Magnetically moderated outbursts of WZ Sagittae

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ABSTRACT

We argue that the quiescent value of the viscosity parameter of the accretion disc in WZ Sge may be $\alpha_{cold} \sim 0.01$, in agreement with estimates of α_{cold} for other dwarf novae. Assuming the white dwarf in WZ Sge to be magnetic, we show that, in quiescence, material close to the white dwarf can be propelled to larger radii, depleting the inner accretion disc. The propeller therefore has the effect of stabilizing the inner disc and allowing the outer disc to accumulate mass. The outbursts of WZ Sge are then regulated by the (magnetically determined) evolution of the surface density of the outer disc at a radius close to the tidal limit. Numerical models confirm that the recurrence time can be significantly extended in this way. The outbursts are expected to be superoutbursts since the outer disc radius is forced to exceed the tidal (3:1 resonance) radius. The large, quiescent disc is expected to be massive, and to be able to supply the observed mass accretion rate during outburst. We predict that the long-term spin evolution of the white dwarf spin will involve a long cycle of spin-up and spin-down phases.

Key words: accretion, accretion discs – binaries: close – stars: individual: WZ Sge – stars: magnetic fields.

1 INTRODUCTION

The SU UMa stars are a subclass of dwarf novae (DN) whose light curves exhibit superoutburst behaviour. Superoutbursts typically occur at intervals of several months, interspersed with normal outbursts every few weeks. In general, superoutbursts are around one magnitude brighter than normal outbursts and last a few weeks rather than 2–3 d. WZ Sagittae is an unusual SU UMa star because the recurrence time is extremely long and no normal outbursts are observed between superoutbursts. Superoutbursts occurred in 1913, 1946, 1978 and 2001, with the last of these coming somewhat earlier than expected after the previously regular recurrence time $t_{rec} \sim$ 33 yr.

Normal outbursts in DN can be explained by a thermal-viscous disc instability due to the partial ionization of hydrogen (see e.g. Cannizzo 1993, for a review). The standard disc instability model (DIM) assumes that the viscosity of the disc material increases by around one order of magnitude during outburst, enhancing the mass-flow rate through the disc. The usual values adopted for the Shakura & Sunyaev (1973) viscosity parameter are $\alpha_{cold} \sim$ 0.01 in quiescence and $\alpha_{hot} \sim 0.1$ in outburst. In an extension to this theory, Osaki (1995) proposed that superoutbursts are due to a thermal-tidal instability. During each normal outburst, only a small

fraction of the total disc mass is deposited on to the white dwarf (WD), leading to an accumulation of disc mass and angular momentum. The outer disc radius therefore expands until the tidal radius is reached. At this point, enhanced tidal interaction with the secondary star is thought to increase dissipation in the disc and so raise the mass accretion rate. A superoutburst may then be triggered. This model successfully explains the superoutburst behaviour of ordinary SU UMa stars using the same viscosity parameters as the standard DIM.

Smak (1993) argued that the viscosity in WZ Sge must be far lower than the standard values (so that $\alpha_{cold} \leqslant 5 \times 10^{-5}$), for the following reasons.

Smak 1. In the standard DIM, the interoutburst recurrence time is governed by the viscous time-scale of the inner accretion disc, which is far shorter than the observed value of ~30 yr. Lowering α_{cold} slows down the viscous evolution of the accretion disc, increasing the interoutburst time.

Smak 2. Integrating the critical surface density using the standard DIM α_{cold} gives a maximum disc mass that is far lower than the observed estimate of the mass accreted during superoutburst, $\Delta M_{acc} \sim 10^{24}$ g. Lowering the viscosity allows more mass to be accumulated in the outer regions of the accretion disc to fuel the outburst accretion rate.

There is, however, no obvious reason why the quiescent accretion disc in WZ Sge should have a much lower viscosity than the other SU UMa stars. This has led a number of authors to suggest other

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reasons to account for the long recurrence time and high disc mass prior to superoutburst.

In order to produce a long recurrence time without requiring a low α_{cold}, the inner, most unstable regions of the disc must be stabilized. Hameury, Lasota & Hure (1997) and Warner, Livio & Tout (1996) find that the disc can be stabilized if the inner regions are removed, either by evaporation into a coronal layer (see also Meyer, Meyer-Hofmeister & Liu 1999), or by the presence of a magnetosphere. This need not necessarily lead to an increased recurrence time, however. Models by Hameury et al. (1997) suggest a marginally stable disc that requires an episode of enhanced mass transfer to trigger a disc outburst. The recurrence time in this model is governed by the mass transfer fluctuation cycle, which has no obvious physical connection with the regular 33-yr outburst cycle observed in WZ Sge. Warner et al. (1996) also find that under certain conditions the disc will be marginally unstable and produce outside-in outbursts with the required recurrence time (we explore the case of a marginally stable disc in Section 3.3). However, neither of these models addresses the problem of how to accumulate enough disc mass during quiescence to explain the mass accreted in outburst (Smak 2). Only $\sim 10^{21}$ g is available in the disc just prior to outburst so that both of the above models require mass to be added to the disc during the outburst. The authors appeal to irradiation of the secondary star to increase the transfer rate during outburst and supply the missing mass. Warner et al. (1996) only produce normal outbursts, not superoutbursts in their model. The reason for this is that their disc radius $R_{\text{disc}} \sim 1.1 \times 10^{10}$ cm is far smaller than the tidal radius, so enhanced tidal interaction with the secondary (required to produce a superoutburst) is not possible.

In this paper, we argue that the value of α_{cold} in WZ Sge may be consistent with standard DIM values ($\alpha_{\text{cold}} \approx 0.01$). In Section 2, we perform a similar calculation to that of Smak (1993) adopting the system parameters suggested by Spruit & Rutten (1998) (hereafter SR). For a fixed disc mass, we find that $\alpha_{\text{cold}} \propto R_{\text{out}}^{3.9}$ (where R_{out} is the outer disc radius), and show that a value of $\alpha_{\text{cold}} \approx 0.01$, and a moderate increase in R_{out} results in a value for ΔM_{acc} consistent with observation. We then obtain a similar value for α_{cold} by estimating the cool viscous time-scale from the observed superoutburst profile.

X-ray observations of WZ Sge by Patterson (1998) have revealed a coherent 27.86-s oscillation in the ASCA 2–6 keV energy band (see also Patterson 1980). The most convincing explanation for these data is that WZ Sge contains a magnetic WD. A paper by Lasota, Kuulkers & Charles (1999) interprets WZ Sge as a DQ Her star and the 27.86-s oscillation as the spin period of the WD (P_{spin}). The authors then suggest that WZ Sge is an ejector system: i.e. most of the material transferred from the secondary is ejected from the system, similar to the case of AE Aqr (Wynn, King & Horne 1997). In this model, an accretion disc is recreated in outburst, which again is triggered by a mass transfer event.

In Section 3, we suggest that if the magnetic torque is not sufficient to eject mass completely from the system, but merely propels it further out into the Roche lobe of the WD (we refer to the system as a weak magnetic propeller), then Smak 1 and Smak 2 can be explained with a standard DIM value for α_{cold}. The injection of angular momentum into the disc by the propeller radically alters the surface density profile, compared with that of a conventional DIM disc. The inner disc in such a system is empty, and outbursts cannot therefore be triggered there in the normal way. The propeller also inhibits accretion, and mass accumulates in the outer disc, increasing the recurrence time significantly. The disc may also be forced to expand to the tidal radius (and a little beyond) which could allow further material to accumulate in the outer regions of the disc before

triggering an outburst. This would increase the recurrence time and outburst mass still further, solving Smak 1 and Smak 2. The outburst is initiated as in the standard DIM, and does not require any episode of enhanced mass transfer. Simulations are performed in one and two dimensions. The former are computationally cheap, so that we can present several outburst cycles, but cannot account for the enhanced mass storage due to tidally drive disc expansion.

In Section 4, we discuss the observational evidence for a truncated accretion disc with reference to observed spin periods and spectral data. Both these results support are in agreement with a truncated disc. In Section 5, we consider the spin evolution of the WD, and whether a spin cycle might exist. Finally, in Section 6 the applications to other CVs are discussed.

2 THE QUIESCENT VALUE OF α IN WZ SGE

2.1 The quiescent disc mass as a limit on α_{cold}

Following Smak (1993), we estimate a value for α_{cold} by considering the disc mass immediately prior to outburst. An outburst is triggered when the surface density at some radius $\Sigma(R)$ becomes greater than the maximum value (Σ_{crit}) allowed on the cool branch of the Σ–T relation (e.g. Cannizzo 1993) at that radius. Therefore, the maximum possible disc mass prior to outburst is given by

$$M_{\text{max}} \simeq \int_{R_{\text{in}}}^{R_{\text{out}}} 2\pi R \, \Sigma_{\text{crit}}(R) \, \text{d}R, \tag{1}$$

where R_{in} and R_{out} are the inner and outer disc radii, respectively. Calculations of the vertical disc structure yield (e.g. Ludwig, Meyer-Hofmeister & Ritter 1994)

$$\Sigma_{\text{crit}}(R) = 670 \, M_1^{-0.37} \, R_{10}^{1.10} \, \alpha_{\text{c,0.01}}^{-0.8} \, \mu^{0.4} \text{ g cm}^{-2}, \tag{2}$$

where M_1 is the mass of the WD in solar masses, $R_{10} = R/10^{10}$ cm, $\alpha_{\text{c,0.01}} = \alpha_{\text{cold}}/0.01$ and μ is the mean molecular weight. Assuming that $\mu \sim 1$ and setting $R_{\text{in}} = 0$, equations (1) and (2) can be combined to give

$$M_{\text{max}} \simeq 1.36 \times 10^{23} M_1^{-0.37} \alpha_{\text{c,0.01}}^{-0.8} \, R_{\text{out,10}}^{3.1}. \tag{3}$$

Although WZ Sge has a long observational history, the values of some of its fundamental parameters are still uncertain. For instance, Smak (1993) deduces that $M_1 = 0.45$ and $q = M_2/M_1 = 0.13$, whereas the spectroscopic observations of SR lead to values of $M_1 = 1.2$ and $q = 0.075$. Lasota et al. (1999) point out that a WD mass of $M_1 = 0.45$ is too small if it is assumed to be rotating with a period of 27.87 s (although $M_1 = 1.2$ is possibly too high), so for the purposes of this paper we shall adopt the system masses suggested by SR. We also adopt an orbital period of 82 min.

Smak (1993) estimated the total mass accreted during outburst to be around $\Delta M_{\text{acc}} \sim 1$–$2 \times 10^{24}$ g. Setting $\Delta M_{\text{acc}} \lesssim M_{\text{max}}$, and using the system parameters $M_1 = 1.2$, $q = 0.075$ and $R_{\text{out}} = 0.37a = 1.7 \times 10^{10}$ cm (SR), we find from equation (3) that $\alpha_{\text{cold}} \lesssim 0.006$. While this value is around a factor of 2 lower than the standard DIM value of $\alpha_{\text{cold}} \sim 0.01$, it is two orders of magnitude higher than the value obtained by Smak. However, setting $M_{\text{max}} \approx \Delta M_{\text{acc}}$, we find $\alpha_{\text{cold}} \propto R_{\text{out}}^{3.9}$, i.e. α_{cold} is very sensitive to disc radius. Therefore, any error in the observationally inferred value will produce a large discrepancy in α_{cold}. The observational estimate in SR ($R_{\text{out}} = 0.37a$) was determined from models of the accretion stream disc impact region, which was found to be rather extended in the case of WZ Sge. A theoretical estimate of the tidal radius can be found by utilizing smoothed particle hydrodynamics (SPH) calculations assuming the system parameters detailed above. This approach yields
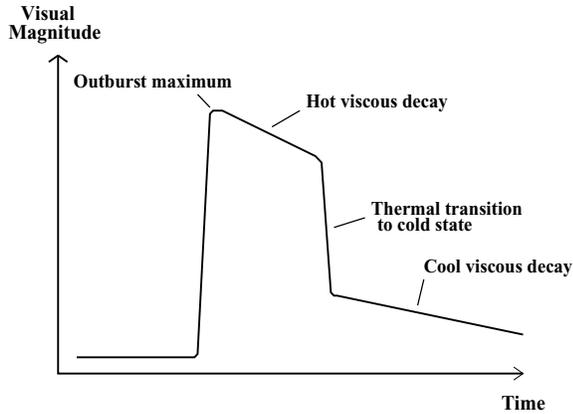
Figure 1. A schematic diagram of a superoutburst profile.

a tidal radius of $R_{out} \simeq 0.5a$ (see Section 3.4). Assuming this value for R_{out} and repeating the above calculation, we find that $\alpha_{cold} \lesssim 0.02$, in good agreement with the standard DIM value. We note that the analytic treatment of Osaki & Meyer (2002) gives a similar R_{out} when the same parameters are used.

2.2 Extracting α_{cold} from the outburst light curve

A second, independent estimate of α_{cold} can be obtained using the observed outburst profiles (Patterson et al. 1981). Superoutbursts have similar observational profiles to the outbursts of X-ray transients (King & Ritter 1998), i.e. they have a rapid rise to outburst maximum, followed by an exponential decay on the hot viscous time-scale until the disc is no longer hot enough to remain fully ionized and falls into quiescence. This produces a steep decline in brightness on a thermal time-scale followed by a slower decline as the disc readjusts on the cool viscous time-scale as shown in Fig. 1.

Fig. 2 (from Kuulkers 2000) presents the visual light curves for the 1913, 1946 and 1978 outbursts. It is possible to determine α_{cold} by considering the visual luminosity as the disc returns to quiescence, i.e. $\sim$30–90 d after the outburst began. This is more difficult in the case of the 2001 outburst since the light curve is complicated by the
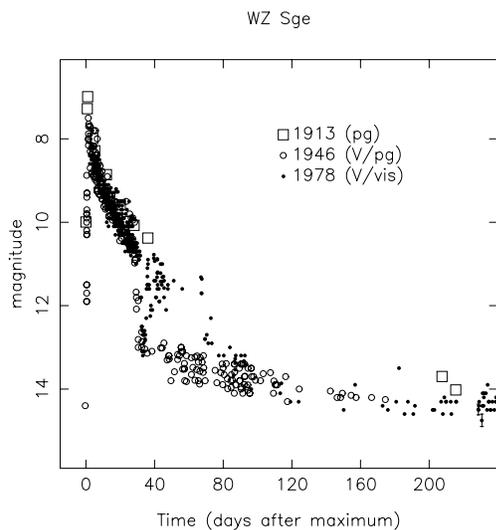
Figure 2. A plot of optical magnitude during the 1913, 1946 and 1978 outbursts of WZ Sge, from Kuulkers (2000). Reprinted with permission from Elsevier.

presence of echo outbursts. The disc brightness decays exponentially on the cool viscous time-scale $t_{visc,c}$ (this is clearly seen in the 1946 outburst, but is less obvious in 1978), such that the visual luminosity is given by

$$F(t) = F_0 \exp\left(\frac{-t}{t_{visc,c}}\right), \tag{4}$$

where t is the time and the cold viscous time-scale is given by

$$t_{visc,c} \sim \frac{R^2}{\nu_c}. \tag{5}$$

The cold state viscosity is given by (Shakura & Sunyaev 1973)

$$\nu_c \sim \alpha_{cold} c_s H. \tag{6}$$

Here, c_s is the adiabatic sound speed and H is the disc scale height. During quiescence, the disc temperature is $T \sim 5000 \pm 1000$ K. If we assume that $\mu \sim 1$, the sound speed is then $c_s \simeq \sqrt{\frac{kT}{\mu m_H}} \sim 6.4 \times 10^5$ cm s^{-1}. We consider the part of the disc from which the majority of the emission should come during the cold viscous decay, namely $R \sim 2 \times 10^{10} \pm 1 \times 10^{10}$ cm. Viscous dissipation is given by

$$D(R) = \frac{9}{8}\nu\Sigma\Omega, \tag{7}$$

where Ω is the disc angular frequency (e.g. Frank, King & Raine 2002). The very inner part of the disc is evacuated, while the outer part has a low angular frequency so that our estimate for R seems sensible. We further assume that $H/R \sim 0.10 \pm 0.05$ for a geometrically thin disc. From the 1946 outburst light curve, it can be seen that, as the disc falls into quiescence, the visual flux drops by one magnitude in $\sim$30 $\pm$ 40 d. From equation (5), we find $\alpha_{cold} \sim 0.028 \pm 0.022$, in reasonable agreement with the estimate in Section 2.1.

3 A MAGNETIC SOLUTION?

3.1 WZ Sge as a magnetic propeller

In Sections 2.1 and 2.2, we have argued that the value of α_{cold} in WZ Sge may not be different from that in other DN. This is consistent with the observed ΔM_{acc}, if $R_{out} \sim 0.5a$, and with the decay time of outbursts. However, the recurrence time problem (Smak 1) still remains unresolved.

The observation of a 27.87-s oscillation in the quiescent X-ray light curve of WZ Sge (Patterson 1998) suggests that the WD is magnetic. The introduction of a magnetic field to a rapidly rotating WD raises the possibility of increasing the recurrence time by removing the inner, most unstable regions of the accretion disc. This is a result of the magnetic torque which can, rather than accreting the inner disc, force mass close to the WD to larger radii. This magnetic propeller effect occurs when the stellar magnetic field moves more rapidly than the disc material so that the disc gains angular momentum at the expense of the star.

The inner disc can be depleted by the propeller and the surface density profile of the disc will then be altered as material is forced to orbit further out in the disc, at radii close to the tidal limit. The magnetic propeller inhibits accretion so that mass is forced to build up in the outer disc, where the critical surface density is higher and the Σ evolution time-scale ($\Sigma_{crit}/\dot{\Sigma}$) is longer. The magnetic propeller effect cannot therefore only increase t_{rec} but can also allow the disc to accumulate enough mass prior to outburst to supply an increased ΔM_{acc}, with the outer disc acting as a large reservoir of mass. In this way, both Smak 1 and Smak 2 can be explained simultaneously

with a standard value for α_{cold}, and without appealing to episodes of enhanced mass transfer.

It should be noted that disc truncation is not normally, in itself, sufficient to increase recurrence times. Menou et al. (2000) show this by considering the evaporation of the inner disc. The fundamental difference between the magnetic propeller and other mechanisms such as evaporation and magnetically enhanced accretion is that the propeller causes a build-up of mass in the outer disc by preventing accretion on to the WD (during quiescence only). The overall profile of the disc is therefore completely altered, permitting much more mass to be stored in the disc. The mechanism presented here is analogous to that which was proposed by Matthews, Speith & Wynn (2004) to explain long recurrence times in FU Ori type young stellar objects.

The magnetic propeller should, in forcing the disc outwards, aid access to the 3:1 resonance. It is likely therefore that further mass will be stored in the outer disc, as proposed by Osaki (1995), providing an even larger reservoir ΔM for mass between outbursts. This tidal effect may therefore amplify the result of the weak propeller driving recurrence times to become even longer.

In the remainder of this section, we outline the theoretical basis for the magnetic propeller model, and present the results of numerical experiments.

3.2 The magnetic model

We adopt a description for the magnetic interaction which assumes that as material moves through the magnetosphere it interacts with the local magnetic field via a velocity-dependent acceleration of the general form:

$$a_{\text{mag}} = -k\,(\boldsymbol{v} - \boldsymbol{v}_{\text{f}})_{\perp}, \tag{8}$$

where $\boldsymbol{v}$ and $\boldsymbol{v}_{\text{f}}$ are the velocities of the material and magnetic field, respectively, and the suffix $\perp$ refers to the velocity components perpendicular to the field lines. A version of this description has been successfully applied to a number of other intermediate polars (IPs) by King (1993), Wynn & King (1995), Wynn et al. (1997) and King & Wynn (1999). It has also been applied to discs around young stars by Matthews et al. (2004, 2005). The magnetic acceleration in equation (8) is intended to represent the dominant term of the magnetic interaction, with k playing the role of a 'magnetic α'. The magnetic time-scale can be written in terms of k as

$$t_{\text{mag}} = k^{-1}\,\frac{|\boldsymbol{v}|}{|\boldsymbol{v} - \boldsymbol{v}_{\text{f}}|_{\perp}}. \tag{9}$$

It is assumed that the magnetic field of the WD is dipolar and that it rotates as a solid body with the star. We refer to a system as a weak magnetic propeller when, in quiescence, the magnetic field is strong enough for the system to behave as a magnetic propeller, but not strong enough to eject a significant fraction of the transferred mass from the Roche lobe of the primary (as is the case in AE Aqr). During outburst, the field should not be strong enough to prevent accretion. This requires the following hierarchy of time-scales:

$$t_{\text{visc,h}} \lesssim t_{\text{mag}} \lesssim t_{\text{visc,c}}, \tag{10}$$

where $t_{\text{visc,h}}$ and $t_{\text{visc,c}}$ are the hot and cold viscous time-scales, respectively. Assuming the temperature of the quiescent disc to be $T \sim 5000$ K and $\alpha_c = 0.01$, and that in outburst we have $T \sim 5 \times 10^4$ K and $\alpha_h = 0.1$, equation (5) gives

$$1 \lesssim t_{\text{mag}} \lesssim 40 \text{ d}. \tag{11}$$

A further condition for a system to act as a weak propeller is that the magnetic interaction takes place at radii exceeding the corotation radius $R_{\text{co}} = (GM_1 P_{\text{spin}}^2/4\pi^2)^{1/2}$, where the field lines rotate with the local (circular) Kepler velocity. A spin period of 27.87 s for the WD in WZ Sge gives $R_{\text{co}} \gtrsim R_{\text{WD}} \sim 10^9$ cm, easily fulfilling this requirement. If, conversely, the interaction were to take place within R_{co} then the effect of the magnetic field would be to enhance accretion.

It now remains to quantify t_{mag}. There are a number of different models for the interaction of the inner disc and the WD magnetosphere which have been applied to magnetic cataclysmic variables (CVs). Here we examine the two extreme cases in which the accretion disc is completely magnetized, and conversely, in which the disc is diamagnetic. In both cases, we still assume that the stellar field has a dipolar geometry, and that any local field distortions may be treated as perturbations on this structure.

When the magnetic field lines permeate the inner disc any vertical velocity shear tends to amplify the toroidal field component B_φ (e.g. Yi & Kenyon 1997). Diffusive losses counteract this amplification and in steady state, these effects balance to give

$$\left| \frac{B_\varphi}{B_z} \right| = \left| \frac{\gamma\,(\Omega_\star - \Omega_{\text{K}})}{\tau\,\Omega_{\text{K}}} \right|. \tag{12}$$

Here γ is a parameter that accounts for the uncertainty in the vertical velocity shear and is of the order of unity, $\tau \lesssim 1$ represents an uncertainty in the diffusive loss time-scale, whilst $\Omega_\star$ and Ω_{K} are the angular velocities of the WD and disc, respectively. The tension in the field lines results in a magnetic acceleration which may be approximated by

$$a_{\text{mag}} \simeq \frac{1}{\rho_{\text{d}} r_{\text{c}}} \left(\frac{B_z^2}{4\pi} \right) \hat{\boldsymbol{n}}, \tag{13}$$

where ρ_{d} is the disc density, $\hat{\boldsymbol{n}}$ is the unit vector perpendicular to the field lines and r_{c} is the radius of curvature of the field lines. It is possible to parametrize r_{c} as

$$r_{\text{c}} \sim \epsilon H \left(\frac{B_z}{B_\varphi} \right), \tag{14}$$

where ϵ is a constant of the order of unity. Combining equations (12), (14) and (13), with $H \sim 0.1R$, the magnetic acceleration is given by

$$a_{\text{mag}} \sim -\frac{5\gamma B_z^2}{2\pi R \tau \epsilon \rho_{\text{d}} v_{\text{K}}} (v_{\text{K}} - v_{\text{f}})_{\perp}, \tag{15}$$

and t_{mag} by

$$t_{\text{mag}} \sim \frac{2\pi R \tau \epsilon \rho_{\text{d}} v_{\text{K}}}{5\gamma B_z^2} \frac{|v_{\text{K}}|}{|v_{\text{K}} - v_{\text{f}}|_{\perp}}. \tag{16}$$

At the circularization radius $R_{\text{circ}} \sim 10^{10}$ cm, the disc density is $\rho_{\text{d}} \sim 10^{-6}$ g cm^{-3} (from equation 2, with $H \sim 0.1R$). Assuming $\tau \sim \epsilon \sim \gamma \sim 1$, we are able to constrain the magnetic moment $\mu \sim B_z R^3$ using equation (11), giving

$$2 \times 10^{32} \lesssim \mu \lesssim 10^{33} \text{ G cm}^3. \tag{17}$$

For the opposite case of a diamagnetic flow, we assume that the inner regions of the disc are broken up into a series of individual filaments, as predicated by the analysis of Aly & Kuijpers (1990). In order to follow the evolution of the diamagnetic gas blobs (which are formed well outside the corotation radius), we follow the approach of Wynn et al. (1997). This approach assumes that the diamagnetic blobs interact with the field via a surface drag force, characterized

by the time-scale

$$t_{\rm mag} \sim c_{\rm A} \rho_{\rm b} l_{\rm b} B^{-2} \frac{|\boldsymbol{v}_{\rm K}|}{|\boldsymbol{v}_{\rm K} - \boldsymbol{v}_{\rm f}|_\perp}, \tag{18}$$

where $c_{\rm A}$ is the Alfvén speed in the medium surrounding the plasma, B is the local magnetic field, $\rho_{\rm b}$ is the plasma density and $l_{\rm b}$ is the typical length-scale over which field lines are distorted. Making the approximations that $\rho_{\rm b} l_{\rm b} \sim \Sigma_{\rm crit}(10^{10}\,{\rm cm})$, $c_{\rm A} \simeq c_{\rm s}$ and ${\rm v} \simeq {\rm v}_{\rm K}$, we find that

$$5 \times 10^{30} \lesssim \mu \lesssim 3 \times 10^{31}\,{\rm G\,cm}^3. \tag{19}$$

3.3 One-dimensional numerical results

In this section, we use a one-dimensional numerical model to confirm that it is feasible to create a depleted region in the centre of a quiescent accretion disc with a propeller. We also show that the propeller can lead to a build up of mass in the outer disc, and to extended recurrence times. The code outlined below is based on that used to similar effect in Matthews et al. (2004), but with the addition of viscosity switching to reproduce the effect of disc outbursts. A thin axisymmetric accretion disc is assumed to be rotating about a central mass M. The hydrodynamical equations are averaged over the azimuthal dimension so that non-axisymmetric effects, such as tidal forces from the secondary star, are not modelled. It is assumed that the disc is sufficiently cold that the dynamical time-scale $t_{\rm dyn} \sim R/v_\varphi$ is much smaller than the viscous or magnetic time-scales so that a Keplerian approximation can then be adopted for the azimuthal motion. The magnetic torque due to the rotating WD is added using a parametrization of the form

$$\Lambda = \frac{l}{t_\Lambda} = \frac{\sqrt{GM} R^{1/2}}{t_\Lambda}, \tag{20}$$

where G represents the universal constant of gravitation, M is the mass of the central star and t_Λ is the time-scale on which the local disc material gains angular momentum, which in this case is given by equation (16). The resulting equation for the evolution of surface density with time (derived in Matthews et al. 2004) is

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{R} \frac{\partial}{\partial R} \left[R^{1/2} \frac{\partial(R^{1/2}\nu\Sigma)}{\partial R} \right]$$
$$- \frac{\beta}{R} \frac{\partial}{\partial R} \left[\frac{1}{R^{7/2}} \left(\left[\frac{R}{R_{\rm co}} \right]^{3/2} - 1 \right) \right], \tag{21}$$

where β is defined as

$$\beta \sim \frac{\mu^2}{2\pi\sqrt{GM}}. \tag{22}$$

We adopt the Shakura & Sunyaev (1973) viscosity prescription (6) and the viscous dissipation is calculated using equation (7). The thermal-viscous disc instability (DIM) must be modelled if disc outburst behaviour is to be reproduced, and the viscosity is therefore permitted also to vary in time. The approach to viscosity switching adopted here is very similar to that used by Truss et al. (2000) in their SPH simulations. Disc annuli are switched between high and low viscosity states, by altering the viscous α parameter, using critical density 'triggers' (Cannizzo, Shafter & Wheeler 1988) which lie at the extremes of the limit cycle. Accordingly, when the local surface density Σ exceeds that of equation (2) the local viscous parameter is altered smoothly on a thermal time-scale $[t_{\rm th} \sim (\alpha\Omega)^{-1}]$ to a value of $\alpha_{\rm hot} = 10\alpha_{\rm cold} = 0.1$. Later, when Σ falls below a lower

trigger density, the viscous parameter is returned to its original value $\alpha_{\rm cold} = 0.01$. In the high state, the accretion disc should also be hotter, so the temperature T is switched between 5000 and 50 000 K in the same way.

The code calculates the diffusive and advective terms separately on a grid of resolution 500 and combines the results using operator splitting. The diffusive term is calculated using a forward in time space-centred scheme (FTSC) and the values of Σ produced here are used as input for a modified two-step Lax–Wendroff scheme (Press et al. 1992) which is used to solve the advective term. The inner boundary, representing the stellar boundary layer, is placed at $R_\star = 1.0 \times 10^9$ cm and an outflow condition such that $\frac{\partial^2 \Sigma}{\partial R^2}$ is constant is applied. An inflow condition is applied at the outer boundary which is placed at the $0.4a$, following SR. The stellar mass is set to $M = 1.2\,{\rm M}_\odot$ and mass is added at the outer boundary at a rate of $\dot{M} = 2 \times 10^{-10}\,{\rm M}_\odot\,{\rm yr}^{-1}$. The WD spin period is set to $P_{\rm spin} = 28$ s which gives a corotation radius of $R_{\rm co} = 1.5 \times 10^9$ cm.

Calculations were performed without a primary magnetic field and with a field of $B_z \sim 1$ kG ($\mu \sim 10^{30}\,{\rm G\,cm}^3$), although the precise value depends on the prescription adopted for the magnetic acceleration. The effect of the magnetic field was to truncate the inner disc, producing a depleted central region and to reduce the peak surface density. The inner disc is likely to be the starting point of any outburst in the non-magnetic case, while in the magnetically truncated case this should instead occur close to the truncation radius. It is important that in neither case is the disc close to marginal stability, where the steady state profile would peak very close to $\Sigma/\Sigma_{\rm crit} = 1$. Such marginal stability is another possible reason for long outburst times, but it would not cause an increase in outburst amplitude.

When the viscosity switching mechanism was enabled, a series of outbursts occurred in both the magnetic and non-magnetic cases. These are plotted in Fig. 3. In the non-magnetic case, the outburst time is $t_{\rm ob} \sim 10$ d and the recurrence time is $t_{\rm rec} \sim 20$ d. This is roughly in agreement with observations of normal DN, although the outbursts are longer than expected. The outbursts vary in amplitude but are of the expected order of magnitude. In the magnetic case, the recurrence time is increased by $\sim$40 times to $t_{\rm rec} \sim 2$ yr, and the outburst amplitude is increased by a factor of $\sim$2. In both cases, the quiescent dissipation is more than two orders of magnitude less than the outburst dissipation. The profile of an individual magnetically moderated outburst is shown in Fig. 4. The outburst lasts for $t_{\rm ob} \sim 10$ d, notably shorter than is observed. It appears, after an initial sharp rise, to rise on a time-scale of a few days. This is close to the hot viscous time-scale, rather than to the thermal time-scale. This is probably because once the disc becomes hot, on a thermal time-scale, mass migrates into the centre of the disc, filling the evacuated central regions on the viscous time-scale. The outburst decays on a similar viscous time-scale, and then much more quickly, on the thermal time-scale, as the disc returns to the cool state.

Fig. 5 shows how the mass of the accretion disc varied during outbursts. The total peak disc mass in the magnetic case was $M_{\rm disc} \sim 8 \times 10^{23}$ g, twice that of the non-magnetic case. Further, the mass consumed in the magnetic outbursts was $\Delta M \sim 4 \times 10^{23}$ g, 10 times that consumed in the non-magnetic outbursts, half of the total accretion disc mass, and approaching the value deduced from observations. We also note that the mass growth rate in the magnetic case is close to linear, this confirms that the disc is not close to marginal stability, as was discussed above. The results from the non-magnetic case show a series of alternate outbursts and mini-outbursts. Mass accretion on to the WD is plotted in Fig. 6, on the same time axis as in Fig. 5. The accretion rate peaks at $5 \times 10^{-9}\,{\rm M}_\odot\,{\rm yr}^{-1}$ in the

Figure 4. A plot of dissipation showing a magnetically moderated outburst in one-dimensional simulations.

Figure 3. Plots of dissipation, showing outburst behaviour, in one-dimensional simulations of the WZ Sge accretion disc. Frame (a) is without a stellar magnetic field and frame (b) has a field of $B \sim 1\,\mathrm{kG}$.
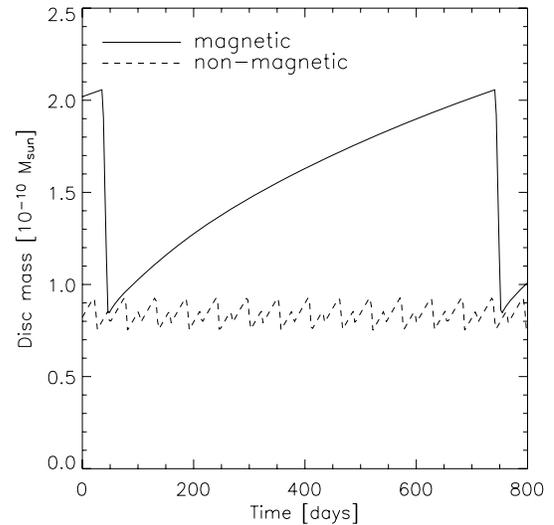
Figure 5. A plot showing evolution of the accretion disc mass as a function of time magnetic and non-magnetic cases. The magnetic outbursts (where $B \sim 1\,\mathrm{kG}$) consume more than half of the accretion disc, about 10 times as much as the non-magnetic outbursts.

magnetic case, which is a factor of 2 more than the non-magnetic case. It is also notable that the cold viscous decay, which is not visible in Fig. 4, can be clearly seen in the magnetic case.

In Fig. 7, the detailed evolution of a magnetically moderated outburst can be followed. During quiescence, the propeller has prevented the disc from spreading in to the star, and by preventing accretion has forced a large build up of mass in the outer disc. The outburst begins when the surface density first exceeds the critical value, close to the truncation radius, rather than near the boundary layer in the usual way. Heating waves then sweep inwards and outwards on a thermal time-scale until the whole disc is in the hot state. The viscosity of the disc is now higher, and the viscous time-scale shorter, allowing mass to spread inwards and quench the propeller. The disc now takes a form close to the standard non-truncated Shakura–Sunyaev solution. Mass accretion on to the WD is high and the density of the disc drops rapidly until the lower trigger is reached at the outer edge of the disc. At this point, a cooling wave sweeps

inwards so that the disc becomes cool again and is gradually shifted outwards as the propeller is re-established. Around half of the total disc mass is consumed in the outburst.

Results from these calculations confirm that a magnetic propeller is capable of truncating the inner accretion disc and may also lead to an extended recurrence time. However, the recurrence time is still an order of magnitude shorter than that observed with these simulation parameters. Two possible mechanisms for further extending the recurrence time suggest themselves. The first requires only a slightly stronger magnetic field. If the magnetic field is stronger then the disc will be truncated at a larger radius, and this can be tuned so that the disc is very close to the steady state when it reaches the outburst trigger. As the disc approaches this steady state, its growth rate slows asymptotically to zero so that the recurrence time in this case can be
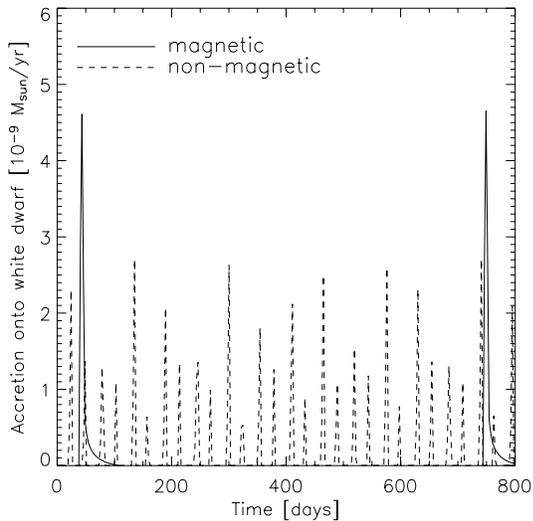
Figure 6. A plot showing accretion on to the WD as a function of time in magnetic and non-magnetic cases on the same time-scale as in Fig. 5. The magnetic outbursts have a higher peak accretion rate than the standard outbursts.

extremely long. This effect was discussed by Warner et al. (1996) and Menou et al. (2000). It is distinct from, and acts in addition to the mechanism described above which is concerned instead with the linear regime of disc growth. Results from a calculation using

this marginal stability effect are shown in Fig. 8. The recurrence time is increased to $t_{\rm rec} \sim 20\,{\rm yr}$, but neither the mass consumed nor the amplitude of the outbursts is greatly altered from the standard magnetic case illustrated in Fig. 3, so that the marginal stability effect would be unable, in itself, to solve Smak 2. This mechanism also requires fine tuning; if the field is slightly too strong outbursts will be prevented entirely.

A second possible explanation for the further extension of the recurrence time, which does not require such fine tuning, is that the size of the mass reservoir may be further increased by tidal effects. The one-dimensional treatment detailed above precludes the consideration of binary tidal forces and resonances. To better model the structure of the accretion disc, including its non-axisymmetric modes, we turn to the complementary two-dimensional model described in Section 3.4.

3.4 Two-dimensional numerical results

Further numerical calculations were performed in two dimensions with SPH simulations (e.g. Kunze, Speith & Hessman 2001; Schäfer et al. 2004). The above prescription (15) for the magnetic acceleration was implemented in an SPH code and calculations were performed with and without a dipolar magnetic field in the plane of the accretion disc. The system parameters were similar those used in the one-dimensional simulations described above, although the mass transfer rate was $\dot{M} = 3 \times 10^{-11}\,{\rm M}_\odot\,{\rm yr}^{-1}$. Each calculation used around 40 000 particles with a fixed smoothing length.
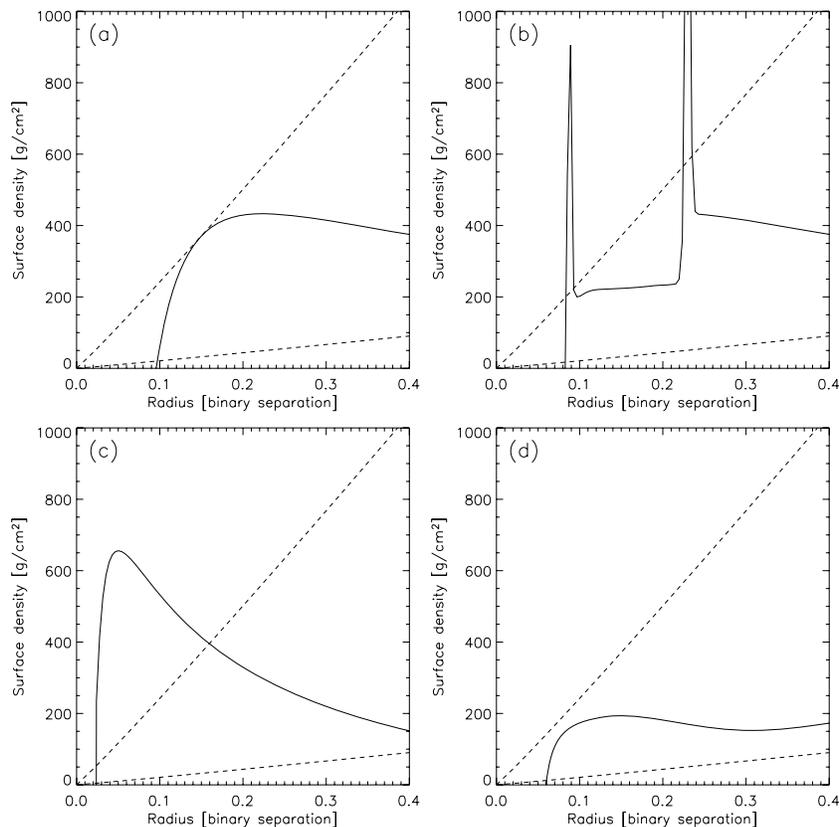
Figure 7. Surface density profiles of the disc in WZ Sge as simulated in the one-dimensional code with a magnetic torque applied. Four snapshots are shown in frames (a)–(d). Frame (a) shows the disc just before the onset of an outburst. Frame (b) is 7 h later showing the heating waves propagating inwards and outwards. Frame (c) shows the disc 4 d later, at the height of the outburst, with the disc in the fully hot state and with no magnetic truncation. Finally, frame (d) shows the early interoutburst period, 18 d later, with a greatly depleted disc. The critical trigger densities are plotted with dashed lines.
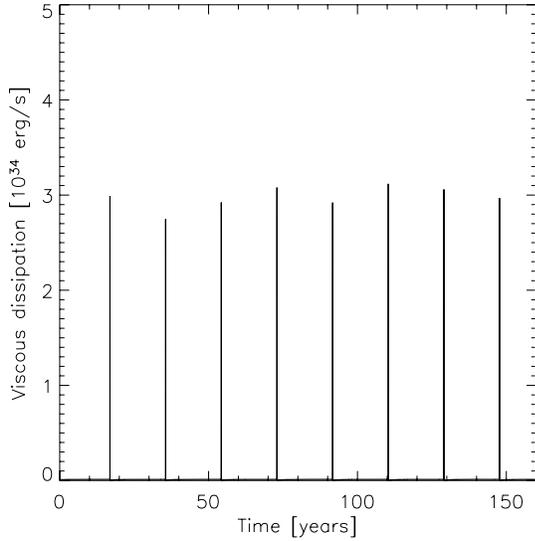
Figure 8. A plot showing viscous dissipation for the case of a magnetically truncated disc close to marginal stability with $B \sim 1$ kG. The recurrence time is increased to $t_{rec} \sim 20$ yr.

Particles were injected at the first Lagrangian point and removed when they approached the primary star.

The resulting discs are shown in Figs 9 and 10, plotted as face-on disc views and radial profiles, respectively. The profiles are somewhat different to those from the one-dimensional simulations, shown in Fig. 8, as a result of the torques from the binary poten-

tial. The effect of the magnetic field in pushing the disc outwards is more pronounced than in the one-dimensional model. In the magnetic case, because matter is driven towards the resonant radii, the disc becomes eccentric and begins to develop some spiral structure. These are clearly seen as two peaks in the second plot of Fig. 10.

This structure also affects the time evolution of the surface density. Fig. 11 shows how the peak surface density as a fraction of the critical surface density grows with time in the magnetic and non-magnetic cases. This can be viewed as a measure of how close the disc is to outburst, since when any substantial part of the disc reaches its local critical density an outburst will be triggered. In the non-magnetic case, there is a simple monotonic growth of this parameter. The noise visible in the otherwise very smooth trend is caused by density enhancements associated with accretion events at the inner boundary, which may be a result of the limited resolution of the simulation. This monotonic growth is as expected from Fig. 10 which shows that the non-magnetic disc profile keeps the same form, with the density peak remaining at the same point around $\sim 0.3a$. If surface density continues to grow at this rate until outburst then this simulation represents around 3 per cent of the duty cycle for a non-magnetic system. In the magnetic case, the same parameter grows monotonically at first, much more slowly than in the non-magnetic case. Its behaviour becomes stochastic once the disc is driven to become eccentric. It is problematic to extrapolate from such a short period to the length of the recurrence time, however, the overall trend remains clear: the approach to outburst is clearly slower in the magnetic case. This result does not contradict that of 3.3. It therefore seems unlikely that effects due to the binary would prevent the propeller from leading to a long recurrence time. It also
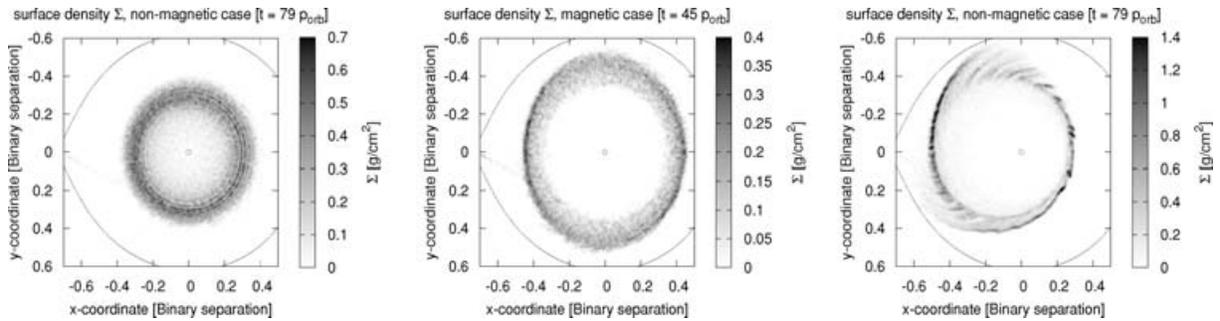
Figure 9. Grey-scale density plots of the accretion disc in WZ Sge, simulated in the two-dimensional SPH code. The first plot shows the disc in the non-magnetic case after 79 orbits. The second and third plots show the disc in the magnetic case, after 45 and 79 orbits, respectively. The first of these two plots is just before the disc begins to become eccentric and the second shows the later, eccentric disc. The primary Roche lobe is plotted as a solid line and the primary star is marked by a circle (not to scale) in all three plots.
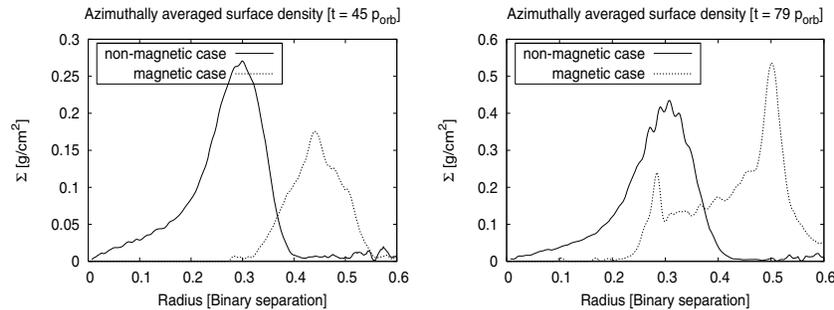
Figure 10. Azimuthally averaged plots of surface density against radius in two-dimensional simulations of WZ Sge. The first plot shows the disc profile in the magnetic and non-magnetic cases after 45 orbits, before the disc becomes eccentric. The second plot shows the disc after 79 orbits, when the disc has become eccentric in the magnetic case. Note that the Σ axis scale is not the same in both plots.

Peak surface density - azimuthally averaged
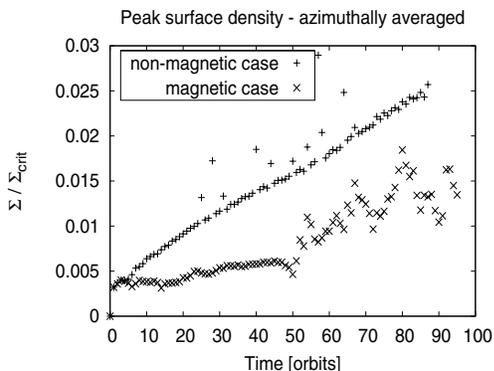
Figure 11. Evolution of the peak surface density in the disc of WZ Sge as a fraction of the critical surface density in non-magnetic and magnetic cases as a function of time. Results are taken from two-dimensional SPH calculations.

remains possible that the tidal forces could lead to the development of a much larger reservoir, and hence a longer recurrence time, than in the simple axisymmetric case.

4 OBSERVATIONAL CONSEQUENCES

The weak magnetic propeller model for the outbursts of WZ Sge results in a number of observationally testable predictions. The major consequence of the model, the existence of a substantial hole in the inner disc, has already been suggested by Mennickent & Arenas (1998). The authors estimate the ratio of the inner and outer disc radii ($R = R_{in}/R_{out}$) from observations of the Hα emission-line widths. WZ Sge was found to show an extremely large value of $R \sim 0.3$. This value agrees reasonably well with the second plot in Fig. 9 where $R_{out} \simeq 0.5a$ and $R_{in} \simeq 0.2a$ for the magnetic case, and is much larger than the value expected if the accretion disc extended all the way to the WD surface ($R \sim 0.03$).

The asynchronous magnetic CVs (the IPs) show multiple periodicities in their light curves. WZ Sge has been observed to display oscillations at 27.87 and 28.96 s as well as weaker oscillations at 28.2 and 29.69 s (Lasota et al. 1999, and references therein). Lasota et al. (1999) interpret WZ Sge as an IP, and identify the 27.87-s oscillation as arising from the spinning WD, and the other oscillations as beat periods between the WD spin and material orbiting within the disc. The beat periods arise from the reprocessing of X-ray emission from the WD on the surface of the disc, the hotspot and/or the secondary star. Assuming the material in the disc is moving in circular, Keplerian orbits the radius associated with these beat periods R_{beat} can be written as

$$R_{beat} = \left(\frac{P_K^2 GM}{4\pi^2} \right)^{1/3}, \tag{23}$$

where

$$\frac{1}{P_K} = \frac{1}{P_{spin}} - \frac{1}{P_{obs}}. \tag{24}$$

Here P_K is the Keplerian period of the disc material, and P_{obs} is the observed beat period. Assuming $M = 1.2$ M$_\odot$, we find $R_{28.96} \simeq 1.3 \times 10^{10}$ cm ($\simeq 0.3a$), $R_{28.2} \simeq 2.8 \times 10^{10}$ cm ($\simeq 0.6a$) and $R_{29.69} \simeq 9.4 \times 10^9$ cm ($\simeq 0.2a$). The radii $R_{28.96}$ and $R_{29.69}$ correlate reasonably well with material orbiting close to the inner edge of the quiescent disc in two-dimensional numerical models (cf. Fig. 9). The radius $R_{28.2}$, on the other hand, is quite close to the hotspot

(stream/disc impact region) which should be located at $\simeq 0.5a$, as seen in the magnetic cases in Fig. 9. Obtaining a closer fit between these radii and the model would require tighter constraints on the WD mass, and q, as well a suitable model for the X-ray reprocessing within the disc. None the less absence of beat radii from the supposed evacuated region is encouraging for the magnetic propeller model. The fact that the oscillations are not observed during outburst is also entirely consistent with this model: the magnetosphere is compressed during outburst, and the system is observationally 'non-magnetic'. The model outlined in this paper predicts that WZ Sge has an eccentric precessing disc, and it might therefore be expected to display permanent superhumps.

5 THE SPIN EVOLUTION OF THE WD

The spin evolution of the WD in WZ Sge is determined by angular momentum transfer in the quiescent and outburst phases. In quiescence, the WD acts as a magnetic propeller, i.e. material approaching the WD is forced to larger radii. We can approximate this interaction by assuming the transferred gas, with initial specific angular momentum $(G M R_{circ})^{1/2}$, is forced to orbit at a mean outer radius $\bar{R}_{out}$. The associated spin-down torque on the WD is then

$$\dot{J}_d = -\zeta \dot{M}_2 (GM)^{1/2} \left(\bar{R}_{out}^{1/2} - R_{circ}^{1/2} \right), \tag{25}$$

where the mass transfer rate across the L1 point $\dot{M}_2 \sim 10^{15}$ gs^{-1}, $\bar{R}_{out} \sim 0.5a$, $R_{circ} \sim 0.35a$, and the parameter $\zeta > 1$ if material at $\bar{R}_{out}$ loses angular momentum to the secondary star via tides (this will be expected to occur once the outer edge of the disc reaches the tidal radius). Conversely, in outburst, the WD gains angular momentum with accreted matter at the rate

$$\dot{J}_u = \dot{M}_{acc}(GM R_{in})^{1/2}, \tag{26}$$

where we adopt an inner disc radius of $R_{in} = 10^9$ cm. From the results of Smak (1993), the average accretion rate during outburst can be estimated as $\dot{M}_{acc} \sim 10^{18}$ g s^{-1}.

The evolution time-scale of the WD spin period can be written in terms of $\dot{J}$ as

$$\tau_{spin} \sim \frac{P_{spin}}{\dot{P}_{spin}} = -\frac{2\pi I}{P_{spin} \dot{J}}, \tag{27}$$

where I is the moment of inertia of the WD. Averaging over a duty cycle, the torque on the WD is given by

$$\langle \dot{J} \rangle \simeq \frac{\dot{J}_u t_{ob} - \dot{J}_d t_{rec}}{(t_{rec} + t_o)}, \tag{28}$$

where $t_{ob} \sim 50$ d is the outburst duration and we take $t_{rec} \sim 30$ yr. From the estimates above (assuming $\zeta \sim 1$), we find $\dot{J}_u t_o / \dot{J}_d t_{rec} \sim 6$. The greater spin-up torque gives a spin-up on the time-scale

$$\tau_d \sim \frac{P_{spin}}{|\dot{P}_{spin}|} \simeq -\frac{2\pi I}{P_{spin}|\langle \dot{J} \rangle|} \sim 10^9 \text{ yr}. \tag{29}$$

The WD in a weak propeller may experience a net spin-up, as above, or a net spin-down, if outbursts are shorter or the mass transfer rate is higher for example. Both these results may lead to a spin-up/spin-down cycle.

Net spin-up. In this case, illustrated by WZ Sge, the net spin evolution is towards shorter periods. Therefore, according to equation (8) the propeller will become stronger with time. The condition $t_{mag} < t_{visc,h}$ will eventually be satisfied. At this point, accretion will be prevented completely and outbursts will cease; the system will become AE Aqr like. This phase should be short lived, however, and it must lead to a spin-down. Therefore, a cycle between WZ Sge and AE Aqr type stars may arise.

Net spin-down. In this opposite case, the net evolution is towards longer WD spin periods. The propeller becomes weaker with time until it is eventually quenched when $t_{mag} > t_{visc,c}$. The system would then look like a normal SU UMa star. This SU UMa phase would usually result in WD spin-up. Therefore, an alternative cycle may exist between WZ Sge and SU UMa type systems. Both of these cycles are a natural consequence of hierarchy (10).

6 DISCUSSION

We have argued that the outbursts of WZ Sge can be explained using the standard DIM value for α_{cold} if we assume that the WD in the system acts as a magnetic propeller. Numerical models of the magnetic disc, in both one and two dimensions, predict an increased interoutburst time t_{rec}, and a disc massive enough to fuel an increased outburst accretion rate. No episode of enhanced mass transfer from the secondary star is required to trigger outbursts, or to supply mass during outburst.

The weak-propeller models assume that the magnetic tension force is the dominant cause of angular momentum transfer in the quiescent disc, and that the force is proportional to the local shear between the disc plasma and the magnetic field. In outburst angular momentum transfer is dominated by viscous diffusion, and mass accretion on to the WD is permitted. This results in an estimated WD magnetic moment in the range

$$2 \times 10^{32} \lesssim \mu \lesssim 10^{33} \text{ G cm}^3 \tag{30}$$

if the disc is assumed to be fully magnetized, or a value in the range

$$5 \times 10^{30} \lesssim \mu \lesssim 3 \times 10^{31} \text{ G cm}^3 \tag{31}$$

if the disc is assumed to be diamagnetic. These values for the WD magnetic moment would suggest that WZ Sge is a short period equivalent of the IPs. Most IPs lie above a period gap ($P_{orb} \gtrsim 3$ h) and have magnetic moments in the range $10^{31} \lesssim \mu \lesssim 10^{34}$ G cm^3. King & Wynn (1999) pointed out that the only two confirmed IPs with $P_{orb} \lesssim 2$ h (EX Hya and RX1238-38) have $\mu \gtrsim 10^{33}$ G cm^3 and long spin periods (67 and 36 min, respectively). WZ Sge would therefore be the first weakly magnetic $\mu < 10^{32}$ G cm^3 CV to be found below the period gap.

In differentiating between the models of the disc-magnetosphere interaction, it is useful to consider the case of AE Aquarii. This system is thought to contain a WD with a magnetic moment $\sim 10^{32}$ G cm^3, which ejects ~ 99 per cent of the mass transfer stream from the system (Wynn et al. 1997). If the WD magnetic moment $\mu \gtrsim 10^{31}$ in WZ Sge then it would satisfy the criteria necessary to become an ejector system. Hence in the case a fully magnetized disc, it would seem that the presence of the disc itself is the only protection the system has from ejecting the transferred mass. If the disc was completely accreted or destroyed for any reason WZ Sge would resemble a short period version of AE Aqr. This would not apply to the case where the WD has a magnetic moment in the range predicted by the diamagnetic model.

A WD spin evolution cycle is a natural consequence of the time-scale hierarchy (10). In the case of WZ Sge, we expect the spin cycle to run between the current state, characterized by long interoutburst times, and that of an AE Aqr-like strong propeller. In some similar systems, however, spin-up phases would look like a normal SU UMa system, whilst the spin-down phases would be WZ Sge like. WZ Sge-like systems following such spin cycles are expected to be relatively rare because of the constraint (equation 10), which results in the restricted range of allowed WD magnetic moments presented above.

Mennickent & Arenas (1998) find evidence for ring-like accretion discs in long supercycle length SU UMa stars from a radial velocity study of the Hα emission lines of these systems. WZ Sge is the most extreme of these objects with a recurrence time of $\sim 12\,000$ d and $R = R_{in}/R_{out} = 0.3$, which is in good agreement with our numerical results. Other SU UMas show a strong positive correlation between supercycle times and the R parameter. This result is in agreement with the model presented in this paper for WZ Sge. The WDs in the other, less extreme, SU UMa stars would simply act as less efficient propellers: the WDs would have weaker magnetic moments, or lower spin rates (which may reflect various stages of the spin-up/spin-down cycles postulated above). Strong candidates for WZ Sge-like systems are RZ Leo ($t_{rec} < 4259$ d, $R \simeq 0.16$), CU Vel ($t_{rec} \simeq 700$–900 d, $R \simeq 0.15$) and WX Cet ($t_{rec} \simeq 1000$ d, $R \simeq 0.12$), (taken from Mennickent & Arenas 1998, table 5).

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