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Changes of Separation Status During Registration and Scattering

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Abstract In our previous work, a new approach to the notorious problem of quantum measurement was proposed. Existing treatments of the problem were incorrect because they ignored the disturbance of measurement by identical particles and standard quantum mechanics had to be modified to obey the cluster separability principle. The key tool was the notion of separation status. Changes of separation status occur during preparations, registrations and scattering on macroscopic targets. Standard quantum mechanics does not provide any correct rules that would govern these changes. This gives us the possibility to add new rules to quantum mechanics that would satisfy the objectification requirement. The method of the present paper is to start from the standard unitary evolution and then introduce minimal corrections. Several representative examples of registration and particle scattering on macroscopic targets are analysed case by case in order to see their common features. The resulting general Rule of Separation Status Changes is stated in the Conclusion.

Keywords Problem of quantum measurement · Cluster separability · Reformed quantum mechanics · Detectors · Probability reproducibility · Objectification requirement

1 Introduction

In several recent papers [1–5], a new understanding of quantum mechanics is proposed, called The Reformed Quantum Mechanics. While the statistical character and non-locality remain unchanged and are considered as facts of life, the main thrust of the reform is aimed at the emergence of classical theories. This is hindered by the three notorious problems: those of realist interpretation, of classical properties

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and of quantum measurement. Objective properties of quantum systems are assumed there to be those that are uniquely defined by preparation rather than values of observables. Then, there are enough objective properties to view quantum systems as physical objects and quantum mechanics becomes as objective as any other physical theory [1, 4]. Classical properties are understood as specific objective properties of high-entropy states of macroscopic quantum systems (including Newtonian mechanics) and classical limit is a suitably taken high-entropy limit [2, 4]. The present paper is a continuation of our work on the measurement [3–5].

It is well known that the quantum theory of measurement is in an unsatisfactory state [6, 7]. In [3] a new idea is described: First, any quantum theory of measurement that disregards the disturbance of registration due to identical particles is proved to be wrong. Second, notions of a D-local observable and a separation status have been introduced and shown how they help to eliminate this disturbance. The reformed theory allows only those operators that satisfy a D-locality condition to be observables. Preparations must separate the quantum systems from the sea of identical particles giving them a non-trivial separation status characterised by some non-empty domain D of space. Third, registration of microsystems must use detectors, in which then the separation status becomes trivial ($D = \emptyset$) again. Let us emphasise that the change of separation status is an objective property of the composite system+apparatus, as it follows from our definition of objective properties in [1, 4].

The standard quantum mechanics turns out to be just a theory of isolated systems ($D = \mathbb{R}^3$) and subsystems of other isolated systems ($D = \emptyset$) allowing only two separation statuses. Hence, it must be supplied by a theory of general separation statuses and an additional rule governing changes of separation status. Finally, fourth, the freedom in the choice of the additional rules allows us to satisfy the existing observational evidence, in particular the objectification requirement. The rule describes an objective process inside a macroscopic detector including a change of kinematic description, a unitary evolution and a state reduction. For details, see [3–5].

The idea that standard quantum theory must be corrected because all standard observables are global while no registration apparatus can control the whole space has also been put forward by Wan and his collaborators [8] and [9], Sect. 2.12. Let us explain briefly what is the difference in the aims for which local observables are used in [3, 8]. Wan's first main idea is to consider measurement as a scattering process. The crucial property of most scattering states is that they become spatially separated from each other in the asymptotic region. Local observables have no correlation between such states. Then, some asymptotic states, even if pure, are equivalent, with respect to the set of all local observables, to mixed states. Wan's second main idea is to apply the corresponding superselection approach to classical observables and measurement problem. For a description of the superselection approach see [6].

Our first main idea is that local observables help to eliminate the disturbance of measurement due to identical particles. To keep the theory sufficiently general, we consider any measurement as a process inside a bounded region of space (e.g., a *finite* laboratory) and so avoid situating registrations in asymptotic regions. This leads to additional requirements on preparation, which must provide a non-trivial separation status for any registrable system and the only observables are then those local ones that are associated with this separation status. The second main idea is that new

dynamical rules governing changes of separation status must be added to standard quantum mechanics, and this provides a natural framework for a direct reduction of state.

An example of an additional rule governing a change of separation status has been described in [3]. In the present paper, we generalise the ideas of [3] and give a systematic theory of such rules. Our leading principles are 1 to preserve as much of standard quantum mechanics as possible, 2 to satisfy the objectification requirement and 3 to have a rule applicable to each process that contains changes of separation status in an unambiguous and observer-independent way.

The plan of the paper is as follows. The first two sections bring some material from the previous papers to make the present paper self-contained. In particular, Sect. 2 recapitulates the quantum model of measurement due to Beltrametti, Cassinelli and Lahti [10] and the first part of Sect. 3 summarises the existing theory of separation status. Section 3.2 introduces important technical tools for study of separation status changes, that of *separated systems* and that of *formal evolution*.

Using these tools, Sect. 4 extends the study of registration processes in [3], where a single registration of a one-particle system in a vector state by an arrangement of ideal detectors with fixed signals were dealt with. Registrations of systems in non-vector states, by detectors with flexible signals and release of particles from detectors are also considered. Section 4.1.4 brings generalisation of the theory to registrations of many-particle systems. It agrees with the correlations between different detectors due to the Hanbury Brown and Twiss effect [16] as well as with those in Einstein, Podolski and Rosen experiment. Section 4.2 discusses the case of non-ideal detectors and introduces the notion of approximate probability reproducibility. Particle tracks in cloud chambers are explained in Sect. 4.3 as multiple registrations. Formulas of Sect. 4 represent different cases of non-unitary evolution. Section 5 studies changes of separation status that occur during scattering on macroscopic systems. A formula that governs these cases is unitary and agrees with the predictions of standard quantum mechanics.

Finally, the account of different cases of separation status changes in Sects. 4 and 5 allows us to exclude some ideas and an analysis of what all the cases have in common suggests how a general rule could look like. The resulting Rule of Separation Status Change is stated and discussed in the Conclusion.

2 Beltrametti-Cassinelli-Lahti Model

In this section, we are going to recapitulate the well-known ideas on measurement that will be needed or criticised later. We describe a quite general measurement process, which we call Beltrametti-Cassinelli-Lahti (BCL) model [10].

Let a discrete observable O of system $\mathcal{S}$ with Hilbert space $\mathbf{H}$ be measured. Let o_n be the eigenvalues and $\{\phi_{nj}\} \subset \mathbf{H}$ be a complete orthonormal set of eigenvectors of O,

$$\mathsf{O}\phi_{nj} = o_n \phi_{nj}.$$

We assume that $n = 1, \ldots, N$ so that there is only a finite number of different eigenvalues o_n. This is justified by the fact that no real registration apparatus can distinguish all elements of an infinite set from each other. It can therefore measure only a function of a general observable that maps the spectrum onto a finite set of real numbers. Our observable O is such a function. The restriction to discrete observables is also valid for real measurements. The continuous eigenvalues can be grouped into small intervals and orthonormal bases can be chosen in the corresponding subspaces [6].

Let the registration apparatus[1] be a quantum system $\mathcal{A}$ with Hilbert space $\mathbf{H}_\mathcal{A}$ and an observable A. Let A be a non-degenerate, discrete observable with the same eigenvalues o_n and with orthonormal set of eigenvectors ψ_n,

$$\mathsf{A}\psi_n = o_n\psi_n,$$

with possible further eigenvectors and eigenvalues. A is the so-called *pointer observable* [6].

Let the measurement start with the preparation of $\mathcal{S}$ in state T and the independent preparation of $\mathcal{A}$ in state $\mathsf{T}_\mathcal{A}$. The initial state of the composite system $\mathcal{S}+\mathcal{A}$ is thus $\mathsf{T}\otimes\mathsf{T}_\mathcal{A}$.

Let $\mathcal{S}$ and $\mathcal{A}$ then interact for a finite time by the so-called *measurement coupling* and let the resulting state be $\mathsf{U}(\mathsf{T}\otimes\mathsf{T}_\mathcal{A})\mathsf{U}^\dagger$, where U is a unitary transformation on $\mathbf{H}\otimes\mathbf{H}_\mathcal{A}$. If $\mathcal{A}$ is to measure O, the probability of $\mathcal{A}$ being in state ψ_n after the interaction must be the same as the probability of eigenvalue o_n being registered on $\mathcal{S}$ as given by Born rule. This is called *probability reproducibility* [6].

The evolution of composite $\mathcal{S}+\mathcal{A}$ due to the measurement coupling can be generalised to be non-unitary by introducing some environment and allowing the system to be only approximately isolated [6], but this would not change the subsequent results in an important way.

Now, there is a theorem [10]:

Theorem 1 *Let a measurement fulfil all assumptions and conditions listed above. Then, for any initial vector state $\psi\in\mathbf{H}_\mathcal{A}$, there is a set $\{\varphi_{nl}\}\subset\mathbf{H}$ satisfying the orthogonality conditions*

$$\langle\varphi_{nl}|\varphi_{nj}\rangle = \delta_{lj} \tag{1}$$

such that U is a unitary extension of the map

$$\phi_{nl}\otimes\psi \mapsto \varphi_{nl}\otimes\psi_n. \tag{2}$$

An observational evidence of many years of quantum experimenting is that each individual measurement process leads to a definite result shown by the apparatus. More precisely, the apparatus must be in one of the states $|\psi_n\rangle\langle\psi_n|$ after each individual registration. This is called *objectification requirement* [6].

[1] In our language, a measurement consists of preparation and registration so that what Ref. [6] often calls "measurement" is our "registration".

Suppose that the initial state of $\mathcal{S}$ is an arbitrary vector state, $\mathsf{T} = |\phi\rangle\langle\phi|$. Decomposing ϕ into the eigenstates,

$$\phi = \sum_{nl} c_{nl}\phi_{nl}, \tag{3}$$

we obtain from (2) and the linearity of U:

$$\Phi_{\text{end}} = \mathsf{U}(\phi \otimes \psi) = \sum_{n} \sqrt{p_n}\varphi_n^1 \otimes \psi_n, \tag{4}$$

where

$$\varphi_n^1 = \frac{\sum_k c_{nk}\varphi_{nk}}{\sqrt{\langle \sum_l c_{nl}\varphi_{nl} | \sum_j c_{nj}\varphi_{nj} \rangle}} \tag{5}$$

and

$$p_n = \sum_l |c_{nl}|^2$$

is the probability that a registration of O performed on vector state ϕ gives value o_k.

Equation (2) implies that the probability of value o_n is p_n if $\mathsf{1} \otimes \mathsf{A}$ is registered on $\mathcal{S} + \mathcal{A}$ in final state (4). Hence, apparatus $\mathcal{A}$ measures observable O. However, this final state is a linear superposition of composite states containing different states ψ_n of the apparatus. This means that the apparatus is simultaneously in all states ψ_n for which coefficients p_n are non-zero. For more discussion, see [3, 6].

Hence, the objectification requirement is not satisfied and the prediction of the theoretical model does not agree with observation. This is called "problem of quantum measurement" or "problem of objectification", John Bell has called it the problem of "and" versus "or" and Schrödinger invented "Schrödinger cat" to visualise it in a provocative way. Von Neumann introduced changes into quantum mechanics, the so-called "first kind of dynamics", which was later called "the collapse of wave function", to obtain agreement with observation. A modified version of von Neumann's solution is adopted in our theory [3].

3 Separation Status

In [3], quantum mechanics is reformed so that the disturbance of measurements due to remote identical particles is avoided and the cluster separability principle is satisfied. Let us briefly recapitulate and further develop this reform. We shall work with Q-representation in this section.

3.1 Basic Definitions and Rules

First, a locality requirement on operators is introduced:

Definition 1 Let $D \subset \mathbb{R}^3$ be open. Operator with kernel $a(\vec{x};\vec{x}')$ is *D-local* if

$$\int d^3x'\, a(\vec{x};\vec{x}')f(\vec{x}') = \int d^3x\, a(\vec{x};\vec{x}')f(\vec{x}) = 0,$$

for any test function f vanishing in D.

An equivalent definition has been given in [8] and [9], Sect. 2.12. For the generalisation to composite system see [4]. All self-adjoint D-local operators of a system $\mathcal{S}$ form an algebra that will be denoted by $\mathbf{A}[\mathcal{S}]_D$.

The key notion of our theory is:

Definition 2 Let $\mathcal{S}$ be a particle[2] and $D \subset \mathbb{R}^3$ an open set such that:

- Registrations of any $\mathsf{A} \in \mathbf{A}[\mathcal{S}]_D$ lead to average $\langle\psi(\vec{x})|\mathsf{A}\psi(\vec{x})\rangle$ for all vector states $\psi(\vec{x})$ of $\mathcal{S}$.[3]
- $\mathcal{S}$ is prepared in state $\psi(\vec{x})$ that does not vanish in D.

Then $\mathcal{S}$ is said to have *separation status* D.

Generalisation to composite $\mathcal{S}$ and its non-vector states are given in [4]. As an example, consider D in which all wave functions of particles identical to $\mathcal{S}$ vanish. A separation status is called *trivial* if $D = \emptyset$.

We require next that any preparation of $\mathcal{S}$ must give it a non-trivial separation status. Then elements of $\mathbf{A}[\mathcal{S}]_D$ are observables of $\mathcal{S}$ and *only these are*. Standard quantum mechanics assumes that all self-adjoint operators on the Hilbert space of $\mathcal{S}$ are observables and can in principle be registered. This is different in the reformed quantum mechanics: only some subset of all self-adjoint operators contains observables and the subset even depends on preparation. Next, exceptions to the standard rule on composition of identical systems are described [5]:

Rule 1 Let $\mathcal{S}$ be prepared in state T and have separation status $D \neq \emptyset$. Then its observables are elements of $\mathbf{A}[\mathcal{S}]_D$ and its state is T independently of any remote system identical to $\mathcal{S}$ that may exist.

Composition of such states and observables satisfy

Rule 2 $\mathcal{S}_1$ and $\mathcal{S}_2$ prepared in states T_1 and T_2 with non-trivial separation statuses D_1 and D_2, $D_1 \cap D_2 = \emptyset$. Then $\mathcal{S}_1 + \mathcal{S}_2$ has state $\mathsf{T}_1 \otimes \mathsf{T}_2$ and its observables are elements of $\mathbf{A}[\mathcal{S}_1]_{D_1} \otimes \mathbf{A}[\mathcal{S}_2]_{D_2}$. This holds even if $\mathcal{S}_1$ and $\mathcal{S}_2$ have particles of the same kind in common.

[2] Particles have wave functions with three arguments, composite systems containing N particles those with $3N$ arguments.

[3] It seems that this can be generalised to approximate, $\pm\epsilon$ say, equality of the average to the expression $\langle\psi(\vec{x})|\mathsf{A}\psi(\vec{x})\rangle$, leading to generalised separation status denoted by (D,ϵ). The corresponding reformulation of the theory will be published in another paper.

For registrations, it is assumed:

Rule 3 Any registration apparatus for *microsystems* must contain at least one detector and every "reading of a pointer value" is a signal from a detector.

What constitutes a detector and its signal may be defined by detector classifications, such as [11, 12]. Rule 3 has many surprising consequences: e.g., a generalisation of separation status [3]. The most important consequence is that by entering the sensitive matter of a detector, a system is transferred from a non-trivial separation status into the trivial one. For more discussion, see [3, 4].

It follows that preparation and registration acquire an additional importance in the reformed quantum mechanics: they necessarily include changes of separation status.

To summarise: Standard quantum mechanics is just a theory of isolated systems and as such it is incomplete:

1. It admits only two separation statuses for any system $\mathcal{S}$:
   (a) $\mathcal{S}$ is isolated. Then $D = \mathbb{R}^3$ and all s.a. operators are observables.
   (b) $\mathcal{S}$ is a member of an isolated system containing particles identical to $\mathcal{S}$. Then $D = \emptyset$ and there are no observables for $\mathcal{S}$.
2. It ignores the existence of separation-status changes and the fact that such processes are objectively different from all other ones. Rules for changes of separation status that can be derived from standard quantum mechanics do not agree with observations in most cases.

This seems to give us an opportunity to introduce new rules that govern processes in which separation status changes. The conditions on such new rules are:

1. They do not contradict the rest of quantum mechanics. That is, all correct results of standard quantum mechanics remain valid.
2. They agree with, and explain, observational facts, in particularly the objectification requirement.
3. They can be applied to any change of separation status in an unambiguous and observer independent way.

A possible new rule for Beltrametti-Cassinelli-Lahti model modified by ionisation gas detectors was proposed in [3]. Points 1 and 2 were then satisfied but point 3 were not.

3.2 Some Technical Tools

This subsection introduces some technical tools to deal with separation status changes. Notions of separated systems and formal evolution will be introduced and their importance for analysis of separation status changes explained.

We start with separated systems. Let $\mathcal{S}$ and $\mathcal{S}'$ be systems with Hilbert spaces $\mathbf{H}$ and $\mathbf{H}'$. The composite $\mathcal{S} + \mathcal{S}'$ can be uniquely decomposed into subsystems,

$$\mathcal{S} + \mathcal{S}' = \mathcal{B}_1 + \cdots + \mathcal{B}_b + \mathcal{F}_1 + \cdots + \mathcal{F}_f,$$

so that $\mathcal{B}_n$ contains only bosons of the same kind for each $n = 1, \ldots, b$ and $\mathcal{F}_n$ contains only fermions of the same kind for each $n = 1, \ldots, f$. The map

$$\mathsf{P}_{as} : \mathbf{H} \otimes \mathbf{H}' \mapsto \mathbf{H}_{as}$$

is defined as the orthogonal projection onto $\mathbf{H}_{as}$, where $\mathbf{H}_{as}$ is the representation subspace of a permutation group representation on $\mathbf{H} \otimes \mathbf{H}'$, the elements of which are symmetric over each set $\mathcal{B}_n$ and antisymmetric over each set $\mathcal{F}_n$. Thus, P_{as} is linear and self-adjoint.

Example 1 Let $\mathcal{S}$ be a fermion particle and $\mathcal{S}'$ a composite of one fermion of the same kind as $\mathcal{S}$ and some particle of a different kind. Let $\phi(\vec{x}_1)$ be an element of $\mathbf{H}$ and $\phi'(\vec{x}_2, \vec{x}_3)$ that of $\mathbf{H}'$, $\vec{x}_2$ being the coordinate of the fermion. Then

$$\Psi(\vec{x}_1, \vec{x}_2, \vec{x}_3) = \mathsf{P}_{as}\big(\phi(\vec{x}_1)\phi'(\vec{x}_2, \vec{x}_3)\big) = \frac{1}{2}\big(\phi(\vec{x}_1)\phi'(\vec{x}_2, \vec{x}_3) - \phi(\vec{x}_2)\phi'(\vec{x}_1, \vec{x}_3)\big).$$

In general, P_{as} is non-invertible and does not preserve norm. Another map we shall need is the normalisation,

$$\mathsf{N} : \mathbf{H}_{as} \setminus \{0\} \mapsto \mathbf{H}_{as},$$

which is, in general, neither linear nor invertible. Its range is the unit sphere in $\mathbf{H}_{as}$.

We show that the maps are invertible in a special case of separation statuses.

Definition 3 Let composite $\mathcal{S} + \mathcal{S}'$ be prepared in state $\bar{\mathsf{T}}$. Let $\mathsf{T} = \mathrm{tr}_{\mathcal{S}'}[\bar{\mathsf{T}}]$ and $\mathsf{T}' = \mathrm{tr}_{\mathcal{S}}[\bar{\mathsf{T}}]$ be states of $\mathcal{S}$ and $\mathcal{S}'$ with separation statuses D and D', respectively, satisfying

$$D \cap D' = \emptyset. \tag{6}$$

Then $\mathcal{S}$ and $\mathcal{S}'$ are called separated.

We limit ourselves to the non-entangled case, $\bar{\mathsf{T}} = \mathsf{T} \otimes \mathsf{T}'$.
Consider first vector states ϕ and ϕ'. Let us define map J by

$$\mathsf{J} = \mathsf{N}|_{\mathbf{H}_{as}\setminus\{0\}} \circ \mathsf{P}_{as}, \tag{7}$$

and let

$$\Phi_{as} = \mathsf{P}_{as}(\phi \otimes \phi'), \qquad \Phi_{asn} = \mathsf{J}(\phi \otimes \phi').$$

If $\mathcal{S}$ and $\mathcal{S}'$ are separated, then ϕ and ϕ' satisfy:

$$\int d^3x_i\, f'(\vec{x}_i)\phi(\vec{x}_1, \ldots, \vec{x}_K) = 0$$

for any $i = 1, \ldots, K$ and for any test function f' with supp $f' \subset D'$, and

$$\int d^3x_i\, f(\vec{x}_i)\phi'(\vec{x}_1, \ldots, \vec{x}_L) = 0$$

for any $i = 1, \ldots, L$ and for any test function f with supp $f \subset D$.

Let f' be a test function such that $f' \in \mathbf{H}'$ with $\operatorname{supp} f \subset (D'\times)^L$, where $(D'\times)^L$ is an abbreviation for the Cartesian product of L factors D'. Let us define map $R[f', D'] : \mathbf{H}_{as} \mapsto \mathbf{H}$ by

$$\begin{aligned}(R[f', D']\Phi_{as})(\vec{x}_1, \ldots, \vec{x}_K) &= \int d^3x_{K+1} \ldots d^3x_{K+L} f'(\vec{x}_{K+1}, \ldots, \vec{x}_{K+L}) \\ &\quad \times \Phi_{as}(\vec{x}_1, \ldots, \vec{x}_K, \vec{x}_{K+1}, \ldots, \vec{x}_{K+L}),\end{aligned}$$

and similarly, for test function $f \in \mathbf{H}$ and $\operatorname{supp} f \subset (D\times)^K$, $R[f, D] : \mathbf{H}_{as} \mapsto \mathbf{H}'$ by

$$\begin{aligned}(R[f, D]\Phi_{as})(\vec{x}_{K+1}, \ldots, \vec{x}_{K+L}) &= \int d^3x_1 \ldots d^3x_K f(\vec{x}_1, \ldots, \vec{x}_K) \\ &\quad \times \Phi_{as}(\vec{x}_1, \ldots, \vec{x}_K, \vec{x}_{K+1}, \ldots, \vec{x}_{K+L}).\end{aligned}$$

Then, we obtain easily:

$$R[f', D']\Phi_{as} = N'_f \phi(\vec{x}_1, \ldots, \vec{x}_K),$$

where

$$N'_f = N_{as} \int d^3x_{K+1} \ldots d^3x_{K+L} f'(\vec{x}_{K+1}, \ldots, \vec{x}_{K+L}) \phi'(\vec{x}_{K+1}, \ldots, \vec{x}_{K+L}),$$

and N_{as} is the normalisation factor defined by P_{as}. N'_f is non-zero for at least some f'. Similarly,

$$R[f, D]\Phi_{as} = N_f \phi'(\vec{x}_{K+1}, \ldots, \vec{x}_{K+L}),$$

where

$$N_f = N_{as} \int d^3x_1 \ldots d^3x_K f(\vec{x}_1, \ldots, \vec{x}_K) \phi(\vec{x}_1, \ldots, \vec{x}_K).$$

Thus, we obtain both functions $\phi(\vec{x}_1, \ldots, \vec{x}_K)$ and $\phi'(\vec{x}_{K+1}, \ldots, \vec{x}_{K+L})$ up to normalisation. As the functions are normalised, they can be reconstructed. Analogous steps work for Φ_{asn}.

For the generalisation of these ideas to state operators, we shall need adjoints of operators $R[f', D']$ and $R[f, D]$. The definition of $R[f, D']^\dagger : \mathbf{H} \mapsto \mathbf{H}_{as}$ is

$$(R[f', D']^\dagger \phi, \Phi) = (\phi, R[f', D']\Phi)$$

and simple calculation yields

$$R[f', D']^\dagger \phi = \mathsf{P}_{as}(\phi \otimes f'^*).$$

Similarly,

$$R[f, D]^\dagger \phi' = \mathsf{P}_{as}(f^* \otimes \phi').$$

Map J can be generalised to tensor product of any two operators T of $\mathcal{S}$ and T′ of $\mathcal{S}'$. Map $\mathsf{T}\otimes\mathsf{T}'\mapsto \mathsf{P}_{as}(\mathsf{T}\otimes\mathsf{T}')\mathsf{P}_{as}$ is linear in both T and T′ and its result is an operator on $\mathbf{H}\otimes\mathbf{H}'$ that leaves $\mathbf{H}_{as}$ invariant. Operator $\mathsf{P}_{as}(\mathsf{T}\otimes\mathsf{T}')\mathsf{P}_{as}:\mathbf{H}_{as}\mapsto\mathbf{H}_{as}$ is self-adjoint and positive if T and T′ are state operators. Let $\{\psi_n\}$ be a basis $\mathbf{H}$ and $\{\psi'_\alpha\}$ that of $\mathbf{H}'$. We can write

$$\mathsf{T}=\sum_{mn}T_{mn}|\psi_m\rangle\langle\psi_n|,\qquad \mathsf{T}'=\sum_{\alpha\beta}T'_{\alpha\beta}|\psi'_\alpha\rangle\langle\psi'_\beta|.$$

Then

$$\mathsf{P}_{as}(\mathsf{T}\otimes\mathsf{T}')\mathsf{P}_{as}=\sum_{mn}\sum_{\alpha\beta}T_{mn}T'_{\alpha\beta}|\mathsf{P}_{as}(\psi_m\otimes\psi'_\alpha)\rangle\langle\mathsf{P}_{as}(\psi_n\otimes\psi'_\beta)|.$$

Operator $\mathsf{P}_{as}(\mathsf{T}\otimes\mathsf{T}')\mathsf{P}_{as}$ is not normalised even if T and T′ are state operators. Let us define

$$\mathsf{J}(\mathsf{T}\otimes\mathsf{T}')=\frac{\mathsf{P}_{as}(\mathsf{T}\otimes\mathsf{T}')\mathsf{P}_{as}}{\mathrm{tr}[\mathsf{P}_{as}(\mathsf{T}\otimes\mathsf{T}')\mathsf{P}_{as}]}.$$

Clearly, J maps states on states. Now, the above proof that vector states ϕ and ϕ' can be reconstructed from $\mathsf{J}(\phi\otimes\phi')$ can be easily extended to general states T and T′.

Moreover, for separated systems, the "individual" observables from $\mathbf{A}[\mathcal{S}]_D$ and $\mathbf{A}[\mathcal{S}']_{D'}$ can be recovered from operators on $\mathbf{H}_{as}$ that are, in turn, constructed from operators either of $\mathbf{A}[\mathcal{S}]_D$ or of $\mathbf{A}[\mathcal{S}']_{D'}$.

For instance, consider systems and states defined in the above Example and let $\mathsf{a}\in\mathbf{A}[\mathcal{S}]_D$. Then A constructed from a is an operator on $\mathbf{H}_{as}$ that is defined by its kernel

$$a(\vec{x}_1;\vec{x}'_1)\delta(\vec{x}_2-\vec{x}'_2)\delta(\vec{x}_3-\vec{x}'_3)+a(\vec{x}_2;\vec{x}'_2)\delta(\vec{x}_1-\vec{x}'_1)\delta(\vec{x}_3-\vec{x}'_3)$$

so that

$$(\mathsf{A}\Psi)(\vec{x}_1,\vec{x}_2,\vec{x}_3)=\frac{1}{2}\big((\mathsf{a}\phi)(\vec{x}_1)\phi'(\vec{x}_2,\vec{x}_3)-(\mathsf{a}\phi)(\vec{x}_2)\phi'(\vec{x}_1,\vec{x}_3)\big).$$

Then,

$$R[f',D'](\mathsf{A}\Psi)=N'_f(\mathsf{a}\phi)(\vec{x}_1),$$

where

$$N'_f=\frac{1}{2}\int d^3x_2d^3x_3\,f'(\vec{x}_2,\vec{x}_3)\phi'(\vec{x}_2,\vec{x}_3).$$

But $\phi(\vec{x}_1)$, $\phi'(\vec{x}_2,\vec{x}_3)$ and $f'(\vec{x}_2,\vec{x}_3)$ are known, hence, as ϕ is arbitrary, a is well-defined.

To summarise: for separated systems $\mathcal{S}$ and $\mathcal{S}'$, there are two equivalent descriptions: the *standard QM description* of $\mathcal{S}+\mathcal{S}'$ on the Hilbert space $\mathbf{H}_{as}$ and the *reformed QM description* on $\mathbf{H}\otimes\mathbf{H}'$ explained in Sect. 3.1.

Now, we come to the notion of formal evolution.

Definition 4 Let system $\mathcal{S}$ be initially ($t = t_1$) prepared in state T and simultaneously another quantum system $\mathcal{S}'$ in state T'. Let composite $\mathcal{S} + \mathcal{S}'$ be isolated and have a time-independent Hamiltonian defining a unitary group $\mathsf{U}(t - t_1)$ of evolution operators on $\mathbf{H}_{as}$. Then, the standard quantum mechanical evolution of $\mathcal{S} + \mathcal{S}'$,

$$\mathsf{T}_{\mathsf{J}}(t) = \mathsf{U}(t - t_1)\mathsf{J}(\mathsf{T} \otimes \mathsf{T}')\mathsf{U}(t - t_1)^{\dagger}, \tag{8}$$

is called *formal evolution* of two interacting systems $\mathcal{S}$ and $\mathcal{S}'$.

The choice of "initial state" $\mathsf{J}(\mathsf{T} \otimes \mathsf{T}')$ for the formal evolution clearly contradicts Rules 1 and 2. The change of kinematic description that accompanies change of separation status [3] has been arbitrarily shifted into the past. This is why the evolution is called "formal". The name is also justified by the fact that this evolution does not agree with observation in many cases of separation status change. In our reformed quantum mechanics, we have to define physical evolution in a different way. However, the formal evolution is our first step in the mathematical analysis of separation status changes. With its help, we can even recognise that a change of separation status has taken place. For example:

Let $\mathcal{S}$ and $\mathcal{S}'$ be two quantum systems, $\mathcal{S}$ containing K particles and $\mathcal{S}'$ containing L particles. Let the systems be prepared, at time t_1, in states T and T' with separation statuses D_1 and D', respectively, so that $D_1 \cap D' = \emptyset$. Let the formal evolution of the composite $\mathcal{S} + \mathcal{S}'$ for the initial state $\mathsf{T}_{\mathsf{J}}(t_1) = \mathsf{J}(\mathsf{T} \otimes \mathsf{T}')$ be described by its kernel in Q-representation:

$$T_{\mathsf{J}}(t)(\vec{x}_1, \ldots, \vec{x}_K, \vec{x}_{K+1}, \ldots, \vec{x}_{K+L}; \vec{x}'_1, \ldots, \vec{x}'_K, \vec{x}'_{K+1}, \ldots, \vec{x}'_{K+L}).$$

1. Suppose that, for some $t_2 > t_1$, supp $T_{\mathsf{J}}(t_2) = (D'\times)^{2(K+L)}$. Then we can say: at time t_2, the separation status of $\mathcal{S}$ is $\emptyset$, that of $\mathcal{S}'$ is D' and that of the composite $\mathcal{S} + \mathcal{S}'$ is also D' or, that $\mathcal{S}$ is *swallowed* by $\mathcal{S}'$.[4]
2. Suppose that, for some $t_3 > t_2$, there is a set D_3, $D_3 \cap D' = \emptyset$, such that the kernel $T_{\mathsf{J}}(t_3)$ has the properties:
   (a) For any test function $f' \in \mathbf{H}'$ and
   $$\text{supp}\, f' = (D'\times)^L, \qquad R[f', D']T_{\mathsf{J}}(t_3)R[f', D']^{\dagger} \neq 0,$$
   $\mathsf{N}(R[f', D']T_{\mathsf{J}}(t_3)R[f', D']^{\dagger})$ is a state of $\mathcal{S}$ independent of f'.
   (b) For any test function $f \in \mathbf{H}$ and
   $$\text{supp}\, f = (D_3\times)^K, \qquad R[f, D_3]T_{\mathsf{J}}(t_3)R[f, D_3]^{\dagger} \neq 0,$$
   $\mathsf{N}(R[f, D_3]T_{\mathsf{J}}(t_3)R[f, D_3]^{\dagger})$ is a state of $\mathcal{S}'$ independent of f.
   (c) For any test function $g \in \mathbf{H}$ and supp $g = (D_3\times)^K$, we have
   $$\mathsf{N}(R[f', D']T_{\mathsf{J}}(t_3)R[f', D']^{\dagger})|g\rangle = 0.$$

[4]This can easily be generalised to a more realistic condition, e.g., $\int_{(D'\times)^{K+L}} d^3x_1 \ldots d^3x_{K+L} \times T_{\mathsf{J}}(t_2)(\vec{x}_1, \ldots, \vec{x}_{K+L}; \vec{x}_1, \ldots, \vec{x}_{K+L}) \approx 1$.

(d) For any test function $g' \in \mathbf{H}'$ and $\operatorname{supp} g' = (D'\times)^L$, we have

$$\mathsf{N}(R[f, D_3]T_{\mathsf{J}}(t_3)R[f, D_3]^\dagger)|g'\rangle = 0.$$

Then we can say: the systems become separated again at time $t_3 > t_2$, system $\mathcal{S}$ being in state $\mathsf{N}(R[f', D']T_{\mathsf{J}}(t_3)R[f', D']^\dagger)$ with separation status D_3 and system $\mathcal{S}'$ in state $\mathsf{N}(R[f, D_3]T_{\mathsf{J}}(t_3)R[f, D_3]^\dagger)$ with separation status D'.

Although we can find the separation statuses of $\mathcal{S}$ and $\mathcal{S}'$ by studying the formal evolution of $\mathcal{S}+\mathcal{S}'$, we cannot claim that this is the physical evolution of the composite. Hence, the next question is how the formal evolution is to be corrected in the case that it leads to separation-status changes. This will be studied in the next section.

4 Reformed Models of Registration

We shall now analyse several cases of registration and try the minimal modifications of standard quantum mechanics so that the objectification requirement could still be satisfied. The modification starts by introducing detectors, the formal evolution and some phenomenological model assumptions analogous to BCL method.

4.1 Ideal Detectors

First, we simplify things by assumption that the detectors are ideal. For an ideal detector, the number of events registered by the detector equals the number of events impinging on it (*intrinsic efficiency* equal to 1). We also restrict ourselves to the sensitive matter of the detector, denote it by $\mathcal{A}$ and speak of it as of the detector. Initially, the registered system $\mathcal{S}$ and $\mathcal{A}$ are separated. We can, therefore, speak of initial states ϕ_{mk} of $\mathcal{S}$ as in Sect. 2 and T of $\mathcal{A}$, where T is assumed to be a stationary, high entropy state.

A *direct signal* of $\mathcal{A}$ is the macroscopic signal available from the sensitive matter of the detector. Its possible transformation into an electronic signal (such as for scintillation detectors) and further amplification by an electronic amplifier connected to the detector is not included in it. If we speak about detector signals, we always mean the direct ones.

Equations (2) has now to be replaced by the formal evolution of $\mathcal{S}+\mathcal{A}$ on $\mathsf{H}_{as} = \mathsf{P}_{as}(\mathsf{H}\otimes\mathsf{H}_{\mathcal{A}})$. Let us write a suitable initial state as follows:

$$\mathsf{T}_{\mathrm{init}}(\mathrm{c}) = \mathsf{J}\left(\sum_{kl} c_k c_l^* |\phi_{mk}\rangle\langle\phi_{ml}|\otimes\mathsf{T}\right) = \mathsf{N}\left(\sum_{kl} c_k c_l^* \mathsf{P}_{as}(|\phi_{mk}\rangle\langle\phi_{ml}|\otimes\mathsf{T})\mathsf{P}_{as}\right), \quad (9)$$

where c_k are components of a unit complex vector c. Its evolution by any unitary map U is

$$\mathsf{U}\mathsf{T}_{\mathrm{init}}(\mathrm{c})\mathsf{U}^\dagger = \mathsf{N}\left(\sum_{kl} c_k c_l^* \mathsf{U}\mathsf{P}_{as}(|\phi_{mk}\rangle\langle\phi_{ml}|\otimes\mathsf{T})\mathsf{P}_{as}\mathsf{U}^\dagger\right). \quad (10)$$

It is, therefore, sufficient to consider operators $\mathsf{P}_{as}(|\phi_{mk}\rangle\langle\phi_{ml}|\otimes\mathsf{T})\mathsf{P}_{as}$ and their evolution for different possible values of m, k, and l.

Let the formal evolution on H_{as} between the initial and an end state be given by unitary map U. It defines operators T'_{mkl} on H_{as}:

$$\mathsf{U}\mathsf{P}_{as}(|\phi_{mk}\rangle\langle\phi_{ml}|\otimes\mathsf{T})\mathsf{P}_{as}\mathsf{U}^{\dagger}=N\mathsf{T}'_{mkl} \tag{11}$$

where N is a normalisation constant due to map P_{as} not preserving norms. It is chosen so that

$$\mathrm{tr}[\mathsf{T}'_{mkl}]=\delta_{kl}.$$

Let us formulate our model assumptions in terms of operators T'_{mkl}.

A For any complex unit vector c, state $\sum_{kl}c_kc_l^*\mathsf{T}'_{mkl}$ includes a direct signal of the detector.

B For any pair of complex unit vectors c and c′, the states $\sum_{kl}c_kc_l^*\mathsf{T}'_{mkl}$ and $\sum_{kl}c'_kc'^*_l\mathsf{T}'_{mkl}$ are not macroscopically different. That is, the signal of the detector depends only on m so that the detector registers O.

C For any complex unit vector c, state $\sum_{kl}c_kc_l^*\mathsf{T}'_{mkl}$ describes system $\mathcal{S}$ being swallowed by $\mathcal{A}$, that is, the separation status of $\mathcal{S}$ changes. Hence, we cannot reproduce any particular state operator on $\mathbf{H}$ as an end state of $\mathcal{S}$ and on $\mathbf{H}_{\mathcal{A}}$ as an end state of $\mathcal{A}$. In general, it is not true that $\mathcal{S}$ and $\mathcal{A}$ are each in a well-defined state at the end.

If the formal evolution were applied to general initial state ϕ of $\mathcal{S}$ with decomposition (3) then the end state of the composite $\mathcal{S}+\mathcal{A}$ would contain linear superposition of different detector signals and the objectification requirement would be violated. We shall therefore try next to weaken the assumption of unitarity.

4.1.1 Flexible-Signal Detectors

Detectors can be divided in *fixed-signal* and *flexible-signal* ones. For a fixed-signal detector, the amplification erases differences of states $\sum_{kl}c_kc_l^*\mathsf{T}'_{mkl}$ so that the signal is independent not only of c but also of m. An example is a Geiger-Mueller counter. A flexible-signal detector, such as a proportional counter, gives different signals for different m.

The minimal change of the unitarity assumption results from the consequence of assumptions A and B that the formal evolution of initial states ϕ constructed from all eigenstates of $\mathcal{S}$ with one fixed eigenvalue,

$$\phi=\sum_k c_k\phi_{mk},$$

does not lead to violation of objectification requirement.

Let us call this part of formal evolution a *channel* or *m-th channel*. For a general initial state ϕ, decomposition (3) can be written as

$$\phi=\sum_m\sqrt{p_m}\,\frac{c_{mk}}{\sqrt{p_m}}\,\phi_{mk}$$

and $(\sqrt{p_m})^{-1}c_{mk}$ is a complex unit vector. Thus, ϕ is now a linear superposition of different channels and we have to put all channels together so that the result agrees with the objectification requirement. The unique possibility is:

$$\mathsf{T}_{flex} = \left(\sum_{m=1}^{N}\right) p_m \sum_{kl} \frac{c_{mk}c_{ml}^*}{p_m}\mathsf{T}'_{mkl}. \tag{12}$$

End state (12) has the form of a convex linear combination of states of the composite $\mathcal{S}+\mathcal{A}$, each of which includes only one detector signal, and the combination is the gemenge structure of the end state. In general, such an additional *reduction* of the end state to a non-trivial gemenge cannot be the result of a unitary evolution. The formal evolution defines the channels but remains valid only within each channel. Observe that this is sufficient to recognise whether the separation status has changed or not. Moreover, we can accept the validity of (11) and all properties A–C of operators T'_{mkl} as model assumptions without requiring full unitarity.

4.1.2 Fixed-Signal Detectors

This is the case considered in [3]. Let state ϕ of particle $\mathcal{S}$ be prepared with separation status D. Let $\mathcal{S}$ be manipulated by fields and screens in D so that beams corresponding to different eigenvalues of O become spatially separated.

Let the detector $\mathcal{A}$ be an array of N fixed-signal sub-detectors $\mathcal{A}^{(m)}$ prepared in initial states $\mathsf{T}^{(m)}$ with separation statuses $D^{(m)}$ where $D^{(n)}\cap D^{(m)}=\emptyset$ for all $n\neq m$ and $D^{(m)}\cap D=\emptyset$ for all m. We assume further that the sub-detectors are placed at the boundary of D in such a way that the beam corresponding to eigenvalue o_m will impinge on sub-detector $\mathcal{A}^{(m)}$ for each m. Each sub-detector $\mathcal{A}^{(m)}$ interacts with $\mathcal{S}$ as a whole and processes running in different sub-detectors do not influence each other.

It has been shown that every observable can in principle be registered by this kind of measurement (see [9], Sect. 3.6). The definition feature of it is that different eigenvalues of the observable are associated with disjoint regions of space and its registration can then be reduced to that of position. However, even if the objectification problem could be solved for such registrations, it still remains unsolved for other kinds of registration (such as that described in the previous section), which no doubts exist and exhibit the objectification effect.

In general, $\mathcal{S}$ hits all sub-detectors simultaneously because it is present in all beams simultaneously. However, $\mathcal{S}$ in initial state $\sum_{kl} c_k c_l^* |\phi_{mk}\rangle\langle\phi_{ml}|$ for any complex unit vector c interacts only with sub-detector $\mathcal{A}^{(m)}$. The formal evolution of $\mathcal{S}+\mathcal{A}^{(m)}$ can then be decomposed into

$$\mathsf{U}\mathsf{P}_{as}(|\phi_{mk}\rangle\langle\phi_{ml}|\otimes\mathsf{T}^{(m)})\mathsf{P}_{as}\mathsf{U}^{\dagger} = N^{-1}\mathsf{T}_{kl}^{(m)\prime}$$

and we adopt assumptions 1–3 for operators $\mathsf{T}_{kl}^{(m)\prime}$.

Again, we have to put all channels together in the correct way. The end state of $\mathcal{S}+\mathcal{A}$ for any initial state ϕ of $\mathcal{S}$ then is

$$\mathsf{T}_{\mathrm{fix}} = \left(\sum_{m=1}^{N}\right) p_m \sum_{kl} \frac{c_{mk}c_{ml}^*}{p_m}\mathsf{T}_{kl}^{(m)\prime} \otimes \prod_{r=1}^{N\setminus m} \otimes \mathsf{T}^{(r)}, \tag{13}$$

where $\prod_{r=1}^{N\setminus m}$ denotes the product of all terms except for that with $r = m$ and coefficients c_{mk} are defined by (3). Again, formula (13) represents a non-trivial reduction, where only the channels evolve unitarily.

4.1.3 Some Comments and Generalisations

To discuss (12) and (13), let us distinguish absorbing and non-absorbing detectors [3]. An absorbing detector does never release a particle which it detects, a non-absorbing one always releases it. We can consider only (13), which will be needed later, the other case is similar. If the detectors are absorbing, then state $\mathsf{T}_{\mathrm{fix}}$ evolves with $\mathcal{S}$ staying inside $\mathcal{A}$. $\mathcal{S}$ is not manipulable and can be considered as lost in the detector.

The case of non-absorbing detectors is more interesting. Extension of the formal evolution in each channel then leads to separation of the two systems at some later time. Further evolution of operator $\mathsf{P}_{as}(|\phi_{mk}\rangle\langle\phi_{ml}| \otimes \mathsf{T})\mathsf{P}_{as}$ depends on the Hamiltonian. The simplest imaginable end result is $\mathsf{P}_{as}(|\varphi_{mk}\rangle\langle\varphi_{ml}| \otimes \mathsf{T}^{(m)\prime\prime})\mathsf{P}_{as}$, where φ_{mk} is a state of a system identical to $\mathcal{S}$ with separation status D_m, $D_m \cap D^{(n)} = \emptyset$ for all m and n, and $\mathsf{T}^{(m)\prime\prime}$ is a state of $\mathcal{A}^{(m)}$ with separation status $D^{(m)}$. Thus, end state $\mathsf{T}_{\mathrm{release}}$ that can be reconstructed from the formal evolution is

$$\mathsf{T}_{\mathrm{release}} = \left(\sum_{m=1}^{N}\right) p_m \sum_{kl} \frac{c_{mk}c_{ml}^*}{p_m} |\varphi_{mk}\rangle\langle\varphi_{ml}| \otimes \mathsf{T}^{(m)\prime\prime} \otimes \prod_{r=1}^{N\setminus m} \otimes \mathsf{T}^{(r)}. \tag{14}$$

System $\mathcal{S}$ has a non-trivial separation status again so that the release in each channel can be understood as an instance of preparation for the composite $\mathcal{S} + \mathcal{A}^{(m)}$ and the whole evolution as a random mixture of these single preparations. The formula (14) preserves the reduction.

The new rules that have been proposed as yet always correct the unitary formal evolution determined by standard quantum mechanics by a reduction of the state operator. The reduced state occurs in the formulas as the so-called "end state". We assume that the time instant at which each end state formula is valid is the time at which the detector gives its macroscopic signal. No details of the time evolution to this end state is given. The end state itself as well as any time evolution to it cannot be derived from quantum mechanics but must simply be guessed and subjected to experimental checks. For the question of detailed time evolution in particular, one had first to find some observable aspects for it to show that the question does make sense.

An interesting case, which has some relevance to the end-time question and which is a hybrid of the registration by non-absorbing and absorbing detector is the Einstein, Podolski and Rosen (EPR) experiment [13]. We consider Bohm's form of it [14]. A spin-zero particle decays into two spin-1/2 ones, $\mathcal{S}_1$ and $\mathcal{S}_2$, that run in two opposite directions. The state of composite $\mathcal{S}_1 \otimes \mathcal{S}_2$ is then

$$\frac{1}{\sqrt{2}}(|1_+\rangle \otimes |2_-\rangle - |1_-\rangle \otimes |2_+\rangle), \tag{15}$$

where $|1_+\rangle$ is the spin-up state of $\mathcal{S}_1$, etc. Finally, the spin of $\mathcal{S}_1$ is registered after some time at which the particles $\mathcal{S}_1$ and $\mathcal{S}_2$ may be far away from each other. Let the

detector be a special case of fixed-signal one, as described in Sect. 4.1.2. Hence, there are two sub-detectors, $\mathcal{A}_1^{(+)}$ and $\mathcal{A}_1^{(-)}$ so that spin up of $\mathcal{S}_1$ is associated with a signal from $\mathcal{A}_1^{(+)}$ and spin down with that from $\mathcal{A}_1^{(-)}$. Let the state of $\mathcal{S}_1 + \mathcal{A}_1^{(+)}$ containing the signal be $\mathsf{T}_1^{(+)\prime}$ and that of $\mathcal{S}_1 + \mathcal{A}_1^{(-)}$ be $\mathsf{T}_1^{(-)\prime}$. Although $\mathcal{S}_1$ will be swallowed by the detector (see Sect. 4.2.1), the left particle may remain accessible to registration. Thus, our new rule is analogous to (14):

$$\mathsf{T}_{\mathrm{EPR}} = \frac{1}{2}|2_+\rangle\langle 2_+| \otimes \mathsf{T}_1^{(+)} \otimes \mathsf{T}_1^{(-)\prime} + \frac{1}{2}|2_-\rangle\langle 2_-| \otimes \mathsf{T}_1^{(+)\prime} \otimes \mathsf{T}_1^{(-)}, \tag{16}$$

where $\mathsf{T}_1^{(+)}$ and $\mathsf{T}_1^{(-)}$ are the non-excited states of the corresponding sub-detectors. The state reduction takes place at the time of the detector signal and has a *non-local* character. We don't see any paradox in it. The only problem comes with the generalisation to a relativistic theory: what is the correct simultaneity plane? This problem has been solved by Keyser and Stodolsky [15], see also the discussion in [4].

Formulas (12), (13) and (14) can readily be generalised to registration on a non-vector state (we avoid the use of the term "mixed state" because it has different meaning for different authors) S of $\mathcal{S}$. First, we have to decompose S into eigenstates of O,

$$\mathsf{S} = \sum_{nkml} S_{nkml}|\phi_{nk}\rangle\langle\phi_{ml}|; \tag{17}$$

the probability to register eigenvalue o_k on $\mathcal{S}$ is

$$p_m = \sum_k S_{mkmk}.$$

Finally, because of the linearity of U, everything we must do is to replace the expressions in formulas (12), (13) and (14) as follows:

$$\frac{c_{mk}c_{ml}^*}{p_m} \mapsto \frac{S_{mkml}}{p_m}. \tag{18}$$

Next, consider the case that the registered particle can miss the detectors and enter into environment. We can use formula (13) again by modelling the part of the environment that the particle must join if it misses the detector by one of the sub-detectors, $\mathcal{A}^{(N)}$, say.

This also explains the fact that Schrödinger cat is never observed in the linear superposition of life and death states. Indeed, in the case of Schrödinger cat, there is a radioactive substance releasing alpha-particles and a detector of alpha-particles, the signal of which leads to the death of the cat. Then, we can decompose the state of an alpha-particle into that of it being in the nucleus or of being released and missing the detector and that of hitting the detector, so that the above analysis is applicable.

4.1.4 Registration of Composite Systems

Formulas (12) and (13) were obtained for registrations of one-particle systems. This section will generalise them to many-particle ones. Composite systems can be classified into *bound* and *unbound*. Bound systems such as atoms and molecules can be

dealt with in an analogous way as particles. The only change is that map P_{as} is more complicated. Then, formulas (12) and (13) are valid for bounded composite systems. Unbounded composite systems are different. A system $\mathcal{S}$ that contains K particles can excite more detectors simultaneously, at most K detectors.

Generalisation to such systems is not completely straightforward because it must achieve, on the one hand, that there can be some non-trivial correlations between the signals from different detectors and, on the other, that the detectors are never in a linear superposition of their different signals, which in turn erases some correlations between different detectors. Of course, for one-particle systems, signals of different detectors are always anti-correlated in a trivial way. Non-trivial correlations that can emerge for unbounded many-particle systems are e.g. Hanbury-Brown-Twiss (HBT) ones [16] or Eistein-Podolski-Rosen (EPR) ones. Let us start with HBT effect.

In the original experiment by Hanbury Brown and Twiss, two photomultiplier tubes separated by about 6 m distance, were aimed at the star Sirius. An interference effect was observed between the two intensities, revealing a positive correlation between the two signals. Hanbury Brown and Twiss used the interference signal to determine the angular size of Sirius. The theory of the effect [17] studies a model in which the signal consists of two photons that impinge simultaneously on two detectors. Our strategy will be to construct a non-relativistic model of Hanbury Brown and Twiss effect following closely Fano's ideas [17] and try then to modify it similarly as the BCL model has been modified for the case of one-particle systems.

Let us limit ourselves to $\mathcal{S} = \mathcal{S}_1 + \mathcal{S}_2$ consisting of two bosons, $K = 2$, with Hilbert spaces $\mathbf{H}_1$ and $\mathbf{H}_2$. To simplify further, let the registered observable be $\mathsf{O}_1 + \mathsf{O}_2$, O_k having only two eigenvalues $+1$ and -1 and eigenvectors $|k_+\rangle$ and $|k_-\rangle$, $k = 1, 2$ satisfying

$$\mathsf{O}_k|k_+\rangle = +|k_+\rangle, \qquad \mathsf{O}_k|k_-\rangle = -|k_-\rangle.$$

Let, moreover, the one-particle Hilbert spaces be two-dimensional, i.e. vectors $|k_+\rangle$ and $|k_-\rangle$ form a basis of $\mathbf{H}_k$. Let the projections onto these states be denoted by P_{k+} and P_{k-} so that we have:

$$\mathsf{P}_{k+}\mathsf{P}_{k+} = \mathsf{P}_{k+}, \qquad \mathsf{P}_{k-}\mathsf{P}_{k-} = \mathsf{P}_{k-}, \qquad \mathsf{P}_{k+}\mathsf{P}_{k-} = 0. \tag{19}$$

The generalisation to more particles of arbitrary kinds, general observables and general Hilbert spaces is straightforward.

The Hilbert space $\mathbf{H}$ of the composite system has then basis $\{|++\rangle, |--\rangle, |+-\rangle\}$, where

$$\begin{aligned} |++\rangle &= |1_+\rangle|2_+\rangle, \\ |--\rangle &= |1_-\rangle|2_-\rangle, \\ |+-\rangle &= \frac{1}{\sqrt{2}}(|1_+\rangle|2_-\rangle + |1_-\rangle|2_+\rangle). \end{aligned}$$

It is the basis formed by eigenvectors of $\mathsf{O}_1 + \mathsf{O}_2$ with eigenvalues 2, -2 and 0, respectively. The corresponding projections are

$$\mathsf{P}_{++} = \mathsf{P}_{1+}\mathsf{P}_{2+},$$

$$\mathsf{P}_{--} = \mathsf{P}_{1-}\mathsf{P}_{2-},$$
$$\mathsf{P}_{+-} = \mathsf{P}_{1+}\mathsf{P}_{2-} + \mathsf{P}_{1-}\mathsf{P}_{2+}.$$

It follows from (19) that these are indeed projections.

To calculate the correlation in a state S of system $\mathcal{S}$ between the values ± 1 of any subsystem $\mathcal{S}_1$ or $\mathcal{S}_2$, which is intended to model the correlation measured by Hanbury Brown and Twiss, we need probability p_+ that eigenvalue $+1$ will be registered at least on one subsystem and similarly p_- for -1. These are given by

$$p_+ = \mathrm{tr}[\mathsf{S}(\mathsf{P}_{++} + \mathsf{P}_{+-})],$$
$$p_- = \mathrm{tr}[\mathsf{S}(\mathsf{P}_{--} + \mathsf{P}_{+-})],$$

respectively. If we define

$$\mathsf{P}_+ = \mathsf{P}_{++} + \mathsf{P}_{+-}, \qquad \mathsf{P}_- = \mathsf{P}_{--} + \mathsf{P}_{+-},$$

we have

$$\mathsf{P}_{+-} = \mathsf{P}_+\mathsf{P}_-.$$

The normalised correlation (see, e.g., [6], p. 50) is then given by

$$C(\mathsf{S}) = \frac{\mathrm{tr}[\mathsf{S}\mathsf{P}_+\mathsf{P}_-] - \mathrm{tr}[\mathsf{S}\mathsf{P}_+]\,\mathrm{tr}[\mathsf{S}\mathsf{P}_-]}{\sqrt{\mathrm{tr}[\mathsf{S}\mathsf{P}_+] - (\mathrm{tr}[\mathsf{S}\mathsf{P}_+])^2}\sqrt{\mathrm{tr}[\mathsf{S}\mathsf{P}_-] - (\mathrm{tr}[\mathsf{S}\mathsf{P}_-])^2}}. \tag{20}$$

For example, let $|\Phi\rangle$ be a general vector state in $\mathbf{H}$:

$$|\Phi\rangle = a|++\rangle + b|--\rangle + c|+-\rangle,$$

where a, b and c are complex numbers satisfying

$$|a|^2 + |b|^2 + |c|^2 = 1.$$

Then,

$$C(\Phi) = -\frac{|a|^2|b|^2}{\sqrt{(|a|^2 - |a|^4)(|b|^2 - |b|^4)}}.$$

The correlation lies, in general, between 0 and -1. The value -1 occurs for $c = 0$, means the strong anti-correlation and is the standard (trivial) case for one-particle systems.

Next, we construct a suitable detector. System $\mathcal{S}$ can be prepared in vector state $|\Phi\rangle$ with separation status D where then fields and screens split the beam B of single particles corresponding to $|\Phi\rangle$ into two beams, B_+ and B_-, each corresponding to an eigenvalue ± 1 of observable O_1 or O_2. Let detector $\mathcal{A}$ consist of two sub-detectors, $\mathcal{A}^{(+)}$ placed in the way of the beam B_+ and $\mathcal{A}^{(-)}$ placed in the way of B_- so that the signal of $\mathcal{A}^{(+)}$ registers eigenvalue $+1$ and that of $\mathcal{A}^{(+)}$ eigenvalue -1 on the registered particle similarly as in our model of fixed signal detector in Sect. 4.1.2. Let the Hilbert spaces of the sub-detectors be $\mathbf{H}_+$ and $\mathbf{H}_-$.

Let the sub-detectors be prepared in initial states $|\mathcal{A}^{(+)}0\rangle$ and $|\mathcal{A}^{(-)}0\rangle$ with separation statuses $D^{(+)}$ and $D^{(-)}$, $D^{(+)} \cap D^{(-)} = \emptyset$, $D \cap D^{(\pm)} = \emptyset$. After the interaction between $\mathcal{S}$ and $\mathcal{A}$, the following states are relevant: $|\mathcal{A}^{(+)}1\rangle \in \mathsf{P}_{as}(\mathbf{H}_1 \otimes \mathbf{H}_+)$, $|\mathcal{A}^{(-)}1\rangle \in \mathsf{P}_{as}(\mathbf{H}_1 \otimes \mathbf{H}_-)$, $|\mathcal{A}^{(+)}2\rangle \in \mathsf{P}_{as}(\mathbf{H}_2 \otimes \mathbf{H}_+)$, $|\mathcal{A}^{(-)}2\rangle \in \mathsf{P}_{as}(\mathbf{H}_2 \otimes \mathbf{H}_-)$, $|\mathcal{A}^{(+)}12\rangle \in \mathsf{P}_{as}(\mathbf{H}_1 \otimes \mathbf{H}_2 \otimes \mathbf{H}_+)$ and $|\mathcal{A}^{(-)}12\rangle \in \mathsf{P}_{as}(\mathbf{H}_1 \otimes \mathbf{H}_2 \otimes \mathbf{H}_-)$. These states describe one or two of the particles being swallowed by one of the sub-detectors, they are associated with changes of their separation status and include detector signals.

Finally, to register $\mathsf{O}_1 + \mathsf{O}_2$, the measurement coupling U must satisfy

$$\mathsf{U}\mathsf{P}_{as}(|++\rangle \otimes |\mathcal{A}^{(+)}0\rangle \otimes |\mathcal{A}^{(-)}0\rangle) = \mathsf{P}_{as}(|\mathcal{A}^{(+)}12\rangle \otimes |\mathcal{A}^{(-)}0\rangle), \tag{21}$$

$$\mathsf{U}\mathsf{P}_{as}(|--\rangle \otimes |\mathcal{A}^{(+)}0\rangle \otimes |\mathcal{A}^{(-)}0\rangle) = \mathsf{P}_{as}(|\mathcal{A}^{(+)}0\rangle \otimes |\mathcal{A}^{(-)}12\rangle), \tag{22}$$

$$\mathsf{U}\mathsf{P}_{as}(|+-\rangle \otimes |\mathcal{A}^{(+)}0\rangle \otimes |\mathcal{A}^{(-)}0\rangle) = \mathsf{P}_{as}(|\mathcal{A}^{(+)}1\rangle \otimes |\mathcal{A}^{(-)}2\rangle). \tag{23}$$

Observe that operator P_{as} also exchanges particles 1 and 2, which is a non-trivial operation on the right-hand side of (23).

Equations (21), (22) and (23) describe the formal evolution defining the three channels of the measurement. Each channel leads to the composite signal due to a registration of one copy of system $\mathcal{S}$. Thus, it can include signals of two detectors (23).

The formal evolution of state Φ would yield for the end state of the system $\mathcal{S} + \mathcal{A}$:

$$\begin{aligned}&\mathsf{U}\mathsf{J}(|\Phi\rangle \otimes |\mathcal{A}^{(+)}0\rangle \otimes |\mathcal{A}^{(-)}0\rangle)\\ &\quad = a\mathsf{J}(|\mathcal{A}^{(+)}12\rangle \otimes |\mathcal{A}^{(-)}0\rangle)\\ &\qquad + b\mathsf{J}(|\mathcal{A}^{(+)}0\rangle \otimes |\mathcal{A}^{(-)}12\rangle) + c\mathsf{J}(|\mathcal{A}^{(+)}1\rangle \otimes |\mathcal{A}^{(-)}2\rangle).\end{aligned} \tag{24}$$

According to our theory, this state must be reduced to a gemenge with component states, each of them corresponding to a single channel. Thus, the correct end state $\mathsf{T}_{\mathrm{comp}}$ of the whole system $\mathcal{S} + \mathcal{A}$ after the measurement process described above is

$$\begin{aligned}\mathsf{T}_{\mathrm{comp}} = {}& |a|^2\mathsf{J}(|\mathcal{A}^{(+)}12\rangle\langle\mathcal{A}^{(+)}12|) \otimes |\mathcal{A}^{(-)}0\rangle\langle\mathcal{A}^{(-)}0|\\ &(+)|b|^2|\mathcal{A}^{(+)}0\rangle\langle\mathcal{A}^{(+)}0| \otimes \mathsf{J}(|\mathcal{A}^{(-)}12\rangle\langle\mathcal{A}^{(-)}12|)\\ &(+)|c|^2\mathsf{J}(|\mathcal{A}^{(+)}1\rangle\langle\mathcal{A}^{(+)}1| \otimes |\mathcal{A}^{(-)}2\rangle\langle\mathcal{A}^{(-)}2|).\end{aligned} \tag{25}$$

We assume that formula (25) describes a special case of the registration of many-particle systems by many detectors and that it illustrates a method that can be used for more general cases. State $\mathsf{T}_{\mathrm{comp}}$ is an operator on $\mathbf{H} \otimes \mathbf{H}_+ \otimes \mathbf{H}_-$ and it is a convex combination of three states each on a different subspace of it. These three states are obtained by reconstruction from the corresponding results of formal evolution in accordance with the separation statuses. For example, the formal evolution gives for the first state

$$\mathsf{J}(|\mathcal{A}^{(+)}12\rangle\langle\mathcal{A}^{(+)}12| \otimes |\mathcal{A}^{(-)}0\rangle\langle\mathcal{A}^{(-)}0|),$$

but both particles are inside $\mathcal{A}^{(+)}$ and are, together with $\mathcal{A}^{(+)}$, separated from $\mathcal{A}^{(-)}$.

One can see that the pair of sub-detectors is in a well-defined signal state after each individual registration on $\mathcal{S}$ and, at the same time, the correlation contained in state $|\Phi\rangle$ that models the HTB correlation is preserved and can be read off the signals of the sub-detectors. This is of course due to the fact that HTB correlation is a function of the absolute values $|a|$, $|b|$ and $|c|$, none of which is erased by reduction of (24) to (25), while the extra correlations due to the linear superposition depend on mixed products such as ab^* etc.

A different but analogous case is the EPR experiment. The composite system of two fermions $\mathcal{S}_1$ and $\mathcal{S}_2$ is in initial state (15). The detector consists of four sub-detectors, $\mathcal{A}_1^{(+)}$, $\mathcal{A}_1^{(-)}$, $\mathcal{A}_2^{(+)}$ and $\mathcal{A}_2^{(-)}$, where the first pair interacts only with $\mathcal{S}_1$ and the second only with $\mathcal{S}_2$. The initial states of the sub-detectors are $\mathsf{T}_k^{(\pm)}$. The symbol $\mathsf{T}_k^{(\pm)\prime}$ denotes the state of system $\mathcal{A}_k^{(\pm)} + \mathcal{S}_k$ in which the sub-detector $\mathcal{A}_k^{(\pm)}$ swallows particle $\mathcal{S}_k$ and sends its signal. Procedure analogous to that leading to formula (25) will now give for the end state

$$\frac{1}{2}\mathsf{T}_1^{(+)} \otimes \mathsf{T}_1^{(-)\prime} \otimes \mathsf{T}_2^{(-)\prime} \otimes \mathsf{T}_2^{(+)\prime} (+) \frac{1}{2}\mathsf{T}_1^{(+)\prime} \otimes \mathsf{T}_1^{(-)\prime} \otimes \mathsf{T}_2^{(-)} \otimes \mathsf{T}_2^{(+)\prime}. \tag{26}$$

Again, EPR anti-correlation of the sub-detector signals is preserved even if the quadruple of the sub-detectors is always in a well-defined signal state at the end.

4.2 Non-ideal Detectors

Non-ideal detectors may be the natural and dominating case, from the experimental point of view. If a non-ideal detector $\mathcal{A}$ is hit by a system $\mathcal{S}$, there is only probability $0 < \eta < 1$, the intrinsic efficiency, that it will give a signal. From the theoretical point of view, they are important examples because our simple method of channels does not work for them.

We restrict ourselves to flexible-signal detectors with possible signals enumerated by $m = 1, \ldots, N$ and suppose that, in general, η_m depends on m. The other cases can be dealt with in an analogous way. Let again the separation status of $\mathcal{A}$ be $D_{\mathcal{A}}$. If $\mathcal{S}$ is prepared in an eigenstate of O with eigenvalue o_m which formally evolves to $\mathcal{S}$ being inside $D_{\mathcal{A}}$ with certainty, then the probability that $\mathcal{A}$ signals is η_m and not 1. Thus, the condition of probability reproducibility is not satisfied in this case. Instead, we introduce the notion of *approximate probability reproducibility*. Its meaning is that the detector does register eigenvalue o_m on $\mathcal{S}$ if it gives m-th signal, but we don't know anything, if it remains silent.

To construct a model of this situation, we must first modify (2) that expresses the idea of probability reproducibility into what expresses the approximate probability reproducibility (within standard quantum mechanics):

$$\mathsf{U}(\phi_{mk} \otimes \psi) = C_m^1 \varphi_{mk} \otimes \psi_m^1 + C_m^0 \phi'_{mk} \otimes \psi_m^0, \tag{27}$$

where ϕ'_{mk} is a suitable time evolution of ϕ_{mk} into $D_{\mathcal{A}}$ and φ_{mk} are states of $\mathcal{S}$, ψ is the initial, ψ_m^1 the signal and ψ_m^0 a no-signal states of $\mathcal{A}$. These states satisfy orthogonality relations

$$\langle\psi|\psi_m^1\rangle = 0, \qquad \langle\psi_m^1|\psi_n^1\rangle = \delta_{mn}, \qquad \langle\psi_m^0|\psi_n^1\rangle = 0,$$
$$\langle\varphi_{mk}|\varphi_{ml}\rangle = \langle\phi'_{mk}|\phi'_{ml}\rangle = \delta_{kl}.$$

The coefficients C_m^1 and C_m^0 are related by

$$|C_m^1|^2 + |C_m^0|^2 = 1, \qquad |C_m^1|^2 = \eta_m.$$

Measurement coupling U commutes with P_{as} because the Hamiltonian leaves $\mathbf{H}_{as}$ invariant and with N because it is a unitary map. We can, therefore, replace (27) by the corresponding formal evolution:

$$\mathsf{UJ}(\phi_{mk}\otimes\psi) = C_m^1\mathsf{J}(\varphi_{mk}\otimes\psi_m^1) + C_m^0\mathsf{J}(\phi'_{mk}\otimes\psi_m^0). \tag{28}$$

This is not a channel because it is not the formal evolution of an initial state into an end state with a single detector signal. Indeed, no signal is also a macroscopically discernible detector state. We have to return to the formal evolution that starts with general state ϕ of $\mathcal{S}$:

$$\begin{aligned}
&\mathsf{UJ}(|\phi\rangle\langle\phi|\otimes|\psi\rangle\langle\psi|)\mathsf{U}^\dagger\\
&\quad= \sum_{mn}\sum_{kl} c_{mk}c_{nl}^*\Big(C_m^1C_n^{1*}|\mathsf{J}(\varphi_{mk}\otimes\psi_m^1)\rangle\langle\mathsf{J}(\varphi_{nl}\otimes\psi_n^1)|\\
&\qquad + C_m^1C_n^{0*}|\mathsf{J}(\varphi_{mk}\otimes\psi_m^1)\rangle\langle\mathsf{J}(\phi'_{nl}\otimes\psi_n^0)| + C_m^0C_n^{1*}|\mathsf{J}(\phi'_{mk}\otimes\psi_m^0)\rangle\langle\mathsf{J}(\varphi_{nl}\otimes\psi_n')|\\
&\qquad + C_m^0C_n^{0*}|\mathsf{J}(\phi'_{mk}\otimes\psi_m^0)\rangle\langle\mathsf{J}(\phi'_{nl}\otimes\psi_n^0)|\Big).
\end{aligned} \tag{29}$$

To obtain a correct end state of a non-ideal detector, we have to discard the cross-terms between ψ_m^1 and ψ_n^1 and between ψ_m^1 and ψ_n^0. This is a general method that works also in the case that there are channels. The result is

$$\begin{aligned}
\mathsf{T}_{\text{nonid1}} = &\left(\sum_{m=1}^N\right) p_m\eta_m\sum_{kl}\frac{c_{mk}c_{ml}^*}{p_m}|\mathsf{J}(\varphi_{mk}\otimes\psi_m^1)\rangle\langle\mathsf{J}(\varphi_{ml}\otimes\psi_m^1)|\\
&(+)\sum_{mn}\sum_{kl} c_{mk}c_{nl}^*C_m^0C_n^{0*}|\mathsf{J}(\phi'_{mk}\otimes\psi_m^0)\rangle\langle\mathsf{J}(\phi'_{nl}\otimes\psi_n^0)|.
\end{aligned} \tag{30}$$

This is not yet a practical formula because the detector is always in a state with high entropy, which is not a vector state. Hence, the initial state is $|\phi\rangle\langle\phi|\otimes\mathsf{T}$, and the end state is

$$\mathsf{T}_{\text{nonid2}} = \left(\sum_{m=1}^N\right) p_m\eta_m\sum_{kl}\frac{c_{mk}c_{ml}^*}{p_m}\mathsf{T}_{mkl}^1(+)\sum_{mn}\sum_{kl} c_{mk}c_{nl}^*\mathsf{T}_{mnkl}^0, \tag{31}$$

where we have made the replacements

$$|\mathsf{J}(\varphi_{mk}\otimes\psi_m^1)\rangle\langle\mathsf{J}(\varphi_{ml}\otimes\psi_m^1)| \mapsto \mathsf{T}_{mkl}^1$$

and

$$C_m^0 C_n^{0*} |\mathsf{J}(\phi'_{mk} \otimes \psi_m^0)\rangle\langle \mathsf{J}(\phi'_{nl} \otimes \psi_n^0)| \mapsto \mathsf{T}^0_{mnkl}.$$

Operators T^1_{mkl} and T^0_{mnkl} are determined by the initial state and the formal evolution and satisfy the conditions:

A′ $$\mathrm{tr}[\mathsf{T}^1_{mkl}] = \delta_{kl}, \qquad \mathrm{tr}[\mathsf{T}^0_{mnkl}] = (1-\eta_m)\delta_{mn}\delta_{kl}.$$

B′ For any unit complex vector with components c_k,

$$\sum_{kl} c_k c_l^* \mathsf{T}^1_{mkl}$$

is a state operator on $\mathbf{H}_{as}$ and the state includes direct m-th signal from the detector.

C′ For any unit complex vector with components c_{mk} (for all m and k)

$$\left(\sum_m p_m(1-\eta_m)\right)^{-1} \sum_{mn}\sum_{kl} c_{mk} c_{nl}^* \mathsf{T}^0_{mnkl}$$

is a state operator on $\mathbf{H}_{as}$ and the state includes no detector signal from the detector.

4.3 Particle Tracks in Detectors

Particle tracks in a Wilson chamber look suspiciously similar to classical trajectories and have been an interesting problem for quantum mechanics since the end of 1920's. There is the classical paper by Mott [18] (see also [19]), which shows by applying Schrödinger equation that there is an overwhelming probability of getting a second scattering event very close to the ray pointing away from the decay centre through the location of a first scattering event. A more rigorous calculation is given in [20], which uses the same idea for a one-dimensional model. The initial situation is spherically symmetric and the interaction between the alpha-particle and the detector also is. Thus, the resulting state must also be spherically symmetric and not just one radial track. A consequence of the linearity of Schrödinger equation then is that the end state is a linear superposition of all possible radial tracks. A way to save one single radial track is the state reduction at least for the first ionisation, which is apparently assumed tacitly. This separation of state reduction and unitary evolution does not exactly correspond to what is going on because we have in fact a chain of state reductions with a unitary evolution in between.

In this section, we apply our theory to the problem, but we simplify it by assuming, instead of the spherical symmetry, that the particle momentum has a large average value $\langle \vec{p} \rangle$ and the detector has the plane symmetry with the plane being perpendicular to $\langle \vec{p} \rangle$.

The registration model studied in Sect. 4.1.2 can be characterised as a single transversal layer of detectors: each beam is registered once. What we now have can be viewed as an arrangement of many transversal detector layers: one beam passes

through all layers successively causing a multiple registration. Examples of such arrangements are cloud chambers or MWPC telescopes for particle tracking [11]. The latter is a stack of the so-called multiwire proportional chambers (MWPC) so that the resulting system of electronic signals contains the information about a particle track. Here, we restrict ourselves to cloud chambers, but the generalisation needed to describe MWPC telescopes does not seem difficult.

Then, a model of a Wilson chamber is a system of sub-detectors $\mathcal{A}^{(nk)}$, where n distinguishes different transversal layers and k different sub-detectors in each such layer. Let the space occupied by $\mathcal{A}^{(nk)}$ be $D^{(nk)}$ and let it be at the same time its separation status. We shall assume that $D^{(nk)}$ are small cubes with edge d that is approximately equal to the diameter of the resulting clouds in the Wilson chamber. We denote the n-th layer by $\mathcal{A}^{(n)}$ so that $\mathcal{A}^{(n)} = \bigcup_{k=1}^{N} \mathcal{A}^{(nk)}$. To simplify the subsequent analysis, we assume that coordinates can be chosen in a neighbourhood of $\mathcal{A}^{(n)}$ so that each $D^{(nk)}$ in the neighbourhood can be described by

$$x^1 \in (u_k^1, u_k^1 + d), \qquad x^2 \in (u_k^2, u_k^2 + d), \qquad x^3 \in (u_n^3, u_n^3 + d).$$

The observable $\mathsf{O}^{(n)}$ that is registered by each layer $\mathcal{A}^{(n)}$ is equivalent to the position within the cubes. The eigenfunctions and eigenvalues are

$$\mathsf{O}^{(n)} \phi^{(nk)}_{l_1 l_2 l_3}(\vec{x}) = k \phi^{(nk)}_{l_1 l_2 l_3}(\vec{x}),$$

where $\{l_1, l_2, l_3\}$ is a triple of integers that replaces the degeneration index l,

$$\phi^{(nk)}_{l_1 l_2 l_3}(\vec{x}) = d^{-3/2} \exp\left(\frac{2\pi l_1 i}{d}(x^1 - u_k^1) + \frac{2\pi l_2 i}{d}(x^2 - u_k^2) + \frac{2\pi l_3 i}{d}(x^3 - u_n^3) \right)$$

for $\vec{x} \in D^{(nk)}$ and $\phi^{(nk)}_{l_1 l_2 l_3}(\vec{x}) = 0$ elsewhere.

The state S_n of $\mathcal{S}$ impinging on $\mathcal{A}^{(n)}$ can be defined as the state $\mathcal{S}$ would have after being released by the layer $\mathcal{A}^{(n-1)}$. The interaction of $\mathcal{S}$ with $\mathcal{A}^{(n)}$ can then be described by formula (14) with replacement (18). The decomposition (17) must, of course, use functions $\phi^{(nk)}_{l_1 l_2 l_3}$ instead of $\phi^{(n)}_k$ and the support of $\varphi^{(nk)}_{l_1 l_2 l_3}$ is $D^{(nk)}$. The procedure can be repeated for all n.

The first layer "chooses" one particular $\varphi^{(1k)}_{l_1 l_2 l_3}$ with the support $D^{(1k)}$ in each individual act of registration even in the case that the state arriving at it is a plane wave. Hence, the "choice" in the next layer is already strongly limited. In this way, a straight particle track of width d results during each individual multiple registration. Formally, of course, the resulting state of $\mathcal{S}$ is a gemenge of all such straight tracks, which would have the plane symmetry if the original wave arriving at the detector stack were a plane wave.

5 Changes of Separation Status in Scattering Processes

It is the existence of separation-status change that allows us to choose the gemenge form, such as (12), of the end states so that the theory agrees with the observational fact of objectification. However, separation-status changes can also occur in processes

that have nothing to do with registrations. Must there be any reduction to gemenge form in such processes, too?

To study this question, let us restrict ourselves to a scattering of a microsystem by a macroscopic target and observe that there can then be separation status changes, one when the system enters the target and other when it is released. First, let us consider no-entanglement processes such as the scattering of electrons on a crystal of graphite with a resulting interference pattern [21] or the splitting of a laser beam by a down-conversion process in a crystal of $KNbO_3$ (see, e.g., Ref. [22]). No-entanglement processes can be described by the following model. Let the initial state of the target $\mathcal{A}$ be T with separation status $D_{\mathcal{A}}$ and that of the microsystem $\mathcal{S}$ be ϕ with separation status D_1, $D_1 \cap D_{\mathcal{A}} = \emptyset$. Let there be two subsequent changes of separation status of $\mathcal{S}$: first, it is swallowed by $\mathcal{A}$ in $D_{\mathcal{A}}$ and, second, it is released by $\mathcal{A}$ in state φ with separation status D_2, $D_2 \cap D_{\mathcal{A}} = \emptyset$. We assume that the end state of the target, T', is independent of ϕ and that we have a unitary evolution:

$$|\phi\rangle\langle\phi| \otimes \mathsf{T} \mapsto |\varphi\rangle\langle\varphi| \otimes \mathsf{T}',$$

which can be reconstructed from the formal evolution because the systems are separated initially and finally. The two systems are not entangled by their interaction, hence there is no necessity to divide the resulting correlations between $\mathcal{S}$ and $\mathcal{A}$ in what survives and what is erased. The end state is in fact of the form (14): it has a trivial gemenge structure.

Another example of this situation is a particle prepared in a cavity D with imperfect vacuum. We can model this situation in the above way and so in effect suppose that the particle has separation status D.

A more interesting case is an *entanglement scattering* during which two subsequent changes of separation status of the scattered particle also occur. The scattering of neutrons on spin waves in ferromagnets or ionising an atom of an ideal gas in a vessel are examples. Let microsystem $\mathcal{S}$ in initial state ϕ with separation status D be scattered by a macrosystem $\mathcal{A}$ in initial state T with separation status $D_{\mathcal{A}}$, $D \cap D_{\mathcal{A}} = \emptyset$. For simplicity, we assume that the formal evolution leads to $\operatorname{supp}\phi \subset D_{\mathcal{A}}$ at some time t_{scatt}. Then, t_{scatt} is not uniquely determined but the subsequent calculations are valid for any possible choice of it. A more general situation can be dealt with by the method applied in the case of a microsystem that can miss a detector.

The experimental arrangement determines two Hilbert spaces $\mathbf{H}$ and $\mathbf{H}_{\mathcal{A}}$ and unitary map

$$\mathsf{U} : \mathbf{H} \otimes \mathbf{H}_{\mathcal{A}} \mapsto \mathbf{H} \otimes \mathbf{H}_{\mathcal{A}} \tag{32}$$

describing the interaction according to standard quantum mechanics.

The experimental arrangement studied in the previous section also determined a basis $\{\phi_{mk}\}$ of $\mathbf{H}$, namely the eigenvectors of registered observable O as well as sets of states $\{\varphi_{mk}\}$ in $\mathbf{H}$ and $\{\psi_m\}$ of $\mathbf{H}_{\mathcal{A}}$. This together with the assumption that $\mathcal{A}$ measures O (with exact or approximate probability reproducibility) restricted the possible U. These particular properties enabled us to choose a unique gemenge structure for the end state. The question is how any gemenge form of the end result can be even formally well-defined for processes described by (32), where the physical situation does not determine any such special sets of states.

To be able to give an account of the situation, let us first introduce the formal evolution U_f on $\mathsf{P}_{as}(\mathbf{H} \otimes \mathbf{H}_{\mathcal{A}})$, from which U can be reconstructed. Second, we decompose map U_f into two steps, $\mathsf{U}_f = \mathsf{U}_{f2} \circ \mathsf{U}_{f1}$, where U_{f1} develops up to t_{scatt} and U_{f2} further from t_{scatt}.

Then, the correct intermediate state $\mathsf{T}_{\text{interm}}$ at t_{scatt} is

$$\mathsf{T}_{\text{interm}} = \mathsf{N}(\mathsf{U}_{f1}\mathsf{P}_{as}(|\phi\rangle\langle\phi| \otimes \mathsf{T})\mathsf{P}_{as}\mathsf{U}_{f1}).$$

Indeed, there is no macroscopic signal from $\mathcal{A}$, only some microscopic degrees of freedom of $\mathcal{A}$ change due to the interaction U_{f1}. The overwhelming part of the degrees of freedom of $\mathcal{A}$ remains intact and just serve as a background of the process. Thus, even if there is a separation status change, there is no necessity for reduction: one can say that there is only one channel.

Further evolution is given by U_{f2} supplemented by reconstruction of the states in $\mathbf{H}$ and $\mathbf{H}_{\mathcal{A}}$ as $\mathcal{S}$ is released by $\mathcal{A}$, and we simply obtain: the formula

$$\mathsf{T}_{\text{end}} = \mathsf{U}(|\phi\rangle\langle\phi| \otimes \mathsf{T}_0)\mathsf{U}^{\dagger} \tag{33}$$

of standard quantum mechanics remains valid. Formula (33) makes clear that a separation status change need not cause any reduction.

6 Conclusion

Standard quantum mechanics does not contain rules governing changes of separation status. We have utilised this opportunity to construct the missing rule so that it satisfies the objectification requirement.

Section 3.2 has introduced a new technical tool, the formal evolution, that enables us to study changes of separation status in detail. In rigorous terms, it describes the modification of kinematics due to separation status change on the one hand and the role of Schrödinger equation in the process of separation status change on the other.

Sections 4 and 5 have discussed all possible kinds of experiments in which a change of separation status occurs. A case by case analysis trying to take into account the idiosyncrasy of each experiment and to isolate the relevant features of its results has lead to formulas (12), (13), (25), (26), (31) and (33). The real purpose of the analysis however was to find a general rule so that each of the formulas would be a special case of it. And indeed, now it is almost obvious how the rule must read:

The Rule of Separation Status Change Let microscopic system $\mathcal{S}$ be prepared in state $\mathsf{T}_{\mathcal{S}}$ with separation status $D_{\mathcal{S}}$ and macroscopic systems $\mathcal{A}$ in state $\mathsf{T}_{\mathcal{A}}$ with separation status $D_{\mathcal{A}}$, where $D_{\mathcal{S}} \cap D_{\mathcal{A}} = \emptyset$. The initial state is then $\mathsf{T}_{\mathcal{S}} \otimes \mathsf{T}_{\mathcal{A}}$ according to Rule 2. Let the formal evolution (defined in Sect. 3.2) describing the interaction between $\mathcal{S}$ and $\mathcal{A}$ lead to separation status change of $\mathcal{S}$. If there are any macroscopic direct signals (defined in Sect. 4.1) from $\mathcal{A}$, then the state of the composite $\mathcal{S} + \mathcal{A}$ given by the formal evolution must be corrected by state reduction to the gemenge structure (defined in [3])

$$\mathsf{T}_{\text{end}} = \left(\sum_m\right) p_m \mathsf{T}'_m, \tag{34}$$

where each state T'_m includes only one (possibly composite) direct signal from the whole detector. States T'_m of $\mathcal{S}+\mathcal{A}$ are determined by the formal evolution. The state $\mathsf{T}_{\mathrm{end}}$ refers then to any time after the signals.

Hence, the evolution during a separation status change brings three changes: first, the change of kinematic description $\mathsf{T}_{\mathcal{S}} \otimes \mathsf{T}_{\mathcal{A}} \mapsto \mathsf{J}(\mathsf{T}_{\mathcal{S}} \otimes \mathsf{T}_{\mathcal{A}})$, second, the standard unitary evolution of state $\mathsf{J}(\mathsf{T}_{\mathcal{S}} \otimes \mathsf{T}_{\mathcal{A}})$, and third, the state reduction of the evolved state into (34). Afterwards, the state evolves unitarily with a possible change of kinematics if $\mathcal{S}$ and $\mathcal{A}$ become separated again. Its form (the gemenge structure) is then uniquely determined by detector signals. It is interesting to observe that the signals result in a process of relaxation, in which the sensitive matter of the detector approach its thermal equilibrium. This seems to be in accord with our theory of classical states in [2].

A tenet adopted for the search of the Rule has been that corrections to standard quantum mechanics ought to be the smallest possible changes required just by the experiments. The Rule is of course guessed and not derived and could yet be falsified in confrontation with further observational evidence concerning different changes of separation status. It could also be further extended, e.g., to describe how the postulated end states evolved in more detail. However, for such an evolution, there does not seem to exist as yet any experimental evidence to lead us. Let us emphasise that the clean decomposition of a separation status change into three steps, viz. change of kinematics, unitary evolution and state reduction, is just a method enabling a mathematically well defined application of the Rule, but it is definitely not a description of the time dependence of the real process.

Finally, we observe that The Reformed Quantum Mechanics returns to von Neumann's "two kinds of dynamics" (see also [3]) but that its notion of state reduction differs from von Neumann's in two points. First, it is less ad hoc because it is justified by the argument of separation status change, which is logically independent from the proper quantum measurement problem, and second, it is more specific because it happens only in a detector and its form is determined by objective processes inside the detector sensitive matter.

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