Review of Economic Studies (2004) **71**, 915–944 0034-6527/04/00370915$02.00


# Money and Information

ALEKSANDER BERENTSEN
*University of Basel*

and

GUILLAUME ROCHETEAU
*Australian National University and Federal Reserve Bank of Cleveland*

*First version received January* 2002; *final version accepted April* 2003 (*Eds.*)

This paper investigates the role of money in markets in which producers have private information about the quality of the goods they supply. When the fraction of high-quality producers in the economy is given, money promotes the production of high-quality goods, which improves the quality mix and welfare unambiguously. When this fraction is endogenous, however, we find that money can decrease welfare relative to the barter equilibrium. The origin of this inefficiency is that money provides consumption insurance to low-quality producers, which can result in a higher fraction of low-quality producers in the monetary equilibrium. Finally, we find that most often agents acquire more costly information in the monetary equilibrium than in the barter equilibrium. Consequently, money is welfare-enhancing because it promotes useful production and exchange, but not because it saves information costs.

> The recognition that information is imperfect, that obtaining information can be costly, that there are important asymmetries of information, and that the extent of information asymmetries is affected by actions of firms and individuals ... has provided explanations of economic and social phenomena that otherwise would be hard to understand. Joseph Stiglitz (2000)

## 1. INTRODUCTION

This paper studies the role of money in decentralized markets in which agents have private information about the quality of the goods they supply. We address three issues that are at the crossroads of the economics of information and the pure theory of money (see the Joseph Stiglitz quotation above). First, we explore how money affects the supply of high-quality goods (the quality mix) when the fraction of high-quality producers in the economy is exogenous. This question is related to Akerlof's (1970) paper on "lemons" and the adverse selection problem. Second, we endogenize the fraction of high-quality producers and consider how money affects agents' incentives to engage in opportunistic behaviour, *i.e.* to become lemon producers. Along the lines of Williamson and Wright (1994), this analysis investigates the role of money in alleviating the moral hazard problem. Third, we consider how money affects agents' decision to acquire information. We investigate the validity of Brunner and Meltzer's (1971) and King and Plosser's (1986) claim that money is a substitute for information acquisition.[1]

We analyse these issues by building a search model of money along the lines of Shi (1999, 2001) and Berentsen and Rocheteau (2003) in which money and goods are perfectly divisible

1. There is a voluminous literature that studies the functioning of markets with asymmetric information. For a survey, see Stiglitz (1987). A recent strand of this literature pioneered by Inderst (2001, 2002) studies adverse selection and signalling in random-matching environments. Among the papers that consider the role of money when the quality of goods is private information are Brunner and Meltzer (1971), Alchian (1977), King and Plosser (1986), Williamson and Wright (1994), Banerjee and Maskin (1996) and Trejos (1999).

and the terms of trade are determined endogenously in bilateral meetings. In contrast to most matching models of money, all agents in the market are endowed with money and production opportunities, which allows them to finance their purchases through real production, money, or both. In order to abstract from the double coincidence problem, we assume that a commodity of a given quality provides the same utility to all agents. This setting allows us to focus on private information, which, according to Alchian (1977), is the principal friction underlying the institution of monetary exchange.

We first show how money affects the terms of trade, defined as the quantities produced and consumed in each meeting. We identify two effects. The first effect is related to the recognizability property of money. With valued money, sellers can ask to be paid with money, an object of universally recognizable quality, instead of with goods of uncertain quality. This possibility crowds out the use of real production as a means to finance consumption. The crowding out of real goods payments is what we call the *recognizability effect* of money.

The second effect is the *insurance effect* of money. With money, buyers consume more relative to what they consume in the barter economy because money disconnects what buyers can buy from how they are perceived. In this sense, money acts as consumption insurance. In particular, this insurance allows low-quality producers to consume even when they are recognized as lemon producers. In contrast, in a barter economy, recognized low-quality producers cannot consume because low-quality goods are worthless.

The first part of the paper investigates the adverse selection problem. We assume that the fraction of high-quality producers is exogenous. Nevertheless, the quality mix, defined as the ratio of the total production of high-quality goods to the total production of all goods, is endogenous because the quantities produced are negotiated in bilateral meetings. The key result of this section is that an increase in the real value of money improves the quality mix and welfare unambiguously because increasing the real value of money promotes the production of high-quality goods and reduces the production of lemons. Thus, money is a device to partially overcome the adverse selection problem that arises in barter.

In the second part of the paper, we endogenize the fraction of high-quality producers to study how money affects agents' incentives to become lemon producers. We find that the recognizability effect raises the benefit of being a high-quality producer, and therefore, induces agents to produce high-quality goods more often. In contrast, the insurance effect increases the benefit of being a low-quality producer. This effect, therefore, induces agents to take more risks, *i.e.* to become lemon producers more often.

The severity of the information problem determines which effect dominates. If the information problem is severe, then the recognizability effect dominates the insurance effect and the fraction of high-quality producers is larger in the monetary equilibrium than in the barter respectively non-monetary equilibrium. With increasing information, the recognizability effect becomes relatively less important and is eventually dominated by the insurance effect. Consequently, if the information problem is not severe, the fraction of high-quality producers is smaller in the monetary equilibrium than in the barter equilibrium. Interestingly, for some parameter values there is a positive fraction of lemon producers in the monetary equilibrium but there are no cheaters in the barter equilibrium. Moreover, welfare can be strictly lower in the monetary equilibrium than in the barter equilibrium.

In the third part of this paper, we endogenize the level of information by allowing agents to invest in a costly inspection technology. This investment improves their ability to recognize the quality of the goods supplied in the market. We find that there is always a positive fraction of cheaters in equilibrium. Hence, heterogeneous quality is a natural outcome when the information structure of the economy is endogenized. We show that for most parameter values agents invest more in information in a monetary economy than in a barter economy.

Moreover, when information costs are high there is no active non-monetary equilibrium, whereas an active monetary equilibrium always exists. Thus, in contrast to Brunner and Meltzer (1971) and King and Plosser (1986), our model suggests that investment in information and money are complements.[2]

Our paper is most closely related to the random-matching models of money of Williamson and Wright (1994), Kim (1996), and Trejos (1999).[3] Like Williamson and Wright (1994), we consider an environment where in the absence of private information all agents produce high-quality goods, which implies that there is a double coincidence of wants in each meeting. This allows us to abstract from the double-coincidence-of-wants problem that is most often used to explain money (*e.g.* Kiyotaki and Wright, 1991, 1993), as well as to focus on asymmetric information as an explanation for why agents use fiat money. In contrast to Williamson and Wright (1994) and Kim (1996), who consider environments where both money and goods are indivisible and where agents can hold at most one object at a time, we have divisible money, divisible goods, and no inventory restrictions on money holdings. In contrast to Trejos (1999), in our analysis money is perfectly divisible and the cost of cheating is endogenous because it depends on the equilibrium terms of trade. Moreover, Trejos (1999) rules out barter trades, which implies that money has a welfare-improving role even in the absence of a private information problem. Banerjee and Maskin (1996) consider the adverse selection problem in a Walrasian framework, where each good can be produced in two qualities. They show that the commodity that has the smallest discrepancy between the two qualities will emerge endogenously as the medium of exchange. In this sense, they derive rather than assume the recognizability property of money.

Finally, our paper complements Berentsen and Rocheteau (2003). In that paper we consider a complete information environment where the traders in bilateral meetings have asymmetric tastes for each other's good. The benefit of money arises because it generates larger gains from trade relative to barter. In this paper, the asymmetries between matched traders arise from the combination of heterogeneous quality and private information. In both models asymmetries are the reason why fiat money can be valued. Unlike early random-matching models of money, our framework does not rely on the indivisibility of money and inventory restrictions. Moreover, divisible money allows us to study the effects of changes in the growth rate of the money supply on the quality mix. Interestingly, it also greatly simplifies the typology of equilibria. In particular, under the Friedman rule we generically find uniqueness of the monetary equilibrium. Moreover, divisible money breaks the artificial link between the quantity of money and the fraction of buyers. In fact, an important characteristic of the model is that money is neutral but not superneutral as in most Walrasian models of money.

The remainder of the paper is organized as follows. In Section 2 we present the environment. Section 3 describes the equilibrium, identifies the recognizability and the insurance effects of money, and addresses the adverse selection problem. In Section 4, the fraction of lemon producers and in Section 5 the information level are endogenized. Section 6 concludes.

## 2. MONEY AND THE TERMS OF TRADE

We consider a random-matching model with divisible goods and divisible money. Time is discrete and goes on for ever. The economy is populated with a large number of infinitely lived

2. Quoting Brunner and Meltzer (1971, p. 799): "for individuals, money is a substitute for investment in information and labour allocated to search. By using money, individuals reduce the amount of information they must acquire, process and store."

3. Private information problems in search models of money have also been studied by Cuadras-Morató (1994), Li (1995), Green and Weber (1996), Haegler (1997), Trejos (1997), and Velde, Weber and Wright (1999). All these papers, however, concentrate on issues quite different from the ones we treat.

agents, which are randomly matched into pairs in each period. Each agent in a match has a production opportunity for a good desired by his trading partner, which he cannot consume himself. However, each agent is either a high or a low-quality producer, where the type of the trader is private information. For the moment we assume that the fraction of market participants that produce high quality, denoted by $\Pi$, is exogenous. Holding $\Pi$ constant allows us to study how money affects the production of low-quality goods through its effect on the terms of trade. Later on we endogenize $\Pi$ to see how money affects the decision to cheat.

In this section we discuss the formation of the terms of trade in the bilateral matches. We first present our assumptions about preferences, the production technology and the information available to the matched agents. We then characterize the incentive-feasible and the socially efficient allocations in the bilateral meetings. Finally, we present the trading mechanism, which we will use to analyse how money affects adverse selection, moral hazard and information acquisition.

### 2.1. *Technology and preferences*

There is a continuum of non-storable goods, where each good can be produced in low or high quality.[4] Each agent has the technology to produce one good, and he derives utility from consuming *all* high-quality goods other than his production good. Producing $q$ units of a good of high quality yields the instantaneous disutility $c(q) = q$. Producing low-quality goods (lemons) costs nothing. The instantaneous utility of consuming $q$ units of a commodity of high quality is $u(q)$, where $u(q)$ is increasing and twice differentiable, and satisfies $u(0) = 0$, $u'(0) = \infty$ and $u''(q) < 0$. Furthermore, there exists $q^* > 0$ such that $u'(q^*) = 1$. Consuming a lemon generates no utility. Goods cannot be stored, and production is instantaneous.

In addition to the consumption goods, there is also an intrinsically worthless, storable and fully divisible object called fiat money. In contrast to real goods, there is no uncertainty about the quality of money, *i.e.* it cannot be counterfeited. At the beginning of each period, each market participant holds $m$ units of money.

### 2.2. *Information*

We adopt the information structure suggested by Alchian (1977) and Williamson and Wright (1994). Each agent is able to recognize the quality of a commodity with probability $\theta$ and cannot assess the quality of a commodity with probability $1 - \theta$.

Consider agent $i$ who is randomly matched to some partner $j$. Let $\varepsilon_i \in [0, 1]$ ($\varepsilon_j \in [0, 1]$) denote the belief of agent $i$ ($j$) about the quality of the good produced by his partner in the match. More precisely, $\varepsilon_i$ is the probability that $i$'s partner is a high-quality producer conditional on the information available to $i$. There are two cases to distinguish. First, agent $i$ recognizes the quality of the good produced by his partner in the match. Then, $\varepsilon_i = 1$ if $i$'s partner is a high-quality producer and $\varepsilon_i = 0$ otherwise. Second, agent $i$ does not recognize the quality of the good produced by his partner in the match. Then, $\varepsilon_i = \Pi$ where $\Pi$ is the fraction of high-quality producers in the economy. In summary, the prior beliefs in a match are $(\varepsilon_i, \varepsilon_j) \in E \equiv \{0, \Pi, 1\}^2$. Thus, some matches will be characterized by complete information, others by one-sided or two-sided incomplete information.

When an agent decides which quality to produce he must anticipate how he will be assessed by his partner in the match. Let $f_H(\varepsilon)$ denote the probability that an agent will be assessed

4. We assume the existence of a large number of goods to formalize the idea that agents cannot recognize the quality of all goods in the economy. Otherwise this assumption is unimportant for our analysis.

according to the belief $\varepsilon$ conditional on being a high-quality producer and let $f_L(\varepsilon)$ denote the probability that an agent will be assessed according to the belief $\varepsilon$ conditional on being a low-quality producer. The distributions of probabilities $f_H$ and $f_L$ satisfy

$$\begin{array}{lllll} f_H(0) & = & 0 & f_H(\Pi) = 1-\theta & f_H(1) = \theta \\ f_L(0) & = & \theta & f_L(\Pi) = 1-\theta & f_L(1) = 0. \end{array} \tag{1}$$

Equations (1) have the following interpretation. Conditional on agent $i$ being a high-quality producer, he is recognized as such with probability $f_H(1) = \theta$ and with probability $f_H(\Pi) = 1-\theta$ the quality of $i$'s output is not recognized. The second line of (1) has a similar interpretation.

The unconditional probability $f(x)$ is defined as follows:

$$f(x) = \Pi f_H(x) + (1-\Pi) f_L(x) \qquad \forall x \in [0,1]. \tag{2}$$

Note from (1) and (2) that $f(\cdot)$ and $f_H(\cdot)$ are related as follows:

$$x f(x) = \Pi f_H(x) \qquad \forall x \in [0,1]. \tag{3}$$

### 2.3. *Incentive-feasible trading mechanisms*

In order to define an incentive-feasible trading mechanism, consider a match between agents $i$ and $j$. The true type of agent $i$ is denoted by $\chi_i \in \{L, H\}$ and the one of agent $j$ by $\chi_j \in \{L, H\}$, where $\chi_i = L$ if agent $i$ is a low-quality producer and $\chi_i = H$, if agent $i$ is a high-quality producer. Let $m_i \in \Re^+$ and $m_j \in \Re^+$ denote the money holdings of agents $i$ and $j$, and let $\omega_i$ and $\omega_j$ denote their marginal value of money, respectively. In all meetings the prior beliefs $(\varepsilon_i, \varepsilon_j) \in E$, the money holdings, and the marginal values of money are common knowledge. A match type is a list $(\varepsilon_i, \varepsilon_j, m_i, m_j, \omega_i, \omega_j, \chi_i, \chi_j)$ containing all this information.

A static trading mechanism is a function $o(\varepsilon_i, \varepsilon_j, m_i, m_j, \omega_i, \omega_j, \chi_i, \chi_j)$ assigning to each match type an outcome:[5]

$$o : E \times \Re^4_+ \times \{L, H\}^2 \to \Re^2_{++} \times \Re.$$

An outcome of a trading mechanism is an allocation $(q_i, q_j, x_{ij})$ where $q_i$ is the quantity consumed by $i$ and produced by $j$, $q_j$ is consumed by $j$ and produced by $i$ and $x_{ij}$ is the monetary transfer from agent $i$ to agent $j$. The outcome has to be physically feasible, *i.e.* the quantities $q_i$ and $q_j$ must be non-negative and the transfer of money must satisfy $x_{ij} \in [-m_j, m_i]$. Agent $i$ cannot transfer more money than what he holds and he cannot receive more money than what his partner holds.

A mechanism is incentive-compatible if no agent has an incentive to misrepresent his type. Let us consider the incentive-compatibility constraints for player $i$. For a given $(\varepsilon_i, \varepsilon_j, m_i, m_j, \omega_i, \omega_j)$, let $(q_i^k, q_j^k, x_{ij}^k)$ be the allocation prescribed by the mechanism when player $i$ reports $k = H, L$, and agent $j$ reports truthfully. Note that the allocation $(q_i^k, q_j^k, x_{ij}^k)$ is a function of the true type of agent $j$, $\chi_j$. Let $\mathbb{E}_{\chi_j}[\mathbf{1}_{\{\chi_j = H\}} u(q_i^k) - q_j^k - x_{ij}^k \omega_i]$ be player $i$'s expected utility if he is a high-quality producer and reports $k = H, L$. Accordingly, let $\mathbb{E}_{\chi_j}[\mathbf{1}_{\{\chi_j = H\}} u(q_i^k) - x_{ij}^k \omega_i]$ be his expected utility if he is a low-quality producer and reports $k = H, L$. Then, the incentive-compatibility constraints for player $i$ are

$$\mathbb{E}_{\chi_j}[\mathbf{1}_{\{\chi_j = H\}} u(q_i^H) - q_j^H - x_{ij}^H \omega_i] \geq \mathbb{E}_{\chi_j}[\mathbf{1}_{\{\chi_j = H\}} u(q_i^L) - q_j^L - x_{ij}^L \omega_i], \tag{4}$$

$$\mathbb{E}_{\chi_j}[\mathbf{1}_{\{\chi_j = H\}} u(q_i^L) - x_{ij}^L \omega_i] \geq \mathbb{E}_{\chi_j}[\mathbf{1}_{\{\chi_j = H\}} u(q_i^H) - x_{ij}^H \omega_i], \tag{5}$$

5. By including the prior beliefs we depart slightly from the standard definition of a static mechanism (*e.g.* Ausubel, Cramton and Deneckere, 2002).

where $\mathbf{1}_{\{\chi_j=H\}}$ is the indicator function that is equal to one if $\chi_j = H$. The inequalities (4) and (5) require that for player $i$ honest reporting is a best response. They imply that the difference of the expected utilities of low-quality and high-quality producers $\mathbb{E}_{\chi_j}[\mathbf{1}_{\{\chi_j=H\}}u(q_i^L) - x_{ij}^L\omega_i] - \mathbb{E}_{\chi_j}[\mathbf{1}_{\{\chi_j=H\}}u(q_i^H) - q_j^H - x_{ij}^H\omega_i]$ is at least equal to $\mathbb{E}_{\chi_j}[q_j^H]$. Thus, any incentive-compatible mechanism allows low-quality producers to extract a rent equal to at least the production cost of high-quality producers.

In the following we will focus on pooling mechanisms for the following reasons. First, the outcome of a separating mechanism may be inconsistent with *ex post* individual rationality because the outcome reveals the true type of the low-quality producers.[6] Second, as mentioned above the rent obtained by low-quality producers in any incentive-compatible mechanism is at least $\mathbb{E}_{\chi_j}[q_j^H]$. Thus, separating mechanisms do not eliminate the incentives to cheat.

Any incentive-feasible pooling mechanism must satisfy the participation constraints for the players. For agent $i$, the participation constraint is

$$\mathbb{E}_{\chi_j}[\mathbf{1}_{\{\chi_j=H\}}u(q_i) - \mathbf{1}_{\{\chi_i=H\}}q_j - x_{ij}\omega_i] \geq 0. \tag{6}$$

According to (6), the expected surplus of player $i$ has to be non-negative where $\chi_j$, the type of the partner in the match, is a random variable and where $(q_i, q_j, x_{ij})$ depends on the match type as previously defined. The participation constraints of the high-quality producers are always more restrictive than the participation constraints of the low-quality producers because low-quality producers have no production cost. Consequently, the outcome of a pooling trading mechanism has to satisfy the two following participation constraints:

$$\varepsilon_i u(q_i) - q_j - \omega_i x_{ij} \geq 0, \tag{7}$$

$$\varepsilon_j u(q_j) - q_i + \omega_j x_{ij} \geq 0. \tag{8}$$

Obviously, there are many pooling mechanisms that satisfy (7) and (8) and we will need to further restrict the mechanisms which we consider in the paper. Before we do so, however, let us describe the socially efficient trading mechanism.

### 2.4. *Socially efficient allocations*

For each match, we assume that the social planner chooses the allocation $(q_i, q_j, x_{ij})$ in order to maximize the sum of the utilities of the matched players minus the costs of production, *i.e.*

$$(q_i, q_j, x_{ij}) = \arg\max\{\mathbf{1}_{\{\chi_i=H\}}[u(q_i) - q_i] + \mathbf{1}_{\{\chi_j=H\}}[u(q_j) - q_j]\}$$
$$\text{s.t. } -m_j \leq x_{ij} \leq m_i.$$

The solution to this programme is $q_i = q^*$ if $\chi_j = H$, and $q_j = q^*$ if $\chi_i = H$, where $q^*$ satisfies $u'(q^*) = 1$, *i.e.* the efficient allocation requires a high-quality producer to produce $q^*$. Note that the production and consumption of low-quality goods and the transfer of money is irrelevant for social welfare.

### 2.5. *The Nash-pooling-mechanism*

In the previous section we saw that the choice of a trading mechanism is arbitrary to a large degree. For meetings with complete information a "reasonable" outcome is pairwise Pareto-efficient and is independent from the identities of the players. In matches where agents are

6. If the traders are able to withdraw from the mechanism at the *ex post* stage that arises after the agents have announced their types and an outcome has been chosen, then the mechanism must satisfy the *ex post* participation constraints.

symmetric (they produce the same quality and have the same money holdings) the unique outcome that satisfies the previous requirements is the symmetric Nash bargaining solution. In contrast, for bilateral meetings with incomplete information, very few guidelines exist for the choice of a trading mechanism.[7] Our approach will be to choose a particular pooling mechanism, which has properties that we find interesting, and then to discuss the robustness of our results with respect to this choice throughout this paper.

We call our choice the Nash-pooling-mechanism because it coincides with the Nash bargaining solution when there is complete information in a match. Consider a symmetric equilibrium in which all agents hold $m$ units of money and where the marginal value of money for all households is $\omega$. If the prior beliefs of the agents are $\varepsilon = (\varepsilon_i, \varepsilon_j)$, then the allocation for the match $(q_i, q_j, x_{ij})$ satisfies the following programme:[8]

$$(q_i, q_j, x_{ij}) = \arg\max[\varepsilon_i u(q_i) - q_j - x_{ij}\omega][\varepsilon_j u(q_j) - q_i + x_{ij}\omega] \quad \text{s.t. } -m \le x_{ij} \le m, \tag{9}$$

where $q_i$ is the quantity of goods delivered by agent $j$, $q_j$ is the quantity of goods delivered by agent $i$ and $x_{ij}$ is the quantity of money exchanged. If $x_{ij} > 0$, agent $i$ delivers $x_{ij}$ units of money to agent $j$, and if $x_{ij} < 0$, he receives $x_{ij}$ units of money. In matches with complete information, the solution to the above problem coincides with the Nash bargaining solution. In matches with (one-sided or two-sided) incomplete information, the offer maximizes the product of the expected surpluses of the two players given their prior beliefs, assuming that they are high-quality producers.

The Nash-pooling-mechanism has the following properties. First, as its name suggests, it is pooling: the outcome depends on the prior beliefs of the agents, but not on their true types. Second, the mechanism is incentive feasible. It is physically feasible and the participation and the incentive-compatible constraints of all player types are satisfied. Third, as for any incentive-feasible static trading mechanism, the gain from cheating is equal to the production cost of high-quality producers. Fourth, in matches where both players are recognized as high-quality producers, *i.e.* $(\varepsilon_i, \varepsilon_j) = (1, 1)$, the allocation is pairwise Pareto-efficient and independent from the identity of the players, *i.e.* $(q_i, q_j, x_{ij}) = (q^*, q^*, 0)$. This allocation is of particular interest to us because many results will depend on these complete information meetings only (see Sections 4.4 and 5.2). Fifth, the bargaining power of the players does not depend on the prior beliefs, which implies that the decision of an agent to produce high- or low-quality goods does not affect his bargaining power in future meetings.

The solution to (9) implies that the terms of trade in an $\varepsilon$-meeting, $(q_i, q_j, x_{ij})$, satisfy the following two equations:

$$\varepsilon_i \varepsilon_j u'(q_i) u'(q_j) = 1 \tag{10}$$

$$\frac{1}{\varepsilon_i u'(q_i)} = \frac{\varepsilon_j u(q_j) - q_i + x_{ij}\omega}{\varepsilon_i u(q_i) - q_j - x_{ij}\omega}. \tag{11}$$

In order to characterize the terms of trade as given by (10) and (11), note first that if neither $i$ nor $j$ is constrained by their money holdings, then $q_i = q_i^*$ and $q_j = q_j^*$, where $q_k^*$ satisfy $\varepsilon_k u'(q_k^*) = 1$, $k = i, j$. In asymmetric meetings ($\varepsilon_i \neq \varepsilon_j$), if $\varepsilon_i > \varepsilon_j$ ($\varepsilon_i < \varepsilon_j$), there is a transfer of money from agent $i$ to agent $j$ ($j$ to $i$) and in symmetric meetings ($\varepsilon_i = \varepsilon_j$) no money is exchanged. There is no need for compensation, and consequently, in symmetric meetings the

7. For a review of bargaining games with incomplete information, see Ausubel *et al.* (2002).

8. The Nash-pooling-mechanism is loosely related to the mechanism proposed in Harsanyi and Selten (1972). Their mechanism is also pooling and it maximizes the product of the surpluses of all player types, where the "bargaining" weights depend on how likely a particular type is.

terms of trade are the same as in the non-monetary equilibrium. Note also that in a monetary equilibrium there is some positive production and consumption in all meetings except when both agents are recognized as lemon producers ($\varepsilon_i = \varepsilon_j = 0$). In contrast, in the non-monetary equilibrium, if one agent is recognized as a low-quality producer (either $\varepsilon_i = 0$ or $\varepsilon_j = 0$), no trade takes place. Finally, the terms of trade in the non-monetary equilibrium are simply obtained by setting $\omega = 0$ in equation (11).

### 2.6. *The role of money*

The goal of this paper is to investigate how money affects the quality mix, the incentive to produce high-quality goods and the incentive to acquire information. Money influences these decisions by modifying the quantities produced and consumed in the bilateral meetings. In order to see how they are affected when the real stock of money $m\omega$ changes, consider a meeting between agents $i$ and $j$, where $j$ is more likely to be a high-quality producer than $i$ ($\varepsilon_i > \varepsilon_j$), and assume that $i$ is constrained by his money holdings.[9] Even though both agents produce and consume in equilibrium, call agent $i$ the buyer because he spends money, and agent $j$ the seller because he receives money. From (10) and (11) we have $\frac{\partial q_i}{\partial m\omega} > 0$, $\frac{\partial q_j}{\partial m\omega} \leq 0$, and $\frac{\partial x_{ij}}{\partial m\omega} = 1$. Thus, increasing $m\omega$ increases the production of those agents who are believed to be more likely to produce high-quality goods (the sellers) and decreases the production of those agents who are believed to be more likely to produce lemons (the buyers). Then rational expectations imply that increasing the real stock of money $m\omega$ promotes the production of high-quality goods and reduces the production of lemons.

In order to disentangle the effects of money on agents' behaviour, we define the recognizability and the insurance effects of money.

**The recognizability effect of money.** In the monetary economy, sellers can ask to be paid with money—an object of universally recognizable quality—instead of goods of uncertain quality. This possibility reduces the use of real production as a means of financing consumption. Crowding out of payments with real production by monetary payments is termed the *recognizability effect* of money.

**The insurance effect of money.** In the monetary economy, buyers can finance their consumption with money. This possibility disconnects what buyers can consume from how they are perceived by their trading partners. In particular, this insurance allows low-quality producers to consume even when they are recognized as lemon producers. In contrast, in a barter economy recognized low-quality producers cannot consume because they cannot acquire consumption goods. The presence of this "consumption" insurance in the monetary equilibrium is termed the *insurance effect* of money.

The recognizability and the insurance effects are the main channels through which money affects the quality mix, the incentive to produce high-quality goods, and the incentive to acquire information. It is therefore important to verify that these effects are not simply artefacts of the Nash-pooling-mechanism. We have investigated alternative mechanisms.[10] One mechanism we have looked at is a take-it-or-leave-it offer game with one-sided incomplete information where agent $i$ (the informed party) makes an offer to agent $j$. This game has the structure of a signalling game as defined by Cho and Kreps (1987). Other mechanisms we looked at are take-it-or-leave-it offer games, where agent $j$ (the uninformed party) makes the offer, and optimal

9. If agent $i$'s constraint on money holdings is not binding, then $\frac{\partial q_i}{\partial m\omega} = 0$, $\frac{\partial q_j}{\partial m\omega} = 0$, and $\frac{\partial x_{ij}}{\partial m\omega} < 0$.
10. Our investigation of these mechanisms is available by request.

mechanisms designed by the uniformed party. In all these trading mechanisms we again find the recognizability and the insurance effects of money.

## 3. MONEY AND THE QUALITY MIX

In this section we investigate how money affects the quality mix and we define the monetary equilibrium. In order to abstract from non-tractable distributional issues of money holdings that arise in random matching models of fully divisible money, we assume that each agent is a member of a large household along the lines of Shi (1997).[11] Each household consists of a continuum of members of measure one who pool their money holdings and regard the household's utility as the common objective. This pooling assumption allows us to analyse the effect of private information on the formation of the terms of trades and on the incentive to produce high-quality goods within a representative household framework.

We consider symmetric equilibria only, where all households consume and produce the same quantities. In the following we refer to an arbitrary household as household $h$. Decision variables of this household are denoted by lowercase letters. Capital letters denote other households' variables, which are taken as given by the representative household $h$. Because we will only consider steady-state equilibria where all real variables are constant, we most often omit the time index. Nevertheless because in a steady state nominal variables are not necessarily constant, the index $+1$ refers to a variable at the following period, and the index $-1$ to the variable at the previous period.

The chronology of events within a period is as follows. First, the money stock is divided evenly among the *ex ante* identical household members, which implies that each member holds $m$ units of money when matched.[12] Second, the household members leave the household to search for trading partners. Prior to the matching stage, an exogenously given fraction $\Pi < 1$ of all household members receives a technology shock that enables them to produce high-quality goods only.[13] Members who receive no shock can produce low-quality goods only. Third, household members are matched and carry out their exchanges according to the Nash-pooling-mechanism described by (9). Within a period, a member of the household cannot transfer money balances to another member of the same household. After trading, members bring back their receipts of money, and each agent consumes the goods he has acquired. At the end of a period, the household receives a lump-sum money transfer $\tau$, which can be negative, and then carries the stock $m_{+1}$ to $t+1$.

The quantity of money in the economy is assumed to grow at the gross growth rate $\gamma$. We restrict $\gamma$ to be larger than the discount factor $\beta$. The (indirect) marginal utility of money is $\omega = \beta V'_{+1}(m_{+1})$, where $V(m)$ is the steady-state lifetime discounted utility of the representative household holding $m$ units of money in the current period.

11. The large household assumption, extending a similar one in Lucas (1990), makes the distribution of money holdings degenerate across households and so allows for a tractable analysis of money growth and inflation; see Shi (1997, 1999, 2001). Lagos and Wright (2002) adopt a different assumption to make the money distribution degenerate and Berentsen and Rocheteau (2002) relates Shi's fully divisible money model to earlier indivisible money search models.

12. We impose this assumption to avoid issues related to signalling through money holdings because this would create an additional complexity that goes beyond the scope of this paper. In a similar spirit, in Williamson and Wright (1994) producers are constrained not to hold money which also prevents this form of signalling. For a search model where money holdings have a signalling role, see Okumura (2001).

13. A monetary equilibrium with $\Pi = 1$ does not exist. There is no need for money in an economy where nobody cheats because in each meeting there is a double coincidence of real wants. Note that in Trejos (1999) and Williamson and Wright (1994), in contrast to our paper, if all agents produce high quality a monetary equilibrium exists. In Trejos (1999), this is so because there are also single coincidence meetings. In Williamson and Wright (1994), there is a monetary equilibrium where sellers are indifferent between trading and not trading the good for money.

### *3.1. Value function and the marginal value of money*

According to equation (9) the terms of trade do not depend on the identities of the players but only on the prior beliefs $\varepsilon = (\varepsilon_i, \varepsilon_j)$ in a match. In order to emphasize this dependence we define in the following $(q_\varepsilon^b, q_\varepsilon^s, x_\varepsilon)$ the terms of trade in a match where the prior beliefs are $\varepsilon = (\varepsilon_i, \varepsilon_j)$. The quantity consumed by agent $i$ is $q_\varepsilon^b$, $q_\varepsilon^s$ is the quantity consumed by player $j$, and $x_\varepsilon$ is the transfer of money. The lifetime utility of a household holding $m$ units of money is given by the following Bellman equation:

$$V(m) = \pi \sum\nolimits_{\varepsilon \in E} [\varepsilon_i u(q_\varepsilon^b) - q_\varepsilon^s] f_H(\varepsilon_j) f(\varepsilon_i) + (1-\pi) \sum\nolimits_{\varepsilon \in E} \varepsilon_i u(q_\varepsilon^b) f_L(\varepsilon_j) f(\varepsilon_i) + \beta V_{+1}(m_{+1}), \tag{12}$$

where the law of motion for money holdings is given by

$$m_{+1} - m = \tau - \pi \sum\nolimits_{\varepsilon \in E} x_\varepsilon f_H(\varepsilon_j) f(\varepsilon_i) - (1-\pi) \sum\nolimits_{\varepsilon \in E} x_\varepsilon f_L(\varepsilon_j) f(\varepsilon_i). \tag{13}$$

The first sum in equation (12) aggregates the net expected utilities of all high-quality members in all meetings and the second sum the net expected utilities of all low-quality members. Equation (13) specifies the law of motion of the household's money balances. The first term on the R.H.S. is the amount of the lump-sum transfer which the household receives each period. The second and third terms are the net amounts of money that high-quality and low-quality producers receive in each period.

In order to derive the marginal value money we need to determine the terms of trade in out-of-equilibrium matches, where one agent holds a different amount of money than what is assumed in (9). Our approach here is to design a simple incentive-feasible trading mechanism that makes money as valuable as possible under the Friedman rule. In particular, with this mechanism, money will be sufficiently valued to allow agents to trade the socially efficient quantities in matches where one agent is recognized as a high-quality producer and the other as a low-quality producer. A simple way to achieve this result is to assume that agents holding a different amount of money than the average level $M$ adjust their consumption so as to maximize their utility subject to the constraint that their trading partner must receive the same expected surplus as in (9).[14] This assumption gives agents full bargaining power on their marginal unit of money. The fact that we do not allow agents to adjust their production gives high and low-quality producers the same incentives so that the trading mechanism is also pooling in out-of-equilibrium matches. Let $m \neq M$ be the level of money holdings of the deviating household. For these out-of-equilibrium matches the terms of trade $(q_\varepsilon^b, q_\varepsilon^s, x_\varepsilon)$ maximize (12) subject to the constraints $q_\varepsilon^s = Q_\varepsilon^s$ and

$$\varepsilon_j u(q_\varepsilon^s) - q_\varepsilon^b + x_\varepsilon \Omega \geq \varepsilon_j u(Q_\varepsilon^s) - Q_\varepsilon^b + X_\varepsilon \Omega, \qquad -M \leq x_\varepsilon \leq m, \tag{14}$$

where capital letters refer to equilibrium variables. First, as for the equilibrium outcome, the outcome in an out-of-equilibrium match is pooling. Second, the expected surplus of the non-deviating agent must be equal to the expected surplus in an equilibrium match. Third, the monetary transfer is constrained by the level of money holdings of the matched agents.

From (14) we deduce the following. An agent in an $\varepsilon$-match holding one additional unit of money can acquire $\Omega$ units of goods. The value of this additional consumption is $\varepsilon_i u'(q_\varepsilon^b)\Omega$. From (12), (13) and (14) we obtain the following dynamic equation for the marginal value of

14. This feature of the model is also present in Shi (1997, 1999, 2001). A consequence of this assumption is that under the Friedman rule agents are never constrained by their money holdings. This simplifies the analysis and allows us to abstract from the hold-up problem that is generated by *ex post* bargaining (see Berentsen and Rocheteau, 2001). This assumption does not affect our results in an important way because we can replicate the effects of any alternative trading mechanism that generates a hold-up problem by simply increasing the inflation rate.

money:

$$\frac{\omega_{-1}}{\beta} = \sum_{\varepsilon \in E} \max(\varepsilon_i u'(q_\varepsilon^b)\Omega, \omega) f(\varepsilon_j) f(\varepsilon_i). \tag{15}$$

The marginal value of money satisfies a standard asset pricing equation (15) which has the following interpretation. For the household, the value of an additional unit of money received at the end of the previous period is $\omega_{-1}$. In the current period, this unit of money can be either spent or saved. If it is saved, the value of this unit of money from the point of view of the previous period is simply $\beta\omega$. If it is spent in an $\varepsilon$-meeting, the additional utility of consumption is $\varepsilon_i u'(q_\varepsilon^b)\Omega$. An additional unit of money is spent if and only if the marginal utility of consumption is larger than the marginal value of money, *i.e.* if $\varepsilon_i u'(q_\varepsilon^b)\Omega > \omega$.

### 3.2. *Equilibrium*

In the following we focus on symmetric equilibria, where all households hold the same amount of money $m = M$, and on steady-state equilibria, where the quantities $q_\varepsilon^b$ and $q_\varepsilon^s$, and the real stock of money $m\omega$ are constant. Using the fact that $\gamma = \frac{m}{m_{-1}} = \frac{\omega_{-1}}{\omega}$, the envelope condition (15) satisfies

$$\frac{\gamma}{\beta} = \sum_{\varepsilon \in E} \max[\varepsilon_i u'(q_\varepsilon^b), 1] f(\varepsilon_i) f(\varepsilon_j). \tag{16}$$

Moreover, from (9), the terms of trade in a symmetric monetary equilibrium satisfy

$$(q_\varepsilon^b, q_\varepsilon^s, x_\varepsilon) = \arg\max[\varepsilon_i u(q_\varepsilon^b) - q_\varepsilon^s - x_\varepsilon \omega][\varepsilon_j u(q_\varepsilon^s) - q_\varepsilon^b + x_\varepsilon \omega] \quad \text{s.t. } -M \le x_\varepsilon \le M. \tag{17}$$

*Definition* 1. A symmetric monetary steady-state equilibrium is a $\omega > 0$ and terms of trades $\{(q_\varepsilon^b, q_\varepsilon^s, x_\varepsilon)\}_{\varepsilon \in E}$ that satisfy (16) and (17).

Note that money is neutral in this model. However, it is not superneutral because changing the gross growth rate of the money supply $\gamma$ affects the real value of money holdings $m\omega$ and the terms of trade. Note also that the Friedman rule $\gamma \to \beta$ maximizes $m\omega$ when $\Pi$ and $\theta$ are given. Consequently, throughout the paper we will often compare the non-monetary equilibrium with the monetary equilibrium under the Friedman rule because they are benchmark cases.[15]

### 3.3. *The quality mix and welfare*

This section investigates how money affects the quality mix of aggregate output. We abstract from incentive considerations by taking the fraction $\Pi$ of high-quality producers as constant and exogenous. The quality mix is defined as the ratio of high-quality output to total output:

$$\phi = \frac{\Pi \sum_{\varepsilon \in E} q_\varepsilon^s f(\varepsilon_i) f_H(\varepsilon_j)}{\sum_{\varepsilon \in E} q_\varepsilon^s f(\varepsilon_i) f(\varepsilon_j)}. \tag{18}$$

We interpret an increase in $\phi$ as a reduction of the adverse selection problem.

Increasing the level of information (*i.e.* $\theta$) has two effects on the quality mix: first, there is a direct effect because it changes the distribution of the types of meetings. Second, there is an indirect effect because a change in $\theta$ modifies the real value of money holdings. The direct effect is strictly positive: more information generates more matches where the producers are

15. At $\gamma = \beta$ there exists a continuum of stationary monetary equilibria with identical terms of trade, which only differ in their stationary value of $m\omega$. By considering the limit when $\gamma \to \beta$, we select the unique value of $m\omega$ that is just sufficient to buy the efficient quantity $q^*$ (see Berentsen and Rocheteau, 2001). Note that throughout the paper when we refer to the monetary equilibrium under the Friedman rule we refer to the equilibrium that is attained when $\gamma \to \beta$.

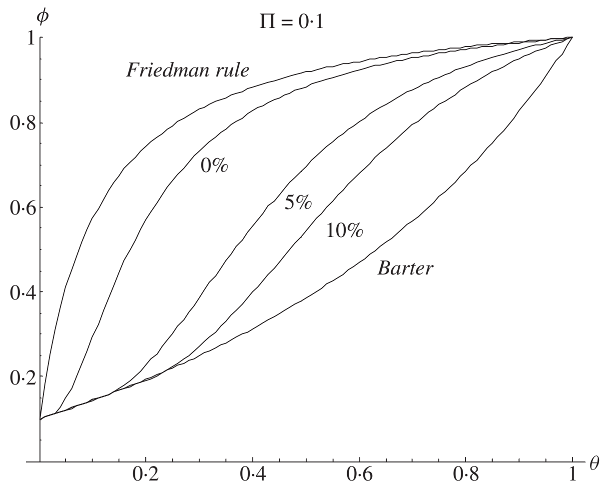


FIGURE 1
Information and the quality mix

recognized, which reduces the production of lemons and increases the production of high-quality goods. The sign of the indirect effect depends on whether an increase in the level of information increases the real value of money or not. If it does, it has a strictly positive impact on $\phi$, too.

Figure 1 displays the fraction of high-quality output $\phi$ as a function of $\theta$ for the Friedman rule (1% deflation), for price stability, for 5 and 10% inflation, and for the barter economy when $\Pi = 0{\cdot}1$.[16] A robust feature of all our simulations is that the ratio $\phi$ is increasing in $\theta$ for any value of $\Pi$ and for any inflation rate. Thus, as expected, if the information problem gets more severe, the quality mix deteriorates. Note that if $\theta = 0$, $\phi = \Pi$: if no information is available, all meetings are of type $\varepsilon = (\Pi, \Pi)$ and the traders exchange the same quantities. Consequently, the ratio of high-quality output to total output is $\Pi$. In contrast, if $\theta = 1$, $\phi = 1$: when lemon producers are recognized with certainty, only high-quality goods will be produced, and consequently the ratio of high-quality output to total output is 1.[17] Finally, from Figure 1 one can see that lowering inflation improves the quality mix $\phi$. Thus, fighting inflation reduces the adverse selection problem.

Figure 2 displays welfare (a household's lifetime utility) as a function of $\theta$ for various rates of inflation when $\Pi = 0{\cdot}1$. If the rate of inflation is not too high, welfare is strictly increasing in the level of information $\theta$. In contrast, in the barter economy, if $\alpha$ is not too large, welfare is decreasing in $\theta$. This can be explained by the fact that under a strictly concave utility function households care about the frequency of trades. As a consequence, welfare can be higher when the information problem is very severe because agents trade frequently even though the quantities exchanged are small.[18] Finally, for any value of $\theta$, the Friedman rule maximizes welfare. Note also that under the Friedman rule, when $\theta$ approaches 1, the economy attains the first best: in

16. For this and all following simulations we have used the iso-elastic utility function $u(q) = \alpha^{-1}q^{\alpha}$.

17. The envelope condition (16) implies that if $\theta = 0$, $m\omega = 0$. If all agents are uninformed, traders are in the same position when they bargain, and they need no money to compensate each other. In contrast, if $\theta = 1$, agents recognize each other in each match, and consequently $\varepsilon_i = 0$ with probability $1 - \Pi$ and $\varepsilon_i = 1$ with probability $\Pi$. There are now asymmetric meetings where agents differ by their willingness to trade.

18. If households are not eager to smooth consumption across their members or across time (if $\alpha$ is large), welfare in the barter equilibrium can be increasing in $\theta$.

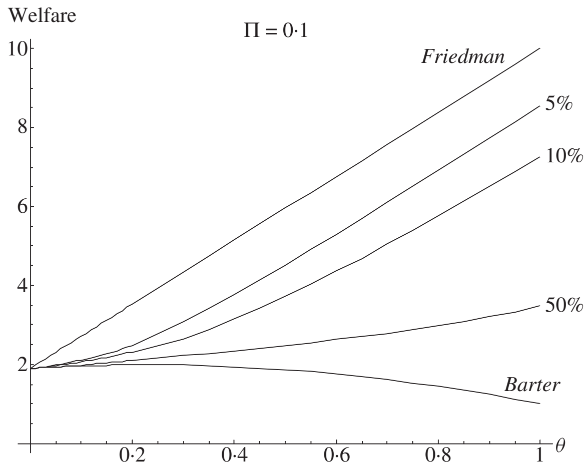


FIGURE 2
Inflation and welfare

each match with a high-quality producer, the high-quality producer trades $q^*$ either for money or for $q^*$ units of another high-quality good.

In summary, there are two ways to increase the quality mix $\phi$: by increasing information for some given inflation rate, which corresponds to a movement along one of the curves in Figure 1, or by decreasing inflation for some given information level, which corresponds to a movement across the curves in Figure 1. Thus, money and information are *substitutes* in the sense that both improve the quality mix.

## 4. MONEY AND MORAL HAZARD

In the previous section we showed that an increase in the real value of money improves the quality mix and welfare when the fraction of high-quality producers is exogenous. In this section we investigate how the presence of valued money affects households' incentives to produce high-quality goods. For this purpose, we consider the two following polar cases that are analytically tractable. Either money is not valued ($\omega = 0$) or money is valued and the Friedman rule ($\gamma \to \beta$) is implemented. As in the previous section, the Friedman rule allows agents to trade the efficient quantity $q^*$ in all matches. Under this rule, the real value of money is

$$m\omega = \frac{u(q^*) + q^*}{2}, \tag{19}$$

where $q^*$ satisfies $u'(q^*) = 1$.

We make the following experiment. We consider an arbitrary period where all households hold the same amount of money and allow each household to choose the fraction of its members that are producing high-quality goods. As in the previous section, the fraction of high-quality producers in the subsequent periods is assumed to be exogenous. This one-time choice allows us to disentangle the effect of valued money on the incentive to produce high-quality goods from the effect of $\Pi$ on the value of money as emphasized in the previous section.[19]

19. For the non-monetary economy, the fact that household's choice for $\pi$ is not repeated is irrelevant: the programme of the household is a succession of static programmes. For the monetary economy, money holdings could potentially reflect households' past decisions. Investigating a non-repeated choice allows us to ignore reputation issues.

Interestingly, under the Friedman rule the real value of money given in (19) is independent from the fraction of high-quality producers in subsequent periods, *i.e.* under the Friedman rule the choice of $\pi$ depends on the value of money but the opposite is not true. This can be explained by the fact that under the Friedman rule agents trade efficient quantities in all matches, in particular in those matches where the asymmetry between the agents is maximal, *i.e.* in matches where one agent is recognized as a low-quality producer and the other as a high-quality producer. This property allows us to identify the effect of valued money on the incentive to produce high-quality goods. For this section, $\pi$ denotes the choice of the representative household and $\Pi$ denotes the average choice of all households in the period under consideration.

As before we assume that members, after having left the household, receive a technology shock with probability $\pi$ that enables them to produce high-quality goods for one period. They cannot produce another quality than the one determined by the shock. When the households choose $\pi$ they do not know the identities of their members who will be high-quality producers in the market. In such an environment, the households treat their members symmetrically and distribute their money holdings evenly across their members. In summary, when choosing $\pi$ the household faces a simple trade-off between the gains from producing goods at no cost when the household members are not recognized and the losses when the household members are recognized.

### 4.1. *The programme of the household*

When the representative household $h$ chooses the fraction of its members that are high-quality producers, $\pi$, it takes the average decision of other households, $\Pi$, as given. If $i$ is a high-quality producer, the prior belief of his partner about the quality of the good he produces is drawn from the distribution $f_H(\varepsilon_j)$, and if $i$ is a lemon producer, it is drawn from the distribution $f_L(\varepsilon_j)$. Note that these two distributions depend on $\Pi$ only because the choice of $\pi$ by a household is assumed to be private information.

The derivative $\mathcal{D}$ of the R.H.S. of (12) with respect to $\pi$ equals

$$\begin{aligned}\mathcal{D}(\Pi) = &\sum\nolimits_{\varepsilon\in E}[\varepsilon_i u(q^b_\varepsilon) - q^s_\varepsilon - x_\varepsilon\omega]\, f(\varepsilon_i)\, f_H(\varepsilon_j)\\ &- \sum\nolimits_{\varepsilon\in E}[\varepsilon_i u(q^b_\varepsilon) - x_\varepsilon\omega]\, f(\varepsilon_i)\, f_L(\varepsilon_j).\end{aligned} \tag{20}$$

Equation (20) is the difference between the expected surpluses of a high-quality producer and low-quality producer. The optimal choice of $\pi$ by the household satisfies

$$\begin{array}{ll}\pi = 1 & \text{if } \mathcal{D}(\Pi) > 0\\ \pi = 0 & \text{if } \mathcal{D}(\Pi) < 0\\ \pi \in [0,1] & \text{otherwise.}\end{array} \tag{21}$$

### 4.2. *Symmetric equilibrium*

We look for symmetric Nash equilibria where all households choose the same fraction of high-quality producers. The value(s) of $\Pi$ that sustain a symmetric Nash equilibrium are defined as follows:

$$\begin{array}{ll}\Pi = 1 & \text{if } \mathcal{D}(1) \geq 0\\ \Pi = 0 & \text{if } \mathcal{D}(0) \leq 0\\ \mathcal{D}(\Pi) = 0 & \text{otherwise.}\end{array} \tag{22}$$

In the following we call an equilibrium with a positive production of high-quality goods, *i.e.* $\Pi > 0$, an *active* equilibrium. An active equilibrium can be either a non-monetary or a monetary equilibrium.

*Definition* 2. An active equilibrium is a $\{(q^b_\varepsilon, q^s_\varepsilon, x_\varepsilon)\}_{\varepsilon \in E}$ satisfying (17) and a $\Pi > 0$ satisfying (22). The equilibrium is monetary if $\omega$ satisfies (19).

Proposition 1 characterizes the non-monetary equilibrium and the monetary equilibrium under the Friedman rule $\gamma \to \beta$.

**Proposition 1.** *Assume that the fraction of high-quality producers,* $\Pi$*, is endogenous. Then the following is true:*

(i) *If* $\theta = 0$*, the unique equilibrium is non-active.*
(ii) *For all* $\theta > 0$*, a non-active and an active non-monetary equilibrium exist. An active non-monetary equilibrium with* $\Pi = 1$ *exists iff* $\theta \geq \theta_B \equiv \frac{q^*}{u(q^*)} < 1$.
(iii) *Under the Friedman rule* $\gamma \to \beta$*, there exists a unique active monetary equilibrium iff* $\theta > 0$*. The fraction of high-quality producers is strictly increasing in* $\theta$ *for all* $0 < \theta < \theta_M \equiv \frac{2q^*}{u(q^*)+q^*} < 1$ *. If* $\theta > \theta_M$*, then* $\Pi = 1$.

*Proof.* See Appendix. ‖

According to Proposition 1, if agents receive no information prior to the bargaining ($\theta = 0$), there is no active equilibrium. This is the standard lemon problem: with no information, the market collapses. Moreover, for all $\theta > 0$ there is a non-active non-monetary equilibrium: if a household expects all other households to produce lemons, a best response is to choose $\pi = 0$. Consequently, for a deviating high-quality producer the probability of finding another high-quality producer is zero.[20]

For all $\theta > 0$ there is an active non-monetary equilibrium. Moreover, if the fraction of informed agents is sufficiently large ($\theta \geq \theta_B$), an active non-monetary equilibrium exists where all agents produce high-quality goods ($\Pi = 1$). Under the Friedman rule there exists a unique active monetary equilibrium when $\theta > 0$. Finally, in the monetary equilibrium, the fraction of high-quality producers is increasing in the level of information $\theta$ if $\theta \in (0, \theta_M)$.[21]

Proposition 2 below ranks the non-monetary and monetary equilibria with respect to the endogenous fraction of high-quality producers in the market.

**Proposition 2.** *Assume* $\gamma \to \beta$*. Then, if* $\theta$ *is close to 0, the fraction of high-quality producers is larger in the monetary equilibrium than in any non-monetary equilibrium. If* $\theta \in (\theta_B, \theta_M)$*, it is lower in the monetary equilibrium than in the non-monetary equilibrium.*

*Proof.* See Appendix. ‖

According to Proposition 2, money has ambiguous effects on the incentives to produce lemons. If the problem of information is severe ($\theta$ is close to 0), the fraction of high-quality producers is larger in a monetary economy. Thus, if information is scarce, valued fiat money disciplines producers. In contrast, if the information problem is not severe ($\theta \in (\theta_B, \theta_M)$), then there is a unique active non-monetary equilibrium where everyone produces high quality,

20. When $\theta > 0$, non-active barter equilibria are unstable in the following sense: If a household anticipates that a small fraction of agents in the market will produce high quality, its best response is to set $\pi = 1$.

21. In Williamson and Wright (1994) and Trejos (1999), an increase in information sometimes decreases the fraction of high-quality producers.

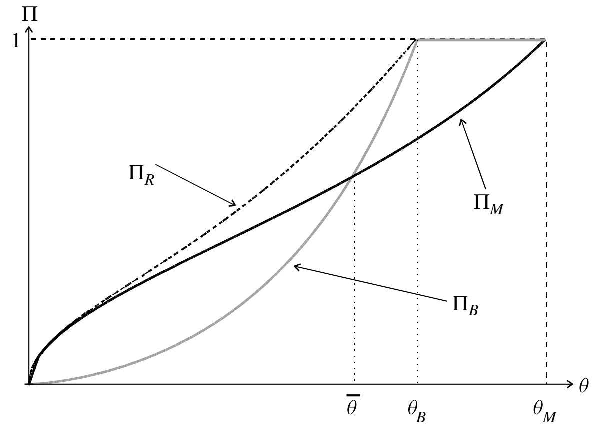


FIGURE 3
Incentive to produce quality

and there is a unique monetary equilibrium where only a fraction of producers produce high-quality goods. Consequently, if $\theta \in (\theta_B, \theta_M)$, money can be valued even though it enhances agents' opportunistic behaviour.

In Figure 3 we plot the endogenous fraction of high-quality producers in the non-monetary $\Pi_B$ and in the monetary economy under the Friedman rule $\Pi_M$ for the iso-elastic utility function $u(q) = \alpha^{-1}q^{\alpha}$ with $0 < \alpha < 1$. For this function we find the following closed form solutions (see the Appendix for the derivation):

$$\Pi_B = \left[\frac{\theta(1-\alpha)}{(1-\theta)\alpha}\right]^{\frac{(1-\alpha)(1+\alpha)}{\alpha}}, \qquad \text{if } \theta < \theta_B \equiv \alpha, \qquad \text{and} \qquad \Pi_B = 1 \qquad \text{if } \theta \geq \theta_B.$$

$$\Pi_M = \left[\frac{\theta(1-\alpha)}{2\alpha(1-\theta)}\right]^{1-\alpha}, \qquad \text{if } \theta < \theta_M \equiv \frac{2\alpha}{1+\alpha}, \qquad \text{and} \qquad \Pi_M = 1 \qquad \text{if } \theta \geq \theta_M.$$

Note that $\Pi_M > \Pi_B$ if $\theta < \overline{\theta} \equiv \frac{\alpha}{2^{\alpha}(1-\alpha)+\alpha}$, and $\Pi_M < \Pi_B$ if $\theta \in (\overline{\theta}, \theta_M)$.

The grey curve labelled $\Pi_B$ represents the fraction of high-quality producers in the non-monetary economy, and the solid black curve labelled $\Pi_M$ the fraction of high-quality producers in the monetary economy under the Friedman rule. Both curves are strictly increasing in the level of information $\theta$ until they eventually hit the upper bound $\Pi = 1$. Moreover, there is a threshold $\overline{\theta}$ such that if $\theta < \overline{\theta}$, money increases the incentive to produce high quality, whereas if $\theta > \overline{\theta}$, it increases the incentive to cheat. The threshold $\overline{\theta}$ is increasing in $\alpha$. Thus, money is a less effective device to alleviate the moral hazard problem in an economy, where households have a high aversion to inequalities across members because the desire to smooth consumption across members already disciplines households. Finally, note that the active non-monetary equilibrium when $u(q) = \alpha^{-1}q^{\alpha}$ is unique—something we were not able to show in Proposition 1.

Finally, if the fraction of high-quality producers is chosen in each period (instead of the one-time choice considered above), then under the Friedman rule a monetary equilibrium cannot exist if $\theta \geq \theta_M$. The reason for this is that when $\theta \geq \theta_M$, then all households choose $\Pi = 1$, which implies that all meetings are symmetric double-coincidence meetings. In such meetings there is no need for a medium of exchange. Consequently, money has no value in an equilibrium where nobody cheats.

### *4.3. Moral hazard*

The key result in the previous subsection is that the fraction of high-quality producers can be smaller in the monetary equilibrium. In this subsection we show that this is so because the *recognizability effect of money* and the *insurance effect of money* have countervailing implications for a household's incentive to cheat.

**The recognizability effect.** The recognizability effect of money raises an agent's incentive to be a high-quality producer. In the monetary economy, when a high-quality producer does not recognize his trading partner, he can ask to be paid with money instead of being paid with a good of uncertain quality. In contrast, in the non-monetary economy high-quality producers are always paid with commodities of uncertain quality. Accordingly, the recognizability effect of money improves the gains from trade for high-quality producers, which induces households to choose a larger fraction of high-quality producers.

**The insurance effect.** In contrast to the recognizability effect, the insurance effect of money raises agents' incentive to engage in opportunistic behaviour because it allows low-quality producers to consume even when they are recognized as lemon producers. This consumption insurance induces households to choose a riskier behaviour by increasing the fraction of low-quality producers compared to their choices in the non-monetary equilibrium.

Which effect dominates depends on the severity of the information problem $\theta$. When $\theta$ is low, the recognizability effect dominates and the fraction of high-quality producers is larger in the monetary equilibrium than in the non-monetary equilibrium. With an increasing level of information, the recognizability effect fades away, and eventually the insurance effect dominates the recognizability effect, which results in a larger fraction of high-quality producers in the non-monetary equilibrium than in the monetary equilibrium for intermediate values of $\theta$. Consequently, the insurance effect of money exacerbates the moral hazard problem.[22]

In order to isolate the positive effect of money on the incentive to produce high quality, we eliminate the insurance effect by analysing the following trading restriction: household members cannot trade if they discover that their partner is a low-quality producer. In the Appendix we show that under this rule, money unambiguously increases the fraction of high-quality producers (see also the dotted curve labelled $\Pi_R$ in Figure 3). Consequently, it is the insurance effect of money that is responsible for the fact that the fraction of lemon producers can be larger in the monetary equilibrium than in the non-monetary equilibrium.

### *4.4. Discussion*

In this subsection we discuss the robustness of our main results. In particular, we address the question of how much our results depend on the particular pricing mechanism we have used. The only assumption we are going to use is the following one. In a match where two agents who hold the same amount of money recognize each other as high-quality producers they both consume and produce the quantity $q^*$ that satisfies $u'(q^*) = 1$. With complete information, the trade $(q^*, q^*)$ is the only trade that is independent on the identity of the players and that is renegotiation proof (*i.e.* is on the Pareto frontier of the bargaining set).

22. It is interesting to relate the previous results with those obtained in Williamson and Wright (1994). In their framework money does not generate a negative moral hazard effect because agents who produce do not hold money, or reciprocally money holders cannot produce. As a consequence, money does not provide insurance services as in our model.

We first discuss the result that the presence of money reduces the incentives to cheat when $\theta$ is close to 0. If $\theta$ is close to 0, $\Pi$ is also close to 0.[23] Consequently, in the non-monetary equilibrium, agents trade small quantities in all matches except in those matches where both agents recognize each other as high-quality producers where the trade is $(q^*, q^*)$. For a household, the fraction of members that produce and consume $q^*$ is $\pi\Pi\theta^2$. In contrast, in the monetary equilibrium under the Friedman rule, agents trade $q^*$ if one or both agents are recognized as high-quality producers. Therefore, in all meetings in which the high-quality producers of the household are recognized they can trade. There is a fraction $\pi\theta \gg \pi\Pi\theta^2$ of such meetings. This explains why there is a higher incentive to produce high quality in the presence of money when the information problem is severe.

The second result we discuss is that the fraction of high-quality producers can be lower in the monetary equilibrium than in the non-monetary equilibrium. Consider a non-monetary equilibrium where $\Pi = 1$. For the household $\pi = 1$ is a best response against $\Pi = 1$ if and only if

$$(1-\theta)u(q^*) \leq u(q^*) - q^*, \tag{23}$$

where the L.H.S. is the expected utility of a lemon producer and the R.H.S. is the expected utility of a high-quality producer. As a consequence, a symmetric non-monetary equilibrium with $\pi = \Pi = 1$ exists if and only if $\theta \geq \theta_B \equiv \frac{q^*}{u(q^*)}$.

Consider next a monetary equilibrium where the fraction of high-quality producers $\Pi$ is very close to 1. Let $S_{1,0}$ denote the surplus of a recognized low-quality producer who trades with a recognized high-quality producer. In a monetary equilibrium with $\Pi < 1$ the pay-off of a high-quality and a low-quality producer must be equal. For $\Pi = 1$, this indifference relation is

$$(1-\theta)u(q^*) + \theta S_{1,0} = u(q^*) - q^*. \tag{24}$$

Denote $\theta_1$ the value of $\theta$ that satisfies (24). This value is the level of information that is consistent with a monetary equilibrium when $\Pi = 1$. Then, inequality (23) and the indifference relation (24) imply that if $S_{1,0} > 0$, then $\theta_1 > \theta_B$. Thus, there are values for the level of information such that a monetary equilibrium with $\Pi_M < 1$ coexists with a non-monetary equilibrium where everybody is a high-quality producer ($\Pi_B = 1$). Note that the argument is true for any rate of inflation even though the inflation rate will reduce the insurance effect by lowering the value of money and thus $S_{1,0}$.

One related way to explain the coexistence of a non-monetary equilibrium in which all agents are high-quality producers and a monetary equilibrium in which a fraction of agents are low-quality producers, when the problem of information is not severe, is as follows. Consider a barter economy, where all agents produce high quality because $\theta = \theta_B$. In this situation, introduce fiat money and assume that it is valued. The presence of this valued asset increases the incentive to cheat because of the consumption insurance it provides. But if some agents begin to produce low-quality goods, some asymmetric matches will occur and fiat money will be indeed valued. Thus, fiat money may create the heterogeneity among producers that is necessary to sustain itself as a medium of exchange.

## 5. INFORMATION ACQUISITION

This section analyses how money affects agents' incentives to acquire information. In particular, we would like to know whether money saves information costs as emphasized by Brunner and

23. This is true for any pricing mechanism which is continuous in $\theta$: if $\theta = 0$ it does not pay to produce high-quality goods because nobody is able to recognize it. Hence, $\Pi = 0$. By continuity, if $\theta$ is close to zero, $\Pi$ must be close to zero, too.

Meltzer (1971). The intuition for this argument is that in a barter economy sellers must spend resources to assess the quality of goods offered to them in exchange. If they are paid with money instead, a good of recognizable quality, they can save this cost. It is often argued that this cost saving is one of the main welfare gains that money provides to society. To evaluate the validity of this argument, we will first treat the fraction of high-quality producers as exogenous. Later on we relax this assumption.

Following Kim (1996), we introduce information acquisition by assuming that each household invests into an inspection technology at cost $C(\theta) = A\frac{\theta^2}{2}$.[24] This technology, which is equally shared by all household members, allows each member to recognize the quality of a fraction $\theta$ of the goods produced in the economy. The parameter $A > 0$ measures the extent of the information problem. As in the previous section, we investigate a non-repeated choice. In the initial period, each household chooses the inspection technology $\theta$. In the subsequent periods, the inspection technology and hence the level of information in the economy is exogenous. For this section, therefore, $\theta$ denotes the choice of the representative household and $\Theta$ denotes the average choice of all households in the period under consideration.

The choice of $\theta$ affects how the members of a household assess the quality of goods produced by their trading partners in the current period. By increasing $\theta$, the household reduces the fraction of the meetings where its members are uninformed. Consider member $i$, and denote by $f(\varepsilon_i, \theta)$ the probability for $i$ to be matched with someone he will assess according to the prior belief $\varepsilon_i$. We have

$$f(0,\theta) = \theta(1-\Pi), \qquad f(\Pi,\theta) = 1-\theta, \qquad f(1,\theta) = \theta\Pi.$$

Denote by $f_\theta(\varepsilon_i, \theta)$ the partial derivative of $f(\varepsilon_i, \theta)$ with respect to $\theta$. Note that $\sum_{\varepsilon_i} f_\theta(\varepsilon_i, \theta) = 0$ and that the beliefs of $i$'s trading partners depend only on their own information level $\Theta$. Accordingly, we denote by $f_L(\varepsilon_j; \Theta)$ and $f_H(\varepsilon_j; \Theta)$ the conditional distributions of the beliefs of $i$'s partners.

### 5.1. *Money and information: substitutes or complements?*

In this subsection we assume that the fraction of high-quality producers is exogenous and equal to $\Pi$. At the beginning of each period, each agent receives a technological shock that determines whether he can produce high-quality goods (with probability $\Pi$) or low-quality goods (with probability $1-\Pi$). This shock is private information. Each household chooses its information $\theta$ by taking the choice of other households $\Theta$ as given. The programme of the household can be written as follows:

$$\begin{aligned}\max\nolimits_\theta \Big\{\Pi \textstyle\sum_{\varepsilon\in E}[\varepsilon_i u(q_\varepsilon^b) - q_\varepsilon^s]\, f(\varepsilon_i;\theta)\, f_H(\varepsilon_j;\Theta)\\ +(1-\Pi)\textstyle\sum_{\varepsilon\in E}\varepsilon_i u(q_\varepsilon^b)\, f(\varepsilon_i;\theta)\, f_L(\varepsilon_j;\Theta) - C(\theta) + \beta V_{+1}(m_{+1})\Big\} \qquad (25)\\ \text{s.t. } m_{+1} - m = \tau - \Pi \textstyle\sum_{\varepsilon\in E} x_\varepsilon\, f(\varepsilon_i;\theta)\, f_H(\varepsilon_j;\Theta)\\ -(1-\Pi)\textstyle\sum_{\varepsilon\in E} x_\varepsilon\, f(\varepsilon_i;\theta)\, f_L(\varepsilon_j;\Theta).\end{aligned}$$

Let $\mathcal{I}(\theta,\Theta)$ denote the derivative of the R.H.S. of (25) with respect to $\theta$.

$$\begin{aligned}\mathcal{I}(\theta,\Theta) = \Pi \textstyle\sum_{\varepsilon\in E}[\varepsilon_i u(q_\varepsilon^b) - q_\varepsilon^s - x_\varepsilon\omega]\, f_\theta(\varepsilon_i;\theta)\, f_H(\varepsilon_j;\Theta)\\ +(1-\Pi)\textstyle\sum_{\varepsilon\in E}[\varepsilon_i u(q_\varepsilon^b) - x_\varepsilon\omega]\, f_\theta(\varepsilon_i;\theta)\, f_L(\varepsilon_j;\Theta) - C'(\theta). \qquad (26)\end{aligned}$$

24. In Kim (1996) low-quality producers, high-quality producers, and money holders differ in their investments in the inspection technology.

Note that $\mathcal{I}(\theta, \Theta)$ is a decreasing function of $\theta$. Accordingly, household $h$'s optimal choice of information is uniquely determined by

$$\begin{array}{ll} \theta = 1 & \text{if } \mathcal{I}(1, \Theta) > 0 \\ \theta = 0 & \text{if } \mathcal{I}(0, \Theta) < 0 \\ \mathcal{I}(\theta, \Theta) = 0 & \text{otherwise.} \end{array} \tag{27}$$

We consider symmetric Nash equilibria where all households choose the same information level: $\theta = \Theta$. The value(s) of $\Pi$ that sustain a symmetric Nash equilibrium are defined as follows:

$$\begin{array}{ll} \Theta = 1 & \text{if } \mathcal{I}(1, 1) \geq 0 \\ \Theta = 0 & \text{if } \mathcal{I}(0, 0) \leq 0 \\ \mathcal{I}(\Theta, \Theta) = 0 & \text{otherwise.} \end{array} \tag{28}$$

As in the previous section, we investigate the two following polar cases. Either money is not valued ($\omega = 0$) or money is valued and the Friedman rule is implemented (*i.e.* $\gamma \to \beta$).

*Definition* 3. An active equilibrium is a $\{(q^b_\varepsilon, q^s_\varepsilon, x_\varepsilon)\}_{\varepsilon \in E}$ satisfying (17) and a $\Theta$ satisfying (28). The equilibrium is monetary if $\omega$ satisfies (19).

Proposition 3 compares the investments in information in the non-monetary equilibrium and the monetary equilibrium under the Friedman rule when the fraction of high-quality producers, $\Pi$, is exogenous.

**Proposition 3.** *Assume that the fraction of high-quality producers,* $\Pi$, *is exogenous and that* $u(q) = \alpha^{-1} q^\alpha$ *and* $C(\theta) = A\frac{\theta^2}{2}$. *Then, if* $\Pi$ *is positive but sufficiently small, investment in information is higher in any monetary equilibrium under the Friedman rule than in the non-monetary equilibrium.*

*Proof.* See Appendix. ‖

Even though Proposition 3 does not provide a general ranking of the level of information in the barter economy and in the monetary economy, it shows that for low values of $\Pi$ households acquire more information in the monetary equilibrium. Thus, if the average quality in the market is low, the presence of money gives higher incentives to be informed.

Figure 4 illustrates the previous proposition for $A = 0{\cdot}1$ and $\alpha = 0{\cdot}5$ (left graph) and $\alpha = 0{\cdot}9$ (right graph), respectively. The dotted curves represent the equilibrium values of $\theta$ as functions of $\Pi$ in the non-monetary equilibrium and the black curves in the monetary equilibrium under the Friedman rule. The left graph shows that if $\alpha$ is not too large ($\alpha = 0{\cdot}5$), the level of information is larger in the monetary equilibrium for all $\Pi \in (0, 1)$. In contrast, as shown in the right graph, if $\alpha$ is large ($\alpha = 0{\cdot}9$), then for large $\Pi$ households invest more in the non-monetary equilibrium than in the monetary equilibrium under the Friedman rule.

In order to improve our understanding of the role of money on the incentive to acquire information, we again look how the *recognizability effect* and the *insurance effect* of money affect the decision to acquire information.

**The recognizability effect.** Consider a high-quality producer (the seller) who is matched with a low-quality producer (the buyer). In the non-monetary equilibrium, for the seller the gain from being informed is the disutility he saves by not producing. In contrast, in the monetary economy the gain is the additional amount of money he receives. Whether the gain from being informed in the monetary equilibrium is larger than the gain in the non-monetary

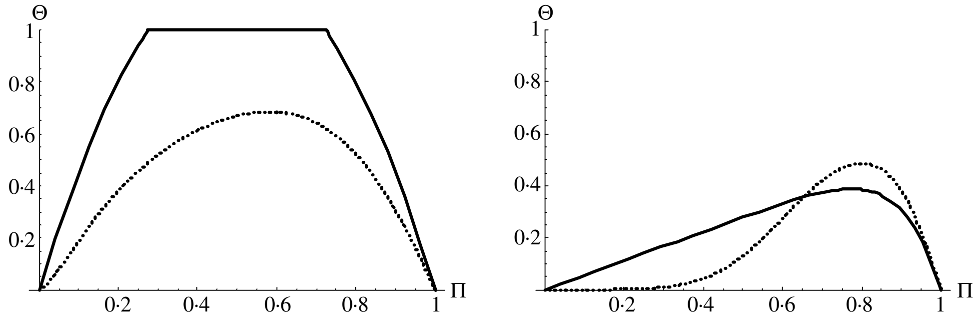


FIGURE 4

Investment in information

equilibrium depends on the specification of the utility function. If households are eager to smooth consumption across their members or across time (if $\alpha$ is small), the additional units of money received in the monetary economy are more valuable than the saved production cost in the barter economy. The opposite is true if the intertemporal elasticity of substitution is large (if $\alpha$ is large). The dependence of this effect on the parameter $\alpha$ leads to the ambiguity of the results.

**The insurance effect.** The "insurance to consume" provides higher incentives to identify the quality of goods. To see this, consider a *recognized* low-quality producer. In the non-monetary equilibrium, he has no benefit of being informed because he cannot trade, and accordingly information is useless to him. In contrast, in the monetary equilibrium information reduces this probability and is therefore valuable.

### *5.2. Discussion*

If we consider the proof of Proposition 3, we can see that it is largely independent on the choice of the pricing mechanism. Consider first the non-monetary equilibrium. If $\Pi$ is small, agents trade very small quantities in all matches except in those matches where both players are recognized as high-quality producers in which case each trader produces and consumes $q^*$. For an agent, the probability to be in such a valuable match is $\theta\,\Theta\,\Pi^2$. Consider next the monetary economy. Now agents will be able to exploit significant gains from trade in each match where one of the two players is recognized as a high-quality producer because they can exchange high-quality goods for money. For an agent, the probability to be in such a valuable match is $\theta\Pi$. Because $\Pi$ is infinitely larger than $\Pi^2$ when $\Pi$ is close to 0, the incentive to invest in information is much greater in the monetary equilibrium if the average quality in the market is low. Note that the previous argument relies more on the *insurance effect* of money whereas Brunner and Meltzer (1971) emphasizes the *recognizability effect* of money.

### *5.3. Choosing $\Pi$ and $\Theta$*

In this subsection, we assume that both $\Pi$ and $\Theta$ are endogenous. Each household chooses simultaneously its fraction of high-quality producers and its investment in information taking the choice of other households as given. The aim is to show how the moral hazard problem and the choice of information acquisition can complement each other in order to explain the welfare gains of money.

**Lemma 1.** *In a symmetric equilibrium, households never invest in a perfect information technology* ($\Theta < 1$)*, and there is always a positive fraction of low-quality producers* ($\Pi < 1$).

*Proof.* See Appendix. ||

If all agents produce high-quality goods, there is no reason to acquire information; hence, households choose to be uninformed. But if everyone is uninformed, nobody is producing high-quality goods.

**Proposition 4.** *Assume that* $\Pi$ *and* $\Theta$ *are endogenous, and that* $u(q) = \alpha^{-1}q^{\alpha}$ *and* $C(\theta) = A\frac{\theta^2}{2}$. *Then, if* $A > \overline{A} \equiv (1-\alpha)^{\frac{1}{\alpha^2}}(1+\alpha)^{\frac{1}{\alpha^2}-1}$, *no active non-monetary equilibrium exists whereas, under the Friedman rule, there exists a unique active monetary equilibrium for all values of A.*

*Proof.* See Appendix. ||

Proposition 4 clarifies why money can be welfare improving in the presence of a private information problem. If the information problem is sufficiently severe ($A > \overline{A}$), there is no active non-monetary equilibrium, whereas a unique active monetary equilibrium exists. This illustrates the strong spillover between the incentive to produce high quality and the information choice in the monetary equilibrium. If the information problem is severe, money mitigates the incentive to cheat and gives higher incentives to acquire information.

If $A < \overline{A}$, a non-monetary equilibrium can exist but there is no general result on how welfare is ranked across the non-monetary and the monetary equilibria. However, our simulations suggest that for most parameter values welfare, the fraction of high-quality producers, and the level of information are higher in the monetary equilibrium under the Friedman rule than in all potential non-monetary equilibria. Nonetheless, this result can be reversed for large values of $\alpha$ and small values of $A$.

## 6. CONCLUSION

We have investigated the role of fiat money in environments in which producers have private information about the quality of the goods they supply. In such environments, three issues are at the centre stage: adverse selection, moral hazard, and the incentive to acquire costly information. In order to discuss these issues, we have first studied how money modifies the quantities produced and exchanged in bilateral meetings. We have identified two effects of money on these quantities. The *recognizability effect* of money states that money crowds out real goods payments. The origin of this effect is that agents prefer to be paid with money—an object of universally recognized quality—rather than with goods of uncertain quality, and this desire gives rise to an endogenous role for money. It is this reduction of uncertainty, that, at least since Menger (1892), has been considered to be an important advantage of monetary exchange over barter. The *insurance effect* of money states that money promotes consumption by disconnecting the quantities that agents can buy from how they are assessed by their trading partners. In particular, this insurance allows agents to consume even when they are recognized as low-quality producers.

Our model supports the notion that money is a device for overcoming the adverse selection problem. When the fraction of high-quality producers and the level of information are exogenous, money promotes the production of high-quality goods and reduces the production of lemons, which increases welfare unambiguously. In contrast, when the fraction of high-quality producers is endogenous, money can be welfare-decreasing because of the *insurance effect* of money.

This effect increases the benefit of being a lemon producer because money enables recognized low-quality producers to consume, something that would never happen in a non-monetary equilibrium since no agent wants to produce in exchange for a lemon. If the information problem is not severe, the insurance effect dominates the recognizability effect, and consequently the fraction of high-quality producers and welfare are lower in the monetary economy.

Our model also suggests that money and investments in information are complements: the endogenous level of information and welfare are larger in the monetary economy if the intertemporal elasticity of substitution is sufficiently small. The intuition for this result is that money increases the return on information by providing consumption insurance, which induces agents to acquire more information than in the non-monetary equilibrium. Thus, contrary to common beliefs, money does not always economize on information costs.

There are several extensions to this paper that are worth considering. First, it would be interesting to introduce signalling into the model. Wealth (money holdings) could be modelled as a signalling device. Costly advertising of product quality is another device. Second, money is only one possible institution to overcome asymmetric information problems. Intermediaries and middlemen are other institutions whose role is to alleviate those problems. Third, other assets could be viewed as alternative to money. The role of credit in such an economy would be worth studying. Fourth, throughout this paper we have assumed that money is an object whose quality is identifiable by everybody. It would be interesting to see how robust our results are in a model that allows for counterfeiting.

## APPENDIX

A.1. *Proof of Proposition* 1

The proof proceeds in six parts.

**Part 1. For all $\theta \geq 0$, a non-active non-monetary equilibrium exists.** If $\Pi = 0$, an agent who is not recognized is perceived as a low-quality producer. Accordingly, beliefs are either 0 or 1. In the barter economy, a trade takes place between agents $i$ and $j$ if and only if $\varepsilon_i = \varepsilon_j = 1$. Thus, according to (20),

$$\mathcal{D}(0) = [u(q^b_{1,1}) - q^s_{1,1}]\, f_H(1)\, f(1) - u(q^b_{1,1})\, f_L(1)\, f(1).$$

From (1), if $\Pi = 0$, $f(1) = 0$. Therefore, $\mathcal{D}(0) = 0$. Consequently, if $\Pi = 0$, the best response of any household is to choose any $\pi \in [0, 1]$. Thus, $\pi = \Pi = 0$ is a fixed point of (22).

**Part 2. If $\theta = 0$, the unique equilibrium is non-active.** According to (1), if $\theta = 0$, then $f(\Pi) = f_H(\Pi) = f_L(\Pi) = 1$ and $f(1) = f_H(1) = f(0) = f_L(0) = 0$. Hence, (20) can be rewritten as follows:

$$\mathcal{D}(\Pi) = \begin{cases} -q^s_{\Pi,\Pi} < 0 & \text{if } \Pi > 0 \\ 0 & \text{if } \Pi = 0. \end{cases}$$

Consequently, if $\theta = 0$, the unique symmetric Nash equilibrium is $\Pi = 0$.

**Part 3. An active non-monetary equilibrium with $\Pi = 1$ exists iff $\theta \geq \theta_B \equiv \frac{q^*}{u(q^*)}$.** In the barter economy, if one of the traders in a match is recognized as a low-quality producer (*i.e.* $\varepsilon_i = 0$ or $\varepsilon_j = 0$), then no trade takes place (*i.e.* $q^b_\varepsilon = q^s_\varepsilon = 0$ ). Hence, using (1), we can rewrite (20) as

$$\mathcal{D}(\Pi) = \theta^2 \Pi [u(q^b_{1,1}) - q^s_{1,1}] + (1-\theta)\theta[\Pi u(q^b_{\Pi,1}) - q^s_{\Pi,1}] \\ - \theta\Pi(1-\theta) q^s_{1,\Pi} - (1-\theta)^2 q^s_{\Pi,\Pi}. \tag{A.1}$$

An equilibrium where all traders are high-quality producers ($\Pi = 1$) exists if and only if $\mathcal{D}(1) \geq 0$. Using the fact that $q^b_{1,1} = q^s_{1,1} = q^*$, this condition yields $\theta \geq \theta_B \equiv \frac{q^*}{u(q^*)}$. Note that $\theta_B < 1$.

**Part 4. For all $\theta \in (0, \theta_B)$, an active non-monetary equilibrium with $\Pi < 1$ exists.** We first establish the following result:

$$\lim_{x\to 0} \frac{u'^{-1}\left(\frac{1}{x}\right)}{x} = 0.$$

From the strict concavity of the utility function and the fact that $u(0) = 0$, we have $\frac{u(x)}{x} > u'(x)$, $\forall x > 0$. Because $u'^{-1}(\cdot)$ is a decreasing function, we have

$$u'^{-1}\left(\frac{u(x)}{x}\right) < x, \qquad \forall x > 0. \tag{A.2}$$

Multiplying each side of the inequality (A.2) by $\frac{u(x)}{x}$, we obtain

$$\frac{u(x)}{x} u'^{-1}\left(\frac{u(x)}{x}\right) < u(x), \qquad \forall x > 0. \tag{A.3}$$

Define $\varphi(x) \equiv \frac{x}{u(x)}$. We have $\varphi'(x) > 0$ for all $x > 0$ and $\lim_{x\to 0} \varphi(x) \le \lim_{x\to 0} \frac{1}{u'(x)} = 0$. Inequality (A.3) can be rewritten as follows:

$$\frac{1}{\varphi(x)} u'^{-1}\left(\frac{1}{\varphi(x)}\right) < u(x) \qquad \forall x > 0.$$

Taking the limit when $x$ approaches 0 and denoting $X = \varphi(x)$ we have

$$\lim_{X\to 0} \frac{1}{X} u'^{-1}\left(\frac{1}{X}\right) = 0. \tag{A.4}$$

From (10) and (11), $q^s_{\Pi,\Pi}$ satisfies $\Pi u'(q^s_{\Pi,\Pi}) = 1$. Hence, $q^s_{\Pi,\Pi} = u'^{-1}\left(\frac{1}{\Pi}\right)$. From (A.4) we deduce that

$$\lim_{\Pi\to 0} \frac{q^s_{\Pi,\Pi}}{\Pi} = 0. \tag{A.5}$$

Since $\omega = 0$ in the non-monetary equilibrium, (11) implies

$$\Pi u(q^b_{\Pi,1}) - q^s_{\Pi,1} = \Pi u'(q^b_{\Pi,1})[u(q^s_{\Pi,1}) - q^b_{\Pi,1}]. \tag{A.6}$$

Consequently, $\mathcal{D}(\Pi)$ given by (A.1) can be rewritten as

$$\mathcal{D}(\Pi) = \Pi \left\{ \theta^2[u(q^b_{1,1}) - q^s_{1,1}] + (1-\theta)\theta u'(q^b_{\Pi,1})[u(q^s_{\Pi,1}) - q^b_{\Pi,1}] \right.$$
$$\left. - \; \theta(1-\theta) q^s_{1,\Pi} - (1-\theta)^2 \frac{q^s_{\Pi,\Pi}}{\Pi} \right\}.$$

The last two terms within the brackets tend to zero when $\Pi$ approaches zero. The first term is strictly positive and independent of $\Pi$. The limit of the second term when $\Pi$ approaches zero is *a priori* indeterminate. Consequently, in the neighbourhood of $\Pi = 0$, $\mathcal{D}(\Pi)$ can be approximated by

$$\mathcal{D}(\Pi) \simeq \Pi\{\theta^2[u(q^b_{1,1}) - q^s_{1,1}] + (1-\theta)\theta u'(q^b_{\Pi,1})[u(q^s_{\Pi,1}) - q^b_{\Pi,1}]\} > 0 \qquad \forall \Pi \in ]0, \xi]$$

where $\xi$ is arbitrarily close to zero.

Furthermore, for all $\theta < \theta_B$, $\mathcal{D}(1) < 0$. Consequently, since $\mathcal{D}(\Pi)$ is continuous if $\theta < \theta_B$, there is a $\Pi \in ]0, 1[$ such that $\mathcal{D}(\Pi) = 0$.

**Part 5. Derivation of $\mathcal{D}(\Pi)$ in the monetary equilibrium when $\gamma \to \beta$.** Let $q^*_{\varepsilon_i}$ denote the value of $q$ such that $\varepsilon_i u'(q) = 1$. Under the Friedman rule ($\gamma \to \beta$), agents in a $(\varepsilon_i, \varepsilon_j)$-match produce and consume $q^*_{\varepsilon_i}$ and $q^*_{\varepsilon_j}$ such that the total surplus of the match is maximized. We have

$$q^b_{1,1} = q^s_{1,1} = q^b_{1,0} = q^s_{0,1} = q^b_{1,\Pi} = q^s_{\Pi,1} = q^*$$
$$q^b_{\Pi,1} = q^s_{1,\Pi} = q^s_{\Pi,\Pi} = q^b_{\Pi,\Pi} = q^b_{\Pi,0} = q^s_{0,\Pi} = q^*_\Pi$$
$$q^b_{0,0} = q^s_{0,0} = q^b_{0,1} = q^s_{1,0} = q^b_{0,\Pi} = q^s_{\Pi,0} = q^*_0.$$

Moreover, when $\gamma \to \beta$, in each meeting agents use a monetary transfer in order to split the total surplus of the match evenly. Hence,

$$\varepsilon_i u(q^b_\varepsilon) - q^s_\varepsilon - x_\varepsilon \omega = \frac{\varepsilon_i u(q^*_{\varepsilon_i}) - q^*_{\varepsilon_i} + \varepsilon_j u(q^*_{\varepsilon_j}) - q^*_{\varepsilon_j}}{2}. \tag{A.7}$$

Using the previous expression, $\mathcal{D}(\Pi)$ can be written as

$$\mathcal{D}(\Pi) = \theta\Big(\frac{u(q^*) - q^*}{2}\Big) - (1-\theta)q^*_{\Pi}. \tag{A.8}$$

**Part 6. Monetary equilibrium under the Friedman rule.** Note, first, that for all $\theta > 0$, we have $\mathcal{D}(0) > 0$, and that $\mathcal{D}(\Pi)$ is strictly decreasing in $\Pi$. Consequently, a monetary equilibrium with $\Pi < 1$ exists if and only if $\mathcal{D}(1) < 0$. From (A.8), this condition yields

$$\theta < \theta_M \equiv \frac{2q^*}{u(q^*) + q^*}.$$

Note that $\theta_M < 1$ because $q^* < u(q^*)$. Because $\mathcal{D}(\Pi)$ is strictly decreasing in $\Pi$, the monetary equilibrium is unique. Note also that (A.8) implies that in the monetary equilibrium one has $\frac{\partial q^*_{\Pi}}{\partial \theta} > 0$ and therefore $\frac{\partial \Pi_M}{\partial \theta} > 0$.

### A.2. *Proof of Proposition* 2

We consider the fraction of high-quality producers ($\Pi_B$) when $\theta$ is close to 0. From (A.1) the condition $\mathcal{D}(\Pi_B) = 0$ can be rewritten as

$$\begin{aligned} q^*_{\Pi_B} &= \theta\{\theta\Pi_B[u(q^b_{1,1}) - q^s_{1,1}] + (1-\theta)[\Pi_B u(q^b_{\Pi_B,1}) - q^s_{\Pi_B,1}] \\ &\quad - \Pi_B(1-\theta)q^s_{1,\Pi_B} + (2-\theta)q^*_{\Pi_B}\}. \end{aligned} \tag{A.9}$$

From (A.8), the fraction of high-quality producers in the monetary equilibrium ($\Pi_M$) satisfies

$$q^*_{\Pi_M} = \theta\left\{\left(\frac{u(q^*) - q^*}{2}\right) + q^*_{\Pi_M}\right\}. \tag{A.10}$$

From (A.9) and (A.10), if $\theta$ is close to 0, $\Pi_B$ and $\Pi_M$ must also be close to zero. Consequently,

$$\begin{aligned} \frac{u(q^*) - q^*}{2} + q^*_{\Pi_M} &\simeq \frac{u(q^*) - q^*}{2} \\ &\gg \theta\Pi_B[u(q^b_{1,1}) - q^s_{1,1}] + (1-\theta)[\Pi_B u(q^b_{\Pi_B,1}) - q^s_{\Pi_B,1}] - \Pi_B(1-\theta)q^s_{1,\Pi_B} + (2-\theta)q^*_{\Pi_B} \simeq 0. \end{aligned}$$

Because $q^*_{\Pi}$ is increasing in $\Pi$, we deduce from (A.9) and (A.10) that $\Pi_M > \Pi_B$.

### A.3. *Derivation of the closed-form solutions* $\Pi_B$ *and* $\Pi_M$

From equations (10) and (11), one can verify that the terms of trade in the non-monetary equilibrium satisfy

$$q^s_{1,\Pi} = \Pi^{\frac{\alpha}{(1-\alpha)(1+\alpha)}}, \qquad q^b_{1,\Pi} = \Pi^{\frac{1}{(1-\alpha)(1+\alpha)}}, \qquad q^s_{1,1} = q^b_{1,1} = q^* = 1, \qquad q^s_{\Pi,\Pi} = q^b_{\Pi,\Pi} = q^*_{\Pi} = \Pi^{\frac{1}{1-\alpha}}. \tag{A.11}$$

Substituting these expressions into (A.1), we obtain

$$\mathcal{D}(\Pi) = \theta^2\Pi\left(\frac{1-\alpha}{\alpha}\right) + (1-\theta)\theta\Pi^{\frac{1}{(1-\alpha)(1+\alpha)}}\left(\frac{1-\alpha}{\alpha}\right) - \theta(1-\theta)\Pi^{\frac{\alpha}{(1-\alpha)(1+\alpha)}+1} - (1-\theta)^2\Pi^{\frac{1}{1-\alpha}}. \tag{A.12}$$

The condition $\mathcal{D}(\Pi) = 0$ can be rewritten as

$$\theta^2\Pi^{\frac{-\alpha^2}{(1-\alpha)(1+\alpha)}}\left(\frac{1-\alpha}{\alpha}\right) + (1-\theta)\theta\left(\frac{1-\alpha}{\alpha}\right) = \theta(1-\theta)\Pi^{\frac{\alpha}{1+\alpha}} + (1-\theta)^2\Pi^{\frac{\alpha}{(1-\alpha)(1+\alpha)}}. \tag{A.13}$$

The L.H.S. of (A.13) is decreasing in $\Pi$ whereas the R.H.S. is increasing in $\Pi$. Hence, for any $\theta$ there is a unique $\Pi$ satisfying (A.13), which is given by

$$\Pi_B = \left(\frac{1-\alpha}{\alpha}\,\frac{\theta}{1-\theta}\right)^{\frac{(1-\alpha)(1+\alpha)}{\alpha}}.$$

The expression for $\Pi_M$ is obtained by replacing $q^*_{\Pi}$ with $\Pi^{\frac{1}{1-\alpha}}$ in the first-order condition (A.8). Finally, note that $\Pi_B < \Pi_M$ if and only if

$$\theta < \overline{\theta} \equiv \frac{\alpha}{\alpha + 2^{\alpha}(1-\alpha)}.$$

A.4. *Restricted trade equilibrium*

**Part 1: Characterization of the restricted trade equilibrium.** The restricted trade rule specifies that when a trader in a match is recognized as a low-quality producer ($\varepsilon_i = 0$ or $\varepsilon_j = 0$), no trade takes place even if the recognized low-quality producer holds money. The programme of the household is given by (12) when all terms with $\varepsilon_i = 0$ or $\varepsilon_j = 0$ are eliminated. The choice $\Pi$ that sustains a symmetric Nash equilibrium is given by

$$\begin{array}{ll} \Pi = 1 & \text{if } \mathcal{D}_R(1) \geq 0 \\ \Pi = 0 & \text{if } \mathcal{D}_R(0) \leq 0 \\ \mathcal{D}_R(\Pi) = 0 & \text{otherwise} \end{array}$$

where $\mathcal{D}_R$ is given by (20) when all terms with $\varepsilon_i = 0$ or $\varepsilon_j = 0$ are eliminated:

$$\mathcal{D}_R(\Pi) = [u(q^b_{1,1}) - q^s_{1,1}]\theta^2\Pi - q^s_{1,\Pi}\theta(1-\theta)\Pi + [\Pi u(q^b_{\Pi,1}) - q^s_{\Pi,1} - x_{\Pi,1}\omega](1-\theta)\theta - q^s_{\Pi,\Pi}(1-\theta)^2. \quad \text{(A.14)}$$

Let us first determine the necessary and sufficient conditions for the existence of an equilibrium without low-quality producers. Using the fact that when $\Pi = 1$ money is not valued, we find that $\mathcal{D}_R(1) = \mathcal{D}_B(1)$. Consequently, a necessary and sufficient condition for an equilibrium without lemon producers is $\theta \geq \theta_B$. Because agents cannot trade when recognized as low-quality producers, the incentive-compatibility condition for $\Pi = 1$ is the same as the one in the barter economy.

The marginal value of money is affected by the restricted trade rule. Nevertheless, under the Friedman rule the constraints on money holdings of households' members are not binding and we have

$$\Pi u(q^b_{\Pi,1}) - q^s_{\Pi,1} - x_{\Pi,1}\omega = \frac{\Pi u(q^*_\Pi) - q^*_\Pi + u(q^*) - q^*}{2}.$$

Consequently, (A.14) can be rewritten as follows:

$$\mathcal{D}_R(\Pi) = \theta^2\Pi[u(q^*) - q^*] + \theta(1-\theta)\left\{\frac{\Pi u(q^*_\Pi) - q^*_\Pi + u(q^*) - q^*}{2}\right\} - (1-\theta)\Pi\theta q^*_\Pi - q^*_\Pi(1-\theta)^2. \quad \text{(A.15)}$$

Let us consider equilibria with $\Pi \in (0, 1)$. Then we have $\mathcal{D}_R(\Pi) = 0$. For all $\theta \in (0, \theta_B)$ we have $\mathcal{D}_R(0) > 0$ and $\mathcal{D}_R(1) < 0$. Consequently, an active monetary equilibrium exists.

**Part 2: Comparison of the non-monetary equilibrium and the restricted trade equilibrium.** According to (A.1), in the non-monetary equilibrium

$$\mathcal{D}_B(\Pi) = \theta^2\Pi[u(q^*) - q^*] + \theta(1-\theta)[\Pi u(q^b_{\Pi,1}) - q^s_{\Pi,1}] - (1-\theta)\Pi\theta q^s_{1,\Pi} - (1-\theta)^2 q^*_\Pi. \quad \text{(A.16)}$$

In order to demonstrate that the fraction of high-quality producers in the restricted trade equilibrium is larger than in any non-monetary equilibrium, we show that $\mathcal{D}_R(\Pi) - \mathcal{D}_B(\Pi) > 0$ for any $\Pi$. From (A.15) and (A.16), we get

$$\mathcal{D}_R(\Pi) - \mathcal{D}_B(\Pi) = \theta(1-\theta)\left\{\frac{\Pi u(q^*_\Pi) - q^*_\Pi + u(q^*) - q^*}{2} - [\Pi u(q^b_{\Pi,1}) - q^s_{\Pi,1}] - \Pi q^*_\Pi + \Pi q^s_{1,\Pi}\right\}. \quad \text{(A.17)}$$

Berentsen and Rocheteau (2003) show that in the non-monetary equilibrium in asymmetric matches, $q^s_{1,\Pi} > q^*_\Pi$. Moreover, for all $\Pi \in (0, 1)$, in a $(\Pi, 1)$-meeting, the buyer surplus is strictly smaller than the seller surplus

$$\Pi u(q^b_{\Pi,1}) - q^s_{\Pi,1} < u(q^s_{\Pi,1}) - q^b_{\Pi,1}, \qquad \forall \Pi \in (0, 1). \quad \text{(A.18)}$$

From (A.18) we have

$$2[\Pi u(q^b_{\Pi,1}) - q^s_{\Pi,1}] < \Pi u(q^b_{\Pi,1}) - q^s_{\Pi,1} + u(q^s_{\Pi,1}) - q^b_{\Pi,1}, \qquad \forall \Pi \in (0, 1). \quad \text{(A.19)}$$

From Berentsen and Rocheteau (2003), we also know that in the barter economy terms of trade in asymmetric matches are socially inefficient in the sense that the total surplus of the match is not maximized. This implies

$$\Pi u(q^b_{\Pi,1}) - q^s_{\Pi,1} + u(q^s_{\Pi,1}) - q^b_{\Pi,1} < \Pi u(q^*_\Pi) - q^*_\Pi + u(q^*) - q^* \qquad \forall \Pi \in (0, 1). \quad \text{(A.20)}$$

Accordingly, (A.19) and (A.20) imply that

$$\Pi u(q^b_{\Pi,1}) - q^s_{\Pi,1} < \frac{\Pi u(q^*_\Pi) - q^*_\Pi + u(q^*) - q^*}{2} \qquad \forall \Pi \in (0, 1).$$

From this we deduce that $\mathcal{D}_R(\Pi) - \mathcal{D}_B(\Pi) > 0$ for all $\Pi \in (0, 1)$.

A.5. *Proof of Proposition* 3

**Part 1. The barter economy.** In the non-monetary equilibrium, there is no trade if one player is recognized as a low-quality producer. Consequently, we can restrict our attention to matches where $(\varepsilon_i, \varepsilon_j) \in \{\Pi, 1\}^2$. Equation (26) can be rewritten as

$$\mathcal{I}(\theta, \Theta) = \Theta\Pi^2[u(q^b_{1,1}) - q^s_{1,1}] - \Theta\Pi[\Pi u(q^b_{\Pi,1}) - q^s_{\Pi,1}] + \Pi(1-\Theta)[u(q^b_{1,\Pi}) - \Pi q^s_{1,\Pi}] - (1-\Theta)\Pi[u(q^b_{\Pi,\Pi}) - q^s_{\Pi,\Pi}] - C'(\theta). \tag{A.21}$$

Because $\mathcal{I}(\theta, \Theta)$ is decreasing in $\theta$, for all $\Theta$, there is a unique $\theta$ that satisfies the first-order condition of the programme of the household. Let us first consider symmetric Nash equilibria where $\theta = \Theta = 0$. The condition $\mathcal{I}(0, 0) \leq 0$ can be rewritten as

$$\Pi[u(q^b_{1,\Pi}) - \Pi q^s_{1,\Pi}] - \Pi[u(q^b_{\Pi,\Pi}) - q^s_{\Pi,\Pi}] \leq 0.$$

For $u(q) = \alpha^{-1}q^\alpha$, we have $q^b_{1,\Pi} = \Pi^{\frac{1}{(1-\alpha)(1+\alpha)}}$, $q^s_{1,\Pi} = \Pi^{\frac{\alpha}{(1-\alpha)(1+\alpha)}}$ and $q^*_\Pi = \Pi^{\frac{1}{1-\alpha}}$. Therefore

$$\Pi[u(q^b_{1,\Pi}) - \Pi q^s_{1,\Pi}] - \Pi[u(q^*_\Pi) - q^*_\Pi] = \Pi\left(\frac{1}{\alpha} - \Pi\right)\left(\Pi^{\frac{\alpha}{1-\alpha^2}} - \Pi^{\frac{\alpha}{1-\alpha}}\right) \geq 0.$$

Consequently, the condition $\mathcal{I}(0, 0) \leq 0$ is satisfied only for $\Pi \in \{0, 1\}$.

Second, consider symmetric Nash equilibria where $\theta = 1$. The condition $\mathcal{I}(1, 1) \geq 0$ can be rewritten as

$$A \leq \Pi^2[u(q^b_{1,1}) - q^s_{1,1}] - \Pi[\Pi u(q^b_{\Pi,1}) - q^s_{\Pi,1}] = \Pi^2\left(\frac{1-\alpha}{\alpha}\right)\left(1 - \Pi^{\frac{\alpha^2}{1-\alpha^2}}\right). \tag{A.22}$$

Third, consider symmetric Nash equilibria where $\theta \in (0, 1)$. The condition $\mathcal{I}(\theta, \theta) = 0$ can be rewritten as

$$\theta = \Pi\frac{[u(q^b_{1,\Pi}) - \Pi q^s_{1,\Pi}] - [u(q^*_\Pi) - q^*_\Pi]}{A - \Pi^2[u(q^*) - q^*] + \Pi[u(q^b_{1,\Pi}) - \Pi q^s_{1,\Pi}] + \Pi[\Pi u(q^b_{\Pi,1}) - q^s_{\Pi,1}] - [\Pi u(q^*_\Pi) - \Pi q^*_\Pi]}. \tag{A.23}$$

If (A.22) is not satisfied, then the value of $\theta$ satisfying (A.23) is between 0 and 1.

Let us recapitulate the previous results. There is a unique symmetric Nash equilibrium. If (A.22) holds, $\theta = 1$. Otherwise, $\theta$ is given by (A.23).

**Part 2. The monetary economy under the Friedman rule.** Under the Friedman rule ($\gamma \to \beta$), in each meeting agents transfer money so that the total surplus of each match is split evenly. This implies that

$$\varepsilon_i u(q^b_\varepsilon) - q^s_\varepsilon - x_\varepsilon\omega = \frac{\varepsilon_i u(q^*_{\varepsilon_i}) - q^*_{\varepsilon_i} + \varepsilon_j u(q^*_{\varepsilon_j}) - q^*_{\varepsilon_j}}{2}. \tag{A.24}$$

Therefore, equation (26) can be rewritten as follows:

$$\begin{aligned}\mathcal{I}(\theta, \Theta) = {} & \tfrac{1}{2}\Pi\textstyle\sum_{\varepsilon\in E}[\varepsilon_i u(q^*_{\varepsilon_i}) - q^*_{\varepsilon_i}]f_\theta(\varepsilon_i)f_H(\varepsilon_j) + \tfrac{1}{2}\Pi\sum_{\varepsilon\in E}[\varepsilon_j u(q^*_{\varepsilon_j}) - q^*_{\varepsilon_j}]f_\theta(\varepsilon_i)f_H(\varepsilon_j)\\ & + \tfrac{1}{2}(1-\Pi)\textstyle\sum_{\varepsilon\in E}[\varepsilon_i u(q^*_{\varepsilon_i}) - q^*_{\varepsilon_i}]f_\theta(\varepsilon_i)f_L(\varepsilon_j) + \tfrac{1}{2}(1-\Pi)\sum_{\varepsilon\in E}[\varepsilon_j u(q^*_{\varepsilon_j}) - q^*_{\varepsilon_j}]f_\theta(\varepsilon_i)f_L(\varepsilon_j)\\ & + (1-\Pi)\textstyle\sum_{\varepsilon\in E} q^*_{\varepsilon_j} f_\theta(\varepsilon_i)f_L(\varepsilon_j) - C'(\theta).\end{aligned} \tag{A.25}$$

Using the fact that $\sum_{\varepsilon_i} f_\theta(\varepsilon_i) = 0$ and $\sum_{\varepsilon_i} f_H(\varepsilon_j) = \sum_{\varepsilon_i} f_L(\varepsilon_j) = 1$, equation (A.25) can be rewritten as

$$\begin{aligned}\mathcal{I}(\theta, \Theta) &= \tfrac{1}{2}\textstyle\sum_{\varepsilon_i\in\{0,\Pi,1\}}[\varepsilon_i u(q^*_{\varepsilon_i}) - q^*_{\varepsilon_i}]f_\theta(\varepsilon_i) - C'(\theta)\\ &= \frac{\Pi[u(q^*) - q^*] - [\Pi u(q^*_\Pi) - q^*_\Pi]}{2} - C'(\theta).\end{aligned}$$

Consider a symmetric Nash equilibrium where $\theta \in (0, 1)$. Then, condition $\mathcal{I}(\theta, \theta) = 0$ for a symmetric Nash equilibrium can be rewritten as

$$A\theta = \Pi\frac{[u(q^*) - q^*] - \left[u(q^*_\Pi) - \frac{q^*_\Pi}{\Pi}\right]}{2}. \tag{A.26}$$

Because of the concavity of the utility function, we have $u(q^*) - u(q^*_\Pi) \geq u'(q^*)(q^* - q^*_\Pi) = q^* - q^*_\Pi$, which implies that $\Pi[u(q^*) - u(q^*_\Pi)] \geq \Pi(q^* - q^*_\Pi) > \Pi q^* - q^*_\Pi$ if $\Pi \in (0, 1)$. The R.H.S. of (A.26) is strictly positive. If $A > \frac{\Pi[u(q^*)-q^*]-[\Pi u(q^*_\Pi)-q^*_\Pi]}{2}$, then $\theta \in [0, 1)$. If $A \leq \frac{\Pi[u(q^*)-q^*]-[\Pi u(q^*_\Pi)-q^*_\Pi]}{2}$ then $\theta = 1$.

**Part 3. Ranking of information acquisition in the monetary economy ($\Theta_M$) and the barter economy ($\Theta_B$).** Let $\Theta_B$ be the value of $\theta$ that satisfies equation (A.23) and let $\Theta_M$ be the value of $\theta$ that satisfies equation (A.26). From (A.23) we have

$$\lim_{\Pi \to 0} \frac{\Theta_B}{\Pi} = 0. \tag{A.27}$$

From (A.26), we have

$$\lim_{\Pi \to 0} \frac{\Theta_M}{\Pi} = \frac{[u(q^*) - q^*]}{2A} > 0. \tag{A.28}$$

Consequently, for $\Pi$ sufficiently close to zero, we have $\Theta_B < \Theta_M$.

A.6. *Proof of Lemma* 1

The proof is by contradiction. Assume that everyone produces high quality: $\Pi = 1$. From (1), we find $f(1; \theta) = 1$ and $f_\theta(1; \theta) = 0$. Equation (26) can then be rewritten as

$$\mathcal{I}(\theta, \Theta) = -C'(\theta).$$

From (27), the optimal choice of information is then $\theta = 0$. But if $\theta = \Theta = 0$, the unique equilibrium is non-active, which contradicts the initial assumption. Note further that $\theta$ cannot be equal to one in equilibrium. Indeed, if $\theta = \Theta = 1$, households would choose $\pi = 1$, but then it would be rational for households to be uninformed ($\theta = 0$).

A.7. *Proof of Proposition* 4

**Part 1. The non-monetary equilibrium.** We look at the candidates for a non-monetary equilibrium. Let $\Pi_B$ denote the fraction of high-quality producers and $\Theta_B$ the level of information at the non-monetary equilibrium. First, from Lemma 1 any candidate must be such that $\Pi_B < 1$ and $\Theta_B < 1$. Second, a candidate must satisfy $\mathcal{I}(\Theta_B, \Theta_B) = 0$ and $\mathcal{D}(\Pi_B) = 0$. From (A.21), the pair $(\Pi_B, \Theta_B)$ must satisfy

$$A\Theta_B = \Theta_B \frac{(1-\alpha)}{\alpha}\left[\Pi_B^2 - (\Pi_B)^{\frac{2-\alpha^2}{1-\alpha^2}}\right] + (1-\Theta_B)\left(\frac{1}{\alpha} - \Pi_B\right)\left[(\Pi_B)^{\frac{1+\alpha-\alpha^2}{1-\alpha^2}} - (\Pi_B)^{\frac{1}{1-\alpha}}\right] \tag{A.29}$$

with

$$\Pi_B = \left[\frac{\Theta_B(1-\alpha)}{(1-\Theta_B)\alpha}\right]^{\frac{(1-\alpha)(1+\alpha)}{\alpha}}.$$

Multiplying each side of (A.29) by $\frac{\alpha}{\Theta_B(1-\alpha)}$, and after some simplification we obtain

$$A\frac{\alpha}{(1-\alpha)} = \frac{1}{\alpha}\Pi_B\left[1 - (\Pi_B)^{\frac{\alpha^2}{1-\alpha^2}}\right]. \tag{A.30}$$

The equilibrium fraction of high-quality producers in the barter economy is a $\Pi_B \in \,]0, 1[$ that satisfies (A.30). It can immediately be checked that the R.H.S. of (A.30) is strictly positive for all $\Pi_B \in \,]0, 1[$ and is equal to 0 for $\Pi_B \in \{0, 1\}$.

Let $\Pi_B^{\max}$ be the value of $\Pi_B \in ]0, 1[$ such that the derivative of the R.H.S. of (A.30) is zero. $\Pi_B^{\max}$ is unique and is given by

$$\Pi_B^{\max} = (1-\alpha^2)^{\frac{1-\alpha^2}{\alpha^2}}.$$

Consequently, there is a threshold $\overline{A}$ for the information cost such that the following is true: if $A = \overline{A}$, $\Pi_B = \Pi_B^{\max}$. If $A > \overline{A}$, there is no active non-monetary equilibrium. If $A < \overline{A}$, there are two potential active non-monetary equilibria. The threshold $\overline{A}$ is the value of $A$ that satisfies (A.30) with $\Pi_B = \Pi_B^{\max}$. Therefore it is equal to

$$\overline{A} = \frac{(1-\alpha^2)^{\frac{1}{\alpha^2}}}{1+\alpha}. \tag{A.31}$$

**Part 2. The monetary equilibrium.** Let $\Pi_M$ denote the fraction of high-quality producers and $\Theta_M$ the level of information at the monetary equilibrium under the Friedman rule. A candidate must satisfy $\mathcal{I}(\Theta_M, \Theta_M) = 0$ and $\mathcal{D}(\Pi_M) = 0$. It could be checked from an equation analogous to (A.25) that the choice of information of each household is independent of its choice of $\pi$. Therefore, each candidate for an equilibrium will in fact be an equilibrium (*i.e.* we don't have to check for a simultaneous deviation for $\theta$ and $\pi$). According to (A.26), a monetary equilibrium is a pair $(\Pi_M, \Theta_M)$ that satisfies

$$A\Theta_M = \left(\frac{1-\alpha}{2\alpha}\right)\left[\Pi_M - (\Pi_M)^{\frac{1}{1-\alpha}}\right] \tag{A.32}$$

with

$$\Pi_M = \left[\frac{\Theta_M(1-\alpha)}{2\alpha(1-\Theta_M)}\right]^{1-\alpha}. \tag{A.33}$$

From (A.33), we can express $\Theta_M$ as

$$\Theta_M = \frac{2\alpha(\Pi_M)^{\frac{1}{1-\alpha}}}{1-\alpha+2\alpha(\Pi_M)^{\frac{1}{1-\alpha}}}. \tag{A.34}$$

Substituting this expression into (A.32) yields

$$A = \frac{(1-\alpha)}{(2\alpha)^2}\left[1-\alpha+2\alpha(\Pi_M)^{\frac{1}{1-\alpha}}\right]\left[(\Pi_M)^{\frac{-\alpha}{1-\alpha}} - 1\right]. \tag{A.35}$$

It can be verified that the R.H.S. of (A.35) is infinite for $\Pi_M = 0$ and is equal to 0 for $\Pi_M = 1$. Consequently, for all $A > 0$, an active monetary equilibrium exists.

We differentiate the R.H.S. of (A.35) to show that it is strictly decreasing in $\Pi_M$:

$$\frac{\partial \text{R.H.S. (A.35)}}{\partial \Pi_M} = \frac{1}{4\alpha}\left\{2(1-\alpha) - 2(\Pi_M)^{\frac{\alpha}{1-\alpha}} - (1-\alpha)(\Pi_M)^{\frac{-1}{1-\alpha}}\right\}.$$

The factor in the braces is equal to $-\infty$ for $\Pi_M = 0$ and to $-(1+\alpha)$ for $\Pi_M = 1$. For $\Pi_M \in [0, 1]$, that factor reaches a maximum for $\Pi_M = \min\left[\left(\frac{1-\alpha}{2\alpha}\right)^{\frac{1-\alpha}{1+\alpha}}, 1\right]$. If $\frac{1-\alpha}{2\alpha} < 1$, this maximum is equal to

$$2(1+\alpha)\left[\frac{(1-\alpha)}{(1+\alpha)} - \left(\frac{1-\alpha}{2\alpha}\right)^{\frac{\alpha}{1+\alpha}}\right] < 0.$$

Consequently, $\frac{\partial \text{R.H.S. (A.35)}}{\partial \Pi_M} < 0$ for all $\Pi_M \in [0, 1]$. Therefore, the monetary equilibrium under the Friedman rule is unique.

*Acknowledgements*. We would like to thank Randall Wright, Kyung-Ha Cho, and two anonymous referees for their very helpful suggestions. This paper has also benefited from the comments of Marius Brülhart, Jean Cartelier, Michael Gratwohl, Jürgen von Hagen, Abhinay Muthoo, Matthew Ryan, Ralph von Selzam, Shouyong Shi, Céderic Wasser, Alexandre Ziegler, and seminar participants at the Universities of Auckland, Berlin (Humboldt-University), Brussels, Canberra (ANU), Essex, Heidelberg, Lausanne, Melbourne, Paris-Nanterre, Paris 2, Regensburg, Singapore (NUS), Southampton, Sydney, and Zürich, and conference participants at the SED-meeting in Stockholm, Konstanz Seminar on Monetary Theory and Policy (2003), SAET-meeting in Ischia, and Monetary Economics meeting at the Federal Reserve Bank of Cleveland (August 2002). The first author gratefully acknowledges financial support from a grant received by the Swiss National Science Foundation. Both authors thank the Federal Reserve Bank of Cleveland for research support.

## REFERENCES

AKERLOF, G. (1970), "The Market for Lemons: Quality Uncertainty and the Market Mechanism", *Quarterly Journal of Economics*, **89**, 488–500.

ALCHIAN, A. (1977), "Why Money?", *Journal of Money, Credit, and Banking, Part 2*, **9** (1), 133–140.

AUSUBEL, L., CRAMTON, P. and DENECKERE, R. (2002), "Bargaining with Incomplete Information", in R. Aumann and S. Hart (eds.) *Handbook of Game Theory*, Vol. 3 (Amsterdam: Elsevier).

BANERJEE, A. and MASKIN, E. (1996), "A Walrasian Theory of Money and Barter", *Quarterly Journal of Economics*, **111**, 955–1005.

BERENTSEN, A. and ROCHETEAU, G. (2001), "On the Friedman Rule in Search Models of Money", *B.E. Journals in Macroeconomics* (forthcoming).

BERENTSEN, A. and ROCHETEAU, G. (2002), "On the Efficiency of Monetary Exchange: How Divisibility of Money Matters", *Journal of Monetary Economics*, **49** (8), 1621–1649.

BERENTSEN, A. and ROCHETEAU, G. (2003), "Money and the Gains from Trade", *International Economic Review*, **44** (1), 263–297.

BRUNNER, K. and MELTZER, A. (1971), "The Uses of Money: Money in the Theory of an Exchange Economy", *American Economic Review*, **61**, 784–805.

CHO, I.-K. and KREPS, D. (1987), "Signalling Games and Stable Equilibria", *Quarterly Journal of Economics*, **102** (2), 179–222.

CUADRAS-MORATÓ, X. (1994), "Commodity Money in the Presence of Goods of Heterogenous Quality", *Economic Theory*, **4**, 579–591.

GREEN, E. and WEBER, W. (1996), "Will the New $100 Bill Decrease Counterfeiting?", *Federal Reserve Bank of Minneapolis Quarterly Review*, **20**, 3–10.

HAEGLER, U. (1997), "Fiat Money and Quality Uncertainty", *Economica*, **64**, 547–565.

HARSANYI, J. and SELTEN, R. (1972), "A Generalized Nash Solution for Two-Person Bargaining Games with Incomplete Information", *Management Science*, **18** (5), 80–106.

INDERST, R. (2001), "Screening in a Matching Market", *Review of Economic Studies*, **68**, 849–868.

INDERST, R. (2002), "Matching Markets with Adverse Selection" (Mimeo, London School of Economics).

KIM, Y. S. (1996), "Money, Barter, and Costly Information Acquisition", *Journal of Monetary Economics*, **37**, 119–142.

KING, R. and PLOSSER, C. (1986), "Money as the Mechanism of Exchange", *Journal of Monetary Economics*, **17**, 93–115.

KIYOTAKI, N. and WRIGHT, R. (1991), "A Contribution to the Pure Theory of Money", *Journal of Economic Theory*, **53**, 215–235.

KIYOTAKI, N. and WRIGHT, R. (1993), "A Search-Theoretic Approach to Monetary Economics", *American Economic Review*, **83**, 63–77.

LAGOS, R. and WRIGHT, R. (2002), "A Unified Framework for Monetary Theory and Policy Analysis" (Mimeo, University of Pennsylvania).

LI, Y. (1995), "Commodity Money under Private Information", *Journal of Monetary Economics*, **36**, 573–592.

LUCAS, R. (1990), "Liquidity and Interest Rates", *Journal of Economic Theory*, **50**, 237–264.

MENGER, K. (1892), "On the Origin of Money", *Economic Journal*, **2**, 239–255.

OKUMURA, T. (2001), "Wealth as a Signal in the Search Model of Money" (Mimeo, Department of Economics, Yokohama National University, Japan).

SHI, S. (1997), "A Divisible Search Model of Fiat Money", *Econometrica*, **65**, 75–102.

SHI, S. (1999), "Search, Inflation and Capital Accumulation", *Journal of Monetary Economics*, **44**, 81–103.

SHI, S. (2001), "Liquidity, Bargaining, and Multiple Equilibria in a Search Monetary Model", *Annals of Economics and Finance*, **2**, 325–351.

STIGLITZ, J. (1987), "The Causes and Consequences of the Dependence of Quality on Prices", *Journal of Economic Literature*, **25**, 1–48.

STIGLITZ, J. (2000), "The Contributions of the Economics of Information to Twentieth Century Economics", *Quarterly Journal of Economics*, **115**, 1441–1478.

TREJOS, A. (1997), "Incentives to Produce Quality and the Liquidity of Money", *Economic Theory*, **9**, 355–365.

TREJOS, A. (1999), "Search, Bargaining, Money and Prices under Private Information", *International Economic Review*, **40**, 679–695.

VELDE, F., WEBER, W. and WRIGHT, R. (1999), "A Model of Commodity Money, with Applications to Gresham's Law and the Debasement Puzzle", *Review of Economic Dynamics*, **2**, 291–323.

WILLIAMSON, S. and WRIGHT, R. (1994), "Barter and Monetary Exchange under Private Information", *American Economic Review*, **84**, 104–123.