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TRIMOR – three-dimensional correlation technique to analyse multi-order spectra of triple stellar systems: application to HD 188753

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ABSTRACT

This paper presents a new algorithm, TRIMOR, to analyse *multi-order* spectra of triple systems. The algorithm is an extension of TRICOR, the three-dimensional correlation technique that derives the radial velocities of triple stellar systems from single-order spectra. The combined correlation derived from many orders enables the detection and the measurement of radial velocities of faint tertiary companions. The paper applied TRIMOR to the already available spectra of HD 188753, a well-known triple system, yielding the radial velocities of the faintest star in the system. This rendered the close pair of the triple system a double-lined spectroscopic binary, which led to a precise mass ratio and an estimate of its inclination. The close-pair inclination is very close to the inclination of the wide orbit, consistent with the assertion that this triple system has a close to coplanar configuration.

Key words: methods: data analysis – techniques: radial velocities – binaries: spectroscopic – star: individual: HD 188753.

1 INTRODUCTION

TwO-Dimensional CORrelation (TODCOR) is a two-dimensional correlation technique to derive the radial velocities of the two components of spectroscopic binaries (Zucker & Mazeh 1994). TODCOR was a natural extension of the one-dimensional correlation technique (Griffin 1967; Simkin 1974; Tonry & Davis 1979) which turned out to be very effective for deriving radial velocities out of spectra with low signal-to-noise ratio (S/N). TODCOR calculates the correlation of an observed spectrum against a combination of two templates with different shifts, resulting in a two-dimensional correlation function, whose peak simultaneously identifies the radial velocities of both primary and secondary components. Various studies have applied TODCOR successfully to many binary spectra, yielding double-lined solutions (e.g. Mazeh et al. 1995; Metcalfe et al. 1996; Torres, Stefanik & Latham 1997; Zucker 2004), some of them with quite faint companions (e.g. Mazeh et al. 1997; Prato et al. 2002; Mazeh et al. 2003).

Zucker, Torres & Mazeh (1995) extended TODCOR to analyse spectra of triple-lined systems by calculating the correlation of an observed spectrum against a combination of *three* templates with three different shifts. The peak of the resulting three-dimensional correlation simultaneously identifies the radial velocities of the primary, secondary and tertiary components. The new algorithm, TRICOR, was successfully applied to the spectra of a few triple systems, yielding triple-lined solutions (e.g. Torres et al. 1995; Jha et al. 2000). Torres, Latham & Stefanik (2007) extended the algorithm to analyse spectra of quadruple systems.

The importance of TRICOR arises from the prevalence of short-period binaries accompanied by a distant companion, as is becoming evident from a few studies (e.g. Mayor & Mazeh 1987, and in particular Tokovinin & Gorynya 2007; see a review by Mazeh 2008). The velocities of the three components contribute to a better understanding of the orbital geometry of the triple system and the dynamics of the three stars (e.g. Torres et al. 2006).

Both TODCOR and TRICOR aim to analyse spectra composed of one order only. However, many of the modern spectrographs produce nowadays multi-order spectra, some of which have as many as a few dozen orders (e.g. Pepe et al. 2000). To optimize the use of multi-order spectra for deriving accurate velocities and enabling us to detect faint companions, we should include in the analysis all the spectral information spread over different orders. Therefore,

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Zucker et al. (2003) developed TODMOR, which can handle double-lined Multi-ORder spectra for the analysis of double-lined systems. Using TODMOR, Zucker et al. (2003, 2004) could discover an evidence for a planet and a brown dwarf hidden in the spectra of the double-lined binary system HD 41004.

The next step is to develop TRIMOR, a three-dimensional correlation function to handle multi-order spectra. This paper explains the basics of TRIMOR, which is conceptually easy to construct, once we have at hand TODMOR and TRICOR. In its second part, the paper applies the algorithm to a set of spectra of HD 188753, which is a well known triple system (Griffin 1977) that attracted new interest lately (Konacki 2005). The spectra analysed here were obtained already by Eggenberger et al. (2007) for a different study that focused on a careful analysis of the radial velocity of the brightest star in the system. Applying TRIMOR to those spectra enabled us to derive the velocities of the faintest companion, rendering the close pair of the system a double-lined binary.

Section 2 presents the principles of TRIMOR, Section 3 applies TRIMOR to the already obtained spectra of HD 188753, while Section 4 derives the relative inclination of the system. Section 5 discusses briefly the results obtained for HD 188753 and future applications of TRIMOR.

2 TRIMOR ALGORITHM

2.1 The three-dimensional correlation

Assume $f(n)$ is an observed single-order spectrum of a triple system, where n is the wavelength. As $f(n)$ is composed of three spectra, we wish to construct a model, $m(n)$, by calculating a spectrum composed of g_1, g_2 and g_3 – three templates for the primary, secondary and tertiary components of the triple system, respectively. Given the templates, the model

$$m(n) = g_1(n - s_1) + \alpha g_2(n - s_2) + \beta g_3(n - s_3) \tag{1}$$

has five basic parameters: s_1, s_2 and s_3, the Doppler shifts of the three templates, and α and β, the flux ratios of g_2 and g_3 relative to g_1, respectively. The correlation, $f(n) * m(n)$, between the observed spectrum $f(n)$ and the model $m(n)$ measures the goodness-of-fit of the model. Note that the correlation is not sensitive to any multiplication factor of the model function, and therefore only the *relative* flux of the three templates is relevant to our analysis.

TRICOR considers the correlation between the observed spectrum and the model as a function of all possible values of s_1, s_2 and s_3, generating a three-dimensional correlation function:

$$\begin{aligned} R^{(3)}(s_1, s_2, s_3; \alpha, \beta) = f * [&g_1(n - s_1) + \alpha g_2(n - s_2) \\ &+ \beta g_3(n - s_3)] \end{aligned} \tag{2}$$

where α and β are considered external parameters that have to be determined a priori. Alternatively, for each set of values of s_1, s_2 and s_3, we can find analytically the values of α and β that maximize the correlation (see appendix A in Zucker et al. 1995 for details). The location of the maximum of $R^{(3)}$ (s_1, s_2, s_3) identifies simultaneously the radial velocities of the three components. To reduce the amount of computation, we avoid direct calculation of the three-dimensional correlation and instead calculate only six one-dimensional correlations as detailed in Zucker et al. (1995).

To consider multi-order spectra, we combine the correlations of the different orders according to the maximum-likelihood approach developed by Zucker (2003):

$$R^2(s_1, s_2, s_3) = 1 - \left\{ \prod_{i=1}^{M} \left[1 - R_i^2(s_1, s_2, s_3) \right] \right\}^{1/M} \tag{3}$$

where R_i is the three-dimensional correlation function of the i th order and M is the number of orders.

When considering multi-order correlation functions, the values of α and β vary from order to order, and therefore α and β are in fact vectors of values, $\{\alpha_i, i = 1, M\}$ and $\{\beta_i, i = 1, M\}$. Finding numerically different α and β values for each order would greatly increase the number of fitted parameters. We, however, can reduce drastically the complexity of the calculation by taking advantage of the fact that for given two stars with known spectral types the flux ratio at a given wavelength is known approximately (e.g. Pickles 1998). We, therefore, pre-calculated tables containing the flux ratio at each order for many possible spectral-type pairs in our template library, using Pickles (1998) tables. In other words, the choice of templates determined the values of $\{\alpha_i; i = 1, M\}$ and $\{\beta_i; i = 1, M\}$ in our analysis.

2.2 Choosing the parameters of the correlation function

To derive the most accurate velocities out of a given set of observed spectra, we have to carefully choose the parameters of the model function. We first search for the best templates to fit the observed spectra. This step depends on the available library of templates that fit the spectral range and resolution of the observed spectra. We first find the template for the primary, as this template has the largest contribution to the model function. We find the best template by running a one-dimensional correlation with each template candidate against all the observed spectra and choose the one that yields the highest correlation peak, averaged over all spectra. We then find the template for the secondary by running TODMOR with all template candidates over all observed spectra, and only at the third stage we choose the template for the tertiary. In many cases, the contribution of the tertiary to the correlation function is minimal and we have to choose the template that leads to the best orbital solution of the tertiary, usually judged by the size of its residual rms.

As explained above, the choice of the three templates determines the flux ratio between the templates that we use. However, to maximize the correlation we allow a multiplicative factors f_α and f_β of the order of unity which we apply to the pre-determined flux ratios of all orders. The factors are determined numerically to maximize the average of the correlation peak.

In some cases, the actual stars in the system have rotational broadening larger than their templates. To correct for that we try to insert numerically some *additional* rotational broadening to the three templates. This is done by convolving the template spectra with a broadening function, based on the Fourier transformation of Smith & Gray (1976). Here, again we first determine the rotational broadening of the primary template, then the secondary and at last the tertiary. The values of the rotation of the three templates are chosen to maximize the averaged correlation peak.

In finding the best parameter values of the model function, we always choose the ones that maximize the correlation peak *averaged over all observed spectra*. Because the values of all these parameters are based on the whole set of observed spectra, we consider these parameters as global ones. In some cases, however, we prefer to perform the averaging only over the high-S/N spectra, as the low-S/N spectra add noise to the process.

To conclude, we fit eight global parameters to the whole set of observed spectra: the best three spectral templates, two multiplicative factors to be applied to the flux tables and three rotational broadening velocities. After finding these eight global parameters, we consider the three-dimensional correlation for each observed spectrum, find its maximum and derive the three velocities of the three stars for each spectrum.

The uncertainties of the three velocities were derived as explained by Zucker (2003), by deriving the inverted Hessian of the likelihood of the velocities. However, these errors were proven to be overestimated in quite a few cases (e.g. Eggenberger et al. 2007), and we had to rescale them by using the rms of the residuals of the orbital solution. In this case, we kept only the relative sense of the derived error estimate, which probably depended mainly on the S/N of the obtained spectra.

3 APPLICATION OF TRIMOR TO THE CASE OF HD 188753

3.1 The system HD 188753

HD 188753 (HIP 98001) is a bright triple system, 7.4 V-mag, at a distance of 46 pc (Söderhjelm 1999). Hough (1899) was the first to discover that the system was a visual binary, with a separation between the two components, A and B, of ~0.3 arcsec. The AB pair is characterized by a period of 25.7 yr, a semimajor axis of 12.3 au and an eccentricity of 0.5 (Söderhjelm 1999, see also Konacki 2005). Later Griffin (1977) discovered that the system contained a spectroscopic binary with an orbital period of 155 d and an eccentricity of 0.26.

Recently, HD 188753 attracted interest when Konacki (2005) reported a radial-velocity modulation of the primary-A, with a period of 3.35 d and an amplitude that implied an unseen companion with a minimum mass of 1.14 Jupiter mass. The detection of a close-in Jupiter-like planet in the triple system HD 188753 posed a problem for the widely accepted planet formation theory (e.g. Pfahl 2005; Jang-Condell 2007). However, the existence of the close-in planet is now a matter of debate, after Eggenberger et al. (2007) obtained their own ELODIE spectra, the analysis of which showed no evidence to support the conjecture of a planet in the system.

Eggenberger et al. (2007) used TODMOR to derive the velocities of A, the bright distant star, and Ba, the brighter star of the close pair. The main goal of Eggenberger et al. (2007) was to study the radial velocity of component A, and to confirm or refute the planet conjecture. Nevertheless, their spectra included information on the close pair, which before this work remained a single-lined spectroscopic binary. To demonstrate the capability of TRIMOR, we re-analysed here the spectra obtained by Eggenberger et al. (2007) and derived the velocities of A, Ba and Bb, the latter being the faintest component of the triple system, rendering the close binary a double-lined spectroscopic system.

3.2 The TRIMOR radial velocities

The data we analysed here comprised the ELODIE spectra of HD 188753, obtained by Eggenberger et al. (2007) between 2005 July and 2006 March. ELODIE is a fibre-fed spectrograph at the Observatoire de Haute-Provence (France) with a fibre of 2 arcsec in diameter (Baranne et al. 1996). With a resolution of $\lambda/\Delta\lambda = 42\,000$, it covers the wavelength range 3850–6800 Å with 67 orders. The analysis used 32 ELODIE orders within the spectral range 4810–6800 Å, after having excluded orders that are heavily polluted by telluric lines. We also excluded the blue orders because in these orders we expected the secondary and tertiary signals to be too weak relative to the signal of the hotter and bluer primary.

Our library of templates included high-S/N ELODIE and CORALIE spectra, with spectral types that run from F8 to M6. To choose the best configuration of templates and rotational broadening, we proceeded as detailed above. In our case, the tertiary template was so faint that its impact on the correlation peaks was minimal. Therefore, we carried out the analysis with the different possible tertiary templates through the orbital solution, and chose the one that minimized the residual rms of the tertiary orbit.

We finally chose a G6 dwarf (HD 224752) for A, a K0V dwarf (HD 225208) for Ba and a K7 dwarf (GL 338B) for Bb. As we do not have a table with flux ratios for every pair of spectral types, we used for α our G5K0 table, for which the flux ratio at 6800 Å was 0.53, and a multiplication factor, f_α, of 1.12. For β, we used the G8M0 table, for which the flux ratio at 6800 Å was 0.06, and a multiplication factor, f_β, of 0.91. In our model, the fluxes of the primary, A, and the secondary, Ba, are comparable, while the tertiary, Bb, is much fainter than the other two. The *additional* rotational broadening velocities chosen for the templates used were 0.0, 2.0 and 2.5 km s^{-1} for A, Ba and Bb, respectively.

As in Eggenberger et al. (2007), we also noted that when two of the three stars had similar velocities, TRIMOR sometimes picked the wrong peak of the correlation, yielding erroneous velocities. Three additional spectra that were analysed by Eggenberger et al. (2007) yielded ambiguous results. We preferred not to include these velocities in our analysis. Thus, we ended up using only 35 spectra, with a typical S/N of 55 per pixel at 5500 Å.

Our radial velocities and their error estimate for the three components of HD 188753 are given in Table 1. The original error estimates for the velocities of Ba and Bb turned out to be larger than the actual residuals of the orbital solution. This effect appeared in a few other stellar systems that we have analysed (see also Eggenberger et al. 2007 for similar effect with TODMOR). We, therefore, adjusted the error estimates by multiplying the derived errors of Ba by 0.34 and those of Bb by 0.26, so the normalized χ^2 value of the solution is unity for each of the three stars.

3.3 The orbital solution

To derive the orbital solution, we solved for double-lined solution of Ba and Bb, *simultaneously* with a linear drift of their centre-of-mass velocity. This added one more linear parameter to the function fitted to the radial velocity measurements. For the A velocities, we solved independently for a linear drift only.

The orbital solution based on the TRIMOR velocities is given in Table 2 and plotted in Fig. 1. The linear drift for A and B could barely be detected, because of the relatively short time span of the observations. All our derived values are consistent with the results of Eggenberger et al. (2007). Our error estimates of these values are smaller than those of Eggenberger et al. (2007), indicating another possible advantage of TRIMOR.

The residuals of the three sets of velocities are plotted in Fig. 2. While the residual rms of A and Ba is at the level of 0.1 km s^{-1}, the rms of the Bb residuals is of the order of 0.5 km s^{-1}. This is consistent with the relative brightness of the three stars.

The new double-lined solution of the B system yields directly its mass ratio:

$$q_{\rm B} = \frac{m_{\rm Bb}}{m_{\rm Ba}} = 0.768 \pm 0.004. \tag{4}$$

Table 1. The derived radial velocities for HD 188753.

JD 2450 000	Primary (km s^{-1})	Secondary (km s^{-1})	Tertiary (km s^{-1})
3575.472	-22.826 ± 0.094	-36.766 ± 0.065	-2.44 ± 0.41
3576.470	-22.811 ± 0.088	-36.632 ± 0.061	-2.66 ± 0.39
3577.478	-22.818 ± 0.096	-36.605 ± 0.065	-2.16 ± 0.42
3585.448	-22.826 ± 0.105	-34.441 ± 0.079	-4.76 ± 0.47
3586.457	-22.874 ± 0.118	-34.079 ± 0.086	-5.84 ± 0.56
3587.405	-22.870 ± 0.104	-33.657 ± 0.076	-5.98 ± 0.49
3588.433	-22.953 ± 0.088	-33.341 ± 0.059	-7.10 ± 0.43
3589.484	-22.916 ± 0.119	-33.026 ± 0.082	-6.68 ± 0.54
3590.467	-22.864 ± 0.108	-32.461 ± 0.077	-7.74 ± 0.48
3591.505	-22.866 ± 0.101	-32.078 ± 0.070	-8.27 ± 0.45
3594.487	-22.861 ± 0.097	-30.890 ± 0.067	-10.27 ± 0.43
3595.420	-22.801 ± 0.092	-30.462 ± 0.065	-10.41 ± 0.43
3596.448	-22.794 ± 0.095	-29.985 ± 0.066	-10.73 ± 0.47
3667.380	-22.782 ± 0.091	-10.066 ± 0.066	-36.27 ± 0.47
3669.299	-22.777 ± 0.080	-10.168 ± 0.058	-35.60 ± 0.44
3686.258	-22.823 ± 0.082	-14.845 ± 0.057	-30.57 ± 0.36
3690.264	-22.971 ± 0.085	-16.871 ± 0.062	-28.18 ± 0.45
3693.256	-23.008 ± 0.095	-18.548 ± 0.068	-26.47 ± 0.47
3694.234	-23.021 ± 0.096	-19.150 ± 0.066	-25.62 ± 0.44
3711.296	-22.977 ± 0.107	-31.103 ± 0.076	-9.27 ± 0.56
3714.245	-23.142 ± 0.123	-33.111 ± 0.087	-7.43 ± 0.64
3715.215	-23.108 ± 0.093	-33.523 ± 0.065	-6.50 ± 0.46
3719.212	-23.107 ± 0.106	-35.348 ± 0.078	-3.58 ± 0.52
3723.247	-23.029 ± 0.087	-36.272 ± 0.063	-2.20 ± 0.41
3724.234	-23.026 ± 0.105	-36.443 ± 0.074	-2.02 ± 0.48
3728.231	-23.039 ± 0.084	-36.768 ± 0.058	-1.75 ± 0.38
3807.689	-22.980 ± 0.123	-10.297 ± 0.087	-35.43 ± 0.59
3809.682	-22.954 ± 0.123	-10.015 ± 0.082	-35.93 ± 0.58
3810.675	-23.002 ± 0.115	-9.921 ± 0.077	-36.45 ± 0.55
3871.585	-23.375 ± 0.091	-34.266 ± 0.062	-4.46 ± 0.39
3873.594	-23.373 ± 0.125	-35.187 ± 0.082	-2.95 ± 0.53
3895.597	-23.438 ± 0.097	-33.774 ± 0.067	-5.04 ± 0.44
3896.581	-23.439 ± 0.091	-33.390 ± 0.063	-5.65 ± 0.40
3899.588	-23.363 ± 0.088	-32.074 ± 0.061	-7.08 ± 0.39
3900.559	-23.423 ± 0.133	-31.911 ± 0.089	-7.16 ± 0.68

Table 2. Orbital parameters for the 154 d spectroscopic orbit of HD 188753 B. The long-period orbital motion of the AB pair was taken into account by including a linear drift.

Parameter	Units	Value
P	(d)	154.448 ± 0.015
T	(JD 2400 000)	$53\,405.007 \pm 0.032$
e		0.175 ± 0.001
γ	(km s^{-1})	-21.623 ± 0.014
ω	(°)	134.87 ± 0.02
K_1	(km s^{-1})	13.479 ± 0.017
K_2	(km s^{-1})	17.63 ± 0.09
Linear drift A	(km s^{-1} yr^{-1})	0.619 ± 0.056
Linear drift B	(km s^{-1} yr^{-1})	0.346 ± 0.039
N_{meas}		35
rms (A)	(m s^{-1})	98.3
rms (Ba)	(m s^{-1})	72.1
rms (Bb)	(m s^{-1})	455.6

The mass ratio of the wide orbit is

$$q_{\text{AB}} = \frac{m_{\text{B}}}{m_{\text{A}}} = 1.79 \pm 0.26. \quad (5)$$

The latter has a large error, because of the short time span of the observations, but still is in agreement with Eggenberger et al. (2007), who derived the linear drift of A with a different way.

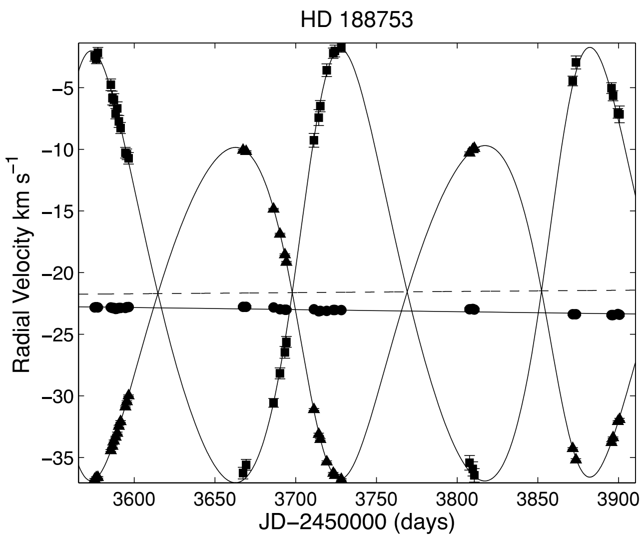

Figure 1. Orbital solution for HD 188753. The circles refer to A, triangles to Ba and squares to Bb. Error bars in A are smaller than the size of the data points. The solid straight line represents the 25.7 yr orbital motion of A and the dashed line that of B.

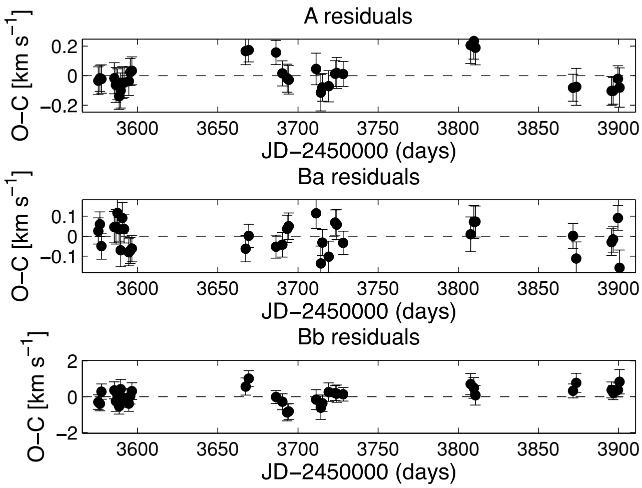

Figure 2. The residuals of the orbital solution in Fig. 1. The rms for A is 98 m s^{-1}, for Ba is 71 m s^{-1} and for Bb is 0.5 km s^{-1}.

3.4 Search for evidence of a planet

To search for an evidence of the planet suggested by Konacki (2005), we performed a search for periodicity in our derived radial velocities of HD 188753 A, as did Eggenberger et al. (2007). In fact, the residuals of A were slightly larger than those of B, and therefore could hide additional real variability. To search for *any* periodicity, we derived a double-harmonic periodogram of the residuals (Shporer et al. 2007), by fitting for each trial period a double-harmonic function. The periodogram value for each trial period was taken as the power (sum of the squared coefficients of the two harmonics) divided by the χ^2 goodness-of-fit parameter of the fitted light curve. We preferred this analysis to the frequently used Lomb–Scargle test (e.g. Scargle 1982), as the radial velocity of the presumed additional planet might not have a perfect circular orbit, and therefore might have a periodic modulation with a few harmonics. Our method is a further generalization of Zechmeister & Kürster (2009) method.

To estimate the statistical significance of a peak with a height S, we analysed 10 000 randomized set of residuals and found the fraction of periodograms that had a peak higher than S. The randomized residuals were derived by taking the actual residuals, with the actual timings randomly permutated. Assuming a negligible red noise in the data (e.g. Pont, Zucker & Queloz 2006), this test gives the statistical significance of any detection.

The computed periodogram of the residuals around the linear trend for component A are plotted in Fig. 3. Our velocities for HD 188753 A show no sign of a short-period signal with the period of ~3 d, although some variability with a period of about 20 d might be present in the data. However, the periodagram peak might reflect the window function of the data, and in any case is not significant enough to draw any conclusion without further observations.

4 THE RELATIVE INCLINATION OF THE TWO ORBITS

HD 188753 is a special case, for which accurate astrometric, photometric and spectroscopic information is available. We wish to use this information and estimate the degree of alignment between the orbital planes of the wide (astrometric) and the close (spectroscopic) orbits. Konacki (2005) shortly addressed this issue, but he did not have yet a direct measurement of the mass ratio of the close pair.

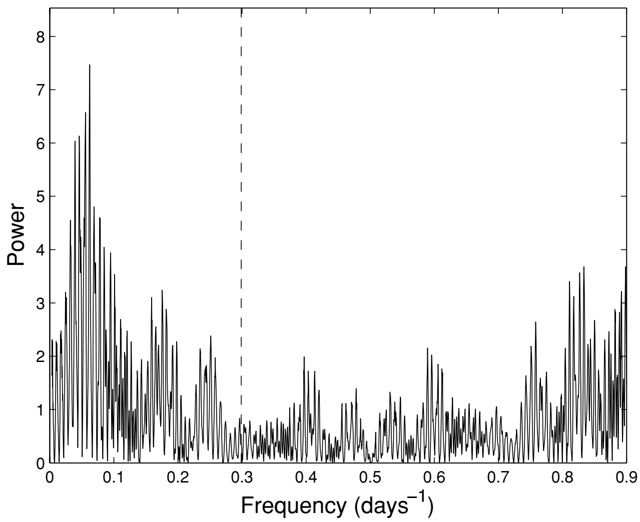

Figure 3. Double-harmonic periodogram of the residuals around the linear trend for HD 188753 A. For this data set, the 1 per cent false alarm probability corresponds to a power of 8.58 and is represented by the top of the box. The dashed line denotes the frequency of the planetary signal reported by Konacki.

Söderhjelm (1999) estimated the inclination of the wide orbit at about 34°. We need now to derive the inclination of the spectroscopic binary, so we can estimate the relative inclination.

For any double-lined spectroscopic binary, the orbital inclination can be written as

$$\sin i = 4.7 \times 10^{-3} \sqrt{1-e^2} \left(\frac{K_1 + K_2}{1\,\mathrm{km\,s^{-1}}}\right) \left(\frac{P}{1\,\mathrm{d}}\right)^{1/3} \times \left(\frac{M_1 + M_2}{\mathrm{M}_\odot}\right)^{-1/3}. \quad (6)$$

In this formula, only the total mass of the spectroscopic binary is not known from the radial-velocity data. Deriving the total mass of the binary from the information not included in the orbital solution leads to an estimate of the binary inclination.

We will use here three approaches to estimate the B-component mass. As in many SB2s, we can derive the total mass, or at least estimate it, through analysing the spectra of the two stars. In our case, we can take advantage of two additional known features of HD 188753. First, the binary is a component of a wide orbit with known relative astrometric orbit. Therefore, the total mass of the triple system is known. To derive the mass of the close pair, we only need the mass ratio of the wide orbit, which we can obtain from the spectroscopic data, provided their time span is long enough (e.g. Eggenberger et al. 2007). Secondly, the parallax of the system is known (Söderhjelm 1999), because A is bright enough to be measured by *Hipparcos*. Using the parallax, we can derive the expected magnitude of HD188753Ba and HD188753Bb for each assumed mass of the Ba component, using the mass ratio, q_{B}, obtained from the radial-velocity solution. We can then compare the expected magnitude of the combined light of Ba and Bb with the observed J, H and K_s Two Micron All Sky Survey (2MASS) magnitudes of B obtained by Konacki (2005). In this way, we can find the actual mass of Ba, the single remaining independent parameter in this exercise.

Table 3 presents the results of the three approaches. For the spectral analysis approach, we utilized the spectral types we used in our TRIMOR analysis (K0V and K7V) and a standard calibration (Habets

Table 3. The total mass of the close binary and its inclination, based on three approaches.

Approach	$M_{\mathrm{Ba}} + M_{\mathrm{Bb}}$ ($\mathrm{M}_{\odot}$)	$\sin i$	i
Spectral types obtained by TRIMOR	1.40 ± 0.15	0.690 ± 0.026	$43\overset{\circ}{.}6 \pm 2\overset{\circ}{.}1$
Astrometry of the visual pair	1.78 ± 0.31	0.637 ± 0.038	$39\overset{\circ}{.}6 \pm 2\overset{\circ}{.}8$
2MASS photometry	1.634 ± 0.019	0.6557 ± 0.0079	$40\overset{\circ}{.}97 \pm 0\overset{\circ}{.}60$

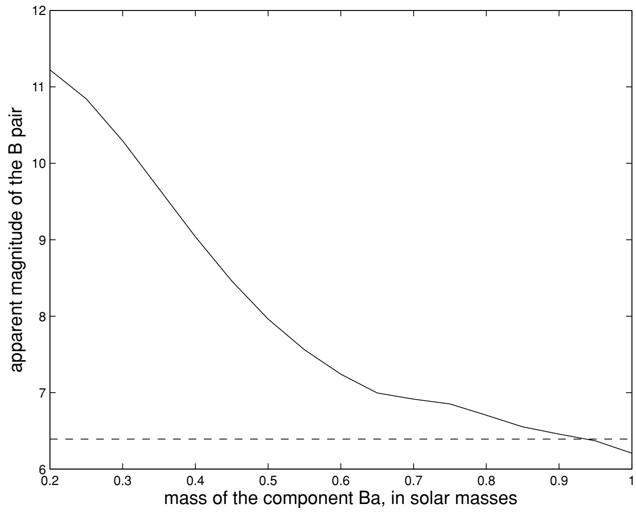

Figure 4. For each assumed mass of the Ba component, the plot shows the derived K_s magnitude (in the 2MASS K_s band), assuming the known mass ratio q_{B}, and the known distance modulus of 3.31. The dashed line represents the observed K_s magnitude.

& Heintze 1981) to convert them to masses. For the astrometric approach, we used the total mass of the system, 2.73 $\mathrm{M}_{\odot}$ (Söderhjelm 1999), and the mass ratio derived by Eggenberger et al. (2007). For the photometric approach, we used a parallax of $\pi = 21.9 \pm 0.6$ mas, the photometric calibration of Bessell & Brett (1988) and the 2MASS correction of Carpenter (2001). Fig. 4 demonstrates this procedure for the K_s band. Error estimates are not readily available for the photometric calibrations, but we can use the variance among the results from the three photometric bands as a proxy to the error.

The results of the three approaches show a remarkable consistency, indicating that the inclination of the close pair is around 41°. It is not easy to estimate the uncertainty of the derived inclination, as the biases of the approaches are not well known. Arbitrarily, we assign an error of 3° to our estimate of the binary inclination.

The close-pair inclination found here, 41° ± 3°, is close to 34°, the value given by Söderhjelm (1999) for the wide orbit inclination. This means that the minimal relative inclination (MRI) between the two orbits is around 7°. Söderhjelm (1999) values are not accompanied by error estimates, and therefore it is difficult to assign an uncertainty to the MRI value of the triple system. However, if we arbitrarily assign an error of 3° to the wide orbit inclination, we can deduce an error of ∼5° to our MRI. Our findings are, therefore, consistent with the MRI being close to zero, and therefore the system might have coplanar configuration.

5 DISCUSSION

In this paper, we laid out the principles of TRIMOR, an algorithm to analyse multi-order spectra of triple systems and deduce the radial velocities of the three components. A few improvements of the algorithm could still be applied, one of which is the derivation of the flux ratios of each order. Instead of relying on pre-determined ratios, based on the template spectral types, one could derive the flux ratios from the observed spectra, as done in TODCOR and TRICOR. The best flux ratios could be derived by maximizing the correlation, using all observed spectra and therefore can, with high enough S/N, yield better results. Another improvement, also associated with the flux ratio, is to apply a ratio that is varying along each order. That is, instead of the model of equation (1) for each order, we can use a modified model:

$$R^{(3)}[s_1, s_2, s_3; \alpha(n), \beta(n)] = f * [g_1(n - s_1) + \alpha(n) g_2(n - s_2) + \beta(n) g_3(n - s_3)]. \qquad (7)$$

The functions $\alpha(n)$ and $\beta(n)$ could be approximated by low-degree polynomials. In fact, such an improvement can be applied to TODCOR, TRICOR and TODMOR as well.

We have applied TRIMOR to the obtained spectra of HD 188753. Without investing any additional observational resources we derived the velocities of the faint component of the triple system and turned the close pair into double-lined spectroscopic binary. The new radial velocities led to an estimate of the close-pair inclination, which depends on the total mass of the close pair. We estimated the total mass of the close pair by three approaches, one that relied on the spectral type of the two stars of the close pair derived here, one that used the astrometry of the wide orbit and the mass ratio of the wide orbit, independent of the present analysis, and finally another approach that depended on the mass ratio of the close pair derived from the new velocities of the present analysis and known photometry of the close pair. All three approaches yielded similar close-pair inclinations that are close to 41°. The consistency of the three estimates of the inclination demonstrated the power of TRIMOR algorithm and established more confidence in the present result.

The derived close-pair inclination suggests that the relative inclination of HD 188753 is close to zero and the motion is close to being coplanar. This result adds HD 188753 to the increasing set of triple systems with known relative inclinations (see reviews by Tokovinin 2004; Tokovinin et al. 2006; Tokovinin 2008). Applying TRIMOR to more close triple or quadruple systems (e.g. Shkolnik et al. 2008), for which we have at hand the astrometric orbit of the wide pair from *Hipparcos*, can enlarge substantially the set of triples with known relative inclinations. This certainly will be true after the new *Global Astrometric Interferometer for Astrophysics* (*GAIA*) satellite all-sky astrometry will be available in the near future (Perryman, de Bruijne & Lammers 2008).

The distribution of the relative inclinations in triple systems can shed some light on formation models of binary and triple systems (e.g. Sterzik & Tokovinin 2002) and can also be relevant to the tidal evolution of close multiple systems (e.g. Mazeh 2008). Binary and multiple system formation models of fragmentation (e.g. Machida 2008), which include a later stage of accretion (e.g. Bate, Bonnell & Bromm 2002; Bonnell & Bate 2006), predict alignment of the rotation stellar axes with the orbital angular vector in binaries, as well as coplanar motion in triple systems. On the other hand, models assuming that binaries and multiple systems are formed by N-body interaction (e.g. Clarke 1996; Delgado-Donate et al. 2004) predict that the stellar rotation might not be aligned with the orbital motion, and triple systems could have high relative inclination.

The relative inclination of triple systems might also have implication for long-term evolution of systems for which tidal evolution of the close pair can circularize the binary orbit during its stellar life time. Mazeh & Shaham (1979) found that the third star can induce almost periodic eccentricity modulation into the binary motion, even if the binary starts with zero eccentricity (see Kozai 1962, who discovered this effect for asteroids with initial eccentricities). Mazeh & Shaham (1979) pointed out that the eccentricity modulation implies that binaries with close enough distant companions never get into a stable circular orbit, which means that the binary tidal dissipation continues to be active forever. The injected binary eccentricity invokes frictional forces inside the two stars that dissipate rotational energy and transfer angular momentum between the stellar rotation and the binary orbital motion. This causes the semimajor axis of the binary to shrink on the circularization time-scale.

The possible impact of the third star on the shrinking of the close-pair separation depends on the amplitude of the eccentricity modulation. Mazeh & Shaham (1979) showed that this amplitude does not depend on the third distant star separation, but strongly depend on the relative inclination between the two orbits. Eggleton & Kiseleva-Eggleton (2001), Eggleton (2006), Borkovits, Forgács-Dajka & Regály (2004) and Fabrycky & Tremaine (2007) further studied the effect found by Mazeh & Shaham (1979), recently termed as the Kozai Cycle with Tidal Friction (KCTF) effect, and showed that the tidal effect that leads to the shrinking of the close-pair separation occurs when the relative angle between the two orbital planes of motion is larger than $\sim 40°$. In fact, Tokovinin et al. (2006) suggested that all or at least most of the very close binaries are found with a distant companion, suggesting that they were formed through the effect suggested by Mazeh & Shaham (see also Tokovinin 2008, for an updated statistical analysis). It is, therefore, important to get the true distribution of relative inclinations of triple systems, with close pairs in particular. Cases like HD 188753, with small relative angle, do not support the shrinking scenario of close binaries in triple systems, as the eccentricity modulation expected in such cases is small. As we have shown here, TRIMOR can be very effective in obtaining the relevant information.

TRIMOR can also be effective in searching spectroscopic evidence for unknown faint third companions in spectra of known double-lined binaries. The accumulated data base of multi-order spectra of binaries can be used for such a project. A somewhat similar approach, with a different algorithm, was taken by D'Angelo, van Kerkwijk & Rucinski (2006), who searched for spectroscopic trace of a faint tertiary in a large set of observed spectra of contact binaries. In about a third of the systems they found an evidence for a tertiary. We note that their spectra are composed of one order centred on 5200 Å, a wavelength for which K and M stars, for example, are quite faint. In our case, where we had the luxury of using CORALIE spectra whose orders get up to 6800 Å, we could detect and measure radial velocities of a late K-type tertiary star. The search for faint companions could be even more efficient with spectra that go further towards the red wavelegths (e.g. Fabricant et al. 2008).

Finally, TRIMOR can help find extra-solar planets in triple systems, in the same way TODMOR was instrumental in discovering the planet in the HD 41004 system (Zucker et al. 2003, 2004).

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