Speckle-Visibility Spectroscopy of Depolarized Dynamic Light Scattering

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Dynamic Depolarized Light Scattering and Particle Systems

DDLS has been used to characterize rod- and ellipsoid-like particles, cellulose whiskers, carbon nanotubes, liquid crystals, particles supporting localized surface plasmon resonances, rotationally asymmetric and Janus particles, aggregates, agglomerates and clusters resulting from particle self-assembly. A partially or entirely crystalline internal structure also breaks the continuous central symmetry (rotational invariance) and brings about intrinsic optical anisotropy that can take over the role of aspect ratio. Therefore, depolarized scattering was used to follow the growth of lamellar grains in a block copolymer, and identify the onset of crystallization of biological macromolecules. A new situation where DDLS also performs better than competing techniques is found in nearly all nanotoxicological assays, and concerns particles dispersed in biological and physiological media, for testing cellular responses.

The importance of anisotropy and depolarized scattering however goes beyond particle sizing. Anisotropy offers the possibility of constructing nano- and meso-structures via self-assembly with an unprecedented variety, and particles with novel surface properties. The so-called patchy particles feature interactions whose range and directions are controlled, and they provide the ability to place and orientate themselves into ordered and hierarchical structures by spontaneously assembling elementary blocks from solution, which is a potentially straightforward and low-cost approach. Shape anisotropy also exhibits unexpected and applicable dynamic behavior in soft-confinement, and the enhanced sensitivity of DDLS towards Brownian dynamics is found to be useful in exploring inter-particle interactions, the influence of confinement, and the viscoelastic properties of soft matrices, such as gels.
Accuracy and Precision of the Coefficient of Variation

The term “accuracy” refers to the closeness of an estimate to the true value (or population value), and the term “precision” refers to the degree of agreement in a series of estimates.

Figure SI1. a) A graph showing the plot of the sampling distribution of the coefficient of variation, illustrating the terms “accuracy” and “precision”. The smaller the bias between the true value and sample mean, the smaller the range of accuracy. The narrower the distribution of the sample estimate, the smaller the range of precision.

We tested the validity of Equation 9 against independent computational experiments. We randomly drew s elements from a population of normally distributed values ($\delta = \sigma/\mu$) and computed the sample coefficient of variation ($\delta_s = \sigma_s/\mu_s$). The details of generating pseudo-random numbers is given elsewhere. To construct sufficiently accurate distributions at each value of s we tested, we repeated each draw $10^3$ times. Figure SI2 shows a sample of Gaussian variables ($s = 10^3$) and its estimated PDF.
Figure SI2 a) Plot showing a sample containing 1000 elements, and b) its probability density function (PDF) estimated via counting. The dashed line is the unadjusted Gaussian curve corresponding to $\mu$ and $\sigma$.

Figure SI 3 shows the results of the computational experiments and the curves given independently by Equation 5.

Figure SI3. The PDFs of the sample coefficient of variation ($\delta_s$) estimated from computational experiments at different sample sizes ($s = 3, 10, 30, 50, 100, and 1000$). The dashed lines are the theoretical predictions given by Equation 5.

The agreement between the two data sets was excellent, and therefore, we were confident in applying it further to estimate the expectable accuracy and precisions as a
function of the sample size. The accuracy and precision intimately relate to the sample mean and variance (Figure SI1), which are defined as:

\[
\text{Mean } \delta_s = \int_0^1 \delta_s p(\delta, s; \delta_s) \, d\delta_s
\]

(1-SI)

\[
\text{Var } \delta_s = \int_0^1 (\delta_s - \text{Mean } \delta_s)^2 p(\delta, s; \delta_s) \, d\delta_s.
\]

(2-SI)

After evaluating Equation 1 and 2, we obtained that

\[
\text{Mean } \delta_s \approx \sqrt{1 - \frac{1}{2s}} \delta
\]

(3-SI)

\[
\text{Var } \delta_s \approx \frac{1}{2s} \delta^2.
\]

(4-SI)

As the sample size increases, the width of the sample PDFs becomes narrower, while the mean value display a less critical dependence.

**Figure SI4.** a) The comparison between theory and computational experiments. The expected sample mean and b) sample variance as a function of the sample size. The solid line are the theoretical curves given by Equation 3-SI and 4-SI, respectively. The
insets show the results on an extended range. c) The expected ranges of accuracy and d) precision of the sample mean as a function of the sample size. The insets show the results on the early narrow range. The solid lines are the theoretical curves given by Equation 5-SI and 6-SI, respectively.

Figure SI4a and b display the results of computational experiments in which the sample mean and sample variance were determined as a function the sample size. Finally, we calculated the ranges of accuracy and precision of determining the sample mean (Figure SI4c and d)

\[
\text{Accuracy } \delta_s \equiv \frac{\text{Mean } \delta_s - \delta}{\delta} \approx \sqrt{1 - \frac{1}{2s}} - 1
\]

\[
\text{Precision } \delta_s \equiv \frac{\sqrt{\text{Var } \delta_s}}{\text{Mean } \delta_s} \equiv \frac{1}{\sqrt{2s-1}}
\]

The agreement between theory and the computational experiment was excellent.

**Figure S15.** The relationship between bias and probability at different ranges of accuracy as a function of the size of one single sample. a) Graph showing the probability as a function sample size at given range of expected accuracy. b) Graph showing the expectable accuracy with a given probability as a function of the sample size.
There are however alternative ways to express the expected outcome of randomness. Relying on Equation 5, we can determine the probability (P) that the bias of the mean of one single sample of s elements is within e.g. 10%, that is, the range of accuracy is within 10%. This means that the difference given by \( \Delta \delta = \delta_s - \delta \) falls within the interval of \( \delta \pm 0.1 \cdot \delta \), and \( |\Delta \delta| \leq 0.1 \cdot \delta \). We estimate this probability by

\[
P(\delta, \omega) = \int_{\delta - \omega \delta}^{\delta + \omega \delta} p(\delta; \delta_s) \, d\delta_s,
\]

where \( \omega \) defines the percentage, for example, \( \omega = 0.1 \) corresponds to 10% accuracy.

**Multiple Scattering**

For larger particles with a relatively high scattering contrast (for silica the refractive index at 660 nm is \( \sim 1.45 \)), multiple scattering may limit the system to very low concentrations. When multiple scattering is significant, the correlation function exhibits a faster decay and shows the features of increased polydispersity. An effective solution of this problem is to suppress multiple scattering in dynamic light scattering experiments using cross-correlation schemes.\(^{69-71}\) This technique is available to us—but only for addressing conventional DLS where one can only probe the polarized component of the scattered light. Figure S3 shows PCS spectra recorded for two samples (SiO\(_2\)-B and D particles). If multiple scattering is significant, we expect that the intensity auto-correlation function recorded by the standard scheme (PCS) decays faster than the one recorded in the 3D-cross correlation setup (3D-Cross PCS) using two beams intersecting one another inside the sample. If multiple scattering is negligible, we expect that there is no practical difference between the decay of the PCS spectra. These results confirms that contributions from multiple scattering was negligible.
Figure S3. Conventional DLS intensity auto-correlation functions recorded in standard scheme (PCS) and 3D-cross correlation scheme (3D-Cross PCS).69

The Influence of Sedimentation of the Particles

Given that the scattering plane and the vector of gravity were perpendicular to each other—as it is the case for most light scattering instrument—, the settling velocity does not shift the phase of the scattered light.72 Therefore, the dynamics of sedimentation—during e.g. 300s, the distance a particle of 400 nm cover is approx. 40 μm, assuming a mass density of 2.65 g/cm³—is not present in the fluctuations of the speckle pattern, and accordingly, it does not affect either the correlation function or the photon count statistics. If particles were highly polydisperse, settling could fractionate the sample during a long measurement, and the ensemble of the particles we observed in a given scattering volume could significantly change.
References


