# The B.E. Journal of Economic Analysis & Policy

## Contributions

Volume 9, Issue 1	2009	Article 53
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## Technological Diversity and Cost Uncertainty

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#### **Recommended Citation**

Frank C. Krysiak (2009) "Technological Diversity and Cost Uncertainty," *The B.E. Journal of Economic Analysis & Policy*: Vol. 9: Iss. 1 (Contributions), Article 53.

## Technological Diversity and Cost Uncertainty\*

Frank C. Krysiak

#### Abstract

In many industries, different technologies are used simultaneously for the production of a homogeneous good. Such diversification is socially beneficial, because it reduces the transmission of factor price volatility, like oil-price shocks, to consumer prices. Therefore, many countries have implemented policies aimed at increasing technological diversification. The question is whether such policies are necessary. We use a two-stage investment model to address this question in the setting of perfect competition and of a monopoly. We show that factor price uncertainty leads to diversification, if capital is not too expensive, and that this diversification is due to each firm investing in a diversified technology portfolio. An important implication of this form of diversification is that technological diversity is socially optimal, even in the case of a monopoly. Thus policy intervention is unnecessary and might even be detrimental.

**KEYWORDS:** technological diversity, uncertainty, policy intervention, investment, electricity industry, monopoly

<sup>\*</sup>I thank the editor and two anonymous reviewers for their helpful comments on an earlier version of this paper.

## **1** Introduction

In many industries, different production technologies are used simultaneously, although these technologies incur substantially differing costs. Sometimes, this diversity is not only due to different firms using different technologies but also to individual firms using several technologies simultaneously.

An important example is electricity generation, where technologies based on different fuels, like lignite, hard coal, or natural gas, are used for the production of a homogeneous good and where individual producers often operate different types of power plants. For example, installed capacity in Germany in 2005 consisted of about 17% natural gas, 22% hard coal, 16% lignite, 4% oil and 17% nuclear energy (Kjärstad and Johnsson, 2007). In the UK, the figures are 37% natural gas, 35% coal, 7% oil and 14% nuclear energy (Kjärstad and Johnsson, 2007).<sup>1</sup> This diversity exists despite substantial differences in the expected costs induced by these technologies, see OECD (2005).<sup>2</sup> To a considerable extent it is the result of a simultaneous use of different technologies by the same firm. The data in Kjärstad and Johnsson (2007) shows that the firms in the electricity industry in the UK and in Germany have invested in a diverse portfolio of technologies in each of the last three decades, and currently planned power stations differ with regard to the technology they use (Kjärstad and Johnsson, 2007).

This type of diversification is also observable in steel production, where large producers operate different types of steel mills (using coal, gas, or electricity as primary energy source). Another example is logistic services, where large companies in Europe typically use several technologies simultaneously (e.g., trucks and railways).

Such technological diversification can be socially desirable. It reduces the transmission of factor price volatility, like oil-price shocks, to consumer prices and thereby increases social welfare. Several studies have shown that the costs of resource price volatility can be substantial and that diversification may help to reduce these costs, see, for example, Ferderer (1996), Awerbuch and Sauter (2006), and Awerbuch (2006). Indeed, many countries pursue policies aimed at increasing technological diversity. Such policies are especially prevalent in the energy sector, where subsidies, tax exemptions, and compensation programs are used to increase the diversity of utilized energy sources. The question is whether such governmental

<sup>&</sup>lt;sup>1</sup>In both countries, the remaining capacity used renewable energy sources.

<sup>&</sup>lt;sup>2</sup> According to the OECD (2005) data for Germany, the total costs of a MWh electricity (calculated for a 10% discount rate) range between 38 and 59 USD for a coal-fired plant, are approximately 50 USD for a gas-fired power plant, and 42 USD for a nuclear plant. Thereby fuel costs account for 30-44% of total costs in the case of coal, for 76% in the case of gas, and for 11% in the case of a nuclear power plant.

intervention is necessary or whether markets provide sufficient incentives to achieve an optimal technological diversity.

Firm level incentives for diversification have been analyzed in a range of studies. Mills (1986) and Lippman et al. (1991) show that demand uncertainty combined with either imperfect competition or capacity constraints can induce firms to choose different technologies. Mills and Smith (1996) prove that strategic interaction among producers can also result in firm heterogeneity. Elberfeld and Nti (2004) show that technological diversity might stem from uncertainty with regard to the costs of new technologies. These studies focus on technological diversity that is due to different firms using different technologies. The case where individual firms hold diversified technology portfolios is not addressed,<sup>3</sup> although it is important in sectors that are primary targets of diversification policies, such as the electricity sector.

Another line of literature uses a mean-variance approach to characterize optimal technological diversification from the social or the firm perspective, see, for example, Awerbuch (2000), Awerbuch and Berger (2003), and Huang and Wu (2008). These studies inquire which mix of technologies leads to the smallest risk while yielding a given expected return. Awerbuch (2000) evaluates the US generation mix with a portfolio approach based on fuel cost uncertainty. Awerbuch and Berger (2003) extend this approach by covering other types of uncertainty and apply it to electricity generation in the EU. Huang and Wu (2008) use a similar approach for Taiwan and calculate the optimal portfolio for a given level of risk aversion.

In this paper, we combine aspects of both types of literature. As in the first line of literature, we use the model of a profit-maximizing firm that has to make an irreversible investment decision under uncertainty. But we consider cost instead of demand uncertainty and analyze technological diversification instead of firm heterogeneity. Our main argument is similar to the second line of literature; diversification is a response to factor price uncertainty. In contrast to this literature, we start with a microeconomic model of profit maximization. That firms choose between risk and return is not an assumption but a result of the time-structure of the investment and production decisions. Indeed, the argument is not that firms are risk-averse and thus balance expected returns against the volatility of returns. Rather, diversification provides firms with higher flexibility to adjust their production to factor price changes. They balance the gain in expected profit resulting from this flexibility against the higher investment costs.

<sup>&</sup>lt;sup>3</sup>Going back to Newbery and Stiglitz (1981), there is literature that covers cost uncertainty and diversification within firms. But it assumes full flexibility in factor employment and is therefore only distantly connected to the investment problem considered here.



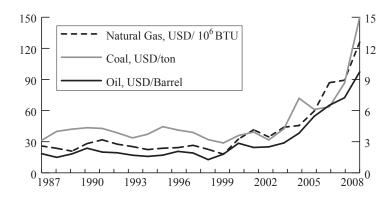


Figure 1: Prices for oil (spot price, Brent, left axis), coal (NW Europe, left axis), and natural gas (EU, right axis), Source: BP (2009).

The cost shocks that are the main driver of diversification in our setup are observable in many industries. For the above example of electricity generation, Figure 1 shows that the prices of the most important fossil fuels have been highly volatile over the last decades. Given that these fuels are specific to particular energy conversion technologies, there is technology-specific uncertainty in the electricity sector.<sup>4</sup> Figure 1 also indicates that the price deviations from the long-term trend are in a range where they could potentially alter the preferability of different types of power plants, according to the data in OECD (2005).<sup>5</sup>

To analyze the diversification incentives resulting from such cost shocks, we use a two-stage model with technology-specific cost uncertainty. We show that cost uncertainty results in technological diversification whenever capital is not too expensive. This diversification is not due to firm heterogeneity but a result of each firm holding a diversified technology portfolio. Furthermore, technological diversity is socially optimal both for perfect competition and for a monopolistic supplier, the latter being a rather surprising result. Consequently, at least in these cases, governmental intervention is unnecessary for assuring a socially optimal diversity. The reason is that diversification is solely a reaction to cost shocks in our setting. A firm that possesses market power will produce less and will thus invest less in total. But it reduces its investment in all technologies proportionally, which leaves the shares of each technology (and thus diversification) unchanged.

This result stands in contrast to those of Mills and Smith (1996), who show that profit maximizing behavior will usually not lead to a socially optimal technological

<sup>&</sup>lt;sup>4</sup>As is apparent from Figure 1, the price movements are correlated. But as long as there is no perfect correlation, diversification can be optimal, as we will show below.

<sup>&</sup>lt;sup>5</sup>See Footnote 2.

diversity. This highlights that the type of uncertainty under consideration (cost or demand uncertainty) and the type of diversification (diversified individual technology portfolios versus firm heterogeneity), is of central importance to the question of whether governments need to intervene to assure optimal diversity.

In the following section, we advance our model and characterize the social optimum. In Section 3, we analyze optimal firm behavior under perfect competition as well as in a monopoly and compare this to the social optimum. In Section 4, we consider a special case to provide more detail concerning the conditions under which diversification occurs. Section 5 concludes the article.

## 2 The Model and Benchmark Cases

We use a two-stage investment model with cost uncertainty to describe a firm's investment in different technologies. We first set up this model. Then we characterize the social optimum as well as firm behavior under certainty to gain two benchmarks to which we compare our later results.

#### 2.1 The Model

We consider a setup with a fixed number of firms (n) that produce a homogeneous good. All firms have the same technological opportunities and have access to the same information. In stage one of our setup, each firm invests in one or several technologies. Investment in a technology is characterized by the amount of production equipment (based on this technology) acquired by the firm. This investment is irreversible and technology-specific, that is, production equipment using one technology cannot be transferred to another technology (a coal-fired power plant cannot be converted to a nuclear power plant). In stage two, the firms decide upon total production and, if they have invested in different technologies, upon the allocation of total production to these technologies. This stage can consist of several independent sub-periods with differing realizations of factor prices and thus of production costs. We will consider variants of the model that differ with regard to the competition on the output market. In all variants, there is perfect competition on all factor markets.

With regard to the informational structure, we assume that, in stage two, all firms have full information concerning current production costs of all technologies. Thus, total production and the allocation of production to the technologies is chosen under perfect information. But, in the investment stage, there is technology-specific cost uncertainty; for example, due to uncertain future prices of technology-specific factors. Given the long planning and construction periods in many industries, espe-

cially in the electricity sector, and the factor price volatility discussed in the introduction, this seems to be a reasonable setup.

To keep the model tractable, we consider only two technologies. Technology s (s = 1, 2) is characterized by a cost function  $\tilde{C}_s(q_{s,i}, k_{s,i}, \eta_s)$  that describes the variable production costs and that depends on the amount produced by firm i with this technology ( $q_{s,i} \in \mathbb{R}_+$ ), the amount of capital invested in this technology ( $k_{s,i} \in \mathbb{R}_+$ ), as well as on a technology-specific, random variable  $\eta_s \in \mathbb{R}$ , which depicts the cost uncertainty. These cost functions are strictly convex and strictly increasing in  $q_{s,i}$ . Furthermore, they are strictly decreasing in  $k_{s,i}$ , with  $\tilde{C}_s(0, k_{s,i}, \eta_s) = 0$ , for all  $k_{s,i}, \eta_s$ , and with  $\tilde{C}_s(q_{s,i}, 0, \eta_s) \to \infty$ , for all  $q_{s,i} > 0$  and for all  $\eta_s$ .

We assume that these cost functions exhibit constant returns to scale, that is, if a production facility is duplicated (by doubling the capital stock  $k_{s,i}$ ) and if the output is doubled, then variable production costs also double. Owing to this assumption, we can write variable costs more compactly as  $k_{s,i}\tilde{c}_s(u_{s,i},\eta_s)$ , where  $u_{s,i} := q_{s,i}/k_{s,i}$  denotes production per unit of capital and where  $\tilde{c}_s(u_{s,i},\eta_s)$  is a reduced cost function that is strictly convex and strictly increasing in  $u_{s,i}$ . Our above assumptions imply  $\tilde{c}_s(0,\eta_s) = 0$  and  $\lim_{u_{s,i}\to\infty} \tilde{c}_s(u_{s,i},\eta_s) \to \infty$ , for all  $\eta_s$ .

The cost shocks are technology-specific, that is, they have identical values for all firms that use the same technology. We assume that the shocks are small, so that they influence only the linear part of the variable costs. Thus we have

$$\tilde{c}_s(u_{s,i},\eta_s) = c_s(u_{s,i}) + \eta_s \, u_{s,i}.$$
 (1)

We normalize the random variable by setting  $\mathcal{E}(\eta_s) = 0$ , so that  $k_{s,i} c_s(u_{s,i})$  are the expected variable costs of technology s. In addition, we define  $\sigma_s^2 = \mathcal{E}(\eta_s^2)$  as well as  $\sigma_{1,2} = \mathcal{E}(\eta_1 \eta_2)$ .

Demand is characterized by the linear inverse demand function

$$P(Q) = a - b Q, \tag{2}$$

where Q is total output, and where a, b > 0 are constant parameters.

Finally, the fixed costs of firm *i* are given by  $rK_i$ , where  $K_i := k_{1,i} + k_{2,i}$  denotes the total capital acquired by firm *i*, and where *r* is the interest rate. We characterize the technological diversification of firm *i* by  $\nu_i := k_{1,i}/K_i$ , that is, by the fraction of the firm's total investment that uses technology 1.

As in Mills (1986), the above model implies that the level of investment does not set fixed limits to production. Rather, for each technology, more investment facilitates a higher production level at the same marginal production costs. Thus, for any given price, a firm with more capital will produce more. Investing in some equipment is necessary to use a technology; for  $k_{s,i} \rightarrow 0$ , marginal production costs become infinite for all  $q_{s,i} > 0$ .

#### 2.2 Benchmark 1: Socially Optimal Diversification

As a first benchmark, we derive the socially optimal technology mix. From a social perspective, technological diversification reduces the transmission of technology-specific cost shocks to consumer prices; the impact of a price increase of natural gas on electricity prices is smaller, if electricity production can be shifted to technologies using other fuels, whenever natural gas becomes expensive. If the consumers' utility functions are strictly concave, such a reduction of consumer price volatility increases expected utility. However, technological diversification might also increase expected production costs, for example, if much capital is invested in technologies that are used only infrequently.

To depict this trade-off, we use the following measure of social welfare.

$$W := \int_{0}^{\sum_{i=1}^{n} (k_{1,i}u_{1,i} + k_{2,i}u_{2,i})} P(\tilde{q})d\tilde{q} - \sum_{i=1}^{n} r(k_{1,i} + k_{2,i})$$

$$- \sum_{i=1}^{n} \left( k_{1,i} \left( c_{1}(u_{1,i}) + \eta_{1}u_{1,i} \right) + k_{2,i} \left( c_{2}(u_{2,i}) + \eta_{2}u_{2,i} \right) \right).$$
(3)

We maintain the information setup of the preceding section by assuming that expected social welfare is maximized in two stages: First, a social planner sets the investment levels  $k_{1,i}, k_{2,i}$  under uncertainty. Second, the social planner chooses optimal production levels  $u_{1,i}, u_{2,i}$  after the uncertainty is resolved.

It suffices to consider a single period of the production stage, because the random variables are intertemporally independent. Optimizing social welfare W with regard to  $(u_{1,i}, u_{2,i})$  shows that the social optimum in the second stage is characterized by  $P = c'_{1,i}(u_{1,i}^{\times}) + \eta_1$ , whenever this implies  $u_{1,i}^{\times} > 0$ , or by  $u_{1,i}^{\times} = 0$ , otherwise. Similarly, we have  $P = c'_{2,i}(u_{2,i}^{\times}) + \eta_2$ , if  $u_{2,i}^{\times} > 0$ , or  $u_{2,i}^{\times} = 0$ . By our convexity assumptions and the linear inverse demand function, these conditions characterize the unique welfare maximum.

We calculate the optimal investment in the first stage by maximizing  $\mathcal{E}(W)$  with regard to total investment  $K_i = k_{1,i} + k_{2,i}$  and with regard to technological diversification  $\nu_i = k_{1,i}/K_i$  taking into account that all  $u_{1,j}^{\times}, u_{2,j}^{\times}$  (for j = 1, ..., n) can depend on these variables. Carrying out this optimization yields<sup>6</sup>

$$\mathcal{E}\left(Pu_{1,i}^{\times} - (c_1(u_{1,i}^{\times}) + \eta_1 u_{1,i}^{\times})\right) = \mathcal{E}\left(Pu_{2,i}^{\times} - (c_2(u_{2,i}^{\times}) + \eta_2 u_{2,i}^{\times})\right) = r.$$
(4)

<sup>6</sup> Differentiating the expected value of W, as defined in Eq. (3), with regard to  $\nu_i$ , we get

$$\mathcal{E}\Big(K_i^{\times}P \cdot (u_{1,i}^{\times} - u_{2,i}^{\times}) - K_i^{\times}(c_1(u_{1,i}^{\times}) + \eta_1 u_{1,i}^{\times}) + K_i^{\times}(c_2(u_{2,i}^{\times}) + \eta_2 u_{2,i}^{\times}) \\ + \sum_{j=1}^n K_j^{\times} \nu_j^{\times} (\partial u_{1,j}^{\times} / \partial \nu_i) (P - c_1'(u_{1,j}^{\times}) - \eta_1) + K_j^{\times} (1 - \nu_j^{\times}) (\partial u_{2,j}^{\times} / \partial \nu_i) (P - c_2'(u_{2,j}^{\times}) - \eta_2) \Big).$$

Thus, in the social optimum, the marginal increase in expected net-welfare caused by an additional unit of investment has to be equal for both technologies and has to equal the marginal capital costs of investment.<sup>7</sup>

Due to the assumption of constant returns to scale, the social planner is indifferent with regard to the allocation of capital to firms. For simplicity, we consider the symmetric case, where all firms have identical capital endowments. In this case, we can use the inverse demand function (2) to characterize the socially optimal diversification and the socially optimal investment more closely:

$$\nu_{i}^{\times} = \frac{1}{nbK_{i}^{\times}\mathcal{E}\left((u_{1,i}^{\times} - u_{2,i}^{\times})^{2}\right)} \left(a\mathcal{E}\left(u_{1,i}^{\times} - u_{2,i}^{\times}\right) - nbK_{i}^{\times}\mathcal{E}\left(u_{2,i}^{\times}(u_{1,i}^{\times} - u_{2,i}^{\times})\right) \quad (5)$$
$$- \left(c_{1}(u_{1,i}^{\times}) + \eta_{1}u_{1,i}^{\times}\right) + \left(c_{2}(u_{2,i}^{\times}) + \eta_{2}u_{2,i}^{\times}\right)\right),$$
$$K_{i}^{\times} = \frac{a\mathcal{E}\left(u_{1,i}^{\times}\right) - \mathcal{E}\left(c_{1}(u_{1,i}^{\times}) + \eta_{1}u_{1,i}^{\times}\right) - r}{nb\mathcal{E}\left(u_{1,i}^{\times}(\nu_{i}^{\times}u_{1,i}^{\times} + (1 - \nu_{i}^{\times})u_{2,i}^{\times})\right)}.$$

These conditions serve as a benchmark for our analysis of firm behavior. We will interpret them in Section 3.

#### 2.3 Benchmark 2: The Case of Certainty

As a second benchmark, we consider investment behavior under certainty. In this case, we always have  $\eta_1 = \eta_2 = 0$ . Thus the firms have no more information in the production stage than in the investment stage. Consequently, the setup can be simplified to a simultaneous choice of investments  $k_i$  and production levels  $u_i$ .

The following lemma shows that diversification is not profit maximizing.

**Lemma 1.** Assume that the technologies have differing minimal average costs. Then, under certainty, both a monopolist and a firm facing perfect competition will invest at most in one technology.

Proof. Under certainty, a monopolist maximizes

$$P(k_1u_1 + k_2u_2) \cdot (k_1u_1 + k_2u_2) - k_1c_1(u_1) - k_2c_2(u_2) - r(k_1 + k_2)$$
(7)

with regard to  $k_1, k_2, u_1, u_2 \in \mathbb{R}_+$ . A firm facing perfect competition will consider P as being fixed in (7). Maximizing (7) with regard to  $k_s, u_s$  shows that in either

Substituting the optimality conditions of stage 2 (including the fact that we have  $\partial u_{s,j}^{\times}/\partial \nu_i = 0$ , if a constraint  $u_{s,i} \ge 0$  is binding) and setting the result equal to zero yields the first part of Eq. (4). The second part follows from doing the same with regard to  $K_i$  and substituting the above result.

<sup>&</sup>lt;sup>7</sup>The second-order conditions are identical to those analyzed in Section 3.1.

case,<sup>8</sup> we have  $c'_s(u^*_s) = (c_s(u^*_s) + r)/u^*_s$ , whenever technology s is actually used (i.e.,  $k_s, u_s > 0$ ). Thus marginal costs equal average costs (both per unit of investment, as we have constant returns to scale), implying that the firm invests and produces so that each utilized technology is used at minimal average costs. As the cost functions are strictly convex, this determines a unique value of  $u^*_s$ .

Assume that the firm would invest in both technologies. Under certainty, this implies that the technologies would be used simultaneously, as production decisions do not vary (there are no shocks), and as it is clearly suboptimal to invest in a technology that is never used. By maximizing Eq. (7) with regard to  $u_1, u_2 > 0$ , we get the additional condition  $c'_1(u_1^*) = c'_2(u_2^*)$ . By  $c'_s(u_s^*) = (c_s(u_s^*)+r)/u_s^*$ , the marginal costs equal the minimal average costs, which differ between the technologies. Thus the additional condition  $c'_1(u_1^*) = c'_2(u_2^*)$  cannot be met; the technologies are not used simultaneously. Consequently, the firm invests only in one technology.

So, if there is no uncertainty, no diversification will occur. This is a consequence of certainty and constant returns to scale. In most cases, technologies will differ with regard to their costs with one technology having smaller minimal average costs. Due to constant returns to scale, it is possible to produce any output at these minimal average costs, if the firm knows at the time at which investment decisions are made how much it will produce later on. Under certainty, the firm has this knowledge. Thus it can produce the desired quantity at the lowest possible costs by investing only in the technology with the lower minimal average costs. As we will show in the following sections, this result rests strongly on the assumption of full information concerning future costs. It also rests on the assumption of constant returns to scale; for decreasing returns to scale, it might be profitable to invest in both technologies.<sup>9</sup>

### **3** Firm Behavior under Uncertainty

We now contrast these benchmarks with the investment behavior of a firm that faces technology-specific cost uncertainty. We first establish that this firm behavior is socially optimal under conditions of perfect competition. Then we show that, albeit total investment is socially suboptimal, technological diversification is still socially optimal in the case of a monopoly. In this section, we focus on the relation between

<sup>&</sup>lt;sup>8</sup>Differentiating (7) with regard to  $k_s$ ,  $u_s$ , we get  $k_s(P + P'Q - c'_s)$  and  $u_s(P + P'Q - (c_s + r)/u_s)$ , where  $Q := k_1u_1 + k_2u_2$ . For perfect competition, we have the same expressions, but without the terms involving P'. Setting these expressions equal to zero and comparing them yields the above condition.

<sup>&</sup>lt;sup>9</sup>In contrast, increasing returns to scale would provide an additional reason to invest only in a single technology.

the socially and the individually optimal diversification. The following section provides a more detailed analysis of the conditions under which diversification occurs.

#### **3.1 Perfect Competition**

Assume that the number of firms n is so large that each firm does not consider the effects that its production and investment decisions have on the product price. In the production stage, firm i thus maximizes its profit

$$P \cdot (k_{1,i}u_{1,i} + k_{2,i}u_{2,i}) - k_{1,i}(c_1(u_{1,i}) + \eta_1 u_{1,i}) - k_{2,i}(c_2(u_{2,i}) + \eta_2 u_{2,i}) - r(k_{1,i} + k_{2,i}),$$

$$(8)$$

with regard to  $u_{1,i}, u_{2,i} \ge 0$  for observed values of  $\eta_1, \eta_2$  and given values of  $k_{1,i}, k_{2,i}$ . As above, the solution of this optimization problem is characterized by  $P = c'_{1,i}(u^*_{1,i}) + \eta_1$  (if this yields a non-negative solution) or by  $u^*_{1,i} = 0$  and, similarly, by  $P = c'_{2,i}(u^*_{2,i}) + \eta_2$  or  $u^*_{2,i} = 0$ . Note that these conditions imply that  $u^*_{1,i}$  and  $u^*_{2,i}$  are functions of  $(\eta_1, \eta_2)$ . Also, by our convexity assumptions, the optimal solution is unique and the second-order conditions are met.

We calculate the optimal investment behavior by maximizing the expected value of (8) with regard to  $(K_i, \nu_i)$ , given this second-stage optimization and taking into account that  $u_{1,i}^*, u_{2,i}^*$  depend on  $K_i$  as well as on  $\nu_i$ . In this way, we get<sup>10</sup>

$$\mathcal{E}\left(P\cdot\left(u_{1,i}^{*}-u_{2,i}^{*}\right)\right)-\mathcal{E}\left(c_{1}\left(u_{1,i}^{*}\right)+\eta_{1}u_{1,i}^{*}-\left(c_{2}\left(u_{2,i}^{*}\right)+\eta_{2}u_{2,i}^{*}\right)\right)=0,$$
(9)

$$\mathcal{E}\left(P \cdot \left(\nu_{i}^{*}u_{1,i}^{*} + (1-\nu_{i}^{*})u_{2,i}^{*}\right)\right) - \mathcal{E}\left(\nu_{i}^{*}(c_{1}(u_{1,i}^{*}) + \eta_{1}u_{1,i}^{*})\right) - \mathcal{E}\left((1-\nu_{i}^{*})(c_{2}(u_{2,i}^{*}) + \eta_{2}u_{2,i}^{*})\right) - r = 0.$$
(10)

Note that rearranging these conditions yields Eq. (4), that is, the firms invest so that the expected marginal net-benefits of the investment in each technology equal each other and the marginal investment costs.

Substituting the inverse demand function and solving for  $\nu_i^*, K_i^*$  gives

$$\nu_{i}^{*} = \frac{1}{nbK_{i}^{*}\mathcal{E}\left((u_{1,i}^{*} - u_{2,i}^{*})^{2}\right)} \left(a\mathcal{E}\left(u_{1,i}^{*} - u_{2,i}^{*}\right) - nbK_{i}^{*}\mathcal{E}\left(u_{2,i}^{*}(u_{1,i}^{*} - u_{2,i}^{*})\right) - (c_{1}(u_{1,i}^{*}) + \eta_{1}u_{1,i}^{*}) + (c_{2}(u_{2,i}^{*}) + \eta_{2}u_{2,i}^{*})\right),$$
(11)

<sup>10</sup>Differentiating the expected profit with regard to  $\nu_i$ , we get

$$\mathcal{E}\Big(K_i^*P \cdot (u_{1,i}^* - u_{2,i}^*) - K_i^*(c_1(u_{1,i}^*) + \eta_1 u_{1,i}^*) + K_i^*(c_2(u_{2,i}^*) + \eta_2 u_{2,i}^*) \\ + K_i^*\nu_i^*(\partial u_{1,i}^*/\partial \nu_i)(P - c_1'(u_{1,i}^*) - \eta_1) + K_i^*(1 - \nu_i^*)(\partial u_{2,i}^*/\partial \nu_i)(P - c_2'(u_{2,i}^*) - \eta_2)\Big).$$

Proceeding in the same way as described in Footnote 6 leads to the above results.

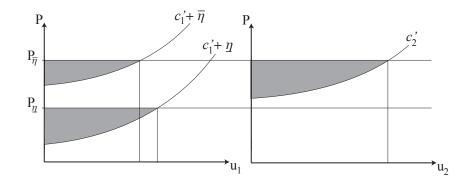


Figure 2: Production decisions for technology 1 and 2 for given investment levels and two cost realizations. The shaded areas correspond to revenues minus variable costs per unit of investment.

$$K_{i}^{*} = \frac{a\mathcal{E}\left(u_{1,i}^{*}\right) - \mathcal{E}\left(c_{1}(u_{1,i}^{*}) + \eta_{1}u_{1,i}^{*}\right) - r}{nb\mathcal{E}\left(u_{1,i}^{*}(\nu_{i}^{*}u_{1,i}^{*} + (1 - \nu_{i}^{*})u_{2,i}^{*})\right)},$$
(12)

as the symmetric solution.<sup>11</sup>

Eq. (11) suggests that we can have  $\nu_i^* \in ]0, 1[$ , that is, it can be optimal for a firm to invest in both technologies. Let us briefly discuss why this might occur; in Section 4, we will provide an in-depth analysis for a special case.

As firms can choose production levels after observing the cost shocks and as the cost functions are strictly convex, the uncertainty increases firm profits compared to a situation with constant, ex-ante known costs.<sup>12</sup> A firm can increase its expected profit by adjusting its production to observed realizations of the cost shocks. The gain in expected profit depends on how much marginal costs change with changing output and on the extent to which the output price varies with the cost shocks. If the marginal cost function is very steep, firms will not adjust their production strongly, and thus gain only little from cost uncertainty. If the output price is perfectly correlated with a firm's cost shocks, the firm will not adjust its output at all and will therefore not gain from the uncertainty.

Diversification reduces the correlation of the output price with a firm's cost changes and thereby increases the firm's expected profit, because a firm with a

<sup>&</sup>lt;sup>11</sup>Due to constant returns to scale and perfect competition, it suffices to consider the symmetric equilibrium.

<sup>&</sup>lt;sup>12</sup>The costs depend linearly on the cost shocks, so that a firm's expected profit would equal the profit under certainty, if the firm would not adjust its production decisions to the shocks. If the firm makes optimal adjustments, it improves upon this and thus receives a higher expected profit.

diversified technology can adjust the allocation of production to the technologies. To see this, assume first that all firms except one firm use only technology 1. Thus the output price will be high, whenever the marginal costs of technology 1 are high. Now, the single firm that has invested in both technologies can shift production to technology 2, whenever technology 1 has high costs and technology 2 has low costs, and increase its expected profit by this behavior. Figure 2 shows this for a simple example, where  $\eta_1$  can take on only two values  $(\bar{\eta}, \underline{\eta})$  and where the costs of technology 2 are certain and higher than the minimal costs of technology 1. Therefore, it can be optimal to invest in both technologies.

If many firms invest in both technologies, the benefit of diversification will be reduced, because the product price will increase less strongly in the case  $\eta = \bar{\eta}$ , as all these firms partially shift production to technology 2. In equilibrium, investments will be chosen so that the marginal benefit of investing in technology 1 equals the marginal benefit of investing in technology 2 as well as the marginal costs of capital. As noted above, rearranging Eqs. (9) and (10) shows that this is what our analysis yields as optimality conditions.

This example also highlights that the assumed timing of investment and production decisions is essential for our diversification result. If the firms would have to choose production levels (i.e.,  $u_{1,i}, u_{2,i}$ ) ex-ante, it would not be profitable to invest in technology 2, because the benefit of being able to use this technology in the case where technology 1 has high marginal costs ( $\eta = \overline{\eta}$ ) is more than compensated by the loss incurred when technology 1 has low marginal costs ( $\eta = \eta$ ).

Unfortunately, it is not possible to provide general conditions under which diversification occurs, because, in Eqs. (11)–(12),  $u_{1,i}^*$ ,  $u_2^*$  depend on  $\nu_i^*$ ,  $K_i^*$ . But it is possible to compare the decentralized outcome to the socially optimal outcome derived above.

**Proposition 1.** Under perfect competition, the firm's investment behavior is socially optimal, both with regard to total investment and with regard to technological diversification.

Furthermore, if Eqs. (11)–(12) have a solution  $K_i^* > 0, \nu_i^* \in ]0, 1[$ , then this solution is the unique profit maximum.

*Proof.* Both, in the firms' and the social planner's optimization, the second-stage production decisions are characterized by the same conditions. Furthermore, Eqs. (11)-(12) are identical to Eqs. (5)-(6). Thus in both stages, the firms' and the social planner's optimality conditions coincide.

The determinant of the Hessian of the firm's optimization problem is

$$\mathcal{E}\left(A^{2}(\eta_{1},\eta_{2}) C(\eta_{1},\eta_{2})\right) \mathcal{E}\left(B^{2}(\eta_{1},\eta_{2}) C(\eta_{1},\eta_{2})\right) - \left(\mathcal{E}\left(A(\eta_{1},\eta_{2}) B(\eta_{1},\eta_{2}) C(\eta_{1},\eta_{2})\right)\right)^{2},$$
(13)

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with

$$A(\eta_1, \eta_2) := K_i^*(u_{1,i}^* - u_{2,i}^*), \tag{14}$$

$$B(\eta_1, \eta_2) := \nu_i^* u_{1,i}^* + (1 - \nu_i^*) u_{2,i}^*, \tag{15}$$

$$C(\eta_1, \eta_2) := -\frac{b^2 K_i^* n^2 c_1'' c_2'' (\nu_i^* c_2'' + (1 - \nu_i^*) c_1'')}{(bn K_i^* (1 - \nu_i^*) c_1'' + (bn K_i^* \nu_i^* + c_1'') c_2'')^2}.$$
(16)

This determinant is strictly positive.<sup>13</sup> The first diagonal element of the Hessian equals  $\mathcal{E}(A^2(\eta_1, \eta_2) C(\eta_1, \eta_2)/K_i^*)$  and thus is strictly negative, because  $C(\eta_1, \eta_2) < 0$ . Therefore, the first-order conditions characterize a maximum of the expected profit. This holds for any solution of the first-order conditions, so that there is no other interior extreme point than a maximum. But no continuous function can have two maxima (interior or on the boundary) without a different type of extreme point in between. Thus, if the solution of Eqs. (11)–(12) is feasible (i.e., if  $K_i^* > 0, \nu_i^* \in ]0, 1[$ ), it is the unique profit maximum.

That firms facing perfect competition will choose a socially optimal level of total investment is a standard result. What Proposition 1 adds is that the firms also invest in the socially optimal technology mix.

From the social perspective, technological diversification is beneficial, because it reduces the transmission of cost shocks to consumer prices and thus the price risk that consumers face. The optimal diversification is achieved, if, for both technologies, the marginal increase in expected consumer plus expected producer surplus caused by investing in one technology equals the marginal investment costs. From the firm perspective, diversification is advantageous, because it increases a firm's ability to react to cost shocks. These reactions increase the firm's expected profit and stabilize the output price. The level of diversification is optimal from the firm perspective, if, for both technologies, the marginal increase in expected profit caused by investing in a technology equals the marginal investment costs. Under perfect competition, the marginal effect of investment on expected profit equals its marginal effect on expected consumer and producer surplus (cf. Eq. (3) and Eq.

$$(1/2) \int \left( A(\eta_1^a, \eta_2^a) B(\eta_1^b, \eta_2^b) - A(\eta_1^b, \eta_2^b) B(\eta_1^a, \eta_2^a) \right)^2 C(\eta_1^a, \eta_2^a) C(\eta_1^b, \eta_2^b) \\ \cdot \phi(\eta_1^a, \eta_2^a) \phi(\eta_1^b, \eta_2^b) d\eta_1^a d\eta_2^a \eta_1^b \eta_2^b$$

and is thus strictly positive, whenever  $\sigma_1 > 0$  or  $\sigma_2 > 0$ . For a discrete distribution, a similar expression can be derived.

<sup>&</sup>lt;sup>13</sup>To see this, consider a continuous probability distribution of  $\eta_1, \eta_2$  that has the density  $\phi(\eta_1, \eta_2)$ . By symmetry, Eq. (13) can be written as

(8)). Thus the firms' and the social planner's calculations coincide with regard to the optimal level of diversification.

This holds also in a corner solution, where no diversification takes place. Such a corner solution will arise, if diversification is costly (e.g., if r is very large) or if the benefits of diversification are small (e.g., if the technologies' cost shocks are highly correlated).<sup>14</sup> In such a case, both profit maximization and welfare maximization imply that only one technology should be used.

#### 3.2 Monopoly

An important question is whether the above result is specific to perfect competition. As is well known, under imperfect competition, a firm chooses a socially suboptimal investment level. Thus it seems likely that diversification will also be suboptimal. Previous studies, like Mills and Smith (1996), support this conjecture.

We consider a monopoly as a simple example of imperfect competition. The setup is the same as in the preceding section, except that we now have n = 1 and that the firm can influence the market price. In contrast to the preceding section, there is a substantial difference between firm heterogeneity and technological diversification in this setting. There can be no heterogeneity, if there is only a single producer. But there can still be technological diversity.

Profit maximization in stage two now yields

$$a - 2bK(\nu u_1^* + (1 - \nu)u_2^*) = c_1'(u_1^*) + \eta_1,$$
(17)

$$a - 2bK(\nu u_1^* + (1 - \nu)u_2^*) = c_2'(u_2^*) + \eta_2,$$
(18)

whenever the constraints  $u_1, u_2 \ge 0$  are not binding. If one of them is binding, the other production level is still described by the corresponding condition above.

We calculate the optimal total investment and diversification as in the preceding section, and get the first-order conditions:<sup>15</sup>

$$K^* = \frac{a\mathcal{E}(u_1^*) - \mathcal{E}(c_1(u_1^*) + \eta_1 u_1^*) - r}{2b\mathcal{E}(u_1^*(\nu^* u_1^* + (1 - \nu^*)u_2^*))},$$
(19)

<sup>14</sup>We provide a more detailed analysis in Section 4.

<sup>15</sup>Differentiating the expected profit w.r.t.  $\nu$  yields

$$\mathcal{E}\Big(K^* \left(P + P'K^*(\nu^* u_1^* + (1 - \nu^*)u_2^*)\right) \cdot (u_1^* - u_2^*) - K^*(c_1(u_1^*) + \eta_1 u_1^*) + K^*(c_2(u_2^*) + \eta_2 u_2^*) \\ + K^*\nu^*(\partial u_1^*/\partial \nu)(P + P'K^*(\nu^* u_1^* + (1 - \nu^*)u_2^*) - c_1'(u_1^*) - \eta_1) \\ + K^*(1 - \nu^*)(\partial u_2^*/\partial \nu)(P + P'K^*(\nu^* u_1^* + (1 - \nu^*)u_2^*) - c_2'(u_2^*) - \eta_2)\Big).$$

Setting this expression equal to zero and substituting Eq. (2) and the first-order conditions of stage two leads to Eq. (20). Differentiating w.r.t. K and proceeding in the same way leads to Eq. (19).

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$$\nu^* = \frac{a\mathcal{E}\left(u_1^* - u_2^*\right) - 2bK^*\mathcal{E}\left(u_2^*\left(u_1^* - u_2^*\right)\right) - \left(c_1(u_1^*) + \eta_1 u_1^*\right) + \left(c_2(u_2^*) + \eta_2 u_2^*\right)}{2bK^*\mathcal{E}\left((u_1^* - u_2^*)^2\right)}.$$
(20)

This yields the following conclusion.

**Proposition 2.** A monopolist's investment behavior is socially optimal with regard to technological diversification but not with regard to the total level of investment.

*Proof.* We first show that Eqs. (19)–(20) characterize a unique profit-maximum, if they have a feasible solution. The determinant of the Hessian of the firm's optimization problem can be written as

$$\mathcal{E} \left( A^{2}(\eta_{1}, \eta_{2}) C_{m}(\eta_{1}, \eta_{2}) \right) \mathcal{E} \left( B^{2}(\eta_{1}, \eta_{2}) C_{m}(\eta_{1}, \eta_{2}) \right)$$

$$- \mathcal{E} \left( A(\eta_{1}, \eta_{2}) B(\eta_{1}, \eta_{2}) C_{m}(\eta_{1}, \eta_{2}) \right)^{2},$$
(21)

with

$$C_m(\eta_1, \eta_2) := -\frac{2bc_1''c_2''}{(2bK^*(1-\nu^*)c_1'' + (2bK^*\nu^* + c_1'')c_2'')},$$
(22)

and  $A(\eta_1, \eta_2), B(\eta_1, \eta_2)$  defined by Eqs. (14)–(15) given in the proof of Proposition 1. Again, this determinant is strictly positive.<sup>16</sup> Furthermore, the first diagonal element of the Hessian equals  $\mathcal{E}(A^2(\eta_1, \eta_2) C_m(\eta_1, \eta_2)/K^*)$  and is thus strictly negative, as  $C(\eta_1, \eta_2) < 0$ . As argued in the proof of Proposition 1, the solution of Eqs. (19)–(20) is thus the unique profit maximum, whenever it is feasible.

Suppose that there is a socially optimal solution  $K^{\times} > 0$ ,  $\nu^{\times} \in ]0, 1[, u_1^{\times}, u_2^{\times} \ge 0$ . Comparing (19) with (6), (20) with (5), and (17)–(18) with the socially optimal production levels shows that, if this solutions fulfills the conditions of social optimality, then  $K^* = K^{\times}/2$ ,  $\nu^* = \nu^{\times}$ , and  $u_1^* = u_1^{\times}, u_2^* = u_2^{\times}$  solves the monopolist's profit maximization conditions. As any feasible solution of Eqs. (19)–(20) is unique, this constructed solution is indeed the profit maximum. By construction, this solution has the property that the diversification is socially optimal whereas total investment is not. So, we have proven that the assertion holds, if an interior social optimum exists.

Now, suppose that there is no interior solution of the social optimality conditions. Then there can also be no interior solution of the monopolist's profit maximization problem. If an interior solution  $(K^*, \nu^*, u_1^*, u_2^*)$  would exist,  $K^{\times} = 2K^*$ ,  $\nu^{\times} = \nu^*$ , and  $u_1^{\times} = u_1^*, u_2^{\times} = u_2^*$  would be a feasible interior solution of the social planner's conditions. Any such solution would be the social optimum.<sup>17</sup> Thus the

<sup>&</sup>lt;sup>16</sup>The proof is identical to the one given in the preceding section and thus not repeated.

<sup>&</sup>lt;sup>17</sup>The second-order conditions of the social planner's problem are identical to those of the firm's problem under perfect competition, so that the proof of Prop. 1 shows that a feasible interior solution of the social planner's problem constitutes the unique welfare maximum.

monopolist's problem has only an interior solution, if an interior social optimum exists.

Finally, a direct comparison shows that if the social planner's problem has a boundary solution, the same boundary solution is obtained in the monopolist's problem with regard to  $\nu$ .

So, a monopolist's investment is socially suboptimal in total. However, the level of diversification is not distorted by market power. This is intuitive. The monopolist produces less than under perfect competition to obtain a higher price. For a homogeneous good, the price depends only on total production, not on the allocation of production to technologies, so that there is no demand-related incentive to distort the allocation of production. Due to constant returns to scale, it is optimal to scale down production without changing this allocation compared to the competitive solution; the allocation of production to technologies is cost minimizing for a firm's total production under perfect competition and, due to constant returns to scale, it is also cost minimizing for any other total production. In fact, our above proof shows that  $u_1, u_2$  remain unchanged in monopoly compared to perfect competition; total production is only reduced by a lower value of K.<sup>18</sup> But if the technologies' relative utilization is unchanged, so is the relative investment in these technologies, as the investment costs are the same as under perfect competition.

This result implies that, even in the case of monopoly, there is no reason to interfere with the firm's diversification decision. Governmental programs that aim at increasing technological diversity, for example, by increasing the attractiveness of "underused" technologies, are likely to be socially detrimental. Both under perfect competition and in a monopoly, firms have socially optimal incentives to diversify their investment. If further diversification is induced, the additional benefit of more stable consumer prices will be smaller than the additional costs of investing in only infrequently used equipment.

However, with imperfect competition, it is optimal to induce higher total investment and thereby more production. To achieve this, a simple product subsidy suffices. It is easy to show that the subsidy  $\omega = -P' \cdot k^{\times} (\nu^{\times} u_1^{\times} + (1 - \nu^{\times}) u_2^{\times})$  corrects the production and investment incentives of the monopolist but does not distort the technological diversification.<sup>19</sup> So, a standard subsidy can implement the socially optimal solution, with regard to production, total investment, and technological diversification. No technology-specific intervention is necessary to achieve the social optimum.

<sup>&</sup>lt;sup>18</sup>Total production equals  $K \cdot (\nu u_1 + (1 - \nu)u_2)$ .

<sup>&</sup>lt;sup>19</sup>Comparing Eqs. (11)–(12) with Eqs. (19)–(20) as well as comparing the optimality conditions of the production stage, with a in the monopolists problem being replaced by  $a + \omega$ , directly yields this assertion.

Altogether, our results suggest that technology-specific subsidies or other programs intended to increase technological diversity are neither necessary nor helpful if diversification is primarily a means to reduce the detrimental effects of technologyspecific cost shocks. Firms already have sufficient incentives to invest in technological diversity without such programs. Of course, there may be other reasons for the support of specific technologies. But especially in the energy industry, where the "insurance motive" of diversity is frequently stressed, it seems questionable whether the wide spread of technology-specific governmental intervention actually contributes to social welfare.

## 4 A Special Case

In the preceding sections, we have used a fairly general cost specification to show that both, under perfect competition and in a monopoly, individual investment decisions lead to a socially optimal technological diversification. Thus if diversification is socially optimal, it will be induced by market forces. However, it is not possible to analyze in detail under which conditions diversification is optimal in this general setup. In this section, we use a more restrictive cost specification to provide such an analysis.

As the result for the case of a monopoly is more surprising and as this case induces the same diversification as perfect competition, we consider only a monopolistic producer. We assume that the expected variable costs  $c_s(u_s)$  can be approximated by a quadratic function:

$$c_s(u_s) \approx \alpha_s \, u_s + \frac{\beta_s}{2} u_s^2. \tag{23}$$

Here,  $\alpha_s \ge 0$  and  $\beta_s > 0$  are constant parameters. For notational simplicity, we assume<sup>20</sup>  $\beta_1 = \beta_2 =: \beta$ , set  $\alpha_1 = 0$ , and define  $\delta := \alpha_2$ , so that  $\delta$  is a measure of the difference of the technologies with regard to their expected variable costs. Without loss of generality, we assume  $\delta \ge 0$ . Also, we assume  $a > \delta$ , which excludes cases where the use of technology 2 is economically unfeasible on average.

As above, we denote the variance of  $\eta_1$  by  $\sigma_1^2$ , that of  $\eta_2$  by  $\sigma_2^2$ , and the covariance by  $\sigma_{1,2}$ . Whereas we have assumed  $\delta \ge 0$ , we do not impose restrictions on the relation between  $\sigma_1$  and  $\sigma_2$ , so that either technology can be subject to higher cost volatility.

<sup>&</sup>lt;sup>20</sup>This assumption is only for presentational ease, our results can be easily adjusted to the case where the technologies also differ with regard to  $\beta_s$ .

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By Eqs. (17), (18), and (23), the optimal production levels are

$$u_1^* = \frac{\beta(a - \eta_1) - 2bK(1 - \nu)(\eta_1 - \eta_2 - \delta)}{\beta(\beta + 2bK)},$$
(24)

$$u_{2}^{*} = \frac{\beta(a - \eta_{2} - \delta) - 2bK\nu(\eta_{2} + \delta - \eta_{1})}{\beta(\beta + 2bK)},$$
(25)

if these values are non-negative. Otherwise, the negative  $u_s^*$  is replaced by zero. For notational simplicity, and in line with our assumption on the cost shocks in Section 2.1, we assume that large cost shocks (i.e., those that lead to a binding constraint  $u_s \ge 0$ ) have a negligible probability.

Calculating the expected profit of the monopolist from Eqs. (24) and (25) under this assumption yields

$$\mathcal{E}(\pi) = \frac{K}{2\beta(\beta + 2bK)} \Big( \beta \left( (1 - \nu)(\delta^2 + \sigma_2^2 - \sigma_1^2) + \sigma_1^2 - 2r\beta \right) + a\beta \left( a - 2(1 - \nu)\delta \right) - 2bK \left( 2r\beta - \nu(1 - \nu)(\delta^2 + \sigma_1^2 + \sigma_2^2 - 2\sigma_{1,2}) \right) \Big).$$
(26)

Optimizing  $\mathcal{E}(\pi)$  with regard to  $\nu$  leads to

$$\nu^* = \frac{1}{2} - \frac{\beta \left(\delta^2 + \sigma_2^2 - \sigma_1^2 - 2\delta a\right)}{4bK \left(\delta^2 + \sigma_1^2 + \sigma_2^2 - 2\sigma_{1,2}\right)}.$$
(27)

By substituting Eq. (27) in Eq. (26) and optimizing with regard to K, we get<sup>21</sup>

$$K^* = \frac{\beta}{2b} \left( -1 \right)$$

$$+ \sqrt{\frac{4 \left( a^2 \left( \sigma_1^2 + \sigma_2^2 - 2\sigma_{1,2} \right) + \sigma_1^2 \left( \delta^2 + \sigma_2^2 \right) - 2a\delta \left( \sigma_1^2 - \sigma_{1,2} \right) - \sigma_{1,2}^2 \right)}{\left( \delta^2 + \sigma_1^2 + \sigma_2^2 - 2\sigma_{1,2} \right) \left( 8r\beta - \delta^2 - \sigma_1^2 - \sigma_2^2 + 2\sigma_{1,2} \right)}} \right).$$
(28)

As can be easily verified, these conditions satisfy Eqs. (19)–(20) of the preceding section. The following proposition provides some characteristics of these optimality conditions.

#### **Proposition 3.** Define

$$\bar{r} := \begin{cases} \frac{\left(a^2 + \sigma_1^2\right)\left(\delta^2 + \sigma_1^2 + \sigma_2^2 - 2\sigma_{1,2}\right)^2}{8\beta\left(\delta a + \sigma_1^2 - \sigma_{1,2}\right)^2} & \text{if } \sigma_2^2 < \sigma_1^2 + 2\delta a - \delta^2, \\ \infty & \text{if } \sigma_2^2 = \sigma_1^2 + 2\delta a - \delta^2, \\ \frac{\left((a - \delta)^2 + \sigma_2^2\right)\left(\delta^2 + \sigma_1^2 + \sigma_2^2 - 2\sigma_{1,2}\right)^2}{8\beta\left(\sigma_2^2 - \sigma_{1,2} - \delta(a - \delta)\right)\right)^2} & \text{if } \sigma_2^2 > \sigma_1^2 + 2\delta a - \delta^2. \end{cases}$$
(29)

<sup>&</sup>lt;sup>21</sup>The second solution of the (quadratic) first-order condition w.r.t. K is always non-positive.

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(a) If

$$\frac{\delta^2 + \sigma_1^2 + \sigma_2^2 - 2\sigma_{1,2}}{8\beta} < r < \bar{r},\tag{30}$$

then there is a unique optimal investment plan for the monopolist that is characterized by Eqs. (27) and (28) and that includes diversification (i.e.,  $\nu^* \in ]0,1[$ ).

- (b) If  $r \geq \bar{r}$ , then the firm will invest solely in technology 1, whenever  $\sigma_2^2 < \sigma_1^2 + 2\delta a \delta^2$ , and solely in technology 2, whenever  $\sigma_2^2 > \sigma_1^2 + 2\delta a \delta^2$ .
- (c) If  $\sigma_2^2 = \sigma_1^2 + 2\delta a \delta^2$ , then the firm will always choose  $\nu = 1/2$ .
- (d) In all cases, the level of diversification is socially optimal.

*Proof.* Let  $\Delta := \sigma_2^2 - \sigma_1^2$ ,  $\sigma := \sigma_1$ , and  $\gamma := 2(\sigma_1^2 - \sigma_{1,2})$ . By Eq. (28),  $K^*$  is a real number and is strictly decreasing in r, if r is larger than the lower boundary in condition (30). Furthermore,  $K^*$  is positive for  $r = \bar{r}$ . Thus, under condition (30),  $K^*$  is feasible. Eq. (27) shows that  $\nu^* \in ]0, 1[$  (and is thus feasible), if, in addition,  $K^* < |(2a\beta\delta - \beta(2\delta + \delta^2 + \Delta))/(2\beta(\gamma + \delta^2 + \Delta))|$ . Substituting Eq. (28) in this condition yields the upper boundary (29) for r.

So, under condition (30), we have a feasible solution  $\nu^* \in ]0, 1[$  and  $K^* \geq 0$ . Furthermore, the determinant of the Hessian of the expected profit at this solution is  $\frac{4((\Delta+\gamma)a^2-\delta\gamma a+\sigma^2(\delta^2+\Delta+\gamma))-\gamma^2}{(2br+\beta)^4}$ , which is strictly positive under condition (30). The first diagonal element of the Hessian equals  $-\frac{2br^2(\gamma+\delta^2+\Delta)}{\beta(2br+\beta)}$ , which is strictly negative. Thus any interior extreme point is a maximum, again implying that there can be no boundary maxima. So, the interior maximum is unique.

If  $r \ge \bar{r}$ , then substituting (28) in (27) yields no solution  $\nu^* \in ]0, 1[$ . For  $\Delta < 2\delta a - \delta^2$ , we get  $\nu^* \ge 1$ , and the optimal solution is  $\nu^* = 1$ . For  $\Delta > 2\delta a - \delta^2$ , we get  $\nu^* \le 0$ , and the optimal solution is  $\nu^* = 0$ . For  $\Delta = 2\delta a - \delta^2$ , condition (27) yields the feasible solution  $\nu^* = 1/2$ .

That the level of diversification is socially optimal follows from Prop. 2.  $\Box$ 

Proposition 3 yields a detailed characterization of the conditions under which diversification occurs. As both the competitive and the monopoly solution are socially optimal with regard to diversification, it also applies to the case of perfect competition. It shows that diversification occurs, if capital is not too expensive.<sup>22</sup> In light of our above discussion of diversification incentives this is intuitive. Diversification is beneficial, because it affords the firm more flexibility in adjusting to cost shocks. It is costly, because capital is less utilized on average, that is, for the

 $<sup>^{22}</sup>$ The lower boundary in Eq. (30) is only necessary to assure that total investment is always finite.

same average level of production, the firm needs to invest more. If capital is not too expensive, the benefit exceeds the costs and the firm will diversify its investment.

The two parts of the upper limit for r correspond to different cases with regard to the preferability of the technologies. If  $\sigma_2^2 > \sigma_1^2 + \delta(2a - \delta)$ , the profit gain due to uncertainty is so much higher for technology 2 that it compensates the (possibly) higher expected costs. Consequently, technology 2 is preferred, so that we have  $\nu^* \in [0, 0.5]$  and the limiting condition for diversification is  $\nu^* > 0$ . In the opposite case, where  $\sigma_2^2 < \sigma_1^2 + \delta(2a - \delta)$ , technology 1 is preferred, and we have  $\nu^* \in [0.5, 1]$ with diversification occurring for  $\nu^* < 1$ .

Note that for  $\sigma_1^2 = \sigma_2^2 = 0$ , there is no diversification; condition (30) cannot be met.<sup>23</sup> Thus diversification is solely a response to cost uncertainty, as our analysis of Section 2.3 has suggested. Also, diversification is not simply due to choosing an optimal portfolio of assets with regard to risks and expected returns, but is driven by the greater flexibility that it affords in production.<sup>24</sup>

Proposition 3 also provides further insight into the benefit of diversification. As we have argued above, and as Eq. (26) clearly shows,<sup>25</sup> the firm gains from the cost uncertainty. Thus, the larger  $(\sigma_2^2 - \sigma_1^2)$  is, the more beneficial is technology 2. In contrast, the higher  $\delta$  is, the less will technology 2 be used and thus the less beneficial is an investment in technology 2. Proposition 3 shows that there is an indifference curve that relates  $(\sigma_2^2 - \sigma_1^2)$  and  $\delta$  in a way that both technologies are equally attractive; the more costly technology 2 is in relation to technology 1 (high  $\delta$ ), the more risky (high  $\sigma_2^2$ ) it has to be for being equally attractive.<sup>26</sup>

An important question left open in Proposition 3 is under which conditions, there are values of r that meet the condition of assertion (a) and thus induce the firm to use a diversified technology portfolio.

#### **Corollary 1.**

- (a) If there is uncertainty, that is, if either  $\sigma_1^2 > 0$  or  $\sigma_2^2 > 0$ , and if the technologyspecific shocks are not perfectly positively correlated, that is, if  $\frac{\sigma_{1,2}}{\sigma_1\sigma_2} < 1$ , then there exist positive values of r that meet condition (30).
- (b) Under these conditions, the range of values of r for which diversification occurs is strictly strictly decreasing in  $\beta$ .

<sup>&</sup>lt;sup>23</sup>The upper and the lower bound become identical.

<sup>&</sup>lt;sup>24</sup>If the firms could not respond to the cost shocks by adjusting production, diversification would not be optimal in our setup.

<sup>&</sup>lt;sup>25</sup>Observe that the expected profit is increasing in  $\sigma_1^2$  and  $\sigma_2^2$ .

<sup>&</sup>lt;sup>26</sup>As we have assumed  $a > \delta$ , the relation between  $\delta$  and  $(\sigma_2^2 - \sigma_1^2)$  implied by assertion (c) of Prop. 3 is strictly monotonic in  $\delta$ .

*Proof.* Any *r* that meets condition (30) is non-negative, because the lower bound specified there is non-negative. Analyzing (30) shows that the lower and the upper bound can become equal only if  $\frac{\sigma_{1,2}}{\sigma_1\sigma_2} = \frac{-(\delta+\psi)\pm\sqrt{(\sigma_2^2+\psi^2)(\sigma_1^2+(\delta+\psi)^2)}}{\sigma_1\sigma_2}$ , where  $\psi := a - \delta$ , which is strictly positive by assumption. The term under the root is greater than or equal to  $(\psi(\delta+\psi)+\sigma_1\sigma_2)^2$ . Thus the lower and upper bound can only become equal for  $\frac{\sigma_{1,2}}{\sigma_1\sigma_2} \ge 1$  or  $\frac{\sigma_{1,2}}{\sigma_1\sigma_2} < -1$ , the latter of which is impossible. Direct inspection shows that, within the thus defined range  $-1 \le \frac{\sigma_{1,2}}{\sigma_1\sigma_2} < 1$ , the upper bound is strictly greater than the lower bound, so that feasible values of *r* can be found. This proves (a). Assertion (b) follows directly from Eq. (30).

So, diversification can occur, whenever the technology-specific shocks are not perfectly positively correlated. This is intuitive: If the shocks are not perfectly positively correlated, diversifying investment improves a firm's ability to react to factor price changes. As the costs of diversification are proportional to r, there will be a value of r for which it is individually optimal to hold a diversified technology portfolio. Furthermore, diversification occurs for a wider range of capital costs, if the marginal cost functions have a smaller slope. Again, this is intuitive; the gains from diversification result from a better ability in adjusting to cost shocks. If the technologies are more flexible (smaller  $\beta$ ), which facilitates larger adjustments, these gains increase, and thus diversification becomes rational for a higher price of capital.

Corollary 1 addresses the question of how the occurrence of diversification depends on the model parameters. The following corollary investigates the influence of the parameters on the level of diversification.

**Corollary 2.** Under the assumptions of part (a) of Proposition 3, the optimal investment has the following properties:

- (a) For given and strictly positive levels of uncertainty, a given cost difference  $\delta$ , and a given correlation  $\sigma_{1,2}/(\sigma_1\sigma_2) < 1$ , every level of diversification is possible for feasible values of the other model parameters.
- (b) Diversification increases (i.e.,  $\nu^*$  gets closer to 1/2) for decreasing values of r or  $\beta$ .

*Proof.* Let  $\delta, \sigma_1, \sigma_2 > 0$  be given and assume  $\sigma_{1,2}/(\sigma_1\sigma_2) < 1$ . By Corollary 1, there are values of r for which diversification occurs. Eqs. (27)–(30) show that choosing r equal to the lower boundary of (30) leads to  $\nu^* \rightarrow 1/2$ , whereas choosing r equal to the upper boundary leads to  $\nu^* = 1$ , for  $\sigma_2^2 < \sigma_1^2 + \delta(2a - \delta)$ , or to  $\nu^* = 0$ , for  $\sigma_2^2 > \sigma_1^2 + \delta(2a - \delta)$ . As  $K^*$  and  $\nu^*$  are continuous functions of r within the range specified by (30), all values of  $\nu_i^* \in [0, 1]$  can be reached by

varying r and by choosing a smaller or larger than  $(\sigma_2^2 - \sigma_1^2 + \delta^2)/(2\delta)$ . This proves (a). Assertion (b) follows directly from (27) and (28).

The most important implication of Corollary 2 is that the extent of uncertainty does not limit the level of diversification, as long as there is some uncertainty. This is important, because we have assumed that the cost shocks are small. The above result shows that this assumption does not restrict the ability of our model to explain substantial diversification.

The corollary also shows that a lower price of capital or a more flexible technology lead to more diversification. Thus flexibility (low  $\beta$ ) does not only increase the number of cases (in terms of possible values of r) in which diversification occurs but also increases the level of diversification. The intuition is the same as above; more flexibility leads to higher gains from uncertainty and thus to more diversification. In contrast, higher values of r imply higher costs of diversification and thus less diversification.

As we have discussed in the introduction, technological diversification is observable in the electricity industry in the UK and in Germany. Our results suggest that technology-specific uncertainty may provide a possible explanation for the observed technological diversity. As Proposition 3 and Corollary 2 show, substantial diversification can occur even if there are large differences in expected costs between the technologies and even if cost shocks are not very large and strongly (but not perfectly) correlated. Furthermore, our model explains why there is not only diversification in the aggregate but also at the firm level, which fits the observations from the electricity industry.

## 5 Conclusions

In this article, we have analyzed the questions of whether technology-specific uncertainty might provide an explanation for observed technological diversity and whether the profit maximizing level of diversification is socially optimal. We have advanced a model of technology choice under cost uncertainty and have shown that such uncertainty can provide an incentive for firms to hold a diversified technology portfolio. The profit maximizing diversification is socially optimal, both under perfect competition and in the case of a monopoly.

Our study complements the literature by considering cost instead of demand uncertainty, by analyzing firm-level incentives for investing in a diversified technology portfolio instead of firm heterogeneity, and by comparing the results of firm behavior to socially optimal outcomes for two market situations. As we have argued, this change in assumptions reflects characteristics observed in important applications, such as the electricity industry.

Our analysis has produced two new results. First, cost uncertainty induces firms to invest in a diversified technology portfolio. Second, albeit total investment can be suboptimal, the diversification of this investment is socially optimal, even in the case of a monopolistic producer.

These results have substantial policy implications. In the cases considered in the literature on firm heterogeneity, technological diversity is a response to demand uncertainty or the outcome of strategic interaction. A socially optimal technological diversity is usually not attained without policy intervention. In contrast, our analysis shows that, in the case of cost uncertainty, such intervention is not needed.

This case is relevant in many applications. Resource price volatility, such as oil-price shocks, is often stated as a major motivation for programs aimed at increasing technological diversity, like technology-specific subsidies, tax reductions, or compensation programs. Such policies may be desirable for other reasons, such as achieving environmental objectives or reducing import dependency in strategically important sectors. But our results suggest that they are dispensable to reduce an economy's vulnerability to resource price shocks.

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