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Bargaining power and equilibrium consumption

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Abstract We examine how a shift of bargaining power within households operating in a competitive market environment affects equilibrium allocation and welfare. If price effects are sufficiently small, then typically an individual benefits from an increase of bargaining power, necessarily to the detriment of others. If price effects are drastic, the welfare of all household members moves in the same direction when bargaining power shifts, at the expense (or for the benefit) of outside consumers. Typically a shift of bargaining power within a set of households also impacts upon other households. We show that each individual of a sociological group tends to benefit, if he can increase his bargaining power, but suffers if others in his group do the same.

1 Introduction

Societies often experience a shift of bargaining power in households. For instance, ceteris paribus, a shift of bargaining power in favor of the female partner has taken

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place over the last decades. Such a shift induces a change in household demand for goods and services. In turn, market clearing might occur at different prices and, consequently, the terms of trade for households might be altered.

It is the consequences, not the causes of shifts in intra-household bargaining power that interest us here. We are concerned with pure economic (positive) effects on the allocation of resources, as well as welfare (normative) effects at both the individual and societal levels. We are going to study those effects in a general equilibrium context. Our study reveals that the magnitude of equilibrium price responses to a shift of intra-household bargaining power matters. If price effects are sufficiently small, then typically an individual benefits from an increase of bargaining power—necessarily to the detriment of others. In particular, the other member(s) of the household will lose. In contrast, if price effects are drastic, then the members of the individual's household all benefit or are all harmed. Typically a shift of bargaining power within a set of households also impacts upon other households. We show that each individual of a sociological group tends to benefit if he can increase his bargaining power, but suffers if others in his group enjoy more bargaining power. For quasi-linear preferences, however, a change of the bargaining power within a particular household only impacts on the distribution of the numéraire in the household under consideration without affecting the consumption of other commodities. A local change of bargaining power has no price effect and does not affect the utility of individuals in other households.

The underlying model of the household satisfies collective rationality in the sense of Chiappori (1988a, 1992).[1] It departs from traditional economic theory which has, for the most part, treated households as if they were single consumers. The model admits households with several, typically heterogeneous members who have individual preferences. A household takes market prices as given and makes an efficient consumption choice (in terms of the preferences of its members) subject to its budget constraint. Different households may use different collective decision mechanisms. This departure from the traditional market model enables us to investigate the interplay of dual roles of households: households as collective decision making units on the one hand and as competitive market participants on the other hand.

The current model starts from the general equilibrium model in Haller (2000) where the household structure is fixed.[2] We specialize by assuming that the efficient collective household decision is the result of (possibly asymmetric) Nash bargaining within the household. This feature allows us to parametrize relative bargaining power, to perform comparative statics and to answer the question at hand, how a shift of bargaining power within households affects equilibrium allocation and welfare.

The model is introduced in the next section. In Sect. 3, we focus on a two-person household embedded in a larger economy and study how a shift of bargaining power within that household affects the consumption and welfare of its members. We decompose the intra-household effects into two relevant effects, a pure bargaining effect and a price effect. We focus in this section on negligible price effects.

[1] See also the surveys by Bourguignon and Chiappori (1992, 1994).

[2] See Gersbach and Haller (2001, 2002) for versions with variable household structure.

In Sect. 4, we exemplify the different scenarios when price effects may be drastic. We go through a sequence of representative examples, with a two-person household and a one-person household, and examine the general equilibrium implications of a shift of bargaining power within the two-person household. We observe that at least one member is always affected by a shift of bargaining power within the two-person household, but that the non-member may be affected as well. We observe further that price effects may be drastic if preferences exhibit little substitutability. We should mention that the findings for these two-household economies are also valid for respective replica economies obtained from the representative examples, provided that each of the two-person households of the replica economy undergoes the same shift of intra-household bargaining power. These shifts constitute a particular instance of a widespread shift of bargaining power in favor of a specific sociological group.

In Sect. 5, we investigate in more detail shifts of bargaining power in favor of a specific sociological group, with added emphasis on inter-household or spill-over effects. We distinguish between "first members" and "second members" of households. With particular consumer characteristics, spill-overs are absent: The effects of a change of bargaining power within a household are confined to that household. With different consumer characteristics, spill-overs can occur exactly as described earlier. For instance, a first member of a household benefits from an increase in own bargaining power, but loses if ceteris paribus first members of other households gain more bargaining power. In Sect. 6, we allow for price-dependent outside options. In Sect. 7, we offer concluding remarks.

2 General equilibrium model

We consider a finite pure exchange economy. The main departure from the traditional model is that a household can have several members, each with their own preferences.

Fixed household structure

The population is divided into finitely many households $h = 1, \ldots, n$, with $n \geq 2$. Each household h consists of finitely many members $i = hm$ with $m = 1, \ldots, m(h)$, $m(h) \geq 1$. Put $I = \{hm : h = 1, \ldots, n;\ m = 1, \ldots, m(h)\}$, the finite population of individuals to be considered.

Commodities, endowments, and individual preferences

The commodity space is $\mathbb{R}^\ell$ with $\ell \geq 1$. Household h is endowed with a commodity bundle $\omega_h \in \mathbb{R}^\ell$, $\omega_h > 0$. The aggregate or social endowment is $\omega = \sum_h \omega_h$. A generic individual $i = hm \in I$ has:

- consumption set $X_i = \mathbb{R}^\ell_+$;
- preferences $\succsim_i$ on the allocation space $\mathcal{X} \equiv \prod_{j\in I} X_j$ represented by a utility function $U_i : \mathcal{X} \longrightarrow \mathbb{R}$.

The consumption bundle of a generic individual i is denoted by x_i. Let $\mathbf{x} = (x_i)$, $\mathbf{y} = (y_i)$ denote generic elements of $\mathcal{X}$. For $h = 1, \ldots, n$, define $\mathcal{X}_h = \prod_{m=1}^{m(h)} X_{hm}$ with generic elements $\mathbf{x_h} = (x_{h1}, \ldots, x_{hm(h)})$. If $\mathbf{x} \in \mathcal{X}$ is an allocation, then for $h = 1, \ldots, n$, household consumption is given by $\mathbf{x_h} = (x_{h1}, \ldots, x_{hm(h)}) \in \mathcal{X}_h$.

We will allow for the possibility of consumption externalities. Following Haller (2000), we shall restrict attention to the case where such consumption externalities, if any, exist only between members of the same household. This is captured by the notion of intra-household externalities where utility functions are restricted to the household consumption $\mathbf{x_h}$, i.e.:

(E1) Intra-household externalities: $U_i(\mathbf{x}) = U_i(\mathbf{x_h})$ for $i = hm$, $\mathbf{x} \in \mathcal{X}$.

A special case is the absence of externalities. When there are no externalities, the utility function of an individual i depends only on his consumption bundle x_i, i.e.

(E2) Absence of externalities: $U_i(\mathbf{x}) = U_i(x_i)$ for $i = hm$, $\mathbf{x} = (x_i) \in \mathcal{X}$.

While our examples in Sect. 4 work with E2 we allow for intra-household externalities in Sects. 3 and 6.

Budget constraints. Now consider a household h and a price system $p \in \mathbb{R}^{\ell}$. For $\mathbf{x_h} = (x_{h1}, \ldots, x_{hm(h)}) \in \mathcal{X}_h$, denote total household expenditure

$$p * \mathbf{x_h} := p \cdot \left(\sum_{m=1}^{m(h)} x_{hm} \right).$$

Then h's *budget set* is defined as $B_h(p) = \{\mathbf{x_h} \in \mathcal{X}_h : p * \mathbf{x_h} \leq p \cdot \omega_h\}$. We define the *efficient budget set* $EB_h(p)$ by: $\mathbf{x_h} = (x_{h1}, \ldots, x_{hm(h)}) \in EB_h(p)$ if and only if $\mathbf{x_h} \in B_h(p)$ and there is no $\mathbf{y_h} \in B_h(p)$ such that

$$\begin{aligned} U_{hm}(\mathbf{y_h}) &\geq U_{hm}(\mathbf{x_h}) \quad \text{for all } m = 1, \ldots, m(h); \\ U_{hm}(\mathbf{y_h}) &> U_{hm}(\mathbf{x_h}) \quad \text{for some } m = 1, \ldots, m(h). \end{aligned}$$

General equilibrium. A competitive equilibrium (among households) is a price system p together with an allocation $\mathbf{x} = (x_i)$ satisfying

(i) $\mathbf{x_h} \in EB_h(p)$ for $h = 1, \ldots, n$, and
(ii) $\sum_i x_i = \omega$.

Thus, in a competitive equilibrium among households $(p; \mathbf{x})$, each household makes an efficient choice under its budget constraint and markets clear.[3] Efficient choice by the household refers to the individual consumption and welfare of its members, not merely to the aggregate consumption bundle of the household.

[3] Negative intra-household externalities allow for the possibility that a household has a bliss point despite the fact that each household member has monotonic preferences with respect to her individual consumption (see Haller 2000 for examples). If this happens, the social feasibility or market clearing condition (ii) has to be replaced by the *free disposal condition* $\sum_i x_i \leq \omega$.

Nash bargaining. An efficient household choice under a budget constraint may be the outcome of maximizing a function of the form

$$W_h(\mathbf{x_h}) = S_h\left(U_{h1}(\mathbf{x_h}), \ldots, U_{hm(h)}(\mathbf{x_h})\right),$$

subject to the budget constraint. A special case thereof is a *Nash-bargained household decision*. In this case, S_h assumes the form

$$S_h\left(U_{h1}, \ldots, U_{hm(h)}\right) = \prod_{m=1}^{m(h)} U_{hm}^{\alpha_{hm}}, \tag{1}$$

with the provision that $\alpha_{hm} \geq 0$ and $U_{hm} \geq 0$ for $m = 1, \ldots, m(h)$. The bargaining weight α_{hm} measures the *relative bargaining power* of individual $i = hm$ within household h. In the sequel, we shall concentrate on two-person households, i.e. $m(h) = 2$. We assume $\alpha_{h1}, \alpha_{h2} > 0$ and $\alpha_{h1} + \alpha_{h2} = 1$.

The assumption of Nash-bargained and, hence, efficient household decisions serves us well for the present inquiry into the consequences of shifts of bargaining power. The empirical question of whether collective household decisions are Nash-bargained, indeed, has gotten a fair amount of attention, in particular in the debate between Chiappori (1988b, 1991) on the one side and McElroy and Horney (1981, 1990) on the other side (see Bergstrom 1997 for discussions). There has been a growing number of empirical studies performing empirical tests of the collective rationality approach which nests Nash bargaining models as particular cases (Udry 1996; Fortin and Lacroix 1997; Browning and Chiappori 1998; Chiappori et al. 2002, among others).

Two qualifying comments are warranted. First, the interpretation of the maximands of S_h as Nash-bargained outcomes assumes that for each member of a multi-person household, the individual's reservation utility level is zero. The choice of disagreement points for intra-household bargaining is somewhat controversial and depends on the assumed inside or outside options of household members. Therefore, we opt here for a price-independent reservation utility which we normalize to zero solely for computational convenience. In Sect. 6, however, we discuss the extension of our model to price dependent outside options and indicate that they do not alter the qualitative implications.

Second, although maximization of the Nash product (1) describes the way in which the household reaches an efficient collective decision, it would be a grave mistake to attribute further meaning to the maximal value of (1) and to changes of it. Normative statements always refer to individuals, either one by one, identifying gainers and losers, or as constituents of society. Pareto-optimality and Pareto-improvements are defined in the standard fashion.

The economies and corresponding examples in the paper all have locally unique competitive equilibria and possess the budget exhaustion property which stipulates that each household's efficient choices under its budget constraint lie on the household's "budget line". Haller (2000) shows the validity of the first welfare theorem for economies with the budget exhaustion property. Therefore, equilibrium allocations are Pareto-optimal and comparative statics moves the economy from one Pareto-optimum to another one. Consequently, if a household member gains from a shift in bargaining power, then someone else inside or outside the household must lose.

3 General comparative statics for a two-person household

In this section, we perform comparative statics with respect to the balance of bargaining power within a two-person household denoted by h. We allow for an arbitrary number of commodities and we consider the general case of intra-household externalities. The entire population consists of an arbitrary number, n of households.

3.1 Preliminaries

We shall perform comparative statics with respect to the bargaining weights within a select two-person household h, with members $h1$ and $h2$. Whenever convenient and unambiguous, we shall drop the household name and simply refer to consumers 1 and 2. Without restriction, we may also assume that our selected household has the lowest number, i.e. $h = 1$ and the other households are labelled $k = 2, \ldots, n$. For the sake of convenience, we shall further adopt the notation $\alpha = \alpha_{h1}$ and $1 - \alpha = \alpha_{h2}$ so that comparative statics can be performed with respect to the parameter $\alpha \in (0, 1)$. Finally, denote $F \equiv \ln S_h$. Explicitly, we obtain

$$F = F(U_1(\mathbf{x_h}), U_2(\mathbf{x_h}); \alpha) = \alpha \ln U_1(\mathbf{x_h}) + (1 - \alpha) \ln U_2(\mathbf{x_h}). \tag{2}$$

While α is treated as variable, the other characteristics of household h as well as all the characteristics of the rest of the households remain fixed. Each household $k \neq h$ is assumed to choose an efficient consumption plan, $\mathbf{x_k} \in EB(p)$. It may, but need not, maximize a Nash product.

We assume sufficient regularity in the sense that for each $\alpha \in (0, 1)$ the economy has an equilibrium $(p(\alpha); \mathbf{x}(\alpha))$ satisfying:

(iii) local uniqueness and
(iv) continuous differentiability in α.

For each α, at the given price system $p(\alpha)$, household h solves the problem

$$\max \; F(U_1(\mathbf{x_h}), U_2(\mathbf{x_h}); \alpha) \; s.t. \; p(\alpha)[(x_1 + x_2) - \omega_h] \leq 0. \tag{3}$$

The corresponding solution is $\mathbf{x_h}(\alpha) = (x_1(\alpha), x_2(\alpha))$. The budget constraint can be rewritten $\mathbf{x_h} \in B_h(p(\alpha))$. In turn the household budget set $B_h(p(\alpha))$ defines a set $\mathcal{V}(\alpha)$ of *feasible utility allocations* for household h, given the price system $p(\alpha)$:

$$\mathcal{V}(\alpha) \equiv \{(V_1, V_2) \in \mathbb{R}^2 : (V_1, V_2) = (U_1(\mathbf{x_h}), U_2(\mathbf{x_h})) \text{ for some } \mathbf{x_h} \in B_h(p(\alpha))\}$$

In the sequel, the term *Pareto frontier* refers to the Pareto frontier of $\mathcal{V}(\alpha)$ in the space of utility allocations for the household. In particular, $(U_1(\mathbf{x_h}(\alpha), U_2(\mathbf{x_h}(\alpha))$ lies on the

Pareto frontier and solves the problem

$$\max \ F(V_1, V_2; \alpha) \ s.t. \ (V_1, V_2) \in \mathcal{V}(\alpha). \tag{4}$$

Finally, for the household under consideration and a given α, the term *α-indifference curve* refers to a locus in $\mathbb{R}^2$ given by an identity $F(V_1, V_2; \alpha) \equiv \text{const}$.

It is instructive to look first at the case $\ell = 1$ of a single good. Assuming that the equilibrium price is positive, the household's budget set and, therefore, its Pareto frontier is price-independent and the household's consumption decision is reduced to the division of a given pie. Consider an increase from α to $\alpha + \epsilon$. Then there are only two possibilities. It can happen that

$$(U_1(\mathbf{x_h}(\alpha)), U_2(\mathbf{x_h}(\alpha))) = (U_1(\mathbf{x_h}(\alpha + \epsilon)), U_2(\mathbf{x_h}(\alpha + \epsilon)))$$

because of a kinked Pareto frontier or a corner solution. But whenever

$$(U_1(\mathbf{x_h}(\alpha)), U_2(\mathbf{x_h}(\alpha))) \neq (U_1(\mathbf{x_h}(\alpha + \epsilon)), U_2(\mathbf{x_h}(\alpha + \epsilon))),$$

consumer 1 benefits from her increased bargaining power to the detriment of consumer 2. This follows from the fact that an increase in 1's bargaining power, that is, in α, renders the household's α-indifference curves steeper.

3.2 Bargaining and price effects

We focus on binding budget constraints. At the solution $\mathbf{x_h}(\alpha) = (x_1(\alpha), x_2(\alpha))$ of the household's problem (4) the equation

$$\frac{\partial F}{\partial U_1} \cdot D_{x_i} U_1 + \frac{\partial F}{\partial U_2} \cdot D_{x_i} U_2 = \lambda(\alpha) p(\alpha), \tag{5}$$

with positive Lagrange multiplier $\lambda(\alpha)$ holds for $i = 1, 2$.
For the value function

$$\Phi(\alpha) \equiv F(U_1(\mathbf{x_h}(\alpha)), U_2(\mathbf{x_h}(\alpha)); \alpha), \tag{6}$$

we obtain

$$\Phi'(\alpha) = \sum_{i=1}^{2} \left[\frac{\partial F}{\partial U_1} \cdot D_{x_i} U_1 + \frac{\partial F}{\partial U_2} \cdot D_{x_i} U_2 \right] \cdot x_i'(\alpha) + \frac{\partial F}{\partial \alpha}. \tag{7}$$

From the budget constraint

$$p(\alpha) \cdot [x_1(\alpha) + x_2(\alpha) - \omega_h] \equiv 0,$$

we obtain

$$p(\alpha)\left[x_1'(\alpha)+x_2'(\alpha)\right]=-p'(\alpha)\,[x_1(\alpha)+x_2(\alpha)-\omega_h]. \tag{8}$$

Substituting (5) and (8) into (7) yields

$$\Phi'(\alpha)=\frac{\partial F}{\partial\alpha}-\lambda(\alpha)p'(\alpha)[x_1(\alpha)+x_2(\alpha)-\omega_h]. \tag{9}$$

Without further qualification, it is impossible to sign $\Phi'(\alpha)$. Under additional assumptions, however, one can gain some detailed insights. To this end, let us decompose the effects of a change of consumer 1's relative bargaining power from α to $\alpha+\epsilon$ into two parts:

1. a *pure bargaining effect* when α is changed to $\alpha+\epsilon$ whereas the price system stays at $p(\alpha)$;
2. a *price effect* when relative bargaining power remains constant at $\alpha+\epsilon$ while the price system adjusts from $p(\alpha)$ to $p(\alpha+\epsilon)$.[4]

In Eq. (9), $p'(\alpha)$ reflects the price effect.

3.3 Negligible price effects

In this subsection, we examine the case when the price effect is negligible, i.e. $p'(\alpha)\approx 0$.[5] Then we obtain from Eq. (9), up to the negligible price effect:

$$\Phi'(\alpha)=\frac{\partial F}{\partial\alpha}=\ln U_1(\mathbf{x_h}(\alpha))-\ln U_2(\mathbf{x_h}(\alpha)). \tag{10}$$

We can exploit the following immediate consequence of Eq. (10):

Fact 1 *The value function increases* (*decreases*) *in* α*, if* $U_1>U_2$ $(U_1<U_2)$.

However, this result alone does not allow the further conclusion that the utility of at least one household member increases (decreases). A look at a more elementary proof of the fact proves instructive. Namely, let without loss of generality $U_1>U_2>0$ and consider α and ϵ with $0<\alpha<\alpha+\epsilon<1$. Then for sufficiently small ϵ, $\mathbf{x_h}(\alpha)\in B_h(p(\alpha+\epsilon))$ and

[4] Of course, the price effect could be further decomposed into a substitution and an income effect. But that is immaterial to our analysis.

[5] Note that the price effect vanishes when the budget constraint is not binding.

$$
\begin{aligned}
&[U_1(\mathbf{x_h}(\alpha+\epsilon))]^{\alpha+\epsilon}\cdot[U_2(\mathbf{x_h}(\alpha+\epsilon))]^{1-(\alpha+\epsilon)}\\
&\quad\geq[U_1(\mathbf{x_h}(\alpha))]^{\alpha+\epsilon}\cdot[U_2(\mathbf{x_h}(\alpha))]^{1-(\alpha+\epsilon)}\\
&\quad=[U_1(\mathbf{x_h}(\alpha))]^{\alpha}\cdot[U_2(\mathbf{x_h}(\alpha))]^{1-\alpha}\cdot(U_1/U_2)^{\epsilon}\\
&\quad>[U_1(\mathbf{x_h}(\alpha))]^{\alpha}\cdot[U_2(\mathbf{x_h}(\alpha))]^{1-\alpha}.
\end{aligned}
$$

The last inequality shows that the shift in bargaining power has a "nominal effect" on the household's Nash product even before reoptimization takes place. For this reason, we cannot conclude from a surge of the household's maximum value of F per se that the utility of at least one household member has increased. The impact of a shift of bargaining power has to be assessed for each household member individually.

In order to sign individual utility changes, we focus on Eq. (5) which is the key to the pure bargaining effect. With $DU_j = (D_{x_1}U_j, D_{x_2}U_j)$ for $j = 1, 2$, let us rewrite (5) as

$$\frac{\alpha}{U_1}\cdot DU_1=-\frac{1-\alpha}{U_2}\cdot DU_2+\lambda(\alpha)(p(\alpha),p(\alpha)). \tag{11}$$

Now consider a change $\Delta\mathbf{x_h}$ away from $\mathbf{x_h}(\alpha)$ while maintaining the budget identity, i.e. $p(\alpha)*(\mathbf{x_h}(\alpha)+\Delta\mathbf{x_h}) = p(\alpha)*\mathbf{x_h}(\alpha) = p(\alpha)\omega_h$. Then $(p(\alpha), p(\alpha))\cdot\Delta\mathbf{x_h} = p(\alpha)*\Delta\mathbf{x_h} = 0$, hence with (11),

$$\left[\frac{\alpha}{U_1}\cdot DU_1\right]\cdot\Delta\mathbf{x_h}=-\left[\frac{1-\alpha}{U_2}\cdot DU_2\right]\cdot\Delta\mathbf{x_h}. \tag{12}$$

We first examine local comparative statics. One possibility is $(U_1(\mathbf{x_h}(\alpha)), U_2(\mathbf{x_h}(\alpha))) = (U_1(\mathbf{x_h}(\alpha+\epsilon)), U_2(\mathbf{x_h}(\alpha+\epsilon)))$. The second possibility is $(U_1(\mathbf{x_h}(\alpha)), U_2(\mathbf{x_h}(\alpha)) \neq (U_1(\mathbf{x_h}(\alpha+\epsilon)),_2(\mathbf{x_h}(\alpha+\epsilon)))$. An increase of α makes the household's α-indifference curves steeper. Hence, as long as $\mathbf{x_h}(\alpha+\epsilon) \in B_h(p(\alpha))$ and $\mathbf{x_h}(\alpha) \in B_h(p(\alpha+\epsilon))$, the revised utility allocation $(U_1(\mathbf{x_h}(\alpha+\epsilon)), U_2(\mathbf{x_h}(\alpha+\epsilon)))$ must lie to the southeast of $(U_1(\mathbf{x_h}(\alpha)), U_2(\mathbf{x_h}(\alpha))$. Thus consumer 1 benefits from a small increase of her bargaining power to the detriment of consumer 2. The foregoing local comparative statics can be easily globalized.

Proposition 1 *Suppose that the price effect is negligible. If* $0 < \alpha_* < \alpha^* < 1$*, then one of the following two assertions holds:*

(i) $U_1(\mathbf{x_h}(\alpha_*)) = U_1(\mathbf{x_h}(\alpha^*)),\ U_2(\mathbf{x_h}(\alpha_*)) = U_2(\mathbf{x_h}(\alpha^*))$.
(ii) $U_1(\mathbf{x_h}(\alpha_*)) < U_1(\mathbf{x_h}(\alpha^*)),\ U_2(\mathbf{x_h}(\alpha_*)) > U_2(\mathbf{x_h}(\alpha^*))$.

The proof of Proposition 1 is given in the appendix. The focus on a particular household h amid many might suggest that shifts of bargaining power are sporadic and therefore price effects are likely to be negligible. Our analysis so far provides valuable insights in case the change of bargaining power is a sporadic event, indeed.

Drastic price effects will prevail for instance, if the economy is replicated and the same shift in bargaining power occurs in all households that are replicas of h. If the price effect is drastic, both utilities may move in the same direction. The price effect also depends on preferences, including the preferences of consumers not belonging to the household as we will discuss next.

4 Comparative statics with drastic price effects

In this section, we allow for drastic price effects and consider a series of examples. The entire population consists of a total of three consumers, two belonging to household h and one forming a one-person household denoted k. To capture widespread shifts in bargaining power in a large finite population, one can consider h as a prototype of a two-person household and k as representative of a one-person household. Literally, one can think in terms of replica economies derived from the basic economies under consideration, with an equal number of two-person households like h and one-person households like k.

Throughout this section, there are always two goods: $\ell = 2$. The second good serves as numéraire. The symbols $x, x_1, x_2, \ldots, x_i, \ldots$ denote quantities of the first good. The symbols $y, y_1, y_2, \ldots, y_i, \ldots$ denote quantities of the second good. c_i^* stands for the equilibrium consumption bundle of a generic person (individual, consumer) i. All consumers fulfill condition E2, i.e., absence of externalities. To simplify the exposition of the later examples, we consider first an auxiliary example of an economy consisting of two one-person households, g and k. The respective consumers are named 0 and 3.

Example 0

The initial endowments are $\omega_0 = (1, 0)$ and $\omega_3 = (0, 1)$. The utility representations are

$$u_0 = u_0(x_0, y_0) = x_0^{\alpha} y_0^{1-\alpha}, \quad \text{with } 0 < \alpha < 1, \quad \text{and}$$
$$u_3 = u_3(x_3, y_3) = x_3^{1/2} y_3^{1/2}.$$

After normalizing the price of the second good, market equilibrium is unique. The equilibrium price system is

$$p^* = \left(\frac{1}{2(1-\alpha)}, 1\right);$$

the equilibrium consumption bundles are $c_0^* = (\alpha, 1/2)$, $c_3^* = (1-\alpha, 1/2)$. □

Now we are prepared to consider the case of three individuals, labelled $i = 1, 2, 3$. Consumers 1 and 2 form the two-person household h. In this household, consumer 1 has bargaining power α and consumer 2 has bargaining power $1-\alpha$. Consumer 3 constitutes the single household k. We are going to scrutinize several representative examples which are almost exhaustive in that they exhibit three possible allocative responses to a shift of bargaining power within the two-person household:

(a) Only one member is affected.
(b) The two members are affected in opposite ways.
(c) Both members are affected in the same way.

The examples differ only in individual consumer preferences. The analysis suggests that less substitutability leads to more drastic price effects. We start with the following example of case (a).

Example 1

Here consumer 1 benefits from more bargaining power, to the detriment of consumer 3 while consumer 2 is unaffected. Household h is endowed with $\omega_h = (1, 0)$. Its two members, $i = 1, 2$ have utility representations

$$u_1(x_1, y_1) = x_1 \quad \text{and} \quad u_2(x_2, y_2) = y_2.$$

The household maximizes

$$S_h = u_1^{\alpha} u_2^{1-\alpha} = x_1^{\alpha} y_2^{1-\alpha}, \quad 0 < \alpha < 1.$$

The characteristics of household k are as in the previous example, that is the endowment is $\omega_3 = (0, 1)$ and the utility representation is

$$u_3(x_3, y_3) = x_3^{1/2} y_3^{1/2}.$$

Since the aggregate demand function of household h coincides with the demand function of consumer 0 in Example 0, the equilibrium quantities are

$$p^* = \left(\frac{1}{2(1-\alpha)}, 1\right);$$
$$c_1^* = (\alpha, 0), \quad c_2^* = \left(0, \frac{1}{2}\right), \quad c_3^* = \left(1-\alpha, \frac{1}{2}\right).$$

Hence as asserted consumer 1 benefits from more bargaining power, to the detriment of consumer 3. Consumer 2 is unaffected. □

In the example, the first good becomes more valuable to the two-person household as the bargaining power of the first consumer increases. This boosts the equilibrium price of the first good and the income of the two-person household endowed with the first good. The household has become richer both in nominal and real terms. Since the expenditure on the second good remains constant, the second consumer is unaffected. But the increase in the residual income to be spent on the first good more than compensates for the higher price: consumer 1 is better off as a consequence of her increased bargaining power. As for consumer 3, his nominal income derived from the possession of the second resource remains constant. Therefore, he has become poorer, has less purchasing power.

From consumer 2's perspective, if bargaining power shifts towards her and prices are fixed, then her welfare is increased. But the resulting price variation offsets her gain. That consumer 2 is unaffected by a change in bargaining power seems to be caused by limited substitutability within the two-person household. This is confirmed by the next example where enhanced bargaining power of consumer 1 translates into improved welfare for this consumer and welfare losses for consumers 2 and 3.

Example 2

Here consumer 1 benefits from more bargaining power to the detriment of consumer 2. Consumer 3 either gains or loses. Household h is still endowed with $\omega_h = (1, 0)$. But now each member $i = 1, 2$ has Cobb–Douglas preferences with utility representation

$$u_i(x_i, y_i) = x_i^{\gamma_i} y_i^{1-\gamma_i}, \quad 0 < \gamma_i < 1.$$

The household maximizes

$$\begin{aligned} u_1^{\alpha} u_2^{1-\alpha} &= (x_1^{\gamma_1} y_1^{1-\gamma_1})^{\alpha} (x_2^{\gamma_2} y_2^{1-\gamma_2})^{1-\alpha} \\ &= x_1^{\alpha\gamma_1} x_2^{(1-\alpha)\gamma_2} y_1^{\alpha(1-\gamma_1)} y_2^{(1-\alpha)(1-\gamma_2)}. \end{aligned}$$

Again, α and $1 - \alpha$ lend themselves as measures of relative bargaining power of consumer 1 and consumer 2, respectively. Household k has the single member 3, with the same consumer characteristics as before. We obtain:

Fact 2 *A shift of bargaining power from consumer 2 to consumer 1 benefits consumer 1 and harms consumer 2, who ends up consuming less of both commodities.*

The proof of Fact 2 is given in the appendix. In Example 2 there is more substitutability in the economy than in Example 1.

Example 3 below exhibits less substitutability than Example 1, because the preferences of consumer 3 will be altered from Cobb-Douglas to Leontieff. It turns out that the lack of substitution by consumer 3 necessitates a major price adjustment to re-equilibrate the market after bargaining power within household h has shifted. As a result, we observe a very drastic price effect: When bargaining power within their household changes, the equilibrium utilities of consumers 1 and 2 are moving in the same direction.

The example further shows that the aggregate equilibrium consumption of a household can be positively affected by a shift of internal bargaining power. This suggests the possibility that a sophisticated household might succeed in an attempt to manipulate the market outcome, not by misrepresenting endowments or individual preferences, but by misrepresenting the internal bargaining power. To illustrate this novel way of manipulation, which is not yet documented in the literature, suppose the household pretends that the bargaining power of the first consumer is higher than it actually is and they submit the corresponding excess demands to the market. If $\gamma_1 > \gamma_2$, i.e. if the first good is relatively more important to the first consumer, they will end up with a higher aggregate amount of the first good and the same amount of the second good in equilibrium. Whether or not both gain from a successful manipulation depends on the internal distribution of aggregate consumption. If they divided the goods in accordance with their pretended bargaining power, put their money where their mouth is, then consumer 1 would gain and consumer 2 would lose from manipulation. If they divide the goods according to the true bargaining power—which fixes a proportional sharing rule for each of the goods—then both gain from manipulation.[6]

Example 3

Here a shift of bargaining power from consumer 2 to consumer 1 benefits both consumers to the detriment of consumer 3. This example is identical with Example 1, except that consumer 3 now has Leontief preferences with utility representation

$$u_3(x_3, y_3) = \min(x_3, y_3).$$

After setting $s = \min(x_3, y_3)$, the utility maximization problem for consumer 3 can be rewritten as

$$\max s \quad \text{s.t. } (p_1 + 1)s = 1$$

with solution $s = 1/(p_1 + 1)$.

Household h's demand is $(\alpha, (1 - \alpha)p_1)$. Therefore, market clearing for the first good requires $1/(p_1 + 1) = 1 - \alpha$. Thus in equilibrium,

$$\begin{aligned} &p^* = (\alpha/(1 - \alpha), 1); \\ &c_1^* = (\alpha, 0), \quad c_2^* = (0, \alpha), \quad c_3^* = (1 - \alpha, 1 - \alpha). \end{aligned}$$

Thus a shift of bargaining power from consumer 2 to consumer 1 benefits both members of the household to the detriment of consumer 3. A reverse shift harms 1 and 2, and leaves 3 better off. □

The examples suggest that comparative statics is sensitive to the degree of substitutability in the economy. Enhanced substitutability appears to mitigate price effects. Indeed, if in a further variation of Example 1, one assumes linear preferences (perfect substitutability) for consumer 3, with utility representation $u_3(x_3, y_3) = x_3 + y_3$, then the price effect is zero. Moreover, for two-good economies exhibiting CES-utility functions for all individuals with the same elasticity of substitution, the magnitude of the price effect can be parameterized by the elasticity of substitution in the economy. The price effect depends negatively on the elasticity of substitution.

[6] We will later show that this kind of manipulation is excluded when all individuals have quasi-linear preferences.

The preceding examples have not contained any sort of externalities. It is straightforward yet space- and time-consuming to analyze the same examples with additive group externalities, i.e. each household member obtains a fixed utility gain from living together, and price-dependent outside options. The qualitative results remain the same.

5 Comparative statics across households

Until now we have focused primarily on intra-household effects, that is, on the utility changes in a particular household when bargaining power shifts within that household. Via a series of examples, we have demonstrated that such a shift of bargaining power can affect the members of the corresponding two-person household in three different ways: Only one member is affected; the two members are affected in opposite ways; both members are affected the same way. We have argued earlier that the above examples can be readily reinterpreted as instances of widespread shifts of bargaining power in a replica economy. In the resulting replica economy, the main focus remains on intra-household effects, on the repercussions on the members of those households in which a shift in bargaining power has occurred. However, we have also seen that third parties can be affected. In this section, we redirect our attention to such inter-household or spill-over effects.

We start with the quasi-linear case that can serve as a benchmark.

5.1 The quasi-linear case

We consider a society with $n > 1$ identical households. Household h ($h = 1, \dots, n$) has members $h1$ and $h2$, called the first member and the second member, respectively. There are ℓ goods ($\ell > 1$). The consumption of good k ($k = 1, \dots, \ell$) by individual hi ($i = 1, 2$) is denoted by x_{hi}^k. Each household h is endowed with $w_h = (w_h^1, \dots, w_h^\ell)$. The two members of household h have quasi-linear utility representations of the form

$$U_{h1}(x_{h1}) = u_{h1}(x_{h1}^1, \dots, x_{h1}^{\ell-1}) + x_{h1}^\ell \tag{13}$$

$$U_{h2}(x_{h2}) = u_{h2}(x_{h2}^1, \dots, x_{h2}^{\ell-1}) + x_{h2}^\ell \tag{14}$$

where u_{hi} is assumed to be strictly concave, strictly increasing and differentiable. Household h maximizes

$$S_h = U_{h1}^{\alpha_h} U_{h2}^{1-\alpha_h} \quad \text{or} \quad \ln S_h = \alpha_h \ln U_{h1} + (1 - \alpha_h) \ln U_{h2} \tag{15}$$

where $0 < \alpha_h < 1$ is the bargaining power of individual $h1$ in household h. We denote equilibrium values by $\hat{x}_{hi}^k$ and equilibrium utilities by $\hat{U}_{hi}$ and $\hat{u}_{hi}$. For the following we assume that for any array of bargaining power parameters $(\alpha_1, \dots, \alpha_n)$ under consideration, each individual consumes a non-negative amount of the natural numéraire good ℓ in every market equilibrium. We also assume that for any array $(\alpha_1, \dots, \alpha_n)$, the corresponding economy has a unique market equilibrium, up to price normalization. These two assumptions are inessential for our argumentation but simplify the exposition considerably. We shall indicate below which modifications are necessary if the two assumptions are removed. We consider a market equilibrium and parametric changes of the bargaining power in household h and obtain:

Proposition 2 (No spill-overs) *With quasi-linear preferences:*

(i) *A change of α_h in a particular household h does not impact on non-members.*
(ii) $\frac{\partial \hat{x}_{h1}^k}{\partial \alpha_h} = \frac{\partial \hat{x}_{h2}^k}{\partial \alpha_h} = 0$ *for all* $k = 1, \ldots, \ell - 1$.
(iii) $\frac{\partial \hat{x}_{h1}^\ell}{\partial \alpha_h} > 0$, $\frac{\partial \hat{x}_{h2}^\ell}{\partial \alpha_h} < 0$.
(iv) *Suppose that households are homogeneous with respect to individual utility representations and household endowments, with $w_h = \overline{w}$, $\forall h = 1, \ldots, n$. Then:*

$$\hat{x}_{h1}^\ell = \alpha_h \overline{w}^\ell + \alpha_h \, \hat{u}_{h2} - (1 - \alpha_h)\hat{u}_{h1};$$
$$\hat{x}_{h2}^\ell = (1 - \alpha_h)\overline{w}^\ell + (1 - \alpha_h) \, \hat{u}_{h1} - \alpha_h \, \hat{u}_{h2}.$$

The proof of Proposition 2 is given in the appendix. Proposition 2 illustrates that with quasi-linear preferences, a change of the bargaining power within a particular household only impacts on the distribution of the numéraire in household h without affecting the consumption of the first $\ell - 1$ commodities. A local change of bargaining power has no price effect and does not affect the utility of individuals in other households. This also means that a household h cannot manipulate outcomes and possibly improve utility of household members at the expense of outsiders by misrepresenting internal bargaining power.

Regarding our simplifying assumptions for the neutrality result, interiority and uniqueness of equilibrium, giving up the first assumption requires to work with Kuhn–Tucker conditions instead of first-order conditions. Without the second assumption, multiple equilibria cannot be ruled out. But a market clearing price system $(p_1, \ldots, p_{\ell-1}, 1)$ with respect to some array of bargaining power parameters is also market clearing with respect to all other arrays. Given any such market clearing price system and the associated equilibrium selection, the conclusion of Proposition 2 continues to hold.

5.2 Individually preferred goods

We next turn to situations where internal bargaining power changes in a particular household have spill-over effects on other households. In particular, we examine how individuals are affected if similar (dissimilar) persons in other households can increase their bargaining power. We examine an economy like in the last subsection, but with different individual preferences. We assume households which are homogeneous at the beginning but undergo large sociological changes thereafter. We assume $\ell = 2$ and that all households have the same endowment $w_h = \overline{w} = (\overline{w}^1, \overline{w}^2) \gg 0$. Individuals in a household h demand different goods, namely:

$$U_{h1}(x^1_{h1}, x^2_{h1}) = U_{h1}(x^1_{h1}),$$
$$U_{h2}(x^1_{h2}, x^2_{h2}) = U_{h2}(x^2_{h2}).$$

Specifically, individual h1 has a preference for good 1 and demands only that good and h2 has a preference for good 2 and demands only good 2. The utility functions are assumed to be strictly increasing, strictly concave and differentiable. The assumption of household members demanding different goods is one convenient way to divide the society into different sociological groups where individuals are similar within a group and dissimilar across groups. Here we have two groups, "first members" (denoted $h1$) and "second members" (denoted $h2$) of households. Again household h maximizes

$$S_h = U_{h1}^{\alpha_h} U_{h2}^{1-\alpha_h}$$

where $0 < \alpha_h < 1$. We obtain, with ˆ denoting again equilibrium values:

Proposition 3 (Spill-overs) *When household members demand different goods, there exists a unique market equilibrium (up to price normalization) for each array $(\alpha_1, \ldots, \alpha_n)$ of bargaining power parameters. Moreover, for any two households $g \neq h$:*

(i) $\alpha_h > \alpha_g \Rightarrow \hat{x}^1_{h1} > \hat{x}^1_{g1}$.
(ii) $\alpha_h = \alpha_g \Rightarrow \hat{x}^1_{h1} = \hat{x}^1_{g1}$.

The proof of Proposition 3 is given in the appendix. In the special case where $U_{h1}(x^1_{h1}) = (x^1_{h1})^{\beta_1}$ and $U_{h2}(x^2_{h2}) = (x^2_{h2})^{\beta_2}$ with $\beta_1, \beta_2 \in (0, 1)$, further conclusions can be drawn: (iii) $\partial\hat{x}^1_{h1}/\partial\alpha_h > 0$, $\partial\hat{x}^1_{g1}/\partial\alpha_h < 0$; (iv) $\partial\hat{x}^2_{h2}/\partial\alpha_h < 0$, $\partial\hat{x}^2_{g2}/\partial\alpha_h > 0$.

Proposition 3 has clear-cut implications. Consider the sociological groups "first-members" and "second-members", defined by similarities with respect to preferences. If all individuals in the first sociological group have the same bargaining power (and as a consequence all "second-members" as well), all households consume their endowments since we are in an equilibrium with no active trade. An identical shift of bargaining power across all households has no effect on utilities of any individual either since we will again arrive at an equilibrium with no trade.

The situation is completely different when only some members of a sociological group enjoy higher bargaining power. For instance, a "first-member" suffers when only other "first-members" gain more bargaining power in their respective households. Conversely, the "first-member" benefits from higher own bargaining power as long as other "first-members" do not experience a change of bargaining power. The analogue holds for the other sociological group. Therefore, the main thrust of Proposition 3 is that an individual tends to benefit if he can increase his bargaining power but tends to suffer if some or all individuals with the same demand are able to do the same.[7]

For economies of the type discussed in the current subsection, we obtain as an immediate consequence a power illusion phenomenon. Consider two economies denoted

[7] Gersbach and Haller (2005, subsection 5.3) illustrate the assertions and implications of Proposition 3 by solving explicitly for the market equilibria of a specific numerical example.

by $E_1(\{\alpha_h^1\}_1^n)$ and $E_2(\{\alpha_h^2\}_1^n)$ with households that are homogeneous with respect to individual utility functions and endowments. Household members demand different goods. Equilibrium utilities are denoted by $\hat{U}_{h1}^1$, $\hat{U}_{h2}^1$ and $\hat{U}_{h1}^2$, $\hat{U}_{h2}^2$, respectively. Then the following holds:

Corollary 1 (Power illusion)

(i) *If* $\alpha_h^1 = \overline{\alpha}^1$ *for all* h *and* $\alpha_1^2 > \max_{h \neq 1}\{\alpha_h^2\}$, *then* $\hat{U}_{11}^1 < \hat{U}_{11}^2$.
(ii) *If* $\alpha_h^1 = \overline{\alpha}^1$ *for all* h *and* $\alpha_1^2 < \min_{h \neq 1}\{\alpha_h^2\}$, *then* $\hat{U}_{11}^1 > \hat{U}_{11}^2$.

The corollary illustrates that a member of a sociological group is better off if he has the highest internal bargaining power even if the level of his power is much smaller than in another economy where all individuals of the group have the same bargaining power, that is $\overline{\alpha}^1 > \alpha_1^2$. The underlying intuition runs as follows: Diversity across households opens trade opportunities. The gains from trade will, as a rule, accrue primarily to the members of a sociological group who have relatively higher bargaining power than other members of the group. The absolute level of bargaining power is not important. When, however, the bargaining power of other individuals in the same sociological group is enhanced as well and all individuals of the sociological group end up enjoying an identical level in bargaining power, the original gain is totally eroded.

5.3 Separable utility

In this section we consider in which way Proposition 3 can be generalized. We again assume $\ell = 2$ and that all households have the same endowment $w_h = \bar{w} = (\bar{w}^1, \bar{w}^2)$. Each household consists of two members: the first member (denoted $h1$) and the second member (denoted $h2$). Individuals in a household h have utility functions

$$U_{h1}(x_{h1}^1, x_{h1}^2) = U_{h1}^1(x_{h1}^1)U_{h1}^2(x_{h1}^2);$$
$$U_{h2}(x_{h2}^1, x_{h2}^2) = U_{h2}^1(x_{h2}^1)U_{h2}^2(x_{h2}^2).$$

The functions $U_{h1}^1(\cdot)$, $U_{h1}^2(\cdot)$, $U_{h2}^1(\cdot)$, and $U_{h2}^2(\cdot)$ are strictly increasing, strictly concave and differentiable. Moreover, the marginal utility of consuming good 1 and good 2 at zero is infinite for both household members. A typical example are Cobb–Douglas utility functions. Household h maximizes

$$S_h = U_{h1}^{\alpha_h} U_{h2}^{1-\alpha_h}$$

where $0 < \alpha_h < 1$. We obtain, with ˆ denoting equilibrium values:

Proposition 4 *There exists a unique market equilibrium for each array* $(\alpha_1, \ldots, \alpha_n)$ *of bargaining parameters. Moreover, for any two households* $g \neq h$:

(i) $\alpha_h > \alpha_g \Rightarrow \hat{x}_{h1}^1 > \hat{x}_{g1}^1$, $\hat{x}_{h1}^2 > \hat{x}_{g1}^2$.
(ii) $\alpha_h = \alpha_g \Rightarrow \hat{x}_{h1}^1 = \hat{x}_{g1}^1$, $\hat{x}_{h1}^2 = \hat{x}_{g1}^2$.

The proof of Proposition 4 is available upon request. Proposition 4 states that if a member of a particular sociological group has higher bargaining power in his household than another member of the sociological group in another household, he will consume more of both commodities.

Notice that Proposition 4 does not imply an analogue of Corollary 1. Instead, we are going to illustrate two other properties of the model with separable utilities. First, suppose all individuals in the first sociological group have the same bargaining power and thus there is no trade. An identical upward shift of their bargaining power will again arrive at an equilibrium with no trade. However, all individuals in the first sociological group benefit from the shift as they will consume more of both goods.[8] Second, spillovers from a upward shift of bargaining power for a "first-member" in one household can affect other first-members in other households in different ways. Let us consider as a benchmark case the following corollary.

Corollary 2 (Absence of spillovers) *Suppose* $\alpha_h = \alpha_g \ \forall h, g \ h \neq g$. *Let* $\bar{w}^1 = \bar{w}^2$ *and* $U^1_{h1}(\cdot) \equiv U^2_{h1}(\cdot) \equiv U^1_{h2}(\cdot) \equiv U^2_{h2}(\cdot)$. *Then a change of the bargaining power in one household does not affect the utilities of other households.*

The corollary follows from the observation that the first good's equilibrium price of this economy before and after the bargaining change is $p_1 = 1$—with the second good as numéraire. At any other price all households would have an excess demand for the first or second good.[9] Given that the equilibrium price is not affected by a change of bargaining power in a particular household, the assertion in Corollary 2 follows immediately. It is straightforward to construct examples using Cobb-Douglas utility functions with different exponents for first- and second-members where first-members are affected negatively or positively by the change of bargaining power in other households.

6 Price-dependent outside options and group externalities

In this section, we discuss how the model can be extended to price-dependent outside options and group externalities. A simple yet very instructive extension is to introduce an exit option, i.e. the possibility that a household member leaves and forms a single household at the going market prices. Such price-dependent exit options can easily be integrated into our analysis and have been pursued in more detail in Gersbach and Haller (2008).[10] To this end we define: a *competitive equilibrium with free exit* (CEFE) is a price system p together with an allocation $\mathbf{x} = (x_i)$ satisfying

(i) $\mathbf{x_h} \in EB_h(p)$ for all $h = 1, \ldots, n$.
(ii) $\sum_i x_i = \omega$.

[8] This follows from the first-order conditions in the proof of Proposition 4.

[9] This follows from inspecting the first-order conditions. Details are available upon request.

[10] Another and more general approach, suggested by a referee, would be to take a collective model in which decisions are assumed to be Pareto efficient, Pareto weights may be price-dependent and a change of such weights is considered (see Blundell et al. 2005).

(v) There are no current household h, household member $i \in h$ and consumption bundle $y_i \in B_{\{i\}}(p)$ such that $U_i(y_i) > U_i(\mathbf{x_h}; h)$.

Here $U_i(y_i)$ denotes i's utility from consuming the bundle y_i as a single individual. Individual i has an endowment $\omega_{\{i\}}$ and budget set $B_{\{i\}}(p) = \{x_i \in X_i \mid px_i \leq p\omega_{\{i\}}\}$ when forming the single household $\{i\}$. In contrast, $U_i(\mathbf{x_h}; h)$ denotes i's utility when i stays in the multi-member household h and household consumption is $\mathbf{x_h} = (x_j)_{j \in h} \in \mathcal{X}_h$. This formulation allows for consumption externalities, where i's welfare depends on the consumption bundles x_j of other household members $j \in h$, $j \neq i$, and for group externalities, where household member i's welfare depends on household composition, that is on h.

In the specific context of *Nash bargained household decisions*, a CEFE amounts to replacing (1) by

$$S_h\left(U_{h1}, \ldots, U_{hm(h)}\right) = \prod_{m=1}^{m(h)} \left(U_{hm} - V_{hm}^o(p)\right)^{\alpha_{hm}}$$

where $V_{hm}^o(p)$ is individual hm's indirect utility in the single person household $\{hm\}$ at price system p. This formulation presumes a non-empty set of feasible utility vectors $(U_{h1}, \ldots, U_{hm(h)}) \gg (V_{h1}^o(p), \ldots, V_{hm(h)}^o(p))$ over which S_h is maximized.

With this formulation, (5) remains unchanged while (7) becomes

$$\begin{aligned}\Phi'(\alpha) = \sum_{i=1}^{2} \left[\frac{\partial F}{\partial U_1} \cdot D_{x_i} U_1 + \frac{\partial F}{\partial U_2} \cdot D_{x_i} U_2\right] \cdot x_i'(\alpha) \\ - \left[\frac{\partial F}{\partial U_1} \cdot \frac{dV_1^o}{dp} + \frac{\partial F}{\partial U_2} \cdot \frac{dV_2^o}{dp}\right] \cdot p'(\alpha) + \frac{\partial F}{\partial \alpha}\end{aligned}$$

and (9) becomes

$$\begin{aligned}\Phi'(\alpha) = \frac{\partial F}{\partial \alpha} - p'(\alpha)\left[\frac{\partial F}{\partial U_1} \cdot \frac{dV_1^o}{dp} + \frac{\partial F}{\partial U_2} \cdot \frac{dV_2^o}{dp}\right] \\ -\lambda(\alpha)p'(\alpha)[x_1(\alpha) + x_2(\alpha) - \omega_h].\end{aligned}$$

It follows that Proposition 1 generalizes to this case of price-dependent reservation utilities of the form $V_i^o(p)$ where the latter is individual i's indirect utility in the single person household $\{i\}$ at price system p. A proof of the more general result is omitted. It is analogous to the proof of Proposition 1 but more tedious.

Now let us specialize and consider two-person households $h = \{h1, h2\}$ with *pure group externalities* as follows: Individual hm, $m = 1, 2$ has utility of the form

$$\begin{aligned} U_{hm}(x_{hm}) &= \mathrm{u}_{hm}(x_{hm}) \quad \text{for } x_{hm} \in X_{hm} \text{ when single;} \\ U_{hm}(x_{hm}; h) &= \mathrm{u}_{hm}(x_{hm}) + v_m \quad \text{for } x_{hm} \in X_{hm} \text{ when a member of } h \end{aligned}$$

where $v_m > 0$. As mentioned at the end of Sect. 4, the qualitative properties of Examples 1–3 remain the same if this type of additive pure group externalities and price-dependent reservation utilities are incorporated. However, the numerical analysis becomes lengthier and more cumbersome.

Next we examine the robustness of the results of Sects. 5.1 and 5.2 vis-à-vis this type of additive pure group externalities and price-dependent reservation utilities. Again, the qualitative results tend to remain the same.

6.1 The quasi-linear case revisited

Here we provide a specific example with quasi-linear utilities and pure group externalities. Like in Sect. 5.1, we consider a society with $n > 1$ identical two-member households $h = 1, \ldots, n$. Household h has members $h1$ and $h2$. There are $\ell > 1$ goods. The consumption of good $k = 1, \ldots, \ell$ by individual hm, $1 \leq h \leq n$, $m = 1, 2$, is denoted x_{hm}^k. The vector $x_{hm} = (x_{hm}^1, \ldots, x_{hm}^\ell)$ denotes hm's consumption bundle. Each household h is endowed with $w_h = (w_h^1, \ldots, w_h^\ell) \in \mathbb{R}_{++}^\ell$. The two members, $h1$ and $h2$, of household h have quasi-linear utility representations of the form

$$U_{h1}(x_{h1}; h) = u_{h1}(x_{h1}^1, \ldots, x_{h1}^{\ell-1}) + x_{h1}^\ell + v_1, \tag{16}$$

$$U_{h2}(x_{h2}; h) = u_{h2}(x_{h2}^1, \ldots, x_{h2}^{\ell-1}) + x_{h2}^\ell + v_2, \tag{17}$$

where u_{hm} is assumed to be strictly concave, strictly increasing and differentiable. The parameters v_1 and v_2 ($v_1 > 0,\ v_2 > 0$) capture the group externalities that individuals $h1$ and $h2$ experience when living together. As single consumer, individual hm has an endowment $w_{hm} \in \mathbb{R}_{++}^\ell$ and the utility function

$$U_{hm}(x_{hm}) = u_{hm}(x_{hm}^1, \ldots, x_{hm}^{\ell-1}) + x_{hm}^\ell.$$

Hence $\mathrm{u}_{hm}(x_{hm}) = u_{hm}(x_{hm}^1, \ldots, x_{hm}^{\ell-1}) + x_{hm}^\ell$ and, with the notation of Sect. 5.1, $V_{hm}^o(p) = U_{hm}(x_{hm}^0(p))$. Household h now maximizes

$$S_h = \{U_{h1}(x_{h1}; h) - U_{h1}(x_{h1}^0(p))\}^{\alpha_h} \cdot \{U_{h2}(x_{h2}; h) - U_{h2}(x_{h2}^0(p))\}^{1-\alpha_h} \tag{18}$$

subject to its budget constraint plus the constraints $U_{h1}(x_{h1}; h) - U_{h1}(x_{h1}^0(p)) > 0$ and $U_{h2}(x_{h2}; h) - U_{h2}(x_{h2}^0(p)) > 0$, where $0 < \alpha_h < 1$ is the bargaining power of individual $h1$ in household h. The functions $x_{h1}^0(p)$ and $x_{h2}^0(p)$ denote consumer $h1$'s and $h2$'s individual demands at the price system p when they are singles.

Price-dependent outside options limit the feasible allocations in a household as members leave if they are better off as singles. However, as shown in Gersbach and Haller (2008) the first three results of Proposition 2 still hold if outside options are price-dependent. It is obvious that group externalities have the following further consequences. If individual $h1$ gains relatively more from living in household h, i.e. when v_1 increases, he receives less of the numéraire good. But the net effect on utility is positive. Hence, both individuals benefit. The same effects occur when v_2 increases.

6.2 Individually preferred goods revisited

Here we consider instances of individually preferred goods and pure group externalities. Like in Sect. 5.2, we assume a society with two goods and $n > 1$ identical two-member households $h = 1, \ldots, n$. Household members hm have a preference for good $m = 1, 2$ only so that

$$\mathrm{u}_{hm}(x^1_{hm}, x^2_{hm}) = U_{hm}(x^m_{hm}).$$

As before, we assume that all households have the same endowment $w_h = \overline{w} = (\overline{w}^1, \overline{w}^2) \gg 0$. With pure group externalities and price-dependent outside options, Proposition 3 and Corollary 1 still obtain, as an inspection of the proof reveals.

However, even though the qualitative effects of certain changes in bargaining power do not depend on the outside options, the quantitative effects can depend on the specific price-dependent reservation utilities V^o_{hm}. To accentuate the differential quantitative impact of a change in bargaining power under different outside options, we consider two specific cases with individually preferred goods and pure group externalities. First let us consider CASE 1 where as a single, an m^{th} household member is endowed with his or her preferred good only, specifically $w_{h1} = (\overline{w}^1, 0)$ and $w_{h2} = (0, \overline{w}^2)$. Then $V^o_{hm}(p) = U_{hm}(\overline{w}^m)$. Hence in this particular case, the price-dependent reservation utilities turn out to be constant. In CASE 2, when single each individual is endowed only with the good preferred by the other group, specifically $w_{h1} = (0, \overline{w}^2)$ and $w_{h2} = (\overline{w}^1, 0)$.

In both cases, Proposition 3 holds. Thus, in CASES 1 and 2 (and in other cases), the qualitative responses to particular changes in bargaining power are identical. However, the quantitative impact of a change of bargaining power can differ. For the sake of direct comparison, assume instances of CASE 1 and CASE 2 with the same primitive data except for the differences in individual endowments. Further assume that there exists an array of bargaining power parameters such that $\alpha_1 = \alpha_2 = \cdots = \alpha_n$ and at the corresponding equilibrium price system $\widehat{p}$ for CASE 2, $V^o_{hm}(\widehat{p}) = U_{hm}(\overline{w}^m)$ holds for all hm—which can be achieved with suitably chosen model parameters. Then for the particular array of bargaining power parameters, CASE 1 and CASE 2 have identical equilibrium outcomes.

Now take one of the two-member households $h = \{h1, h2\}$ and ceteris paribus consider an increase of α_h. Then in CASE 1 and in CASE 2, consumption of $h1$ increases and consumption of $h2$ decreases. But maintaining market clearing while $\hat{x}^1_{h1}$ increases requires that some other first household member $g1$ consumes less. By assumption, α_g has not changed. Hence, the relative price of good 1 must have risen. Consequently, in CASE 2, V^o_{h1} must have fallen (increasing the utility differential $U_{h1} - V^o_{h1}$) and V_{h2} must have risen (reducing $U_{h2} - V^o_{h2}$). In contrast, the reservation utilities are constant in CASE 1. This implies that the equilibrium consumption $\hat{x}^1_{h1}$ increases less and the equilibrium consumption $\hat{x}^2_{h2}$ decreases less in CASE 2 than in CASE 1. A fortiori, the impact of the change in α_h on all equilibrium values is less in CASE 2 than in CASE 1.

The foregoing argument reflects the fact that the left-hand side of the tangency condition $\mathrm{MRS}_h = p_1$ contains the price-dependent utility differentials $U_{h1} - V^o_{h1}$

and $U_{h2} - V^o_{h2}$. Namely, with price-dependent outside options and prevailing price system $p = (p_1, 1)$, the household's marginal rate of substitution assumes the form

$$\text{MRS}_h = \frac{\alpha_h \cdot U'_{h1}}{(1-\alpha_h) \cdot U'_{h2}} \cdot \frac{U_{h2} - V^o_{h2}(p_1)}{U_{h1} - V^o_{h1}(p_1)}.$$

7 Concluding remarks

The current analysis is confined to a general equilibrium model of a pure exchange economy with a fixed household structure and Nash-bargained household decisions for a select two-person household. General comparative statics as well as numerical examples lend support to the following conclusions. As a rule, a consumer benefits from more bargaining power at the expense of her fellow household member and the other consumer(s). However, in a closed economy, a shift of bargaining power within a significant number of two-person households may cause drastic price effects. As a consequence, both members of such a household may benefit from or both members may be harmed by a shift of internal bargaining power. In exceptional cases, it can happen that a household member is unaffected.

To reiterate, the current model assumes a fixed household structure and pure exchange. Removing any of these restrictions leads to a host of new important issues, which are left to future research.

Appendix

Proof of Proposition 1 Suppose that the price effect is negligible. For every $\alpha \in (0, 1)$, we can choose an $\epsilon(\alpha) > 0$ so that the local comparative statics prevail in the open neighborhood $N(\alpha) \equiv (\alpha - \epsilon(\alpha), \alpha + \epsilon(\alpha))$. Each set $C(\alpha) = N(\alpha) \cap [\alpha_*, \alpha^*]$ is relatively open in the interval $[\alpha_*, \alpha^*]$. The family $C(\alpha)$, $\alpha \in [\alpha_*, \alpha^*]$, is an open covering of the compact set $[\alpha_*, \alpha^*]$. It has a finite subcovering. Let us fix a minimal finite subcovering $C(\alpha_k)$, $k = 1, \ldots, K$. Without loss of generality, assume $\alpha_1 < \alpha_2 < \cdots < \alpha_K$. We claim that:

(A) If $\alpha_* < \alpha_1$, then $\alpha_* \in C(\alpha_1)$.

(B) If $\alpha_K < \alpha^*$, then $\alpha^* \in C(\alpha_K)$.

(C) For each $k \leq K - 1$, there exists β_k with $\alpha_k < \beta_k < \alpha_{k+1}$ and $\beta_k \in C(\alpha_k) \cap C(\alpha_{k+1})$.

To show (A), suppose it were false, i.e. $\alpha_* < \alpha_1$ and $\alpha_* \notin C(\alpha_1)$. Then there exists $k > 1$ with $\alpha_* \in C(\alpha_k)$ and, consequently, $C(\alpha_1) \subset C(\alpha_k)$, contradicting the minimality of the covering. Claims (B) and (C) are shown by similar reasoning.

Now fix $\beta_1, \ldots, \beta_{K-1}$ according to (C) and let us go from α_* to α^* taking small steps, namely

$$\begin{array}{ll} \text{from } \alpha_* \text{ to } \alpha_1, & \text{from } \alpha_1 \text{ to } \beta_1, \\ \text{from } \beta_1 \text{ to } \alpha_2, & \text{from } \alpha_2 \text{ to } \beta_2, \\ \quad \cdots \quad \cdots & \quad \cdots \quad \cdots \\ \text{from } \beta_{K-1} \text{ to } \alpha_K, & \text{and } \alpha_K \text{ to } \alpha^*. \end{array}$$

During each step, either the utilities remain unchanged or consumer 1's utility goes up and consumer 2's utility goes down. Hence the assertion. □

For convenient reference, we state an obvious auxiliary result before proceeding to the proof of Fact 2.

Lemma 1 *Let real numbers $\sigma, \tau, z > 0$ be given. The solution of the problem*

$$\max z_1^{\sigma} z_2^{\tau} \text{ s.t. } z_1 \geq 0, \quad z_2 \geq 0, \quad z_1 + z_2 = z$$

is $z_1 = \frac{\sigma}{\sigma+\tau} \cdot z$, $z_2 = \frac{\tau}{\sigma+\tau} \cdot z$, with value

$$\left(\frac{\sigma}{\sigma+\tau}\right)^{\sigma} \cdot \left(\frac{\tau}{\sigma+\tau}\right)^{\tau} \cdot z^{\sigma+\tau}.$$

Proof of Fact 2 Let $x = x_1 + x_2$ and $y = y_1 + y_2$ denote the total amounts purchased by household h. By Lemma 1, maximization of the Nash product $u_1^{\alpha} u_2^{1-\alpha}$ requires

$$\begin{aligned} x_1 &= \frac{\sigma}{\sigma+\tau} \cdot x, \quad x_2 = \frac{\tau}{\sigma+\tau} \cdot x, \\ y_1 &= \frac{\sigma^*}{\sigma^*+\tau^*} \cdot y, \quad y_2 = \frac{\tau^*}{\sigma^*+\tau^*} \cdot y \end{aligned}$$

where

$$\begin{aligned} \sigma &= \alpha\gamma_1, \quad \tau = (1-\alpha)\gamma_2, \\ \sigma^* &= \alpha(1-\gamma_1), \quad \tau^* = (1-\alpha)(1-\gamma_2). \end{aligned}$$

Moreover, at the maximum,

$$u_1^{\alpha} u_2^{1-\alpha} = \left(\frac{\sigma}{\sigma+\tau}\right)^{\sigma} \left(\frac{\tau}{\sigma+\tau}\right)^{\tau} \left(\frac{\sigma^*}{\sigma^*+\tau^*}\right)^{\sigma^*} \left(\frac{\tau^*}{\sigma^*+\tau^*}\right)^{\tau^*} x^{\delta} y^{1-\delta}$$

with

$$\begin{aligned} \delta &= \sigma + \tau = \alpha\gamma_1 + (1-\alpha)\gamma_2 = \gamma_2 + \alpha(\gamma_1 - \gamma_2); \\ 1 - \delta &= \sigma^* + \tau^* = \alpha(1-\gamma_1) + (1-\alpha)(1-\gamma_2) = 1 - \gamma_2 - \alpha(\gamma_1 - \gamma_2). \end{aligned}$$

Therefore, in equilibrium, the aggregate consumption for household h is $(x, y) = (\delta, \frac{1}{2})$. The associated individual shares are

$$\begin{aligned} x_1 &= \frac{\sigma}{\sigma+\tau} x &&= \sigma; \\ x_2 &= \frac{\tau}{\sigma+\tau} x &&= \tau; \\ y_1 &= \frac{\sigma^*}{\sigma^*+\tau^*} y &&= \frac{1}{2} \frac{\sigma^*}{\sigma^*+\tau^*}; \end{aligned}$$

$$y_2 = \frac{\tau^*}{\sigma^* + \tau^*} y = \frac{1}{2} \frac{\tau^*}{\sigma^* + \tau^*}.$$

As a function of α, consumer 1 achieves

$$\begin{aligned} u_1 &= \sigma^{\gamma_1} \left(\frac{1}{2}\right)^{1-\gamma_1} \left(\frac{\sigma^*}{\sigma^* + \tau^*}\right)^{1-\gamma_1} \\ &= \text{const}_1 \cdot \alpha^{\gamma_1} \left(\frac{\alpha}{1 - \gamma_2 - (\gamma_1 - \gamma_2)\alpha}\right)^{1-\gamma_1} \end{aligned}$$

which is increasing in α. Consumer 2 achieves

$$\begin{aligned} u_2 &= \tau^{\gamma_2} \left(\frac{1}{2}\right)^{1-\gamma_2} \left(\frac{\tau^*}{\sigma^* + \tau^*}\right)^{1-\gamma_2} \\ &= \text{const}_2 \cdot (1-\alpha)^{\gamma_2} \left(\frac{1-\alpha}{1 - \gamma_1 - (\gamma_2 - \gamma_1)(1-\alpha)}\right)^{1-\gamma_2} \end{aligned}$$

which is decreasing in α. Hence a shift of bargaining power from consumer 2 to consumer 1 benefits consumer 1 and harms consumer 2, who ends up consuming less of both commodities. □

Proof of Proposition 2 Good ℓ serves as a numéraire so that the price system assumes the form $(p_1, \ldots, p_{\ell-1}, 1)$. We consider the first-order conditions of maximizing S_h in household h:[11]

$$\begin{aligned} &\alpha_h \frac{1}{U_{h1}} \frac{\partial u_{h1}}{\partial x_{h1}^k} - \lambda_h p_k = 0, \quad k = 1, \ldots, \ell - 1; \\ &\alpha_h \frac{1}{U_{h1}} - \lambda_h = 0; \\ &(1 - \alpha_h) \frac{1}{U_{h2}} \cdot \frac{\partial u_{h2}}{\partial x_{h2}^k} - \lambda_h p_k = 0, \quad k = 1, \ldots, \ell - 1; \\ &(1 - \alpha_h) \frac{1}{U_{h2}} - \lambda_h = 0. \end{aligned}$$

Therefore

$$\lambda_h = \alpha_h \frac{1}{U_{h1}} = (1 - \alpha_h) \frac{1}{U_{h2}}. \tag{19}$$

$$\frac{\partial u_{h1}}{\partial x_{h1}^k} = \frac{\partial u_{h2}}{\partial x_{h2}^k} = p_k, \quad k = 1, \ldots, \ell - 1. \tag{20}$$

Because of differentiability and strict concavity, the demand of household h for commodities $k = 1, \ldots, \ell - 1$ is independent of the bargaining power α_h and $1 - \alpha_h$

[11] Note that our assumption of sufficient endowments with the numéraire good in all households allows us to work with the entire set of first-order conditions.

of individual $h1$ and $h2$, respectively. Hence, by the budget constraint and budget exhaustion also the aggregate household demand for commodity ℓ is independent of α_h. Therefore, the market clearing price system $(p_1, \ldots, p_{\ell-1}, 1)$ does not depend on internal bargaining power of households and, hence, changes of bargaining power in household h have no effect on other households. This establishes assertions (i) and (ii).

In contrast to all other goods, a shift of power in household h affects the distribution of the numéraire good in household h, as we shall establish next. Using the notation for equilibrium values we obtain from Eq. (19):

$$\frac{\alpha_h}{\hat{u}_{h1} + \hat{x}_{h1}^{\ell}} = \frac{1 - \alpha_h}{\hat{u}_{h2} + \hat{x}_{h2}^{\ell}} \tag{21}$$

Since $\hat{u}_{h1}$ and $\hat{u}_{h2}$ are independent of α_h and $\hat{x}_{h1}^{\ell} + \hat{x}_{h2}^{\ell}$ does not depend on α_h either, assertion (iii) follows.

Using again the fact that variations in α_h have no effect on aggregate excess demand, we conclude that if households are completely homogeneous with respect to U_{hi} and w_h, then a market equilibrium does not exhibit any positive net trades. Therefore, $\hat{x}_{h1}^{\ell} + \hat{x}_{h2}^{\ell} = \overline{w}_h^{\ell}$ and via Eq. (21), we obtain the expressions in (iv). □

Proof of Proposition 3 We normalize prices by $p_2 = 1$. Then the problem of household h is given by:

$$\max\{\ln S_h = \alpha_h \ \ln U_{h1}(x_{h1}^1) + (1 - \alpha_h) ln\, U_{h2}(x_{h2}^2)\}$$
$$s.t. \ \ x_{h1}^1 \, p_1 + x_{h2}^2 = w_h^1 \, p_1 + w_h^2$$

The first-order conditions amount to:

$$\alpha_h \frac{1}{U_{h1}(x_{h1}^1)} U_{h1}'(x_{h1}^1) - \lambda_h \, p_1 = 0;$$
$$(1 - \alpha_h) \frac{1}{U_{h2}(x_{h2}^2)} U_{h2}'(x_{h2}^2) - \lambda_h = 0.$$

Using the budget constraint and the first-order conditions yields

$$\alpha_h \frac{1}{U_{h1}(x_{h1}^1)} U_{h1}'(x_{h1}^1) - (1 - \alpha_h) \frac{U_{h2}'\left(w_h^1 \, p_1 + w_h^2 - x_{h1}^1 \, p_1\right)}{U_{h2}\left(w_h^1 \, p_1 + w_h^2 - x_{h1}^1 \, p_1\right)} \, p_1 = 0$$

or

$$\frac{\alpha_h}{1 - \alpha_h} F_1'(x_{h1}^1) = F_2'\left(w_h^1 \, p_1 + w_h^2 - x_{h1}^1 \, p_1\right) \cdot p_1 \tag{22}$$

where $F_1 \equiv \ln U_{h1}$ and $F_2 \equiv \ln U_{h2}$. F_1' and F_2' are strictly decreasing functions. Hence, for a given p_1, a higher (equal) α_h requires a higher (identical) consumption of good 1 to preserve (22). This shows (i) and (ii). □

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