on chemical freeze-out of hadron yields and thermal freeze-out of their momentum spectra at different collision energies. The hadrons emitted in relativistic heavy-ion collisions show thermal characteristics both in their abundances and in the shapes of their transverse momentum spectra. The status of these analyses shortly after RHIC turned on is depicted in Fig. 2.

A short summary of the observations made by the various groups that contributed to this compilation is as follows: (i) Above AGS energies, the extracted chemical and thermal freeze-out temperatures clearly differ from each other, with chemical decoupling occurring at higher temperature [13–19], just as predicted by Jozsó’s dynamical freeze-out criterion. (ii) For $\sqrt{s} \gtrsim 10 A \text{ GeV}$, one observes the same “universal” chemical freeze-out temperature T_{chem} in e^+e^- , pp , $p\bar{p}$, and AA collisions [20, 21] (the only difference between small and large collision systems being the level of strangeness saturation). (iii) For $\sqrt{s} \gtrsim 10 A \text{ GeV}$, T_{chem} agrees with the critical temperature T_c for the color-confining quark-hadron phase transition predicted by lattice QCD for (approximately) baryon-free hot hadronic matter [22, 23]. (Recent developments along this front will be discussed further below.) (iv) Hadronic cascades (RQMD, UrQMD, ...) show that hadronic rescattering after the QGP hadronization alters the momentum distributions and resonance populations of the hadrons (thereby cooling the system while keeping it – at

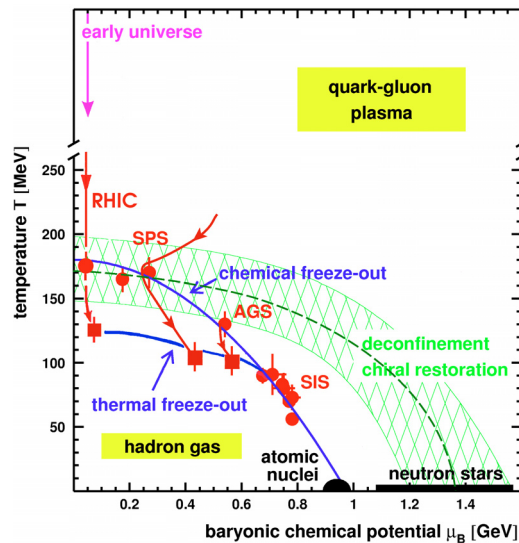


Fig. 2. Chemical and thermal freeze-out points extracted from heavy-ion collisions at the GSI SIS, BNL AGS, CERN SPS and RHIC. The shaded area indicates the likely location of the quark-hadron phase transition as extracted from lattice QCD and theoretical models. An updated version of the chemical freeze-out points can be found in [12].

least for a while – close to local thermal equilibrium [24]), but not the stable hadron yields [25,26], again in agreement with the above considerations based on Jozsó’s dynamical freeze-out criterion. [Hadronic rescattering leads to the loss of a fraction of the baryon-antibaryon pairs, but this can be at least partially traced back to the absence of multi-hadron collision channels so that detailed balance is violated in baryon-antibaryon annihilation [27].]

3.2 The controversy: Kinetic freeze-out of chemical reactions or statistical hadronization?

These empirical facts have split the heavy-ion theory community into two camps which offer different interpretations of the observations. The philosophy of **Camp I** is laid out in Refs. [20, 26,28,29] and holds that hadron production is a statistical process associated with a phase transition, proceeding through very many different possible microscopic channels constrained only by energy, baryon number and (both net and total) strangeness conservation, thereby leading to a maximum entropy (i.e. statistically most probable) configuration described by a thermal distribution of hadron yields. In this interpretation the extracted “thermodynamic” parameters T_{chem} , μ_B and γ_s play the role of Lagrange multipliers to ensure these conservation law constraints while maximizing the entropy [30]. The value of T_{chem} is *not* established by inelastic reactions among hadrons proceeding until chemical equilibrium is reached – rather, the hadrons are directly “born” into a maximum entropy state of apparent chemical equilibrium [29], with the parameter T_{chem} defining the critical energy density e_c at which the hadronization process happens [20]. T_{chem} is thus conceptually different from the thermal (or kinetic) decoupling temperature $T_{\text{therm}} \equiv T_{\text{kin}}$ which *is* the result of quasi-elastic rescatterings among the hadrons (which also contribute to their collective flow).

Camp II includes the followers of Refs. [27,31,32] who hold that chemical freeze-out is a kinetic process within the hadronic phase, conceptually equivalent with kinetic freeze-out, the only difference being the quantitative values of the corresponding freeze-out temperatures which reflect the fact that the inelastic cross sections driving chemical equilibration constitute only a small fraction of the total scattering cross section contributing to momentum exchange. The hadrons are not born into chemical equilibrium, but driven into such a state kinetically by inelastic multi-hadron processes (which, according to Refs. [27,31,32], become crucial near T_c due to high hadron densities) and frozen out by global expansion. Accordingly, T_{chem} is

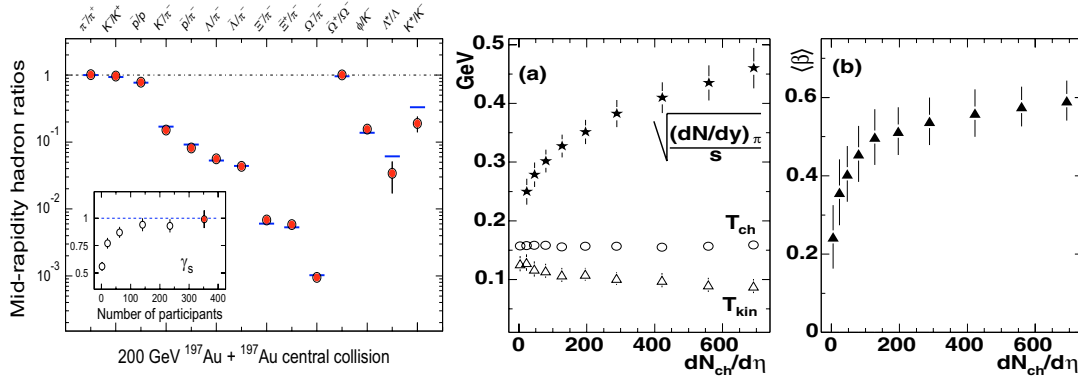


Fig. 3. Left: abundance ratios of stable hadrons from central 200 A GeV Au+Au collisions at RHIC [15]. The inset shows the centrality dependence of the strangeness saturation factor γ_s . Right: centrality dependence (with centrality measured by charged hadron rapidity density $dN_{ch}/d\eta$) of (a) the thermal freeze-out temperature $T_{kin} \equiv T_{therm}$ (open triangles), the chemical freeze-out temperature T_{chem} (open circles), and the square root of the transverse areal density of pions $(dN_{\pi}/d\eta)/S$ (solid stars), and (b) the average transverse flow velocity $\langle\beta\rangle \equiv \langle v_{\perp}\rangle$ (solid triangles), for the same collision system [19].

the “real” temperature describing the latest point at which forward and backward chemical reactions balance each other. (In contrast, for Camp I, there are no “backward” reactions involving hadrons in both initial and final states.)

3.3 How to resolve the controversy: RHIC precision data

Is this more than a philosophical difference of opinions? We think so – this controversy can be resolved unambiguously [34]. To better explain our argument let us first cast a more detailed look at the recent precision data collected at RHIC. Figure 3 shows the results from thermal model fits to hadron yield ratios and transverse momentum spectra from Au+Au collisions at $\sqrt{s} = 200$ A GeV. The final hadron abundances from central collisions can be described by a hadron resonance gas in a state of approximate chemical equilibrium at $T_{chem} = 163 \pm 4$ MeV, $\mu_B = 24 \pm 4$ MeV, and a strangeness saturation factor $\gamma_s = 0.99 \pm 0.07$ [15]. The quality of the statistical model fit is impressive. The STAR collaboration also studied the dependence of the fit parameters on the collision centrality and found that neither the temperature T_{chem} nor the baryon chemical potential μ_B depend appreciably on the impact parameter [19,33]; only the strangeness suppression factor exhibits centrality dependence, beginning at impact parameters $> 8-9$ fm, and drops to values around 0.55 in the most peripheral Au+Au collisions [15]. The centrality independence of T_{chem} (open circles in the middle panel of Fig. 3) is in stark contrast to the behavior observed in the same experiment for the kinetic (thermal) decoupling temperature $T_{kin} \equiv T_{therm}$, which is extracted together with a value for the average radial flow velocity $\langle\beta\rangle$ of the fireball at thermal freeze-out from the shape of the transverse momentum spectra of identified pions, kaons and (anti-)protons [19]: This right two panels in Fig. 3 show that T_{kin} increases significantly with increasing impact parameter, from $T_{kin} = 89 \pm 12$ MeV in the most central to $T_{kin} = 127 \pm 13$ MeV in the most peripheral collisions, while at the same time the average radial flow decreases from $\langle\beta\rangle = 0.59 \pm 0.05$ in the most central to $\langle\beta\rangle = 0.24 \pm 0.08$ in the most peripheral Au+Au collisions. This last observation demonstrates a strong centrality dependence of the fireball expansion dynamics.

Returning to the controversy between Camps I and II described in the preceding subsection, we note that Camp II has to cope with an intrinsic tension between two observations: The high quality of the thermal model fit to the observed hadron yields at RHIC requires sufficient time for inelastic reactions to establish a good chemical equilibrium, whereas the proximity of the fitted chemical freeze-out temperature T_{chem} to the critical temperature T_c of the quark-hadron phase transition from lattice QCD, together with the rapid cooling of the fireball by collective expansion, don’t provide much of a time window for these processes to play out. In essence, to

make the kinetic chemical equilibration scenario work one needs *very* large scattering rates right near T_c which then drop to negligible values just below T_c . [This would be easier to understand if there were a larger gap between T_c and T_{chem} , as suggested by the recent upward revision of T_c from Lattice QCD advocated in [35], but this problem appears serious if the lower $T_c(\chi_{\bar{\psi}\psi})$ from Ref. [23] turns out to be correct.]

In addition, we point out a second conceptual problem with the Camp II interpretation: If freeze-out is a kinetic process, it is controlled by the competition between local scattering (moving the system towards equilibrium) and global expansion (driving the system out of equilibrium). The resulting freeze-out temperature is therefore sensitive to the fireball expansion rate which (as the right panel in Fig. 3 shows) depends on collision centrality. Thus the extracted kinetic decoupling temperature should also depend on centrality. While such a centrality dependence is empirically observed for the kinetic decoupling temperature T_{kin} , the chemical freeze-out temperature does not appear to vary with collision centrality (middle panel in Fig. 3). Hence it cannot be the result of a kinetic decoupling process from inelastic hadronic scattering.

According to Jozsó Zimányi's dynamical freeze-out criterion, dependence of the freeze-out temperature on the collective expansion rate, and through this rate on the collision centrality, is a *tell-tale signature for a kinetic decoupling process*. In the rest of this paper we show that the observed centrality dependences of the average radial flow velocity and thermal freeze-out temperature are consistent with hydrodynamic behaviour of the fireball medium followed by kinetic decoupling of the hadrons from microscopic scattering processes, driven by the collective expansion. We will then show that a centrality independent freeze-out temperature is inconsistent with a kinetic decoupling process unless the chemical scattering rates have an extremely (i.e. almost infinitely) strong temperature dependence. We therefore interpret the observed centrality independence of T_{chem} as evidence that chemical decoupling of the hadron abundances is driven by a *phase transition* during which the chemical reaction rates decrease precipitously, leaving the system in a chemically frozen-out state at the end of the transition. Only in this way is it possible to obtain a universal chemical freeze-out temperature that is *insensitive* to the (centrality dependent) collective dynamics, and only depends on the thermodynamic parameters of the phase transition. Obviously, the chemical processes happening during the hadronization process itself involve colored degrees of freedom and can thus not be efficiently described in hadronic language. We also address the centrality dependence of the strangeness saturation factor and comment on how our picture also reproduces chemical abundance data measured in pp and e^+e^- collisions.

4 Kinetic freeze-out from a hydrodynamically expanding system

In this section we show that the hydrodynamic model can quantitatively reproduce the observed centrality dependence of the kinetic decoupling temperature extracted from hadron momentum spectra at RHIC. We then show that an analogous centrality dependence of the chemical freeze-out temperature cannot be avoided if the hadron yields are similarly controlled by kinetic freeze-out from inelastic hadronic rescattering.

4.1 Kinetic thermal freeze-out from hydrodynamics

We use our (2+1)-dimensional longitudinally boost-invariant hydrodynamic code AZHYDRO [36] with standard initial conditions [37] to generate the flow pattern for 200 A GeV Au+Au collisions. This code has been previously shown to successfully reproduce the measured single particle hadron p_T -spectra and their elliptic flow (for details see [38]). Here, however, we modify the freeze-out criterion for thermal decoupling to account for its kinetic nature: Instead of requiring freeze-out on a surface of constant energy density $e_{\text{dec}} = 0.075 \text{ GeV}/\text{fm}^3$ (corresponding to a fixed temperature $T_{\text{kin}} = 100 \text{ MeV}$ [37]), we define the kinetic freeze-out surface as the set of points satisfying Eq. (1) [1, 39, 40]. In a first attempt, the proportionality constant is set to $\xi = 0.35$, yielding an average temperature along the freeze-out surface for central Au+Au

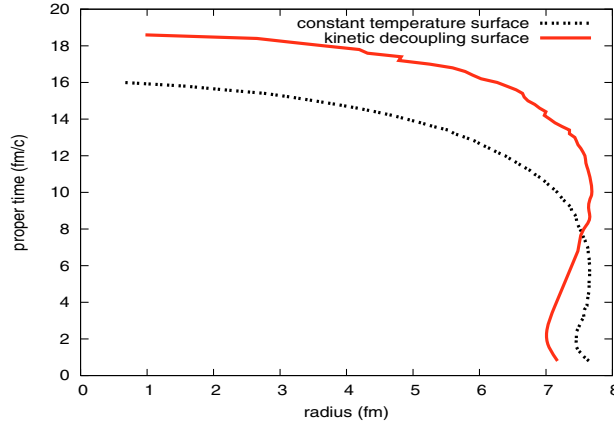


Fig. 4. Kinetic (thermal) freeze-out surface $\tau_{\text{kin}}(r)$ for central ($b = 0$) 200 A GeV Au+Au collisions, computed from Eq. (1) with $\xi = 0.35$ (red solid line) and for a constant freeze-out temperature $T_{\text{kin}} = 115 \text{ MeV}$ (dotted black line). Both surfaces have the same average temperature of $\langle T \rangle = 115 \text{ MeV}$, using the energy density as weight function.

collisions of $\langle T_{\text{kin}} \rangle \simeq 115 \text{ MeV}$. Having fixed ξ in central collisions, Eq. (1) is taken to define the freeze-out surface also at other impact parameters. Inside the freeze-out surface the scattering rate exceeds ξ times the expansion rate, and the matter is thermalized; outside the surface the expansion rate exceeds ξ^{-1} times the scattering rate – there the hadrons are assumed to be decoupled from the fluid, streaming freely into the detector.

The expansion rate $\partial \cdot u = \gamma_{\perp} \left(\frac{1}{\tau} + \nabla_{\perp} \cdot \mathbf{v}_{\perp} \right) + (\partial_{\tau} + \mathbf{v}_{\perp} \cdot \nabla_{\perp}) \gamma_{\perp}$ is computed from the hydrodynamic output for the transverse flow velocity $\mathbf{v}_{\perp}(x)$ and $\gamma_{\perp} = (1 - v_{\perp}^2)^{-1/2}$. Since at RHIC energies hadron production is dominated by pions, we assume for simplicity that all hadrons decouple when pions freeze out. The pion scattering rate is taken from the numerical results presented in Ref. [40] which we parametrize as

$$\frac{1}{\tau_{\text{scatt}}^{\pi}} = (59.5 \text{ fm}^{-1}) \left(\frac{T}{1 \text{ GeV}} \right)^{3.45}. \quad (3)$$

This defines the rate for momentum changing collisions, to be used for the calculation of thermal freeze-out, and will need to be modified when discussing chemical freeze-out below.

In Fig. 4 we plot the kinetic freeze-out surface for central Au+Au collisions computed from Eq. (1) with $\xi = 0.35$ (solid red line) and from the condition $T_{\text{kin}} = 115 \text{ MeV}$ (dotted black line). Both have the same average kinetic freeze-out temperature, but for the kinetic freeze-out criterion (1) the middle of the fireball freezes out a bit later at lower temperature and larger flow whereas the edge decouples earlier at higher temperature and with less flow than the constant- T surface. This is caused by the larger expansion rate near the edge of the fireball.

Figure 5 shows the impact parameter dependence of the average kinetic decoupling temperature and the associated average radial flow calculated from the kinetic freeze-out criterion (1) with $\xi = 0.35$. Central collisions are seen to decouple at relatively low temperatures with large average flow whereas peripheral collisions freeze out earlier when the fireballs are still hotter and less flow has developed. [Note that the average flow velocity is *smaller* in peripheral collisions, but the expansion rate (i.e. the flow velocity gradient) is *larger*!] This is in good qualitative agreement with the STAR data [19], although their freeze-out temperatures are generally a bit lower, with slightly larger average radial flow velocities than seen in Fig. 5.

We adjust for this by fine-tuning the phenomenological parameter ξ in Eq. (1) to $\xi = 0.295$ ($\xi^{-1} = 3.4$). The corresponding freeze-out temperatures are shown as a function of impact parameter b in Fig. 6, together with the STAR data. Now the agreement is also quantitatively acceptable. We conclude that the measured centrality dependence of T_{kin} can be completely understood in terms of a hydrodynamic model for the fireball expansion, coupled to a kinetic freeze-out criterion with realistic temperature dependence of the microscopic scattering rate.

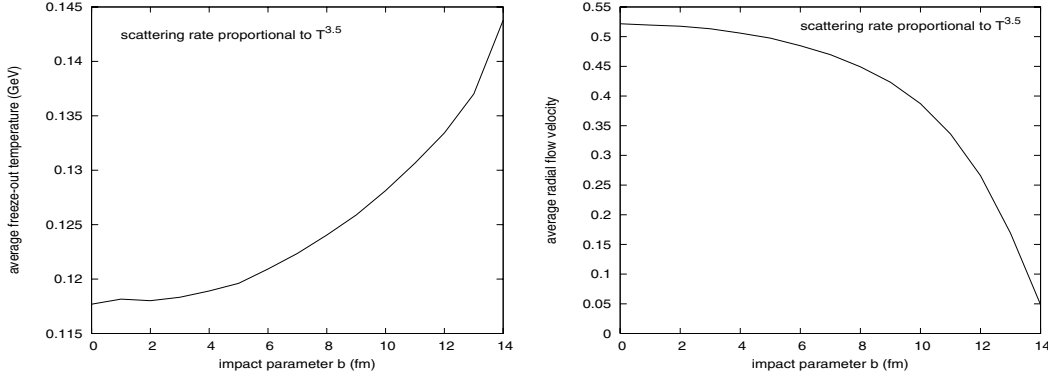


Fig. 5. Impact parameter dependence of the average kinetic decoupling temperature $\langle T_{\text{kin}} \rangle$ (left) and average radial flow velocity $\langle v_{\perp} \rangle$ along the freeze-out surface computed from Eq. (1) with $\xi = 0.35$ and a scattering rate $\sim T^{-3.45}$ as in Eq. (3), for 200 A GeV Au+Au collisions.

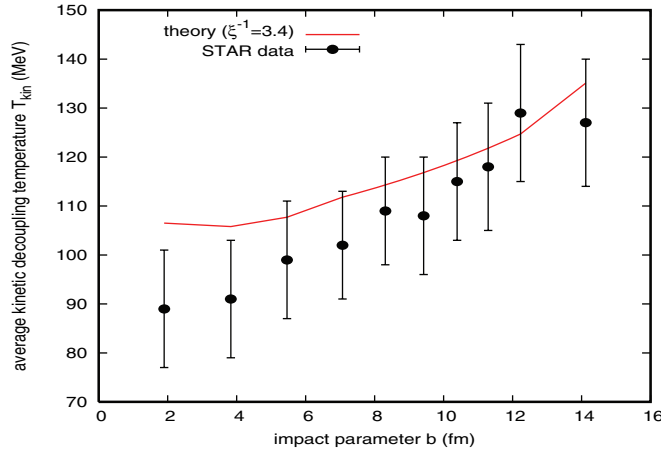
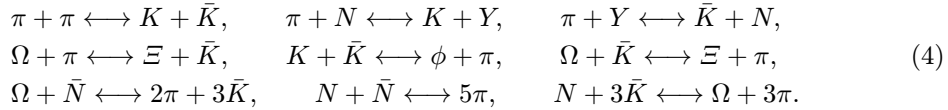


Fig. 6. Impact parameter dependence of the average kinetic decoupling temperature $\langle T_{\text{kin}} \rangle$ computed from hydrodynamics with kinetic freeze-out criterion (1) using $\xi = 0.295$, compared with STAR data [19] for 200 A GeV Au+Au collisions.

4.2 Kinetic chemical freeze-out from hydrodynamics?

Let us now see whether we can similarly understand chemical freeze-out as a kinetic decoupling process from *inelastic* hadronic scattering. A few typical processes relevant for chemical equilibration are



The last line shows so-called multi-hadron collision channels which, in at least one direction, require collisions between more than two hadrons. Rates for processes involving n_{in} incoming hadrons are proportional to the product of their densities $\sim \prod_{i=1}^{n_{\text{in}}} n_i(T)$ where each factor $n_i(T)$ grows with T at least as T^3 (even much more rapidly for hadrons with masses $> T$). At low temperatures, multi-hadron collision processes as well as collisions between very massive hadrons are therefore strongly suppressed. Consequently, particle yields for hadrons requiring collisions of many abundantly available particles for their production or destruction (such as \bar{p}, Ω, \dots) thus tend to freeze out at higher T than particle yields for hadrons whose abundances can be efficiently changed by two-body reactions (π, K, ϕ, \dots).

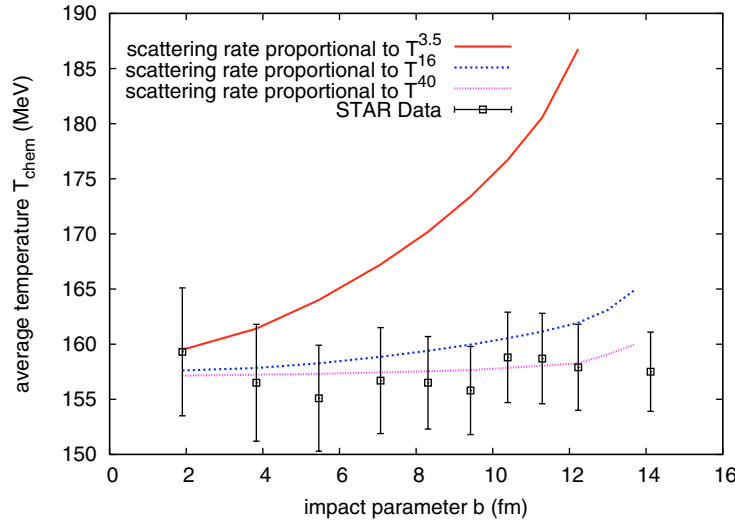


Fig. 7. Impact parameter dependence of the average chemical decoupling temperature $\langle T_{\text{chem}} \rangle$ computed from hydrodynamics with kinetic freeze-out criterion (1) using $\xi = 0.95$ and reaction rates with different temperature dependences as listed, compared with STAR data [19] for 200 A GeV Au+Au collisions.

In an expanding, cooling system, simultaneous freeze-out of all hadron yields at a *common* temperature therefore requires a *conspiracy of rates* with widely differing T -dependences. Indeed, thermal model fits to hadron abundances with a single common temperature are usually not perfect [41], and individual fits to subsets of yields measured in lower-energy collisions at the SPS and AGS tend to lead to a significant spread of chemical freeze-out temperatures [42]. So far, only at RHIC does the single-temperature chemical equilibrium fit give an almost perfect description of the data [14,42].

One way to achieve the conspiracy of different chemical equilibration rates that is required for a good fit with a single freeze-out temperature is to postulate that at chemical freeze-out *all* chemical reactions are completely dominated by multi-hadron collisions and that at any temperature below T_{chem} the medium is so rarefied and so rapidly expanding that even the simplest two-body reactions among the most abundantly produced hadrons (such as those listed in the first line of Eq. (4)) have essentially stopped. As long as collision channels with widely different temperature dependences compete with each other, chemical freeze-out of all hadron species at a single temperature appears to be impossible.

Even more importantly, even if it were possible at one fixed impact parameter to arrange for common freeze-out of all hadron species in spite of a competition of scattering rates with different temperature dependences, such a conspiracy would be impossible to maintain, *with the same value for the freeze-out temperature* T_{chem} , over the entire impact parameter range. Figure 7 shows the centrality dependence of the average chemical freeze-out temperatures along hydrodynamic decoupling surfaces computed with the kinetic freeze-out criterion (1), using $\xi = 0.95$ to adjust the value of $\langle T_{\text{chem}} \rangle$ in central Au+Au collisions to the STAR data [19] and exploring different possible temperature dependences of the dominant inelastic scattering rate. One sees that approximate impact parameter independence of $\langle T_{\text{chem}} \rangle$ can only be achieved if *all* inelastic scattering rates grow with T as T^n with a power $n \gtrsim 20$!

Basically, Fig. 7 tells us that the observed centrality independence of T_{chem} requires chemical freeze-out to happen in a region of parameter space where *all* chemical reaction rates exhibit extremely steep temperature dependence, dropping like a stone as the system cools through the decoupling temperature. It is hard to understand such a behavior within a hadron rescattering picture unless one assumes that *all* relevant chemical reactions involve multi-particle channels involving many hadrons. Making such an assumption clearly pushes the hadronic rescattering model towards breakdown because its chemical kinetics would essentially be controlled by

interactions among clusters of particles involving an unspecifiable number of hadrons. *It is much more natural to associate this kind of behavior with the quark-hadron phase transition* where densely spaced and strongly interacting quarks and gluons provide the necessary multi-particle clusters, and where the dramatic change in number and quality of the effective degrees of freedom within a narrow temperature interval generates the dramatic temperature dependence of the chemical reaction rates at decoupling which seem to be phenomenologically required.

In such a picture, hadrons are not really well-defined states until after the quark-hadron phase transition is complete and, at the same time, chemical reactions among hadrons have ceased. Hadrons are thus indeed “born into chemical equilibrium” [29] in a process that can be rightfully called “statistical hadronization” [20, 26, 28]. If hadrons are formed in this fashion, their measured abundances provide a window with a direct view of the QCD quark-hadron phase transition.

5 Conclusions

We have shown that the observed impact parameter dependence of the average temperature and radial flow velocity at *kinetic (thermal) freeze-out* (i.e. at the point where the hadron momentum distributions decouple) can be quantitatively understood as a kinetic decoupling process in a hydrodynamically expanding source, with freeze-out being driven by the global expansion of the collision fireball. As Jozsó taught us 30 years ago [1], any such kinetic decoupling process is controlled by the local competition between temperature dependent scattering and hydrodynamic expansion rates, and since the latter change with impact parameter as a result of the varying initial energy density and size of the nuclear collision zone, the resulting average freeze-out temperature is necessarily impact parameter dependent. The strength of this impact parameter dependence (i.e. the sensitivity of the freeze-out temperature to the fireball expansion rate) is inversely related to the strength of the temperature dependence of the local scattering rate. To obtain approximate centrality independence of the freeze-out temperature, the scattering rate must exhibit an almost infinitely steep temperature dependence.

From this it follows that the observed impact parameter independence of the chemical freeze-out temperature in Au+Au collisions at RHIC (i.e. of the temperature where the abundances of stable hadron species decouple) cannot be consistently described as the result of a kinetic decoupling process from inelastic hadronic interactions. To obtain the necessary extremely steep temperature dependence of the inelastic scattering rate ($\sim T^n$ with $n \gtrsim 20$) requires that at the freeze-out point *all* chemical reactions are dominated by multi-hadron interactions involving many more than two colliding particles, in which case it seems unlikely that one will ever be able to describe this process quantitatively in hadronic language.

In our opinion the only theoretically consistent interpretation of the STAR data on chemical freeze-out is to associate the steepness of the the temperature dependence of chemical equilibration rates with a phase transition (in this case the quark-hadron transition). In this transition the hadrons are produced statistically and distributed among different species according to the principle of maximum entropy, via a multitude of complicated microscopic channels involving large numbers of strongly interacting quarks and gluons. In this sense the hadrons are “born into chemical equilibrium” in an environment that is too dilute and expands too rapidly to allow for *any* further inelastic reactions among the hadrons.

T_{kin} and T_{chem} thus stand on conceptually different footings. T_{chem} is a Lagrange multiplier related by the Maximum Entropy Principle to the critical energy density e_c for hadronization. Its universality in e^+e^- , pp , and AA collisions of all centralities shows that at e_c a phase transition occurs. Hadrons are formed during this transition in a statistical process subject to the Principle of Maximum Entropy.

The absence of inelastic *hadronic* rescattering processes allows the direct measurement of T_c through T_{chem} and thus the experimental observation of the phase transition. In this context the question arises which of the different definitions of the critical temperature T_c from lattice QCD that were studied in [23] is most closely related to the chemical freeze-out temperature T_{chem} extracted from hadron yield data. It seems unlikely that hadron yields can

be considered frozen out before the hadrons have more or less recovered their full vacuum masses, and this is related to the restoration of the chiral condensate $\langle\bar{\psi}\psi\rangle$ to its vacuum value. We therefore suggest that $T_c(\chi_{\bar{\psi}\psi})$ [23] should be the LQCD number most closely related to the phenomenological value T_{chem} . This seems to be consistent with the actual values extracted in [23] from LQCD and in [19] from hadron yields at RHIC (both are between 150 and 160 MeV).

The increase of the strangeness saturation factor γ_s from e^+e^- and pp to heavy-ion collisions and from peripheral to central Au+Au collisions at RHIC shows that the lifetime of the QGP (and thus the time for chemically equilibrating strange with light quarks) is still limited. Only for midcentral to central Au+Au collisions γ_s has sufficient time to saturate. (Qualitatively similar tendencies are seen in Pb+Pb collisions at lower SPS energies [14].) The primary parton production process at the beginning of the collision apparently suppresses the production of strange quarks, and it also produces s and \bar{s} locally in pairs, thereby generating spatial correlations among s and \bar{s} which ensure strangeness conservation *locally*. In a grand canonical description such correlations induce a strangeness suppression factor $\gamma_s < 1$ [21]. It takes time to diffuse the strange quarks over the entire fireball volume to decorrelate them and adjust their abundance to equilibrium values. Larger initial energy densities in central Au+Au collisions provide more time until the point of hadronization at $e_c \simeq 0.7 \text{ GeV/fm}^3$ is reached than peripheral Au+Au or e^+e^- and pp collisions.

We close by pointing out that our conclusions about the nature and origin of T_{chem} can be put to a relatively easy experimental test: It is well known [13, 14] that at low SPS and AGS energies, where the net baryon density of the matter created in the collision is much larger than at RHIC, the measured chemical decoupling temperatures are well below generally accepted estimates for the phase transition temperature, $T_{\text{chem}} < T_c$. In that case the phase transition can not be the origin of the observation of chemical equilibrium yields; hadronic chemical reactions must be responsible for lowering the chemical freeze-out temperature to values significantly below T_c . Since the present work has shown that the kinetic decoupling of hadronic chemical reaction rates is influenced by the fireball expansion rate, which again depends on collision centrality, *we expect to see impact parameter dependence of T_{chem} whenever its value is measured to be well below T_c* . This conclusion would also apply to RHIC collisions if lattice QCD would eventually converge to T_c values above 190 MeV as proposed in [35]. In this case we would definitely expect T_{chem} to depend on collision centrality. We therefore propose a reanalysis of chemical decoupling data at RHIC with higher statistics in order to unambiguously settle this question.

It may be possible to reanalyze existing SPS data to confirm or falsify our prediction of centrality dependence of T_{chem} at these energies. If not, this will be a worthwhile point to address within the planned low-energy collision program at RHIC. Clarification of this point will be of utmost importance for establishing the observed chemical decoupling temperature at RHIC as a direct measurement of the critical temperature of the quark-hadron phase transition in QCD.

Epilogue

Through his enthusiasm and openness in personal interaction, his frequent hospitality, and his brilliant students whom he sent out into the world already at early stages of their developing scientific careers, József Zimányi has had a lasting influence on the research of one of us (UH). The present work is a living testimony to this, and we therefore dedicate it to his memory.

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