Vera Baranouskaya

Three Essays in Real Options

Submitted for the degree of Ph.D. in Economics at

Faculty of Economics
University of Lugano
Lugano, Switzerland

Thesis Committee:
Prof. G. Barone-Adesi, advisor, University of Lugano
Prof. M. Chesney, University of Zurich
Prof. F. Franzoni, University of Lugano

27th September 2010

Acknowledgments

I would like to thank my thesis supervisor Prof. Giovanni Barone-Adesi for his continuous support and guidance during my doctoral studies.

I am also grateful to Prof. Marc Chesney, Prof. Erwan Morellec, Prof. Aditya Kaul, Prof. Francesco Franzoni, Marina Druz, Tamara Nefedova, Graziano Mirata, Hakim Dall'O, Giuseppe Corvace and all the PhD students of the Swiss Finance Institute whom I have met in these four great years for their valuable comments, feedback and support.

I should separately mention the administrative support provided by the outstanding administrative assistant of the Institute of Finance Dimitri Loringett.

Last but not the least, I express my extreme gratitude to my parents for having faith in me.

Contents

Introduction

Real options refer to the investment, entry, exit and other strategic decisions of the firm that share three important characteristics: they are irreversible, they are made under uncertainty, and their timing is chosen by the firm. The term 'real options' was introduced in 1977 by Stewart Myers in his paper 'Determinants of corporate borrowing' that related risky debt holdings to the future investment policy of the firm.

The literature on real options has since been active and growing with seminal works by Brennan and Schwartz (1985) on the valuation and optimal timing of the natural resource investments; McDonald and Siegel (1986) on general approach to investment timing and scrapping; Margrabe (1978) on the asset exchange options; Fudenberg and Tirole (1985) on the preemption and equilibrium in the technology adoption games; Pindyck (1988) on capacity choice, and Kulatilaka and Perotti (1998) on strategic growth options under imperfect competition.

In the 1990's and 2000's, a number of classical textbooks in real options appeared in print: Dixit and Pindyck (1994), Trigeorgis (1996), Amram and Kulatilaka (1998), and Vollert (2003). In its development the real options literature combines the option pricing framework introduced in Black and Scholes (1973) and Merton (1973) with the research in the specific fields of economics and finance such as capital budgeting and investment policy, corporate debt and agency problems, mergers & acquisitions or game theory.

The present work illustrates the application of the real options approach to three economic areas: strategic competition, mergers & acquisitions and international trade.

The first chapter discusses the optimal timing of the technology adoption, entry and merger decisions in the industry producing a vertically differentiated product. I solve the model for the monopoly, duopoly and merger (which is equivalent to a monopoly with two products) and outline the equilibrium strategies of the Incumbent and the Entrant. In particular, I demonstrate that the Incumbent under the threat of entry always invests in the cost-reducing technology later as compared to the no-threat-of-entry (monopoly) case. This results in the lower option value to the Incumbent under duopoly.

The monopolistic problem is investigated using two alternative specifications for the price of the new technology: in the first case, the price of the technology is constant, whereas in the second, more general case, the price of the new technology is increasing in the amount of cost reduction it provides. I demonstrate that the increasing investment cost (price of the technology) makes the Incumbent postpone the technology adoption as compared to the constant price situation. Besides, increasing investment cost reduces the option value to the Incumbent.

I demonstrate that the merger generates the endogenous synergy in this model. I prove that the merger may not only increase the option value to the Incumbent as compared to the monopoly case, but it may also give the Entrant the opportunity to finally enter the market as a part of the merged firm whereas its independent entry would be unprofitable.

The second chapter investigates the connection between the market valuation and a type of the merger (stock, cash). I solve explicitly for the timing and terms of cash mergers in two different settings to demonstrate that cash mergers generally take place at low market valuations, whereas stock mergers may be observed at both low and high valuations. This result holds with some differences for both dynamic settings and conforms to the empirical evidence on mergers. In the second setting I solve the model employing the Least Squares Monte Carlo simulation approach developed by Longstaff and Schwartz (2001).

I also study the dynamics of the intra-industry mergers within the first setup. I solve for the optimal order of mergers inside the industry. Using three different initial capital allocations within the industry, I demonstrate that stock mergers in more concentrated industries occur at higher market valuation (i.e. later) as compared to mergers in less concentrated industries.

In the third chapter I investigate the problem of the exporter who faces the exchange rate appreciation and, consequently, decrease in profits since its export prices are denominated in the currency of the country of destination. The exporter has a number of opportunities to renegotiate the export prices paying fixed menu costs each time.

I solve the problem employing the real options approach for both infinite and finite horizon. I demonstrate that even small menu costs may contribute to the export price stickiness. I provide the closed-form expressions for the export price adjustment thresholds and explicitly compute the option value of performing an infinite number of export price adjustments for the infinite horizon problem. For the finite horizon problem, I use the binomial tree method to approximately estimate option values and derive the exercise boundaries of the first export price adjustment. I also provide simulation results to show that the aggregate price in the market of destination is much less volatile than the exchange rates for the respective exporters.

Chapter 1

Real options approach to vertical product differentiation: from monopoly to merger

1.1 Introduction

Tastes differ. This universal truth has long been acknowledged by the marketing departments in the industries ranging from beauty products to cars. Consumers are diverse, moreover, they believe themselves to be unique, and companies are doing their best to respond to these unique needs offering a variety of differentiated products. These products may be offered either within one product line or under different brand names within the same company or by competing producers. The market for laptops provides a good example: the same producer offers both high-priced sophisticated laptops with powerful processor and cheaper and simpler lightweight netbooks.

The vertical product differentiation models introduced by Mussa and Rosen (1978), Shaked and Sutton (1982) and Shaked and Sutton (1983) consider the industries producing vertically differentiated products with the aim to investigate the entry, exit, pricing and other strategic decisions of the firms.

In these models products differ in both price and quality; consumers have different tastes, some of them prefer expensive and high-quality products, others are satisfied with cheaper products of lower quality. The market is served either by a monopolist, or by a monopolist under the threat of entry of potential competitors, or by a certain number of competing firms[1].

Mussa and Rosen (1978) demonstrated that the monopolist tends to offer the product of lower quality as compared to the product that would be offered under competition; besides, consumers with low taste for differentiated product sometimes decide not to buy it at all (they are priced out of the market).

Shaked and Sutton (1982) showed that in the case of two firms[2] and relatively small entry cost both firms enter the market offering two distinct products in terms of both quality and price and earn positive profits.

More recently, Motta (1993) examines the vertical product differentiation model comparing price competition to quantity competition. The model encompasses both fixed costs of quality improvement (as in Shaked and Sutton (1982) and Shaked and Sutton (1983)) and variable costs of quality improvement (as in Mussa and Rosen (1978)). Motta (1993) proves that independently of the assumptions about the nature of costs of quality improvement and type of competition firms always produce substantially differentiated products with the degree of differentiation being higher for price competition.

[1]For the modern textbook treatment of vertical price differentiation see, for example, Tirole (1988), Shy (1996) or Motta (2004)

[2]Having exactly two firms in the market is the necessary condition for the existence of the perfect Nash equilibrium.

Acharyya (1998) studies the conditions influencing the choice between the pooling and separating menus by the monopolist[3]; he demonstrates that the heterogeneity of consumers by itself does not imply that the separating menu is selected by the monopolist.

Noh and Moschini (2006) discuss the problem of entry deterrence and entry accommodation in the traditional setup of Mussa and Rosen (1978) with the marginal cost of production increasing in quality of the product; the paper also examines the welfare implications of the accommodated entry.

This paper contributes to the literature on vertical product differentiation, optimal technology adoption and real options. It builds on the classical model by Mussa and Rosen (1978) with quadratic per unit cost as in Noh and Moschini (2006) and Motta (1993) (quadratic cost is a special case of the increasing and convex cost as in Acharyya (1998)).

The model considers the Incumbent active in the market and offering a product of certain quality and price. The Entrant is initially out of the market contemplating either an independent entry with its own version of the differentiated product (duopoly) or an entry followed by the merger with the Incumbent. The model also features consumers differing in their tastes as described by the taste parameter θ, so that consumers with higher θ are willing to pay more for the products of higher quality.

There exist a third-party technology that allows the Incumbent to reduce manufacturing costs. The technology improves over time and can be viewed as a series of patents that appear discontinuously one after another, so that every new patent provides more cost-reduction than the existing one. The Incumbent has an opportunity (an option) to buy this technology; therefore, it needs to determine the optimal timing of technology adoption. The Incumbent faces a trade off between early adoption of technology to start benefiting from lower manufacturing costs and later adoption to wait for even better patent to arrive.

As soon as the cost-reducing technology is adopted, it becomes profit-maximizing for the Incumbent to switch to the product of higher quality and higher price because of the decrease in manufacturing costs. This new product of the Incumbent is tailored for the consumers with higher taste parameter θ than the original product. In the duopoly, reduction in the Incumbent's manufacturing cost also leads to the higher quality and higher price of both products in equilibrium; profits of both players increase too. This leaves more strategic space for the Entrant who may now offer its product to the consumers though in was unprofitable before. Therefore, when deciding on the optimal timing of technology adoption, the Incumbent has to take into account the

[3]Under the pooling menu all consumers are offered the same product, whereas under the separating menu a number of distinct products tailored for different groups of consumers are offered.

actions of the Entrant in order to develop the optimal strategies of entry deterrence and entry accommodation. The model in this paper also provides the Incumbent and the Entrant with the merger opportunity. This allows to compare the optimal dynamic strategies of the players in non-cooperative setting (duopoly) with the strategies in the cooperative setting (merger).

The limitation of the traditional models of vertical product differentiation is that, to the best of my knowledge, they are mostly static games. Since I would like to investigate the properties of the dynamic equilibrium, I need to combine the static setup of vertical product differentiation with the dynamics of the real options approach. This paper continues a line of research that uses real options approach to model and explore the technology adoption and strategic investment decisions of the firms.

The literature on strategic real options considers mostly the markets for homogeneous products with a common stochastic shock factor; firms are usually assumed to be identical with symmetric strategies. A survey by Boyer, Gravel, and Lasserre (2004) provides an extensive discussion of the existing literature.

For example, Huisman and Kort (1999) and Huisman, Kort, Pawlina, and Thijssen (2003) consider a duopoly of identical firms (of asymmetric firms as an extension), with symmetric strategies and stochastic shock factor following a geometric Brownian motion (shock factor is introduced as a demand multiple). The firms may invest to increase their stochastic payoffs; since the investment results in market share increase, there is a trade off between investing earlier to catch larger market share (or to preempt a competitor) and investing later to get higher payoff.

Grenadier and Weiss (1997) develop a model of the optimal investment in technological innovation with the 'technology level' following a geometric Brownian motion. In their model, a new generation of technology arrives when the 'technology level' first hits a certain threshold from below, though the authors also mention the possibility of using a jump process to describe the arrival of innovations. Their model describes the behavior of a single firm with no competition and provides four distinct strategies of technology adoption.

Leahy (1993) and Grenadier (2002) discuss the optimality of myopic investment strategies in the environment with many identical firms, homogeneous product and stochastic shock as above.

Weeds (2002) considers a model with two competing firms willing to invest in the same patent; after the investment, the arrival of discoveries follows a Poisson process which represents technological uncertainty, whereas the value of the patent is stochastic reflecting economic uncertainty. Weeds (2002) shows that in the non-cooperative equilibrium investment takes place later than under cooperation.

Farzin, Huisman, and Kort (1998) develop a model of the optimal timing of technology adoption by a perfectly competitive firm with the technology parameter following a jump process with random size of jumps. Farzin, Huisman, and Kort (1998) show how to obtain the thresholds for both single and multiple technology switches, thought their results for the case of multiple switches are corrected and improved by Doraszelski (2001).

This paper follows Farzin, Huisman, and Kort (1998) and Weeds (2002) in adopting the Poisson (jump) process specification to describe the arrival of new, more advanced technology. It also relates to Huisman and Kort (1999) and Huisman, Kort, Pawlina, and Thijssen (2003) as it investigates the strategic interactions between the players: the Incumbent and the Entrant. It contributes to the literature on strategic real options as it dynamically investigates the technology adoption, entry and merger strategies of the players in the specific vertical product differentiation environment.

This paper also relates to the literature that employs the real options approach to dynamically investigate merger decisions. Lambrecht (2004), Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008) model mergers as dynamic option exercise games between the acquirer and the bidder(s) in which both timing and terms of mergers are determined endogenously.

In particular, Lambrecht (2004) studies mergers motivated by economies of scale in the complete and symmetric information setting and explains the procyclicality of merger waves. On the contrary, Morellec and Zhdanov (2005) relate to Shleifer and Vishny (2003) in assuming that outside investors have imperfect information about the parameters of the model (namely, about the synergy created by the merger); thus, both models generate abnormal returns that conform to empirical evidence.

Morellec and Zhdanov (2005) also introduce competition between the bidders resulting in negative abnormal returns to the winning bidder. Moreover, they explain how outside investors update their information about perceived synergy of the merger observing actions (or, rather, inaction) of bidder(s); learning is also discussed in Grenadier (1999) and Lambrecht and Perraudin (2003). Shleifer and Vishny (2003) develop a one-period static model with exogenous merger timing explaining the choice of the medium of payment (cash, stock).

In this paper the terms and timing of the merger between the Incumbent and the Entrant arise endogenously as in Lambrecht (2004), Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008). Moreover, under the vertical product differentiation, the synergy from the merger is also endogenous as the profits of the merged entity with two products are quite naturally higher than the sum of the profits of two competing firms under duopoly.

In Lambrecht (2004), the synergy comes from the production function displaying increasing returns to scale, whereas in Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008) the merger surplus is linear in the pre-merger valuations of the firms and depends on the exogenous synergy parameters. Thus, as to the source of merger synergy, the present model is closer to the one in Lambrecht (2004).

The research questions of this paper are: What is the optimal timing of technology adoption by the Incumbent in the monopoly case? How does it change under the threat of possible entry? How does the Entrant decide on possible entry? How does competition influence the option value to the Incumbent? How does the opportunity to merge change the optimal dynamic strategies of the firms?

I solve the model for monopoly, duopoly and merger cases and outline the equilibrium strategies of both players. In particular, I demonstrate that the Incumbent under the threat of entry always invests in cost-reducing technology later as compared to the no-threat-of-entry (monopoly) case. This results in the lower option value to the Incumbent under duopoly than under monopoly.

The monopolistic problem is investigated using two alternative specifications for the price of the new technology: in the first case, the price of the technology is constant, whereas in the second, more general case, the price of the new technology is increasing in the amount of cost reduction it provides. I demonstrate that the increasing investment cost (price of the technology) makes the Incumbent postpone the technology adoption as compared to the constant price situation. Besides, increasing investment cost reduces the option value to the Incumbent.

I also demonstrate that merger may not only increase the option value to the Incumbent as compared to the monopoly case, but it may also give the Entrant the opportunity to finally enter the market as a part of the merged firm whereas its independent entry would still be unprofitable.

The paper is organized as follows: Section 1.2 discusses static equilibrium, Section 1.3 discusses dynamic equilibrium and solves for the trigger values of the players, Section 1.4 concludes. Detailed derivations for Sections 1.2 and 1.3 are provided in Appendix A.

1.2 Static equilibrium

This section provides the static solutions for the monopoly, duopoly and merger. In particular, it gives the closed-form expressions for the optimal price and quality of the differentiated product along with the expressions for the maximized profits of the firms. Following the seminal work by Mussa and Rosen (1978), the indirect utility function V_i of consumer i is given by:

$$V_i(p, q) = \theta_i q - p, \tag{1.1}$$

where θ_i is a taste parameter of consumer i distributed uniformly over the interval $[\theta_L, \theta_H]$;
q is the quality of the product;
p is the price of the product.

Utility from buying nothing is zero; each consumer buys either one unit of the product or nothing. Consumers maximize their indirect utility functions V_i subject to the participation constraint $V_i \geq 0$ and, if more than one product is available in the market, buying the product with the highest utility.

Consider a firm j offering a differentiated product of quality q_j and price p_j. Following Noh and Moschini (2006) and Motta (1993), I assume that the cost function of firm j is quadratic in quality and takes the form $\frac{1}{2}c_j q_j^2$ where c_j is the cost parameter specific to firm j. As we shall see later, this parameter plays a central role in this model.

Firm j generates profits π_j:

$$\pi_j = \int_{\theta_{Lj}}^{\theta_{Hj}} \frac{p_j - \frac{1}{2}c_j q_j^2}{\theta_H - \theta_L} d\theta = \frac{\theta_{Hj} - \theta_{Lj}}{\theta_H - \theta_L}(p_j - \frac{1}{2}c_j q_j^2) = \frac{\Delta\theta}{\theta_H - \theta_L}(p_j - \frac{1}{2}c_j q_j^2), \tag{1.2}$$

where $\Delta\theta = \theta_{Hj} - \theta_{Lj}$ is the range of θ such that consumers with θ in this range buy from firm j;
p_j is the price of the product produced by firm j;
q_j is the quality of the product produced by firm j.

Firm j maximizes its profits choosing an optimal price-quality combination (p_j, q_j). Each firm produces only one distinct product; if the firms decide to merge, the new merged entity will produce two products inherited from the parent firms.

1.2.1 Monopoly with one product

Initially, there is only one firm in the market - the Incumbent. The Incumbent is the monopolist and is free to choose any price-quality combination for its product. For any pair (p_M, q_M) chosen by the monopolist with $p_M > 0$ and $q_M > 0$, there exist a consumer M such that $V_M(p_M, q_M) = \theta_M q_M - p_M = 0$ and, consequently, $\theta_M = \frac{p_M}{q_M}$ is the threshold that separates the consumers in two groups in the following way: consumers with $\theta \in [\theta_M, \theta_H]$ derive non-negative utility from the product (p_M, q_M) and buy it, whereas consumers with $\theta \in [\theta_L, \theta_M)$ derive negative utility from the product and do not buy it.

The cost parameter of the Incumbent is c_I. The Incumbent maximizes its profits (based on equation 1.2):

$$\pi_M = \frac{\theta_H - \theta_M}{\theta_H - \theta_L}(p_M - \frac{1}{2}c_I q_M^2) \to \max \tag{1.3}$$

choosing optimal (p_M, q_M) or, given that $p_M = \theta_M q_M$, choosing (θ_M, q_M)[4].

The solution is as follows[5]:

$$\theta_M^* = \frac{2\theta_H}{3} \tag{1.4}$$

$$q_M^* = \frac{2\theta_H}{3c_I} \tag{1.5}$$

$$p_M^* = \frac{4\theta_H^2}{9c_I} \tag{1.6}$$

$$\pi_M^* = \frac{2\theta_H^3}{27c_I\left(\theta_H - \theta_L\right)}. \tag{1.7}$$

It follows from (1.4)-(1.7) that the decrease in the cost parameter of the Incumbent c_I results in the product of higher quality and higher price; profits of the Incumbent increase too.

1.2.2 Duopoly

Assume that at some moment, the Entrant decides to enter the market with the product (p_E, q_E); then the Incumbent shifts from (p_M, q_M) to a different price-quality combination (p_I, q_I). Assume that due to the Incumbent's experience in the field, it manages to capture the niche for the product of higher quality (and of higher price too): $q_E < q_I$ and $p_E < p_I$.

Recall that the cost parameter of the Incumbent is c_I, and the cost parameter of the Entrant is c_E with $c_E \geq c_I > 0$ or, equivalently, $0 < \frac{c_I}{c_E} \leq 1$ implying that the per unit manufacturing cost of the Incumbent does not exceed the cost of the Entrant.

Firms decide on (p_E, q_E) and (p_I, q_I) such that each consumer either buys the product from one of the firms or does not buy it at all. There are two threshold levels of θ associated with (p_E, q_E) and (p_I, q_I): consumers with θ_E are indifferent between buying from the Entrant and not buying at all; consumers with θ_I are indifferent between buying either from the Entrant or from the Incumbent.

Thus, the market is served as follows:

- Consumers with $\theta \in [\theta_L, \theta_E)$ do not buy the product;

[4]It is technically easier to solve in terms of (θ, q) than in terms of (p, q) in case of two products.

[5]Detailed derivations are provided in Appendix A.1.

- Consumers with $\theta \in [\theta_E, \theta_I)$ buy from the Entrant;
- Consumers with $\theta \in [\theta_I, \theta_H]$ buy from the Incumbent.

Firms have to simultaneously decide on (p_E, q_E) (low-quality product) and (p_I, q_I) (high-quality product).

The Nash equilibrium consists of (p_E, q_E) and (p_I, q_I) that maximize profits of both firms:

$$\pi_E = \frac{\theta_I - \theta_E}{\theta_H - \theta_L}\left(p_E - \frac{1}{2}c_E q_E^2\right) \to \max \tag{1.8}$$

$$\pi_I = \frac{\theta_H - \theta_I}{\theta_H - \theta_L}\left(p_I - \frac{1}{2}c_I q_I^2\right) \to \max\,. \tag{1.9}$$

The solution is as follows[6]:

$$\theta_E^* = \frac{4\theta_H(3c_E - 2c_I)}{27c_E - 16c_I} \tag{1.10}$$

$$\theta_I^* = \frac{6\theta_H(3c_E - 2c_I)}{27c_E - 16c_I} \tag{1.11}$$

$$q_E^* = \frac{4\theta_H(3c_E - 2c_I)}{(27c_E - 16c_I)c_E} \tag{1.12}$$

$$q_I^* = \frac{6\theta_H(3c_E - 2c_I)}{(27c_E - 16c_I)c_I}. \tag{1.13}$$

The profits are:

$$\pi_E^* = \frac{16\theta_H^3(3c_E - 2c_I)^3}{(27c_E - 16c_I)^3 c_E(\theta_H - \theta_L)} \tag{1.14}$$

$$\pi_I^* = \frac{2\theta_H^3(9c_E - 4c_I)^2(3c_E - 2c_I)^2}{c_E(27c_E - 16c_I)^3 c_I(\theta_H - \theta_L)}. \tag{1.15}$$

with the total duopolistic profits $\pi_D^* = \pi_E^* + \pi_I^*$ being equal to:

$$\pi_D^* = \frac{6\theta_H^3(3c_E - 2c_I)^2}{(27c_E - 16c_I)^2 c_I(\theta_H - \theta_L)}. \tag{1.16}$$

Thus, in the duopoly the profits of the Incumbent equal π_I^*, whereas in the monopoly they would equal π_M^*. In order to understand to what extent the competition erodes the monopolistic profits π_M^*, introduce the ratio $R = \frac{\pi_I^*}{\pi_M^*}$ that is illustrated in Figure 1.1. It is straightforward to

[6]Detailed derivations are provided in Appendix A.2.

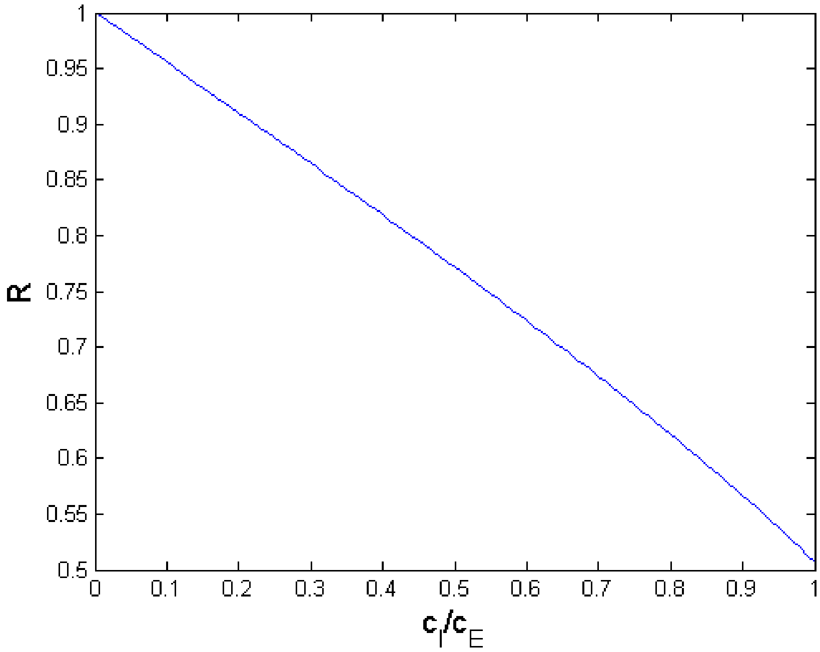


Figure 1.1: The ratio $R = \frac{\pi_I^*}{\pi_M^*}$ as a function of $\frac{c_I}{c_E}$

compute that at $c_I = c_E$ this ratio equals $R = 0.51$, and that $\lim_{\frac{c_I}{c_E} \to 0} R = 1$.

Figure 1.1 shows that the ratio $R = \frac{\pi_I^*}{\pi_M^*}$ is decreasing in $\frac{c_I}{c_E}$. It also demonstrates that the duopolistic profits π_I^* constitute approximately $50-100\%$ of the monopolistic profits π_M^* depending on the ratio of the cost parameters of the players $\frac{c_I}{c_E}$. It implies that the Incumbents accomodates the entry more easily when its cost parameter c_I is small as compared to the cost parameter of the Entrant c_E.

It is easy to show that both $\frac{\partial \pi_E^*}{\partial c_I} < 0$ and $\frac{\partial \pi_I^*}{\partial c_I} < 0$ implying that the decrease in c_I results in the increase in the profits of both the Incumbent and the Entrant. Though not surprising for the profits of the Incumbent, this result may seem unexpected for the profits of the Entrant. Nevertheless, there exist an explanation for it.

It follows from (1.10)-(1.13) that the decrease in the cost parameter of the Incumbent c_I leads to higher quality of both products in equilibrium as $\frac{\partial q_E^*}{\partial c_I} < 0$ and $\frac{\partial q_I^*}{\partial c_I} < 0$. Threshold taste parameters θ_E^* and θ_I^* behave in the same way implying that both the Incumbent and the Entrant shift upwards along the θ-axis. This means that both of them move towards the consumers with higher taste for the differentiated product; these consumers are willing to purchase goods of higher quality paying higher price, hence increase in the profits of both firms.

1.2.3 Merger and monopoly with two products

Assume that instead of entering the market on its own, the Entrant decides to enter the market and merge with the Incumbent to form a monopoly producing two goods. In terms of competition it is equivalent to the collusion between the players in the market: consumers still have the choice between two products as in the duopolistic case, but the total profits are expected to be higher.

As there is only one firm in the market, the same cost parameter c_I applies to both products. Following the same logic as in Section 1.2.2, the solution to the merger problem is given by[7]:

$$\theta_1^* = 0.6\theta_H \tag{1.17}$$

$$\theta_2^* = 0.8\theta_H \tag{1.18}$$

$$q_1^* = \frac{0.4\theta_H}{c_I} \tag{1.19}$$

$$q_2^* = \frac{0.8\theta_H}{c_I}, \tag{1.20}$$

and the profits are:

$$\pi_{MG}^* = \frac{2\theta_H^3}{25c_I\left(\theta_H - \theta_L\right)}. \tag{1.21}$$

Thus, a decrease in the cost parameter c_I leads to an increase in the profits of the merged firm. Comparing merger profits π_{MG}^* with the monopolistic profits π_M^* (1.7) yields:

$$\frac{\pi_{MG}^*}{\pi_M^*} = \frac{27}{25} = 1.08. \tag{1.22}$$

It implies that the merged firm, which is in fact a monopoly with two products, generates slightly higher profits than the monopoly with one product.

I have conjectured in the beginning of this section that the profits of the merged firm should be higher than the sum of the profits of the players in the duopoly. Figure 1.2 supports this claim plotting the ratio $R_{MG} = \frac{\pi_{MG}^*}{\pi_D^*}$ as a function of $\frac{c_I}{c_E}$.

It demonstrates that the merger profits π_{MG}^* are always higher than the total duopolistic profits π_D^* implying that there exists positive synergy from the merger. It is easy to compute that R_{MG} at $c_I = c_E$ equals $R_{MG} = 1.61$, whereas $\lim_{\frac{c_I}{c_E}\to 0} R_{MG} = \frac{27}{25} = 1.08$. Therefore, merger results in approximately $10-60\%$ of the increase in the total profits of the players as compared to the benchmark duopoly case. To summarize, the following relationship between the monopolistic

[7]Detailed derivations are provided in Appendix A.3.

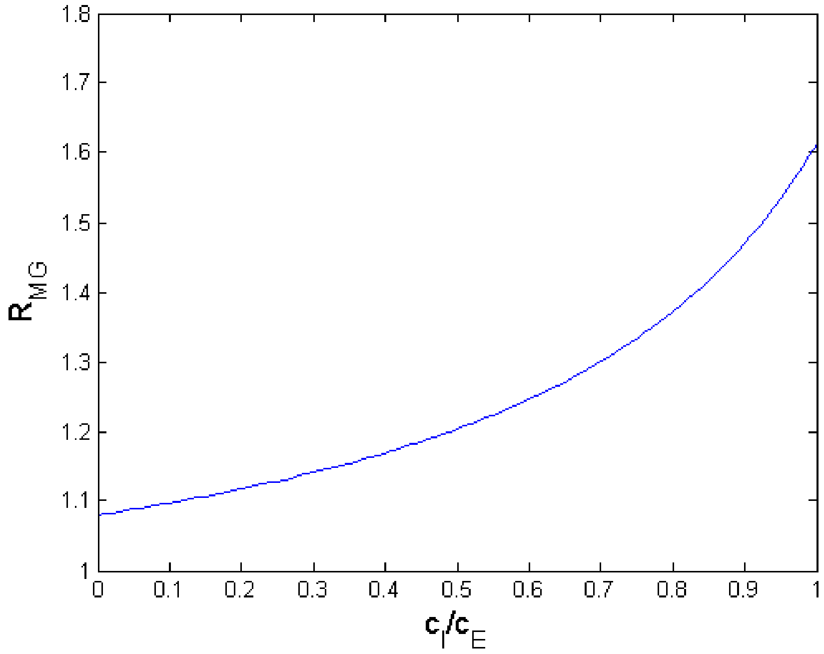


Figure 1.2: The ratio $R_{MG} = \frac{\pi^*_{MG}}{\pi^*_D}$ as a function of $\frac{c_I}{c_E}$

profits π^*_M, duopolistic profits π^*_D and the merger profits π^*_{MG} exists:

$$\pi^*_D \leq \pi^*_M < \pi^*_{MG} \tag{1.23}$$

for $0 < c_I \leq c_E$.

The synergy of the merger comes from the collusion between the players as they choose their prices and qualities not independently, but in cooperation with each other. Therefore, the present model generates the endogenous synergy. It relates, for example, to Lambrecht (2004) where merger synergy is also endogenous, and it results from the production function exhibiting increasing returns to scale.

So far, I have provided the static solutions for the monopoly, duopoly and merger. Now I proceed to the dynamic part of the model that will make use of these results.

1.3 Dynamic equilibrium

At time $t = 0$ the Incumbent is the only player in the market. Its product is characterized by the quality q^*_M and the price p^*_M, its profits equal π^*_M (see equations 1.5-1.7). The cost parameter of the Incumbent at time $t = 0$ equals c_{I0}; recall, that the cost function of the Incumbent is increasing is c_I.

Assume that there exist a third-party technology that allows the Incumbent to reduce its manufacturing costs. At any time $t \geq 0$ this technology is characterized by the cost parameter c_t, and c_t is non-increasing over time. The technology can be thought of as a series of patents that appear one after another so that every new patent provides a better technology than the existing one. At the moment the new patent arrives to substitute for the old one, a downward jump in c_t occurs.

Following Farzin, Huisman, and Kort (1998) and Weeds (2002), I adopt the jump (Poisson) process specification to describe the arrival of patents. In particular, the cost parameter c_t follows:

$$dc_t = c_t dq, \tag{1.24}$$

with dq being a Poisson process with parameter λ:

$$dq = \begin{cases} -g & \text{with probability } \lambda dt \\ 0 & \text{with probability } 1 - \lambda dt. \end{cases} \tag{1.25}$$

Thus, a jump is associated with a fraction $g \in (0; 1)$ of the decrease in c_I. Assume that the initial level of the cost parameter under this technology equals $c_0 = c_{I0}$, so that at time $t = 0$ this technology offers no amelioration over the existing technology of the Incumbent.

As the third-party technology improves over time and the associated level of the cost parameter decreases, the Incumbent may be interested in buying this technology at some time $t = \tau$ in order to instantly reduce its cost parameter from c_{I0} to c_τ. The price of the technology equals RD[8]. To simplify the exposition, I assume that the Incumbent may buy this technology only once.

The Incumbent has a trade off between buying this technology earlier to start benefiting from lower manufacturing costs and buying it later to wait for even better technology to arrive.

To summarize, the Incumbent has an opportunity (an option) to invest in the cost-reducing technology at the price RD. It needs to solve the dynamic optimization problem to find the value of c_t at which it should buy the technology. Therefore, the cost parameter of the Incumbent c_I takes two values: it is equal to c_{I0} before the investment, and to $c_\tau < c_{I0}$ after the investment that occurs at some time $t = \tau$. As to the cost parameter of the Entrant, it remains constant throughout the model: $c_E = c_{I0}$.

[8] RD is an acronym for 'research and development'.

Introduce a new variable $x_t = \frac{c_t}{c_{I0}}$ with the dynamics

$$dx_t = x_t dq, \tag{1.26}$$

where dq as in (1.25), and $x_0 = \frac{c_0}{c_{I0}} = \frac{c_{I0}}{c_{I0}} = 1$.

Since the technology cost parameter c_t is less or equal to the initial value of the cost parameter of the Incumbent c_{I0}, then $x \in (0,\, 1]$. Now the problem of the Incumbent can be solved in terms of the state variable x_t. The Incumbent needs to find the trigger value of x for the investment in the cost-reducing technology that would allow to instantly decrease the cost parameter of the Incumbent from c_{I0} to xc_{I0}.

One can rewrite the expressions for the profits of the players derived in Section 1.2 using the normalization $\frac{2\theta_H^3}{c_{I0}(\theta_H - \theta_L)} = 1$ and the fact that $c_E = c_{I0}$ as:

$$\pi_M^* = \frac{1}{27x} \tag{1.27}$$

with $\pi_{M0}^* = \frac{1}{27}$,

$$\pi_E^* = \frac{8(3-2x)^3}{(27-16x)^3} \tag{1.28}$$

with $\pi_{E0}^* = \frac{8}{1331}$,

$$\pi_I^* = \frac{(9-4x)^2(3-2x)^2}{(27-16x)^3 x} \tag{1.29}$$

with $\pi_{I0}^* = \frac{25}{1331}$,

$$\pi_{MG}^* = \frac{1}{25x} \tag{1.30}$$

with $\pi_{MG0}^* = \frac{1}{25}$.

In its turn, the Entrant has an option to enter the market at any time at a lump-sum entry cost INV. Profits of the Entrant depend both on its cost parameter $c_E = c_{I0}$ and the cost parameter of the Incumbent c_I. At time $t = 0$ the cost parameter of the Incumbent equals c_{I0}, the new state variable x equals $x_0 = 1$, and the profits of the Entrant equal $\pi_{E0}^* = \frac{8}{1331}$ as follows from (1.28).

The Entrant does not control the evolution of the cost parameter of the Incumbent; the entry decision made by the Entrant is based solely on the net present value at the time of entry. Let r be the risk-free rate; then the NPV to the Entrant at time $t = 0$ is given by:

$$NPV_{E0} = \frac{\pi_{E0}^*}{r} - INV = \frac{8}{r1331} - INV. \tag{1.31}$$

Let $NPV_{E0} < 0$ so that it is not profitable for the Entrant to enter at time $t = 0$. It is equivalent to saying that the profitability index of the entry at time $t = 0$ is less than one: $PI = \frac{\pi^*_{E0}}{rINV} < 1$.

However, as it was demonstrated in Section 1.2.2, profits of the Entrant are decreasing in c_I and, consequently, in x:

$$\frac{\partial \pi^*_E}{\partial x} = \frac{-144(3-2x)^2}{(27-16x)^4} < 0. \tag{1.32}$$

Then it may be profitable for the Entrant to enter later, namely immediately after the Incumbent invests in the cost-reducing technology at some time $t = \tau$ and its cost parameter c_I decreases from c_{I0} to c_τ with $x_\tau = \frac{c_\tau}{c_{I0}}$. The NPV to the Entrant in the general case of $x \in (0; 1]$ is given by:

$$NPV_E = \frac{\pi^*_E}{r} - INV = \frac{8(3-2x)^3}{r(27-16x)^3} - \frac{1}{PI}\frac{\pi^*_{E0}}{r}. \tag{1.33}$$

Thus, for any $x \in (0; 1]$ the Entrant decides whether to enter or not based on the NPV_E as in (1.33). The Incumbent is aware that its decision about investing in the cost-reducing technology may trigger the entry by the Entrant.

To summarize, the Incumbent's initial profits are π^*_{M0}, and it enjoys the monopolistic position. The Entrant is out of the market. There are three possible scenarios for the future:

1. The Incumbent invests RD at the monopolistic trigger x_M to obtain the stream of profits $\pi^*_M(x_M)$ in exchange for π^*_{M0}, no subsequent entry happens. The conditions for this are:
   - either no threat of competition (the Entrant does not exist at all);
   - or negative NPV_E to the Entrant at x_M. It means that the investment INV required from the Entrant to enter the market is too high for its entry to be profitable. In this case the entry deterrence by the Incumbent takes place.
2. The Incumbent invests RD at the duopolistic trigger x_D to obtain the stream of profits $\pi^*_I(x_D)$ in exchange for π^*_{M0} followed by the immediate entry of the Entrant that obtains profits of $\pi^*_E(x_D)$. For this to happen, NPV_E to the Entrant at x_D should be non-negative. In this situation, the Incumbent accommodates the entry by adopting the technology at x_D.
3. Instead of acting in the market independently, the Incumbent and the Entrant decide to merge. Thus, the Incumbent invests RD at the merger trigger x_{MG}, the Entrant immediately enters provided its NPV_E is non-negative at x_{MG}, and each firm receives its share of the profits of the merged firm π^*_{MG}. The share of each firm is determined endogenously together with the merger trigger x_{MG}. This is another strategy of entry accommodation by

the Incumbent which may increase its option value as compared to the duopolistic entry accommodation in the previous scenario.

The Incumbent needs to solve three dynamic programming problems to find the triggers x_M, x_D and x_{MG}. The remainder of Section 1.3 provides these solutions; in particular, Section 1.3.1 discusses the general approach to this type of problems, Section 1.3.2 solves for x_M, Section 1.3.3 solves for x_D, and, finally, Section 1.3.4 solves for x_{MG}.

1.3.1 General solution to a dynamic programming problem

The Bellman equation in the continuation region is as follows (see, for example, Dixit and Pindyck (1994)):

$$F(x) = \pi_0(x_0)\,dt + \frac{E(F(x+dx))}{1+rdt}, \tag{1.34}$$

where $F(x)$ is the continuation value;
r is the risk-free (discount) rate;
x_0 is the initial value of the state variable x;
$x_0 = 1$ by construction;
$\pi_0(x_0)$ is the profits flow per unit time at x_0;
$\pi_0(x_0) = \pi^*_{M0}$ as at time $t = 0$ the Incumbent enjoys the monopolistic position in the market;
$E(F(x+dx)) = F(x) + \lambda(F(x(1-g)) - F(x))\,dt$.

Then, the Bellman equation can be rewritten as[9]:

$$F(x) = \frac{\pi_0(x_0)}{r+\lambda} + \frac{\lambda}{r+\lambda}F(x(1-g)). \tag{1.35}$$

As already described above, x follows a jump process with the downward jumps of fixed size xg. Then after k jumps x takes the value $x_k = x_0(1-g)^k$ with $x_0 = 1$, and the Bellman equation (1.35) becomes:

$$F(x_0) = \frac{\pi_0(x_0)}{r} + \left(\frac{\lambda}{r+\lambda}\right)^k \left(F\left((1-g)^k\right) - \frac{\pi_0(x_0)}{r}\right). \tag{1.36}$$

In the stopping region (at the trigger x^*) I impose the ‘value-matching condition’:

$$F(x^*) = V(x^*) \tag{1.37}$$

[9] Complete derivations for Section 1.3.1 are provided in Appendix A.4.

with the stopping value $V(x)$ as follows:

$$V(x_\tau) = \int_\tau^\infty \pi(x_\tau)e^{-r(t-\tau)}dt - I = \frac{\pi(x_\tau)}{r} - I, \tag{1.38}$$

where I is the investment required (it equals RD in the monopoly scenario 1 and duopoly scenario 2, and $RD + INV$ in the merger scenario 3);
$\pi(x)$ is the profits flow per unit time in the stopping region (for example, it equals π_M^* in the monopoly scenario 1).

Note that the uncertainty about the future cash flows of the players is resolved at the time of the technology adoption τ, so that once the firms know the value of x_τ, their future profits are deterministic and known.

Combining (1.36) with (1.37) yields:

$$F(x_0) = \frac{\pi_0(x_0)}{r} + \left(\frac{\lambda}{r+\lambda}\right)^{k^*} \left(\frac{\pi\left((1-g)^{k^*}\right) - \pi_0(x_0) - Ir}{r}\right), \tag{1.39}$$

where $x^* = (1-g)^{k^*}$ is the continuous trigger value of x;
k^* is the continuous number of jumps needed to reach x^*.

It is clear from (1.39) that $\frac{\lambda}{r+\lambda}$ plays the role of discount factor in this model.

Since $F(x_0)$ is the option value to the Incumbent at time $t = 0$, I need to maximize it with respect to k^* to find the number of jumps needed to reach the trigger value x^*. At the moment, I do not consider the problem of overshooting, I discuss it later taking the monopoly case as an example.

1.3.2 Monopoly with one product

In this section I solve the simplest monopoly case. The Incumbent needs to find the monopolistic trigger x_M at which it is optimal to invest in the cost-reducing technology to decrease the value of the cost parameter c_I from c_{I0} to $x_M c_{I0}$.

Assume first that the price of the technology is constant and equal to RD. This is a common assumption in the literature that I will later relax.

Under no threat of entry, the Incumbent needs to maximize its option value F_M based on

(1.39):

$$\begin{aligned}\max_k F_M &= \frac{\pi^*_{M0}}{r} + \left(\frac{\lambda}{r+\lambda}\right)^k \left(\frac{\pi^*_M\left((1-g)^k\right)}{r} - \frac{\pi^*_{M0}}{r} - RD\right) = \\ &= \frac{1}{27r} + \frac{1}{r}\left(\frac{\lambda}{r+\lambda}\right)^k \left(\frac{1}{27(1-g)^k} - \frac{1}{27} - RDr\right),\end{aligned} \tag{1.40}$$

since $\pi_0(x_0) = \pi^*_{M0} = \frac{1}{27}$;
$\pi(x) = \pi^*_M(x) = \frac{1}{27x}$;
$I = RD$;
and $x = (1-g)^k$.

The continuous solution[10] to (1.40) consists of the continuous monopolistic trigger x^c_M (presented in Figure 1.3) and k^c_M which is the continuous number of jumps needed to reach the trigger x^c_M (presented in Figure 1.4):

$$x^c_M = (1-g)^{k^c_M} = \frac{\ln\frac{\lambda}{r+\lambda} - \ln(1-g)}{(1+r\frac{RD}{\pi^*_{M0}})\ln\frac{\lambda}{r+\lambda}} \tag{1.41}$$

$$k^c_M = \ln\frac{\ln\frac{\lambda}{r+\lambda} - \ln(1-g)}{(1+r\frac{RD}{\pi^*_{M0}})\ln\frac{\lambda}{r+\lambda}} / \ln(1-g). \tag{1.42}$$

Throughout the paper the following values of parameters are used to plot the graphs: $g = 0.1$, $\frac{1}{1+r\frac{RD}{\pi^*_{M0}}} = 0.8$.

Some properties of the monopolistic trigger x^c_M are:

$$\lim_{\frac{\lambda}{r+\lambda}\to 0} x^c_M = \frac{1}{1+r\frac{RD}{\pi^*_{M0}}} < 1 \tag{1.43}$$

$$x^c_M\Big|_{\frac{\lambda}{r+\lambda}=1-g} = 0 \tag{1.44}$$

and $x^c_M \in (0; 1)$ for $\frac{\lambda}{r+\lambda} < 1-g$.

Because of the discrete nature of x, there exists a problem of overshooting. Therefore, either $[k^c_M]$ or $[k^c_M]+1$ should be used instead of k^c_M for the number of jumps to be a natural number[11].

[10] Complete derivations for Section 1.3.2 are provided in Appendix A.5.
[11] $[x]$ stands for 'integer part of x'

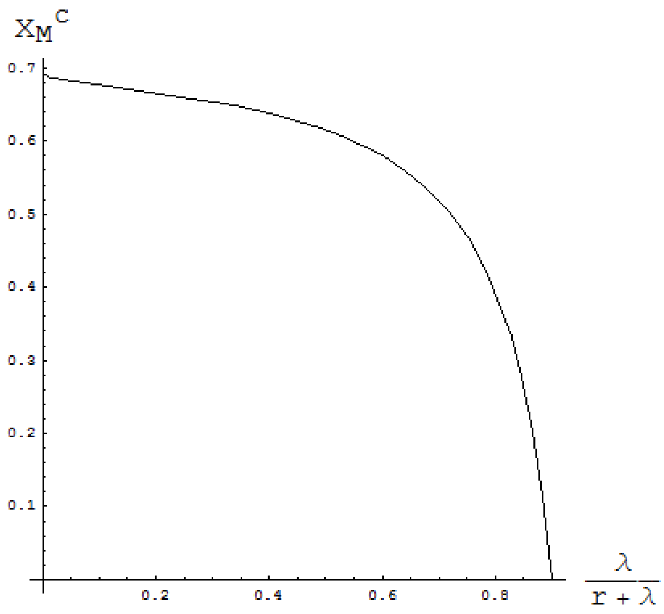


Figure 1.3: Continuous monopolistic trigger x_M^c

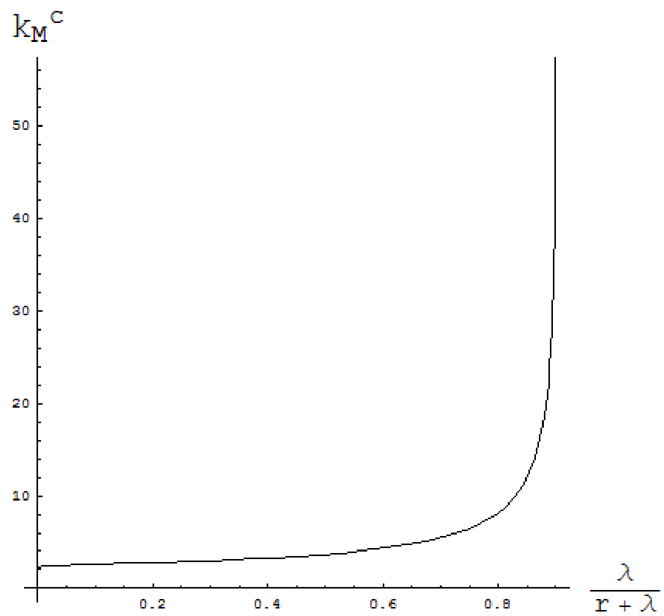


Figure 1.4: Continuous number of jumps k_M^c needed to reach the trigger x_M^c

Monopolistic trigger x_M is then defined as $x_M = (1-g)^{k_M}$ with

$$k_M = \arg\max\left(F_M\left((1-g)^{[k_M^c]}\right), F_M\left((1-g)^{[k_M^c]+1}\right)\right), \tag{1.45}$$

where F_M is the option value to the Incumbent at time $t = 0$ as in (1.40).

For example, at $g = 0.1$, $\frac{1}{1+r\frac{RD}{\pi^*_{M0}}} = 0.8$ and $\frac{\lambda}{r+\lambda} = 0.7$ the solution is:

- $k_M^c = 5.44$;
- $F_M\big|_{k=5} > F_M\big|_{k=6}$;
- then $k_M = 5$ with $x_M = (1-g)^{k_M} = 0.59$.

The same algorithm should be applied to transform any continuous trigger in the discrete one.

Now I relax the assumption of the cost-reducing technology price RD being constant. Let RD be a linear decreasing function of x:

$$RD(x) = RD_0 + \beta(1-x) = RD_0 + \beta\left(1-(1-g)^k\right), \tag{1.46}$$

where RD_0 is equal to the constant technology price RD above and $\beta > 0$. This functional form of RD ensures that the investment cost (technology price) is increasing in the amount of cost reduction it provides.

Then the Incumbent needs to solve the following problem:

$$\max_k F_M^g = \frac{1}{27r} + \frac{1}{r}\left(\frac{\lambda}{r+\lambda}\right)^k\left(\frac{1}{27(1-g)^k} - \frac{1}{27} - r\left(RD_0 + \beta\left(1-(1-g)^k\right)\right)\right), \tag{1.47}$$

where F_M^g is the option value to the Incumbent in the general case of increasing technology price (investment cost).

The solution to (1.47) is given by[12]:

$$x_M^{cg} = \frac{E - \sqrt{E^2 - \frac{4}{K_\beta}}}{2}, \tag{1.48}$$

[12]Detailed derivations are provided in Appendix A.5.

where x_M^{cg} is the continuous monopolistic trigger in the general case of non-constant RD;
E is as follows:

$$E = \frac{\left(1 + r\frac{RD_0}{\pi^*_{M0}}\right)\ln\frac{\lambda}{r+\lambda}}{K_\beta\left(\ln\frac{\lambda}{r+\lambda} - \ln(1-g)\right)} + \frac{\ln\frac{\lambda}{r+\lambda}}{\ln\frac{\lambda}{r+\lambda} + \ln(1-g)}, \tag{1.49}$$

K_β is as follows:

$$K_\beta = \frac{\beta}{\frac{\pi_{M0^*}\left(\ln\frac{\lambda}{r+\lambda} - \ln(1-g)\right)}{r\left(\ln\frac{\lambda}{r+\lambda} + \ln(1-g)\right)}}, \tag{1.50}$$

and provided β satisfies the following condition:

$$0 < \beta < \frac{\pi^*_{M0}\left(\ln\frac{\lambda}{r+\lambda} - \ln(1-g)\right)}{r\left(\ln\frac{\lambda}{r+\lambda} + \ln(1-g)\right)}, \tag{1.51}$$

so that $K_\beta \in (0;\, 1)$.

Proposition 1 *The continuous monopolistic trigger in the general case of increasing investment cost x_M^{cg} is always smaller than the continuous monopolistic trigger in the case of constant cost x_M^c. This means that the Incumbent always invests later under increasing cost than under constant cost.*

The option value to the Incumbent under increasing cost $F_M^g\left(x_M^g\right)$ is always lower than the option value to the Incumbent under constant cost $F_M\left(x_M\right)$.

Proof. The fact that $x_M^{cg} < x_M^c$ is proven in Appendix A.5.
Then $F_M^g\left(x_M^g\right) < F_M\left(x_M^g\right) < F_M\left(x_M\right)$, where the first inequality follows from the fact that $F_M^g\left(x\right) < F_M\left(x\right)$ for $x \in (0;\, 1]$ because of increasing investment cost, and the second inequality follows from the fact that F_M is maximized at x_M. ■

Figure 1.5 plots both x_M^c and x_M^{cg} at $K_\beta = .8$ and $g = 0.1$, $\frac{1}{1+r\frac{RD}{\pi^*_{M0}}} = 0.8$ (as before). Both triggers demonstrate the same behavior (as expected), with x_M^{cg} being always smaller than x_M^c; besides, both of them decrease in $\frac{\lambda}{r+\lambda}$ meaning that higher probability of a jump λ allows the Incumbent to wait longer in order to invest at lower trigger and obtain higher profits.

1.3.3 Duopoly

In this section I relax the assumption of no competition and solve the dynamic problem for the duopoly. Recall that under duopoly, the Incumbent invests in the cost-reducing technology at the duopolistic trigger x_D followed by the immediate entry of the Entrant.

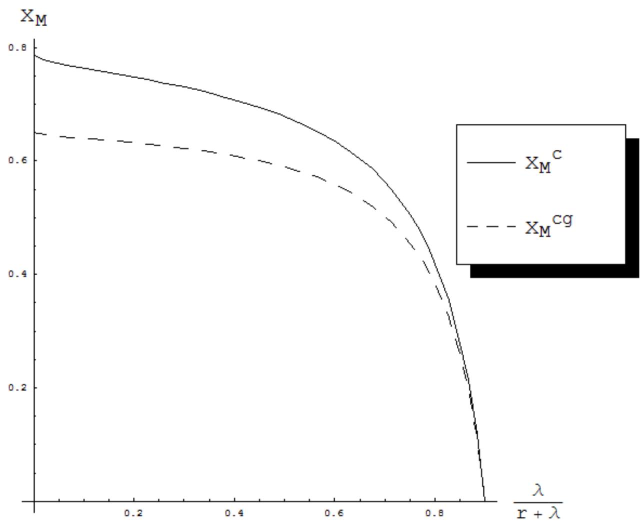


Figure 1.5: Monopolistic triggers for both constant and increasing investment cost

First, I solve the problem of the Entrant. As it has been already demonstrated in the beginning of Section 1.3, the Entrant computes its NPV to decide on entry. Then, given the values of the input parameters, it is possible to compute the threshold value x_E such that:

$$\begin{aligned} NPV_E\left(x_E\right) &= \frac{\pi_E^*\left(x_E\right)}{r} - INV = \\ &= \frac{8(3-2x_E)^3}{r(27-16x_E)^3} - \frac{1}{PI}\frac{\pi_{E0}^*}{r} = 0. \end{aligned} \tag{1.52}$$

Therefore, for any $x \leq x_E$ the entry is profitable as it has non-negative NPV. Figure 1.6 displays x_E as a function of the profitability index PI. It demonstrates that the solution exist only for $PI > 0.55$. The intuition behind is quite straightforward: when entry cost is too high, the entry never happens.

Now I solve the problem of the Incumbent who expects the Entrant to enter immediately after the Incumbent's investment at the duopolistic trigger x_D. From (1.29), the Incumbent's profits are:

$$\pi_I^* = \frac{(9-4x)^2(3-2x)^2}{(27-16x)^3 x}. \tag{1.53}$$

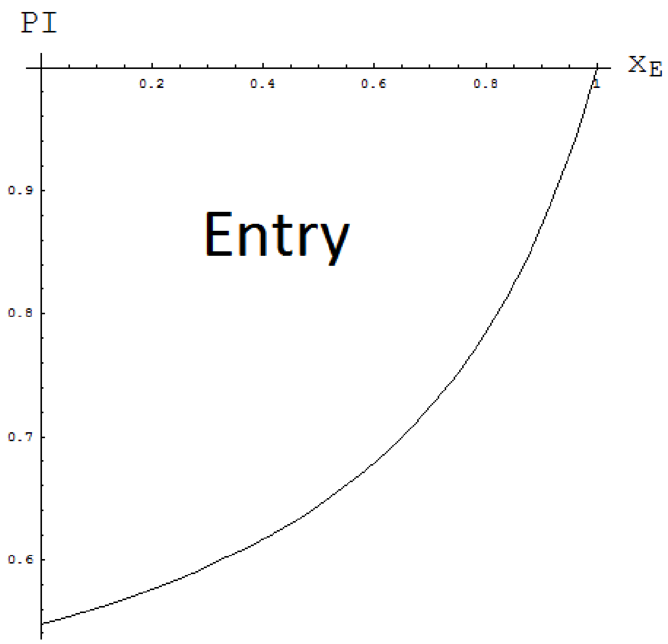


Figure 1.6: Entry region of the Entrant

Define a function $R(x)$ as:

$$R(x) = \frac{\pi_I^*}{\pi_M^*} = \frac{27(9-4x)^2(3-2x)^2}{(27-16x)^3}. \tag{1.54}$$

It is straightforward to demonstrate that the profits of the Incumbent π_I^* are always smaller than the profits of the monopolist π_M^*, or, in other words, that $0 < R(x) < 1$. In particular, $R'(x) < 0$ and $R(x) \in [\frac{675}{1331}; 1)$ for $x \in (0; 1]$.

Therefore, the duopolistic problem of the Incumbent is just a slightly modified monopolistic problem[13] presented in (1.40). The Incumbent needs to maximize its option value under duopoly F_D:

$$\begin{aligned} \max_k F_D &= \frac{1}{27r} + \frac{1}{r}\left(\frac{\lambda}{r+\lambda}\right)^k \left(\pi_I^*\left((1-g)^k\right) - \frac{1}{27} - RDr\right) = \\ &= \frac{1}{27r} + \frac{1}{r}\left(\frac{\lambda}{r+\lambda}\right)^k \left(\frac{R((1-g)^k)}{27(1-g)^k} - \frac{1}{27} - RDr\right). \end{aligned} \tag{1.55}$$

The continuous solution to (1.55) consists of the continuous duopolistic trigger x_D^c and k_D^c

[13]The present problem is solved for the constant investment cost RD. Derivations concerning the second order condition are presented in Appendix A.6.

which is the continuous number of jumps needed to reach the trigger x^c_D:

$$x^c_D = (1-g)^{k^c_D} = \frac{\left(\ln\frac{\lambda}{r+\lambda} - \ln(1-g)\right) R((1-g)^{k^c_D}) + \frac{\partial R}{\partial k}\,(k^c_D)}{(1+r\frac{RD}{\pi^*_{M0}})\ln\frac{\lambda}{r+\lambda}} \tag{1.56}$$

$$k^c_D = \ln\frac{\left(\ln\frac{\lambda}{r+\lambda} - \ln(1-g)\right) R((1-g)^{k^c_D}) + \frac{\partial R}{\partial k}\,(k^c_D)}{(1+r\frac{RD}{\pi^*_{M0}})\ln\frac{\lambda}{r+\lambda}} / \ln(1-g). \tag{1.57}$$

Comparing the results for the monopoly from Section 1.3.2 with the above results for the duopoly yields the following proposition:

Proposition 2 *The duopolistic trigger x^c_D is always smaller than the monopolistic trigger x^c_M. Thus, under the threat of entry the Incumbent always invests later as compared to the no-threat-of-entry situation.*

The option value to the Incumbent under the threat of entry $F_D(x_D)$ is always lower than the option value to the Incumbent under no threat of entry $F_M(x_M)$; therefore, competition erodes the option value to the Incumbent.

Proof. It follows from (1.57) that:

$$x^c_D = x^c_M R((1-g)^k) + \frac{\frac{\partial R}{\partial k}}{(1+r\frac{RD}{\pi_{M0}})\ln\frac{\lambda}{r+\lambda}} < x^c_M \tag{1.58}$$

since $R((1-g)^k) \in (0;\,1)$, $\frac{\partial R}{\partial k} > 0$ for $g \in (0;\,1)$ and $\ln\frac{\lambda}{r+\lambda} < 0$ for $\frac{\lambda}{r+\lambda} \in (0;\,1)$.
Then $F_D(x_D) < F_M(x_D) < F_M(x_M)$, where the first inequality follows from the fact that $\pi^*_I < \pi^*_M$ for $x \in (0;\,1)$, and the second inequality follows from the fact that F_M is maximized at x_M. ■

The following proposition summarizes the optimal strategy of the Incumbent absent merger opportunities:

Proposition 3 *The optimal strategy of the Incumbent is:*

1. *Compute x_M, x_D and x_E;*

2. *In case inequality $x_M > x_E$ holds, invest at x_M to obtain the stream of profits $\pi^*_M(x_M)$, the Entrant will not enter ('first-best' choice); otherwise, invest at x_D to obtain the stream of profits $\pi^*_I(x_D)$, the Entrant enters immediately to obtain the stream of profits $\pi^*_E(x_D)$('second-best' choice for the Incumbent). The former does not erode the option value to the Incumbent, whereas the latter does.*

The Entrant does not enter at all in the first case when $x_M > x_E$, so its NPV is zero, and it enters in the second case with NPV at the moment of entry equal to $NPV_E\,(x_D)$.

1.3.4 Merger and monopoly with two products

Assume that the Incumbent and the Entrant are given the opportunity to merge. It implies that at the merger trigger x_{MG} several actions take place simultaneously:

- The Incumbent invests RD in the cost-reducing technology;
- The Entrant enters with its product at a lump-sum entry cost INV;
- Firms merge, each firm receives its share of the total profits π^*_{MG}.

In order to persuade the Entrant to merge, the Incumbent should offer to the Entrant the share of the total profits s_E such that NPV to the Entrant would be at least equal to its NPV without the merger opportunity.

There exist two mutually exclusive situations described in Proposition 3:

1. In case $x_M > x_E$, the Entrant would never enter the market without the merger opportunity; therefore, it is sufficient to offer to the Entrant NPV equal to zero:
$$NPV_{E1} = \left(\frac{\lambda}{r+\lambda}\right)^{k_{MG1}} \left(s_E \frac{\pi^*_{MG}(x_{MG1})}{r} - INV\right) = 0, \tag{1.59}$$
where NPV_{E1} is NPV to the Entrant in the first case (discounted to time $t=0$);
x_{MG1} is the merger trigger in the first case;
k_{MG1} is the number of steps needed to reach x_{MG1}.

2. On the contrary, in case $x_E \geq x_M$ the Entrant would enter the market at x_D to obtain non-negative NPV; then it should be offered the following:
$$\begin{aligned} NPV_{E2} &= \left(\frac{\lambda}{r+\lambda}\right)^{k_{MG2}} \left(s_E \frac{\pi^*_{MG}(x_{MG2})}{r} - INV\right) = \\ &= \left(\frac{\lambda}{r+\lambda}\right)^{k_D} \left(\frac{\pi^*_E(x_D)}{r} - INV\right) = NPV_{ED}, \end{aligned} \tag{1.60}$$
where NPV_{E2} is NPV to the Entrant in the second case (discounted to time $t=0$);
NPV_{ED} is the NPV to the Entrant at the duopolistic trigger x_D (discounted to time $t=0$);

x_{MG2} is the merger trigger in the second case;
k_{MG2} is the number of steps needed to reach x_{MG2}.

Solve (1.59) first. The Entrant is entitled to the share s_E of the merged entity:

$$s_E = \frac{rINV}{\pi^*_{MG}}, \tag{1.61}$$

and the Incumbent is left with V_I:

$$V_I = \frac{\pi^*_{MG}}{r}(1 - s_E) - RD = \frac{\pi^*_{MG}}{r} - RD - INV. \tag{1.62}$$

Therefore, the Incumbent needs to maximize its option value F_{MG1} based on (1.39):

$$\begin{aligned} \max_k F_{MG1} &= \frac{1}{27r} + \frac{1}{r}\left(\frac{\lambda}{r+\lambda}\right)^k \left(\frac{1}{25(1-g)^k} - \frac{1}{27} - r(RD + INV)\right) = \\ &= \frac{1}{27r} + \frac{27}{25r}\left(\frac{\lambda}{r+\lambda}\right)^k \left(\frac{1}{27(1-g)^k} - \frac{25}{27}\left(\frac{1}{27} + r(RD+INV)\right)\right). \end{aligned} \tag{1.63}$$

The structure of this problem is identical to that of the monopolistic problem; the continuous solution[14] to (1.63) consists of the continuous merger trigger x^c_{MG1} and k^c_{MG1} which is the number of steps needed to reach x_{MG1}:

$$x^c_{MG1} = (1-g)^{k^c_{MG1}} = \frac{\left(\ln \frac{\lambda}{r+\lambda} - \ln(1-g)\right)}{\frac{25}{27}(1 + r\frac{RD+INV}{\pi^*_{M0}}) \ln \frac{\lambda}{r+\lambda}} \tag{1.64}$$

$$k^c_{MG1} = \ln x^c_{MG1} / \ln(1-g). \tag{1.65}$$

Comparing the monopolistic trigger x^c_M derived in (1.41) with the merger trigger x^c_{MG1} (1.65) yields that $x^c_{MG1} > x^c_M$ if the following inequality holds:

$$\frac{25}{27}\left(1 + r\frac{RD + INV}{\pi_{M0}}\right) < 1 + r\frac{RD}{\pi_{M0}}. \tag{1.66}$$

Proposition 4 *In case $x_M > x_E$ and provided that (1.66) holds, it follows that the merger trigger x^c_{MG1} is always greater than the monopolistic trigger x^c_M. This means that the merger occurs earlier than the cost-reducing investment in the absence of the merger opportunity would occur.*

[14]Since $\pi_{MG^*} = \frac{27}{25}\pi^*_M$, the second order condition is satisfied for π^*_{MG} also.

Besides, option value to the Incumbent with the merger opportunity $F_{MG1}(x_{MG1})$ is higher than the option value under monopoly $F_M(x_M)$.

Proof. $x^c_{MG1} > x^c_M \Leftrightarrow$ (1.66).
Then $F_{MG1}(x_{MG1}) > F_{MG1}(x_M) > F_M(x_M)$, where the first inequality follows from the fact that F_{MG1} is maximized at x_{MG1}, and the second inequality follows from the fact that $F_{MG1} > F_M$ for $x \in (0; 1]$ when (1.66) holds. ■

In case (1.66) does not hold, then $x_M \geq x_{MG1}$ and $F_{MG1}(x_{MG1}) > F_M(x_M)$ for some relatively large x_{MG1}, whereas $F_{MG1}(x_{MG1}) \leq F_M(x_M)$ for relatively small x_{MG1}. As INV increases, the difference $F_{MG1}(x_{MG1}) - F_M(x_M)$ decreases to becomes negative at some point, and the merger becomes less attractive than monopoly to the Incumbent. In this case, the Incumbent does not offer the merger to the Entrant.

Now solve (1.60). The share of the merged entity accruing to the Entrant equals:

$$s_E = \frac{r}{\pi^*_{MG}} \left(\frac{NPV_{ED}}{\left(\frac{\lambda}{r+\lambda}\right)^{k_{MG2}}} + INV \right), \tag{1.67}$$

and the Incumbent obtains V_I:

$$V_I = \frac{\pi^*_{MG}}{r}(1 - s_E) - RD = \frac{\pi^*_{MG}}{r} - \frac{NPV_{ED}}{\left(\frac{\lambda}{r+\lambda}\right)^{k_{MG2}}} - INV - RD \tag{1.68}$$

The Incumbent needs to maximize its option value F_{MG2} based on (1.39):

$$\begin{aligned}\max_k F_{MG2} &= \frac{1}{27r} + \frac{1}{r}\left(\frac{\lambda}{r+\lambda}\right)^k \left(\frac{1}{25(1-g)^k} - \frac{rNPV_{ED}}{\left(\frac{\lambda}{r+\lambda}\right)^k} - \frac{1}{27} - r(RD+INV) \right) = \\ &= \frac{1}{27r} - NPV_{ED} + \frac{1}{r}\left(\frac{\lambda}{r+\lambda}\right)^k \left(\frac{1}{25(1-g)^k} - \frac{1}{27} - r(RD+INV) \right).\end{aligned} \tag{1.69}$$

It follows from (1.63) and (1.69) that:

$$F_{MG2} = F_{MG1} - NPV_{ED} \tag{1.70}$$

implying that the merger trigger is the same in both cases:

$$x^c_{MG} = x^c_{MG1} = x^c_{MG2} = (1-g)^{k^c_{MG}} = \frac{\left(\ln \frac{\lambda}{r+\lambda} - \ln(1-g)\right)}{\frac{25}{27}\left(1 + r\frac{RD+INV}{\pi^*_{M0}}\right) \ln \frac{\lambda}{r+\lambda}}. \tag{1.71}$$

Besides, option value to the Incumbent is lower in the second case exactly by NPV_{ED}. Thus, NPV_{ED} is the price the Incumbent needs to pay in order to persuade the Entrant to merge.

The following proposition summarizes the optimal dynamic strategy of the Incumbent:

Proposition 5 *The optimal strategy of the Incumbent is as follows:*

1. *Compute x_M, x_E, x_D and x_{MG}.*

2. *In case $x_M > x_E$, compare $F_{MG1}(x_{MG})$ with $F_M(x_M)$. If $F_{MG1}(x_{MG}) > F_M(x_M)$, invest at x_{MG}; otherwise, invest at x_M. Recall that $x_M > x_E$ implies that without the opportunity to merge (in the duopolistic setup), the Incumbent would invest at the monopolistic trigger x_M, the Entrant would never enter (entry cost INV is too high).*

3. *In case $x_M \leq x_E$, compare $F_{MG2}(x_{MG})$ with $F_D(x_D)$. If $F_{MG2}(x_{MG}) > F_D(x_D)$, invest at x_{MG}; otherwise, invest at x_D. In the duopolistic setup the Incumbent would invest at x_D followed by the immediate entry.*

The shares of the merging firms in the total profits π^*_{MG} are determined endogenously together with the timing of the merger. This accords, for example, with Lambrecht (2004) that discusses the mergers motivated by economies of scale.

1.4 Conclusion

In this paper I investigated the technology adoption, entry and merger decisions in the dynamic model of vertical product differentiation. I have solved the model for the monopoly, duopoly and merger (which is equivalent to a monopoly with two products) and outlined the equilibrium strategies of the Incumbent and the Entrant.

I have demonstrated that the Incumbent under the threat of entry always invests in the cost-reducing technology later as compared to the no-threat-of-entry (monopoly) case. Besides, the option value to the Incumbent under the threat of entry is lower as compared to the monopoly.

I have also shown that relatively high cost of entry may prevent the Entrant from entering the market even after the cost-reducing investment by the Incumbent. The Incumbent, being aware of this fact, behaves as under no threat of entry and invests in cost reduction at the monopolist trigger. On the contrary, when the cost of entry is relatively low, the Entrant enters as soon as the Incumbent invests which takes place at the duopolistic trigger. In the first case, the competition does not reduce the option value to the Incumbent as the Entrant does not exercise its option to enter. In the second case, the competition erodes the option value to the Incumbent since the Entrant exercises its option.

The monopolistic problem has been investigated using two alternative specifications for the price of the new technology: in the first case, the price of the technology was constant, whereas in the second, more general case, the price of the new technology was increasing in the amount of cost reduction it provides. I have demonstrated that the increasing investment cost (price of the technology) makes the Incumbent postpone the technology adoption as compared to the constant price situation. Besides, increasing investment cost reduces the option value to the Incumbent.

I have demonstrated that the merger generates the endogenous synergy in this model. I have proven that the merger may not only increase the option value to the Incumbent as compared to the monopoly case, but it may also give the Entrant the opportunity to finally enter the market as a part of the merged firm whereas its independent entry would be unprofitable.

When the firms are given the opportunity to merge, the Incumbent can choose between the monopolistic and merger trigger or between the duopolistic and merger trigger. Clearly, it chooses the trigger that enhances the option value to the Incumbent more. Thus, the Incumbent is always better off in the result of the merger; the Entrant is at least as well off as before merging.

Chapter 2

Mergers and market valuation: real options approach

2.1 Introduction

Since the beginning of the ongoing financial crisis, the world has witnessed many once-strong firms being fire-sold to their former competitors, and quite often these deals were for cash. Cash takeovers normally follow the typical scheme: bidder makes an offer specifying price per target's share, takeover period etc. Target's shareholders either accept this offer agreeing to sell their shares at the price offered, or reject it. The next round of takeover negotiations may then follow.

A recent all-cash takeover of *BG Group* over *Pure Energy Resources Limited* is a telling example: on February 9, 2009 *BG Group* announced all-cash offer for *Pure* of A$6.40 per share which was at that time superior to the offer made in December by a competing bidder *Arrow Energy Limited* (*Arrow*'s offer was A$2.70 in cash and 1.21 *Arrow* shares for each *Pure* share, being worth A$5.39 per Pure share on February 6, 2009).

On February 18, *Pure* recommended *BG Group*'s offer of A$8 per share (this price increase by *BG Group* was a response to an earlier *Arrow*'s offer update of A$3.00 in cash and 1.57 *Arrow* shares for each *Pure* share).

On April 6, 2009 the takeover offer was closed; at that moment, *BG Group* owned 99.74% shares of *Pure* with the final price being A$8.25.[1]

As *BG Group* stated itself[2]:

> BG Groups Offer gives Pure shareholders the certainty of cash at a time of heightened uncertainty in world equity and financial markets.

Thus, both bidders and targets understand the superiority of cash deals over stock or stock-and-cash ones at the times of low market valuations.

The fact that periods of high market valuations often coincide with periods of intense merger activity (especially stock merger activity) - the so called 'merger waves' - has been extensively documented in merger literature: see, for example, Andrade, Mitchell, and Stafford (2001) and Martynova and Renneboog (2008) for surveys on mergers.

The starting point of this paper can be formulated as follows: out of the last three completed merger waves examined in the literature (the 1960s, 1980s and 1990s), the waves of the 1960s and 1990s were characterized by high market valuations and dominance of stock as preferred medium of payment, whereas market valuations in the 1980s were lower with larger fraction of deals being paid by cash. The research questions is: Is it possible to build a dynamic model of mergers that

[1] See `http://www.bg-group.com/MediaCentre/Press/Pages/Releases.aspx` for more information on the deal.
[2] `http://www.bg-group.com/MediaCentre/Press/Pages/9Feb2009.aspx`

would agree with existing empirical evidence on merger waves and market valuation? The answer is yes.

This paper investigates the connection between the market valuation and a type of the merger (stock, cash) using real options setup. The study relates to the literature that uses real options approach to dynamically investigate merger decisions, in particular timing and terms of mergers. Lambrecht (2004), Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008) model mergers as dynamic option exercise games between target and bidder(s) in which both timing and terms of mergers are determined endogenously.

In particular, Lambrecht (2004) studies mergers motivated by the economies of scale under complete, symmetric information and explains the procyclicality of merger waves.

On the contrary, Morellec and Zhdanov (2005) relate to Shleifer and Vishny (2003) in assuming that outside investors have imperfect information about the parameters of the model (namely, about the synergy created by the merger); thus, both models generate short-run abnormal returns that conforms to empirical evidence.

Morellec and Zhdanov (2005) allow for competition between the bidders resulting in negative abnormal returns to the winning bidder; besides, they explain how outside investors update their information about perceived synergy of merger observing actions (or, rather, inaction) of bidder(s); learning is also discussed in Grenadier (1999) and Lambrecht and Perraudin (2003).

In Lambrecht (2004), merger synergy comes from the production function that must display increasing returns to scale, whereas in Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008) merger surplus is linear in the pre-merger values of the firms and depends on the exogenous synergy parameter(s).

While Lambrecht (2004), Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008) consider mergers for stock only (with Lambrecht (2004) examining both friendly and hostile stock mergers), this paper aims at analyzing both stock and cash mergers. Though neither Lambrecht (2004), nor Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008) do not explicitly label mergers modeled in their papers as stock mergers, I believe that this is the case: in this type of merger each firm obtains shares in the new entity in exchange for the shares in the stand-alone firms (one risky asset is exchanged for another one), whereas in the cash merger the target is paid a lump-sum cash price (risky asset is exchanged for risk-free one). Literally, bidder in the cash merger is entitled to 100% of shares of the merged enrity; this situation can not be modeled within the original setup of Lambrecht (2004), Morellec and Zhdanov (2005) or Hackbarth and Morellec (2008) because in those models terms of merger are solved for endogenuously.

Thus, for the two setups considered (the first one by Lambrecht (2004) and the second one by

Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008)), I extend the original model offering the opportunity of a cash merger to the players and then solve cash merger problem.

The model of Lambrecht (2004) depends on one stochastic process only and allows to obtain closed-form solutions. On the contrary, the setup of Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008) depending on two correlated stochastic processes requires numerical solution; to this end, I use the Least Squares Monte Carlo approach (LSM) by Longstaff and Schwartz (2001).

In both setups, I solve for terms and timing of cash mergers. I compute option values to the players and introduce a measure of market valuation as weighed average of individual firm valuation in the second setup. I am able to demonstrate that in both setups, stock mergers should occur at high market valuation and at times of low market valuation cash mergers (or both types of mergers) should be observed. Thus, my conclusion agrees with existing empirical evidence on dominance of stock mergers at times of high market.

My results also partially accord with the prediction proposed in Shleifer and Vishny (2003) that one should observe more stock mergers at times of high markets and more cash takeovers at times of low markets.

I also investigate the dynamics of the intra-industry mergers within the first setup. I solve for the optimal order of mergers inside an industry for different initial capital allocations; I demonstrate that stock mergers in more concentrated industries occur at higher market valuation (i.e. later) as compared to mergers in less concentrated industries.

The paper is organized as follows: Section 2.2 examines cash mergers in the Lambrecht (2004) setup, Section 2.3 discusses cash merger in the Morellec and Zhdanov (2005) setup, Section 2.4 investigates the dynamics of the intra-industry mergers, Section 2.5 summarizes the results.

2.2 Stock vs. cash mergers under increasing returns to scale

This part of the paper is based on Lambrecht (2004) that examines the timing and terms of stock mergers (both friendly mergers and hostile takeovers) in partial equilibrium framework under complete information, increasing returns to scale (which are the only source of merger synergies) and risk-neutral firms. Lambrecht (2004) also assumes that mergers aim at maximizing shareholder value, thus avoiding the discussion of agency problem.

Lambrecht (2004) demonstrates that stock mergers are procyclical and provides closed-form solutions for the timing and terms of stock mergers. He also shows that stock mergers happen at globally efficient threshold.

In Section 2.2.1 I briefly re-state the setup and results of Lambrecht (2004); next, in Section 2.2.2, I augment the original model of Lambrecht (2004) with cash mergers. The aim is to demonstrate that cash mergers happen at lower market valuations than stock ones.

2.2.1 Stock mergers

In Lambrecht (2004), price-taking firm's instantaneous profits π_t are:

$$\pi_t = p_t L^a K^b - w_L L, \tag{2.1}$$

where p_t is the stochastic output price;
L and K are labor and capital inputs respectively;
w_L is the unit cost of labor;
a and b are positive constants such that $a < 1$ and $a + b > 1$, so that there are increasing returns to scale when both inputs are considered to be variable (as in the case of merger).

Thus, stochastic shock (output price) p_t is common for all the firms in the industry (as opposed to Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008) where firms face correlated stochastic shocks) and is governed by the following geometric Brownian motion:

$$dp_t = \mu p_t dt + \sigma p_t dW_t, \tag{2.2}$$

where W_t is the standard Brownian motion;
μ and σ are constants such that $\mu < r$ and $\sigma > 0$ and r is the risk-free interest rate.

Firm's instantaneous profits maximized with respect to labor input are:

$$\pi_t^* = f(w_L, a) K^\theta p_t^\gamma \tag{2.3}$$

where $f(w_L, a)$ is a known function of w_L and a;
$\theta = \frac{b}{1-a} > 1$ and $\gamma = \frac{1}{1-a} > 1$.

Then the value of the firm equals to:

$$V\left(p_t\right) = \mathbb{E}_Q \left[\int_t^\infty \pi_t^* e^{-rt} dt = \frac{f(w_L, a) K^\theta p_t^\gamma}{r - \mu\gamma - \frac{\sigma^2 \gamma(\gamma - 1)}{2}} \right] = c p_t^\gamma K^\theta, \tag{2.4}$$

where $\mathbb{E}_Q$ is the expectation under is risk-neutral measure Q;
$c = \frac{f(w_L,a)}{r-\mu\gamma-\frac{\sigma^2\gamma(\gamma-1)}{2}}$ subject to $\gamma < \beta_2$;
β_2 is the positive root of the equation:

$$\frac{1}{2}\sigma^2\beta(\beta-1)+\mu\beta-r=0. \tag{2.5}$$

Thus, in the case of two firms with the capital inputs equal to K_1 and K_2 and the lump-sum merger costs equal to M_1 and M_2, the total merger surplus is equal to:

$$\begin{aligned} S(p_t) &= \max\left(V_M\left(p_t\right)-V_1\left(p_t\right)-V_2\left(p_t\right)-M_1-M_2,\, 0\right) \\ &= \max\left(cp_t^\gamma\left(\left(K_1+K_2\right)^\theta-K_1^\theta-K_2^\theta\right)-M_1-M_2,\, 0\right), \end{aligned} \tag{2.6}$$

where V_M is the post-merger value of the new firm;
V_1 and V_2 are pre-merger values of firm 1 and firm 2.

In (2.6), the total benefits of the merger equal to $cp_t^\gamma\left(\left(K_1+K_2\right)^\theta-K_1^\theta-K_2^\theta\right)$; they are positive since $\theta=\frac{b}{1-a}>1$, i.e. for increasing returns to scale.

After the merger the firm i ($i=1,\,2$) is entitled to the fraction s_i of the new entity with $s_1+s_2=1$. The surplus accruing to firm i equals:

$$\begin{aligned} S_i(p_t) &= \max\left(s_iV_M\left(p_t\right)-V_i\left(p_t\right)-M_i,\, 0\right) \\ &= \max\left(cp_t^\gamma\left(s_i\left(K_1+K_2\right)^\theta-K_i^\theta\right)-M_i,\, 0\right). \end{aligned} \tag{2.7}$$

Since the merger surplus of each of the merging firms in a convex increasing function of the stochastic output price p_t, the merger option is exercised by firm i the first time process p_t reaches the threshold p_i^* from below.

It is demonstrated in Lambrecht (2004) that the in the continuation region (for $p_t < p_i^*$) the option to merge of firm i OM_i satisfies:

$$rOM_i=\mu p_tOM_i'+\frac{\sigma^2}{2}p_t^2OM_i'' \tag{2.8}$$

with the general solution being:

$$OM_i=B_1^ip_t^{\beta_1}+B_2^ip_t^{\beta_2} \tag{2.9}$$

where β_1 and β_2 are negative and positive root of (2.5).

Since $\lim_{p_t\to 0}OM_i=0$, then $B_1^i=0$; the value matching and smooth pasting conditions at p_i^* are:

$$OM_i(p_i^*) = c\,(p_i^*)^{\gamma}\left(s_i\,(K_1 + K_2)^{\theta} - K_i^{\theta}\right) - M_i \tag{2.10}$$

$$OM_i'(p_i^*) = c\gamma\,(p_i^*)^{\gamma-1}\left(s_i\,(K_1 + K_2)^{\theta} - K_i^{\theta}\right). \tag{2.11}$$

The solution is:

$$OM_i(p_t) = \left(cp_t^{\gamma}\left(s_i\,(K_1 + K_2)^{\theta} - K_i^{\theta}\right) - M_i\right)\left(\frac{p_t}{p_i^*}\right)^{\beta_2} \tag{2.12}$$

with the merger threshold for firm i being:

$$p_i^* = \left(\frac{\beta_2}{\beta_2 - \gamma}\frac{M_i}{c\left(s_i\,(K_1 + K_2)^{\theta} - K_i^{\theta}\right)}\right)^{\frac{1}{\gamma}}. \tag{2.13}$$

Taking into account that merger threshold of firms should be equal $p_1^* = p_2^* = p^*$ and the fact that $s_1 + s_2 = 1$ allows to solve for the merger threshold p^* and for optimal shares s_1 and s_2:

$$p^* = \left(\frac{\beta_2}{\beta_2 - \gamma}\frac{M_1 + M_2}{c\left((K_1 + K_2)^{\theta} - K_1^{\theta} - K_2^{\theta}\right)}\right)^{\frac{1}{\gamma}} \tag{2.14}$$

$$s_i = \frac{M_i\left((K_i + K_j)^{\theta} - K_j^{\theta}\right) + M_j K_i^{\theta}}{(M_i + M_j)\,(K_i + K_j)^{\theta}}. \tag{2.15}$$

It is demonstrated in Lambrecht (2004) that threshold p^* coincides with the socially optimal threshold derived from the point of view of social maximizer and based on total surplus $S(p_t)$ rather than on individual surplus of each firm $S_i(p_t)$; this means that the merger described by p^* and (s_1, s_2) is Pareto optimal and constitutes Nash equilibrium.

Merger threshold (2.14) will serve as benchmark for analysis of cash mergers in Section 2.2.2.

The choice of roles of bidder and target for this type of merger is completely immaterial not only for merger terms and timing, but also for welfare consequences (surplus distribution) of the merger. The solution does not directly involve 'the bidder' and 'the target'; it is enough to have two firms willing two merge.

2.2.2 Market valuation of stock vs. cash mergers

In the cash merger bidder buys the firm of target paying a lump-sum price P (and not a share of the merged entity as in the stock merger in Section 2.2.1).

The solution for the target is as follows: in the continuation region of the target (for $p_t > p_T$), differential equation (2.8) for the option of the target OT (instead of OM_i) holds with the general solution (2.9) and $B_2 = 0$ since $\lim_{p_t \to \infty} OT = 0$. Usual value-matching and smooth pasting conditions apply:

$$OT(p_T) = P - cp_T^{\gamma} K_T^{\theta} - M_T \tag{2.16}$$

$$OT'(p_T) = -c\gamma p_T^{\gamma-1} K_T^{\theta}. \tag{2.17}$$

The solution is:

$$OT(p_t) = \left(P - cp_T^{\gamma} K_T^{\theta} - M_T\right) \left(\frac{p_t}{p_T}\right)^{\beta_1}, \tag{2.18}$$

where β_1 is the negative root of (2.5);
p_T is the cash merger threshold of the target:

$$p_T = \left(\frac{\beta_1}{\beta_1 - \gamma} \frac{P - M_T}{cK_T^{\theta}}\right)^{\frac{1}{\gamma}}. \tag{2.19}$$

The solution for the bidder is as follows: in the continuation region of the bidder (for $p_t < p_B$), differential equation (2.8) for the option of the bidder OB (instead of OM_i) holds with the general solution (2.9) and $B_1 = 0$ since $\lim_{p_t \to 0} OB = 0$. Usual value-matching and smooth pasting conditions are:

$$OB(p_B) = cp_B^{\gamma} \left((K_B + K_T)^{\theta} - K_B^{\theta}\right) - M_B - P \tag{2.20}$$

$$OB'(p_B) = c\gamma p_B^{\gamma-1} \left((K_B + K_T)^{\theta} - K_B^{\theta}\right) \tag{2.21}$$

The solution is:

$$OB(p_B) = \left(cp_B^{\gamma} \left((K_B + K_T)^{\theta} - K_B^{\theta}\right) - M_B - P\right) \left(\frac{p_t}{p_B}\right)^{\beta_2}, \tag{2.22}$$

where p_B is the cash merger threshold of the bidder:

$$p_B = \left(\frac{\beta_2}{\beta_2 - \gamma} \frac{M_B + P}{c\left((K_B + K_T)^{\theta} - K_B^{\theta}\right)}\right)^{\frac{1}{\gamma}}. \tag{2.23}$$

The bidder is willing to exercise a cash merger option when the state variable p_t first hits the threshold p_B from below, whereas the target is willing to exercise when p_t first hits p_T from above; thus, for the merger to be exercised, the following condition should hold:

$$p_B \leq p_t \leq p_T, \tag{2.24}$$

or, after simplifications,

$$\frac{\beta_2}{\beta_2 - \gamma} \frac{M_B + P}{(K_B + K_T)^\theta - K_B^\theta} < \frac{\beta_1}{\beta_1 - \gamma} \frac{P - M_T}{K_T^\theta}. \tag{2.25}$$

Solving (2.25) for P yields:

$$P \geq \frac{\left((K_B + K_T)^\theta - K_B^\theta\right) M_T \frac{\beta_1}{\beta_1 - \gamma} + K_T^\theta M_B \frac{\beta_2}{\beta_2 - \gamma}}{\frac{\beta_1}{\beta_1 - \gamma} (K_B + K_T)^\theta - \frac{\beta_1}{\beta_1 - \gamma} K_B^\theta - \frac{\beta_2}{\beta_2 - \gamma} K_T^\theta} \tag{2.26}$$

provided the following inequality holds:

$$\frac{\beta_1}{\beta_1 - \gamma} (K_B + K_T)^\theta - \frac{\beta_1}{\beta_1 - \gamma} K_B^\theta - \frac{\beta_2}{\beta_2 - \gamma} K_T^\theta > 0. \tag{2.27}$$

Since $\frac{\beta_2}{\beta_2 - \gamma} > \frac{\beta_1}{\beta_1 - \gamma} > 0$ by the properties of the solution, inequality does not always hold implying that cash merger equilibrium does not always exists. Thus, depending on the model parameters, one can distinguish between two types of outcomes:

1. (2.27) holds; both stock and cash merger equilibria exist;

2. (2.27) does not hold; only stock merger equilibrium exists.

Assume that (2.27) holds i.e. cash merger equilibrium exists; to determine the relationship between the stock merger trigger p^* in (2.14) and the cash merger corridor $[p_B, p_T]$ solve $p^* < p_B$ to obtain:

$$P \geq \frac{\left((K_B + K_T)^\theta - K_B^\theta\right) M_T + K_T^\theta M_B}{(K_B + K_T)^\theta - K_B^\theta - K_T^\theta}. \tag{2.28}$$

The fact that $\frac{\beta_2}{\beta_2 - \gamma} > \frac{\beta_1}{\beta_1 - \gamma} > 0$ means that (2.26) implies (2.28) and, consequently:

$$p^* < p_B < p_T. \tag{2.29}$$

Consider an example with $a = 0.4$, $b = 1.9$, $\mu = 0.01$, $r = 0.08$, $\sigma = 0.2$, $K_B = K_T = 100$, $M_B = M_T = 3$, $c = 1$; (2.27) holds and price P should satisfy $P \geq 73.5$.

Setting $P = 200$, one obtains the following values for bidder's and target's threshold: $p_B = 0.00245471$, $p_T = 0.002532$. The stock merger threshold p^* (see 2.14) equals $p^* = 0.000321576$ and (2.29) holds.

Inequality (2.29) suggests that when market valuation (as measured by the state variable p_t) is in the interval $[p_B, p_T]$, both cash and stock mergers are observed; as p_t increases, only stock mergers should be observed. This conclusion agrees quite well with empirical evidence on procyclicality of merger waves and dominance of stock mergers at high market valuations.

Now I proceed to a more complicated setup of Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008) that employs two correlated stochastic processes (instead of one in this setup) and is based on linear synergy instead of synergy stemming from economies of scale.

2.3 Stock vs. cash mergers under linear merger synergy

I follow Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008) in the setup of my model. Consider an industry consisting of two firms (bidder and target) with capital stock K and Q; present value of the cash flows of the firms are X and Y that are governed by the stochastic differential equations:

$$dX_t = \mu_X X_t dt + \sigma_X X_t dW_t^X \tag{2.30}$$

$$dY_t = \mu_Y Y_t dt + \sigma_Y Y_t dW_t^Y \tag{2.31}$$

where W_t^X and W_t^Y are standard correlated Brownian motions with correlation coefficient ρ; μ_X, μ_Y, σ_X and σ_Y are constants such that $\mu_X < r$, $\mu_Y < r$, $\sigma_X > 0$ and $\sigma_Y > 0$ and r is the risk-free interest rate.

Assume also that investors are risk-neutral.

In case of a merger, combined value of the merged firms equals:

$$V(X, Y) = KX + QY + \alpha (K + Q)(X - Y), \tag{2.32}$$

where KX is the pre-merger value of the bidder;
KY is the pre-merger value of the target;
α is positive and reflects merger synergy;
$\alpha (K + Q)(X - Y)$ is the merger surplus.

It follows from (2.32) that for the merger to be profitable, the bidder should have higher valuations per unit capital than the target; it means that the roles of bidder and target are predetermined as opposed to Lambrecht (2004) where any of the firms can act as a bidder. 'Valuation per unit capital' may be thought of as Tobin's q or M/B ratio.

I choose the complete information setup of Hackbarth and Morellec (2008) as opposed to incomplete information with learning as in Morellec and Zhdanov (2005) for better comparison with the results from the previous section; both Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008) assume that the option to merger has infinite horizon.

First I briefly repeat the results of the stock merger as in Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008), and then I solve this model for the cash takeover game (I follow the same order as in Section 2.2).

2.3.1 Stock mergers

Stock merger is modeled as a simultaneous game with bidder and target giving up their pre-merger values of the firms to get a share in the new merged entity.

Payoffs to the bidder P_B^s and to the target P_T^s at the stock merger are as follows:

$$\begin{aligned} P_B^s(X,Y) &= \max\left(s_B V(X,Y) - KX, 0\right) \\ P_T^s(X,Y) &= \max\left((1-s_B)V(X,Y) - QY, 0\right), \end{aligned} \tag{2.33}$$

where s_B is the share of the merged entity accruing to the bidder.

In the continuation region option to the bidder O^{Bs} and to the target O^{Ts} satisfy the following differential equations:

$$rO^{Bs} = \frac{1}{2}\sigma_X^2 X^2 O_{XX}^{Bs} + \frac{1}{2}\sigma_Y^2 Y^2 O_{YY}^{Bs} + \rho\sigma_X\sigma_Y XY O_{XY}^{Bs} + \mu_X O_X^{Bs} + \mu_Y O_Y^{Bs} \tag{2.34}$$

$$rO^{Ts} = \frac{1}{2}\sigma_X^2 X^2 O_{XX}^{Ts} + \frac{1}{2}\sigma_Y^2 Y^2 O_{YY}^{Ts} + \rho\sigma_X\sigma_Y XY O_{XY}^{Ts} + \mu_X O_X^{Ts} + \mu_Y O_Y^{Ts} \tag{2.35}$$

subject to the following value-matching conditions:

$$O^{Bs}(X^s, Y^s) = s_B V(X^s, Y^s) - KX^s \tag{2.36}$$

$$O^{Ts}(X^s, Y^s) = (1-s_B)V(X^s, Y^s) - QY^s, \tag{2.37}$$

where X^S and Y^S is the stock exercise bound.

Though options to both bidder and target depend on two stochastic processes X (2.30) and Y (2.31), but since the payoffs P_B^s and P_T^s are both linear in X and Y, it is demonstrated in Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008) that terms and timing of the mergers can be solved in terms of the ratio $R = \frac{X}{Y}$.

In particular, option values to bidder and target satisfy:

$$\begin{aligned} O^{Bs}(X,Y) &= Y\left(s_B V(R^*,1) - KR^*\right)\left(\tfrac{R}{R^*}\right)^{\lambda_2} \\ O^{Ts}(X,Y) &= Y\left((1-s_B)V(R^*,1) - Q\right)\left(\tfrac{R}{R^*}\right)^{\lambda_2}, \end{aligned} \tag{2.38}$$

where R^s is the stock merger threshold:

$$R^s = \frac{\lambda_2}{\lambda_2 - 1}, \tag{2.39}$$

s_B is the share of the merged firm accruing to the bidder:

$$s_B = \frac{K}{K+Q}, \tag{2.40}$$

and λ_2 is the positive root of the equation:

$$\frac{1}{2}\left(\sigma_X^2 - 2\rho\sigma_X\sigma_Y + \sigma_Y^2\right)\lambda(\lambda-1) + (\mu_X - \mu_Y)\lambda = r - \mu_Y. \tag{2.41}$$

Merger occurs as soon as process $R = \frac{X}{Y}$ first hits the threshold $R^s = \frac{\lambda_2}{\lambda_2 - 1}$ from below. This result is similar in spirit to the one in Lambrecht (2004) where the state variable p_t also needs to hit the threshold p^* from below.

Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008) also demonstrate that the stock merger equilibrium coincides with the central-planner equilibrium where the central planner is maximizing merger surplus. It means that the payoff of the central planner equals:

$$\begin{aligned} P_{CP}^s(X,Y) &= \max\left(V(X,Y) - KX - QY, 0\right) = \\ &= \max\left(\alpha(K+Q)(X-Y), 0\right), \end{aligned} \tag{2.42}$$

and the option to the central planner O^{CPs} equals the sum of bidder's O^{Bs} and target's O^{Ts} options:

$$\begin{aligned} O^{CPs}(X,Y) &= Y\left(V(R^*,1) - KR^* - Q\right)\left(\tfrac{R}{R^*}\right)^{\lambda_2} = \\ &= Y\left(\alpha(K+Q)(R^*-1)\right)\left(\tfrac{R}{R^*}\right)^{\lambda_2}. \end{aligned} \tag{2.43}$$

Solution to the stock merger problem summarized in this section will provide the benchmark for the cash merger problem presented in the next section.

2.3.2 Cash mergers and market valuation

The bidder offers a lump-sum price P for the whole firm of the target. Payoffs to the bidder P_B^c and to the target P_T^c at the cash merger are as follows:

$$\begin{aligned} P_B^c(X,Y) &= \max\left(V(X,Y) - KX - P, 0\right) = \\ &= \max\left(QY + \alpha\left(K+Q\right)\left(X-Y\right) - P, 0\right) \\ P_T^c(X,Y) &= \max\left(P - QY, 0\right) \end{aligned} \tag{2.44}$$

In the continuation region options to the bidder O^{Bc} and to the target O^{Tc} satisfy the following differential equations:

$$rO^{Bc} = \frac{1}{2}\sigma_X^2 X^2 O_{XX}^{Bc} + \frac{1}{2}\sigma_Y^2 Y^2 O_{YY}^{Bc} + \rho\sigma_X\sigma_Y XY O_{XY}^{Bc} + \mu_X O_X^{Bc} + \mu_Y O_Y^{Bc} \tag{2.45}$$

$$rO^{Tc} = \frac{1}{2}\sigma_Y^2 Y^2 O_{YY}^{Tc} + \mu_Y O_Y^{Tc} \tag{2.46}$$

subject to the following value-matching conditions:

$$O^{Bc}\left(X^c, Y^c\right) = QY^c + \alpha\left(K+Q\right)\left(X^c - Y^c\right) - P \tag{2.47}$$

$$O^{Tc}\left(X^c, Y^c\right) = P - QY^c, \tag{2.48}$$

where X^c and Y^c is the exercise boundary.

Since the value function of the bidder $O^{Bc}\left(X^c, Y^c\right)$ (2.47) is not homogeneous neither in X, nor in Y, it is not possible to reduce the solution to the ratio $\frac{X}{Y}$ as it was done for the stock mergers in Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008) (see Section 2.3.1) and I have to rely on numerical methods to solve this problem.

I use Longstaff and Schwartz (2001) least squares Monte Carlo (LSM) approach that is relatively simple and convenient for multi-factor models[3]. This method was also used in another real options paper by Cortazar, Gravet, and Urzua (2008) to solve the three-factor model for the copper mine valuation based on traditional Brennan and Schwartz (1985) setup.

[3]MATLAB codes for LSM estimation are available from the author on demand.

Parameter calibration for LSM Least squares approach requires setting a finite time horizon (as opposed to infinite horizon in the original papers); for the major part of the solution I choose a horizon of 5 years (T=5). Remaining parameters are set as follows:

- risk-free interest rate $r = 0.06$, dividend payout rates for the bidder $\delta_X = 0.005$ and for the target $\delta_Y = 0.035$ implying drifts of $\mu_X = 0.055$ and $\mu_Y = 0.025$, volatilities $\sigma_X = 0.2$ and $\sigma_Y = 0.2$, correlation between stochastic processes of the firms $\rho = 0.75$ are set as in Hackbarth and Morellec (2008) (see Table I on page 1227);

- estimation is based on $N = 100,000$ paths as in Longstaff and Schwartz (2001) with $y = 10$ exercise points per year for the sample simulation in Table 2.5 and interchangeably $y = 10$ and $y = 50$ otherwise[4];

- though Hackbarth and Morellec (2008) consider mergers of equals with $K = Q$, Andrade, Mitchell, and Stafford (2001) report that the median relative size of the target was 11.7% in 1973-1998; that is why I set the capital stock of the bidder $K = 100$ and of the target $Q = 12$;

- initial values of X and Y are set to $X_0 = Y_0 = 1$ implying that 1) both bidder and target are neither undervalued, nor overvalued and 2) initial merger synergy computed as $\alpha\,(X_0 - Y_0)\,(K + Q)$ is zero;

- lump-sum price P offered for the whole firm of the target is set to $P = 12$ implying zero merger premium for the target;

- synergy parameter α is set to $\alpha = 0.4$ resulting in reasonable merger premium of 22% for cash merger and 52% for stock merger over a 5-year horizon (see Table 2.2).

Since the solution for the cash merger based on LSM hinges on the assumption about chosen finite horizon, it is not directly comparable to the infinite-horizon solution derived in Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008) and presented in Section 2.3.1. Thus, I need to solve stock merger problem using LSM with finite horizon too.

LSM: short algorithm description for both cash and stock merger problems

1. simulate X and Y obtaining N simulation paths for y exercise points per year;

[4] y=50 was used in the original paper by Longstaff and Schwartz (2001).

2. compute state variables and payoffs:
   cash: state variables for the bidder $S_B = V(X,Y) - KX = QY + \alpha(K+Q)(X-Y)$, for the target $S_T = QY$;
   payoffs to the bidder $P_B^c = \max(QY + \alpha(K+Q)(X-Y) - P, 0)$ and to the target $P_T^c = \max(P - QY, 0)$;
   stock: state variable for the central planner $S_{CP} = \alpha(X-Y)(K+Q)$;
   payoff to the central planner $P_{CP} = \max(\alpha(K+Q)(X-Y), 0)$.

   Appendix B.1 explains in detail why it is possible to solve the stock merger problem from the point of view of central planner setting $s_B = \frac{K}{K+Q}$ and provided $\alpha(K+Q) > Q$ (that can be rewritten as $\frac{K}{K+Q} > 1 - \alpha$) is satisfied; in this paper $\frac{K}{K+Q} = 0.8929 > 1 - \alpha = 0.6$.

3. apply LSM as follows[5]:
   cash: for the merger option to be exercised at some point both bidder and target should independently prefer immediate exercise to option continuation at this point;
   stock: central planner should prefer immediate exercise to continuation;

4. compute option values:
   cash: to the bidder O_{LSM}^{Bc} and to the target O_{LSM}^{Tc} as sample mean;
   stock: to the central planner O_{LSM}^{CPs} as sample mean; separately to the bidder O_{LSM}^{Bs} and to the target O_{LSM}^{Ts} as sample mean of discounted payoffs at exercise to the bidder $(s_B V(X^s, Y^s) - KX^s) e^{-rt_{ex}}$ and to the target $((1 - s_B) V(X^s, Y^s) - QY^s) e^{-rt_{ex}}$ (t_{ex} is the time of option exercise);

5. compute average market valuation[6] $MARKET_{LSM}^c$ ($MARKET_{LSM}^s$), average merger premium $PREM_{LSM}^c$ ($PREM_{LSM}^s$), and average ratio R_{LSM}^c (R_{LSM}^s) at the time of exercise using the sub-sample of paths where the merger is exercised at some point as a mean of the following quantities: $\frac{KX_{LSM}^c + QY_{LSM}^c}{K+Q}$ $\left(\frac{KX_{LSM}^s + QY_{LSM}^s}{K+Q}\right)$, $\frac{P}{QY^c} - 1$ $\left(\frac{(1-s_B)V(X^s,Y^s)}{QY^s} - 1\right)$ and $\frac{X^c}{Y^c}$ $\left(\frac{X^s}{Y^s}\right)$.

Table 2.1 presents the results of LSM simulations over different time horizons: 1, 5, 25 and 50 years together with the result for infinite horizon based on Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008) (see Section 2.3.1 for detailed derivations).

Option values to the players increase as time horizon becomes longer; for the stock merger case, option values converge monotonically to the infinite horizon option which is perfectly intuitive.

[5] As regressors, I use a constant and the first three powers of the state variable.
[6] There are only two firms in the model and, consequently, market valuation depends on X and Y only.

'Total' for cash merger options is always lower than for respective stock merger options reflecting the fact that stock merger is the 'first-best' choice as shown in Morellec and Zhdanov (2005), Hackbarth and Morellec (2008) and, though for a different setup, in Lambrecht (2004).

Market valuation as measured by $MARKET^c_{LSM}$ for cash merger and $MARKET^s_{LSM}$ for stock mergers demonstrates desired behavior: for all of the estimated time horizons, market valuation for cash merger is always lower than the market valuation for the stock merger; to prove that this relationship holds generally, I will simulate a cross-section of mergers and conduct regression analysis later in the paper.

The behavior of merger premium of the stock merger $PREM^s_{LSM}$ has one quite striking property: while the size of the premium remains quite reasonable over 1-year and 5-year horizon (18% and 52% respectively), it becomes very high over 50-year horizon (689%) and reaches even higher level of 1066% over an infinite horizon.

Average ratio $\frac{X^s}{Y^s}$ for the stock merger (R^s_{LSM}) also climbs very high over an infinite horizon reaching the level of 9.24, whereas the same ratio for the cash merger remains reasonable.

These large (and unrealistic) magnitudes may suggest that firms do not really consider horizons of such length; that is why the choice of 5-year horizon seems quite appropriate.

Table 2.2 compares the results of LSM estimation over 1-year and 5-year horizon for different number of exercise point per year: 10 and 50. Results suggest that loss in computational accuracy when switching from 50 to 10 exercise points per year is acceptable, whereas gains in computational speed are significant; henceforth, I conduct LSM estimation based on 10 exercise points per year ($y = 10$).

Finally, Figures 2.1 illustrates exercise boundaries for the stock and cash mergers separately for the benchmark example with $T = 5$ and $y = 10$ (for estimation results see row 4 of Table 2.2), whereas Figure 2.2 puts them together for better comparison.

Figure 2.2 demonstrates that when capital valuations of both bidder and target are relatively high, only stock mergers should be observed; on the contrary, when valuations are relatively low, both cash and stock merger may be observed. Taking into account the fact that correlation between X and Y is positive in this example ($\rho = 0.75$), one concludes that when market valuation (as measured by weighted average of firm's valuations) is high, stock mergers should be observed, whereas at low market valuations both types of merger may be observed.

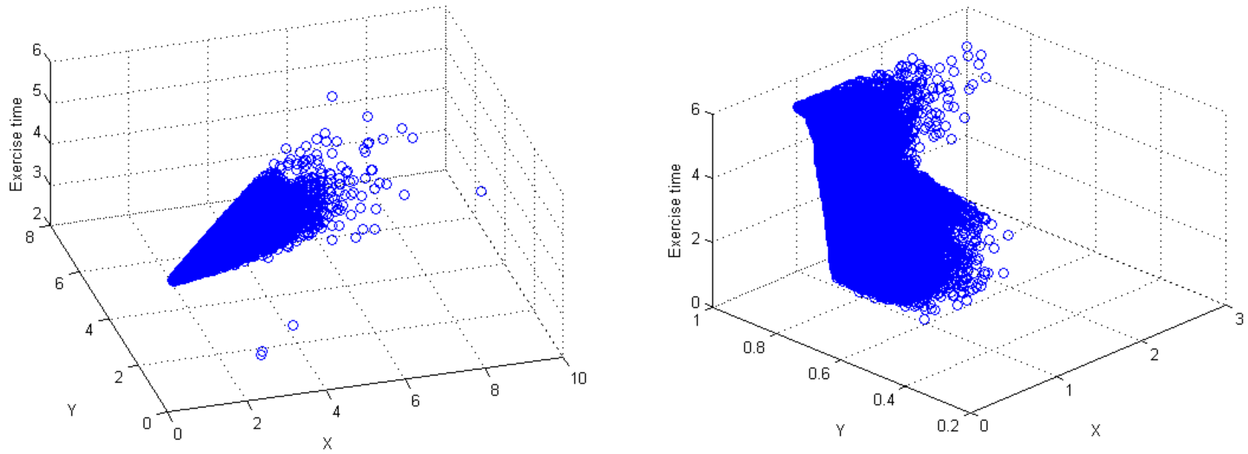
Tables 2.3 and 2.4 provide estimation results for the price P computed as $P = -0.6 + 1.35QY$ with parameters estimated from a sample of cash mergers; one can see that for this functional form of P cash mergers demonstrate the same long-run behavior as stock mergers: $MARKET^c_{LSM}$ and R^c_{LSM} increase significantly over 50-year horizon; on the contrary, cash merger premium

Table 2.1: LSM simulation results over different time horizons

T	y	stock			cash			$MARKET_{LSM}$		$PREM_{LSM}$		R_{LSM}	
		O^{Bs}_{LSM}	O^{Ts}_{LSM}	total	O^{Bc}_{LSM}	O^{Tc}_{LSM}	total	cash	stock	cash	stock	cash	stock
1	10	2.09	1.10	3.19	1.43	0.49	1.92	0.99	1.09	0.11	0.18	1.11	1.14
5	10	5.71	3.03	8.74	3.47	0.94	4.40	1.02	1.38	0.22	0.53	1.26	1.41
25	10	14.43	7.64	22.07	7.64	1.08	8.72	1.25	3.82	0.44	2.56	1.77	2.98
50	10	18.02	9.55	27.57	9.00	0.97	9.97	1.54	13.21	0.63	8.65	2.40	7.69
infinite horizon		19.94	10.56	30.51							10.66		9.24

Table 2.2: Comparison of LSM simulation results over 1-year and 5-year time horizon for 10 and 50 exercise points per year

T	y	stock			cash			$MARKET_{LSM}$		$PREM_{LSM}$		R_{LSM}	
		O^{Bs}_{LSM}	O^{Ts}_{LSM}	total	O^{Bc}_{LSM}	O^{Tc}_{LSM}	total	cash	stock	cash	stock	cash	stock
1	50	2.09	1.11	3.20	1.42	0.50	1.92	0.99	1.09	0.10	0.18	1.09	1.14
1	10	2.09	1.10	3.19	1.43	0.49	1.92	0.99	1.09	0.11	0.18	1.11	1.14
5	50	5.67	3.00	8.67	3.46	0.94	4.40	1.02	1.38	0.20	0.52	1.23	1.40
5	10	5.71	3.03	8.74	3.47	0.94	4.40	1.02	1.38	0.22	0.53	1.26	1.41



(a) Stock merger (b) Cash merger

Figure 2.1: Exercise boundaries for the stock and cash mergers separately at $T = 5$, $y = 10$ and remaining parameters as above

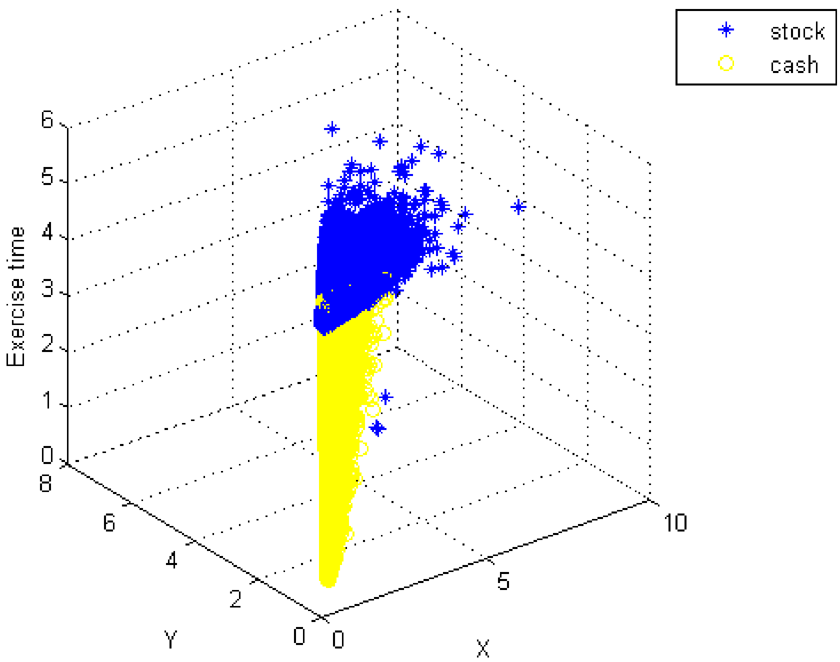


Figure 2.2: Exercise boundaries for the stock and cash mergers at $T = 5$, $y = 10$ and remaining parameters as above

Table 2.3: LSM simulation results over different time horizons: P=-0.6 + 1.35QY

T	y	stock			cash			$MARKET_{LSM}$		$PREM_{LSM}$		R_{LSM}	
		O_{LSM}^{Bs}	O_{LSM}^{Ts}	total	O_{LSM}^{Bc}	O_{LSM}^{Tc}	total	cash	stock	cash	stock	cash	stock
1	10	2.09	1.10	3.19	1.53	1.24	2.77	1.09	1.09	0.30	0.18	1.18	1.14
5	10	5.71	3.03	8.74	6.27	1.69	7.96	1.31	1.38	0.29	0.53	1.37	1.41
25	10	14.43	7.64	22.07	20.32	1.25	21.57	3.37	3.82	0.28	2.56	2.50	2.98
50	10	18.02	9.55	27.57	26.91	0.66	27.57	11.92	13.21	0.29	8.65	5.48	7.69
infinite horizon		19.94	10.56	30.51							10.66		9.24

Table 2.4: Comparison of LSM simulation results over 1-year and 5-year time horizon for 10 and 50 exercise points per year: P=-0.6 + 1.35QY

T	y	stock			cash			$MARKET_{LSM}$		$PREM_{LSM}$		R_{LSM}	
		O_{LSM}^{Bs}	O_{LSM}^{Ts}	total	O_{LSM}^{Bc}	O_{LSM}^{Tc}	total	cash	stock	cash	stock	cash	stock
1	50	2.09	1.11	3.20	1.46	1.29	2.75	1.08	1.09	0.30	0.18	1.17	1.14
1	10	2.09	1.10	3.19	1.53	1.24	2.77	1.09	1.09	0.30	0.18	1.18	1.14
5	50	5.67	3.00	8.67	6.09	1.71	7.81	1.29	1.38	0.29	0.52	1.35	1.40
5	10	5.71	3.03	8.74	6.27	1.69	7.96	1.31	1.38	0.29	0.53	1.37	1.41

Table 2.5: Parameters calibration

Parameter	Benchmark example	Sample simulation rule		
		$U[a,b]$		Other
		a	b	
T	5			$= 5$
y	10			$= 10$
r	0.06	0.04	0.08	
δ_X	0.005	0	0.01	
δ_Y	0.035	0.03	0.04	
σ_X	0.2	0.15	0.25	
σ_Y	0.2	0.15	0.25	
ρ	0.75	-1	1	
K	100	75	125	
Q	12	9	15	
α	0.4	0.35	0.45	
X_0	1	0.5	2.5	
Y_0	1			$= X_0$
P	12			$= QY_0$

$U[a,b]$ stands for uniform distribution with parameters a and b.
Parameters Y_0 and P are set so as to ensure that initial synergy and initial merger premium are both equal to zero (as in estimations in Tables 2.1 and 2.2).
Drifts are $\mu_X = r - \delta_X$, $\mu_Y = r - \delta_Y$.

$PREM^c_{LSM}$ stays under 35% due to the scific functional form of P. However, on 5-year horizon there is no huge qualitative difference between constant price P as in Tables 2.1 and 2.2 and linear price P as in Tables 2.3 and 2.4; for the rest of the Section, I use the first setup with constant price P.

In order to conduct more general test of a hypothesis that stock mergers should be observed at high market valuations, and cash mergers should occur at low market valuations, I simulate a sample of 1000 merger situations (without initially specifying the type of a merger) with majority of input parameters drawn from independent uniform distributions (see Table 2.5 for details).

A fragment of 15 simulated merger situations is presented in Appendix B.2: Table B.1 presents estimation results, and Table B.2 provides input parameters.

One can easily see that though the sum of bidder's and target's stock merger options is always greater than the sum of cash merger options (reflecting the fact that stock merger is the 'first-best'), but in some cases both bidder's and target's stock merger options are greater than respective cash merger options, and in the remainder of cases target's stock option is greater than target's cash

Table 2.6: Payoff matrix

		Target	
		CASH	STOCK
Bidder	CASH	O^{Bc}_{LSM} , O^{Tc}_{LSM}	0 , 0
	STOCK	0 , 0	O^{Bs}_{LSM} , O^{Ts}_{LSM}

options, whereas bidder's stock option is smaller than bidder's cash option.

Generally, when both stock and cash merger options are available to the players, the payoff matrix of the game looks like the one presented in Table 2.6. Each player has two pure strategies: play $CASH$ or play $STOCK$; if players' strategies do not match, then payoffs to both players are zero.

It is easy to see that there are two Nash equilibria in this game: play $CASH, CASH$ and play $STOCK, STOCK$. Depending on the relative size of payoffs, one needs to distinguish between two following situations in order to formulate rules of equilibrium selection:

1. $O^{Bs}_{LSM} \geq O^{Bc}_{LSM}$ and $O^{Ts}_{LSM} > O^{Tc}_{LSM}$; see, for example, row 2 of Table B.1. It means that Nash equilibrium $STOCK, STOCK$ is both payoff and risk dominant over Nash equilibrium $CASH, CASH$; thus, rational players should both agree on playing $STOCK, STOCK$.

2. $O^{Bs}_{LSM} < O^{Bc}_{LSM}$ and $O^{Ts}_{LSM} > O^{Tc}_{LSM}$; see, for example, row 1 of Table B.1. It means that neither of equilibria is payoff dominant; thus, Nash equilibria $CASH, CASH$ and $STOCK, STOCK$ are played with the same probability.

Having established equilibrium selection rules, I proceed to regression analysis using the simulated sample.

Table 2.7 demonstrates that $MARKET_{LSM}$ has negative effect on probability of a cash merger in both probit and logit models: coefficient on $MARKET_{LSM}$ is negative and statistically significant at the 1% significance level.

Thus, regression analysis in Table 2.7 shows that in this setup, cash mergers should be observed at low market valuations, and stock mergers should be observed at high market valuations agreeing with empirical evidence on dominance of stock mergers at high market valuations.

Table 2.7: Regression analysis

	Probit	Logit
constant	-10.24256**	-16.82325**
	(-7.97)	(-7.92)
$MARKET_{LSM}$	-11.72748**	-21.26603**
	(-13.18)	(-11.99)
r	76.8797**	138.9899**
	(8.23)	(7.91)
δ_X	-111.3395**	-198.6983**
	(-4.00)	(-3.97)
σ_X	24.03141**	42.40887**
	(7.54)	(7.29)
σ_Y	-6.76513*	-11.16768*
	(-2.38)	(-2.23)
ρ	-3.874487**	-7.021367**
	(-13.32)	(-12.12)
Q	.0093608	-
	(1.70)	
X_0	16.77087**	30.33026**
	(12.85)	(11.78)
Pseudo R^2	0.6747	0.6726
LR χ^2	755.82	753.5
p-value of LR	0.0000	0.0000
Number of observations	1000	1000
Goodness-of-fit Pearson test	OK	OK

Only last specification shown; z-score values in parentheses; 5% and 1% significance levels denoted by * and ** respectively.

2.4 Dynamics of the intra-industry mergers

This section is based on the results for stock mergers obtained in Lambrecht (2004) and uses the same definitions as Section 2.2, in particular:
option value to firm 1 equals:

$$OM_1(p_t) = \left(c\,(p^*)^{\gamma} \left(s_1\,(K_1+K_2)^{\theta} - K_1^{\theta} \right) - M_1 \right) \left(\frac{p_t}{p^*} \right)^{\beta_2} \tag{2.49}$$

option value to firm 2 equals:

$$OM_2(p_t) = \left(c\,(p^*)^{\gamma} \left((1-s_1)\,(K_1+K_2)^{\theta} - K_2^{\theta} \right) - M_2 \right) \left(\frac{p_t}{p^*} \right)^{\beta_2} \tag{2.50}$$

globally optimal threshold p^* is:

$$p^* = \left(\frac{\beta_2}{\beta_2-\gamma} \frac{M_1+M_2}{c\left((K_1+K_2)^{\theta} - K_1^{\theta} - K_2^{\theta} \right)} \right)^{\frac{1}{\gamma}} \tag{2.51}$$

share of firm 1 equals:

$$s_1 = \frac{M_1\left((K_1+K_2)^{\theta} - K_2^{\theta} \right) + M_2 K_1^{\theta}}{(M_1+M_2)\,(K_1+K_2)^{\theta}}. \tag{2.52}$$

Consider an industry consisting of three firms that differ in capital stock (K_1, K_2 and K_3) and merger costs (M_1, M_2 and M_3). Assume that only two firms can merge at a time, but later this combined entity may merge with the third firm. The questions are: What is the optimal order in which firms should merge? How does it change with changes in initial capital allocation? How does market valuation influence merger process in the industry?

Without loss of generality, assume that in the first step firm 1 merges with firm 2 creating firm 12; in the second step, the merged firm 12 merges with firm 3.

Solving backwards, one needs first to determine the terms and timing of the merger between firm 12 and firm 3; using the formulas above yields:

$$p_2 = \left(\frac{\beta_2}{\beta_2-\gamma} \frac{M_1+M_2+M_3}{c\left((K_1+K_2+K_3)^{\theta} - (K_1+K_2)^{\theta} - K_3^{\theta} \right)} \right)^{\frac{1}{\gamma}} \tag{2.53}$$

for the optimal timing of merger p_2;

$$\begin{aligned} OM_{12}(p_t) &= \left(cp_2^{\gamma}\left(s_{12}\left(K_1+K_2+K_3\right)^{\theta}-\left(K_1+K_2\right)^{\theta}\right)-\right. \\ &\left.-M_1-M_2\right)\left(\tfrac{p_t}{p_2}\right)^{\beta_2}=Bp_t^{\beta_2} \end{aligned} \tag{2.54}$$

for the option value to firm[7] 12 OM_{12};

$$OM_3(p_t)=\left(cp_2^{\gamma}\left(\left(1-s_{12}\right)\left(K_1+K_2+K_3\right)^{\theta}-K_3^{\theta}\right)-M_3\right)\left(\frac{p_t}{p_2}\right)^{\beta_2} \tag{2.55}$$

for the option value to firm 3 OM_3;

$$s_{12}=\frac{\left(M_1+M_2\right)\left(\left(K_1+K_2+K_3\right)^{\theta}-K_3^{\theta}\right)+M_3\left(K_1+K_2\right)^{\theta}}{\left(M_1+M_2+M_3\right)\left(K_1+K_2+K_3\right)^{\theta}} \tag{2.56}$$

for the share of firm 12 in the new entity s_{12}.

Now we are back to the first stage: firm 1 is merging with firm 2 to create a new firm 12. The benefit from merging is twofold: first, participating firms share synergy stemming directly from the merger; second, they acquire the opportunity to merge with the firm 3 later to get even more benefits from this new merger.

The option to merge of firm 1 OM_1 reflects this twofold benefit and satisfies the following value matching and smooth pasting conditions:

$$\begin{aligned} OM_1(p_1) &= cp_1^{\gamma}\left(s_1\left(K_1+K_2\right)^{\theta}-K_1^{\theta}\right)-M_1+s_1OM_{12}= \\ &= cp_1^{\gamma}\left(s_1\left(K_1+K_2\right)^{\theta}-K_1^{\theta}\right)-M_1+s_1Bp_1^{\beta_2} \end{aligned} \tag{2.57}$$

$$OM_1'(p_1)=c\gamma p_1^{\gamma-1}\left(s_1\left(K_1+K_2\right)^{\theta}-K_1^{\theta}\right)+s_1B\beta_2p_1^{\beta_2-1} \tag{2.58}$$

where p_1 is the merger threshold for the merger between firm 1 and firm 2;
s_1 is the share of the new merged entity accruing to firm 1.

Applying the same logic as in Section 2.2.1 yields that the option to firm 1 is of the form $OM_1=Ap_t^{\beta_2}$. Writing down corresponding conditions for firm 2 and solving for OM_1 and for

[7] $B=\frac{cp_2^{\gamma}\left(s_{12}(K_1+K_2+K_3)^{\theta}-(K_1+K_2)^{\theta}\right)-M_1-M_2}{p_2^{\beta_2}}$

OM_2 yields:

$$OM_1(p_t) = \left(cp_1^{\gamma}\left(s_1\left(K_1+K_2\right)^{\theta}-K_1^{\theta}\right)-M_1+s_1Bp_1^{\beta_2}\right)\left(\frac{p_t}{p_1}\right)^{\beta_2} \tag{2.59}$$

$$\begin{aligned} OM_2(p_t) &= \left(cp_1^{\gamma}\left(\left(1-s_1\right)\left(K_1+K_2\right)^{\theta}-K_2^{\theta}\right)-M_2+ \right. \\ &\left. +\left(1-s_1\right)Bp_1^{\beta_2}\right)\left(\tfrac{p_t}{p_1}\right)^{\beta_2} \end{aligned} \tag{2.60}$$

where the merger threshold p_1 equals:

$$p_1 = \left(\frac{\beta_2}{\beta_2-\gamma}\frac{M_1+M_2}{c\left(\left(K_1+K_2\right)^{\theta}-K_1^{\theta}-K_2^{\theta}\right)}\right)^{\frac{1}{\gamma}} \tag{2.61}$$

and the share of the new firm 12 accruing to firm 1 is:

$$s_1 = \frac{M_1\left(\left(K_1+K_2\right)^{\theta}-K_2^{\theta}\right)+M_2K_1^{\theta}}{\left(M_1+M_2\right)\left(K_1+K_2\right)^{\theta}}. \tag{2.62}$$

It is important to notice that neither merger threshold p_1, nor share of firm 1 s_1 change as compared to the baseline model without an option to merge with firm 3 later (compare p_1 to p^* in (2.51) and s_1 to s_1 in (2.52)). This means that the extension of the original model with an extra option does not drive away the equilibrium from being Pareto-optimal.

The strategy of the players now can be summarized as follows:

1. $p_1 < p_2$ means that both mergers occur at optimal thresholds:
   - Firm 1 merges with firm 2 at p_1 to establish a new firm 12;
   - Firm 12 merges with firm 3 at p_2.
2. $p_1 \geq p_2$ means that one of the mergers happens at sub-optimal threshold[8]:
   - Firm 1 merges with firm 2 at p_1 to establish a new firm 12;
   - Firm 12 merges with firm 3 at the same (sub-optimal in this stage) threshold p_1. Option value of this merger is computed based on (2.54) and (2.55) using p_1 rather than p_2 as it would be at the optimal threshold.

[8] I assume here that this is the merger in the second stage that occurs at sub-optimal threshold, but it can also be vice versa.

Option values to the firms are OM_1, OM_2 and OM_3 as in (2.59), (2.60) and (2.55) respectively.

Table 2.8 presents numerical examples for different initial capital allocations in the industry: equal firms with $K_1 = K_2 = K_3 = 33$ (Panel A, least concentrated industry), firms of comparable size with $K_1 = 20$, $K_2 = 30$, $K_3 = 50$ (Panel B), one large firm with $K_3 = 80$ and two small firms with $K_1 = K_2 = 10$ (Panel C, most concentrated industry).

The main conclusion drawn from Table 2.8 is that stock mergers in more concentrated industries (Panel C) occur at higher market valuation (i.e. later) as compared to mergers in less concentrated industries (Panel A). Table 2.8 also demonstrates that total value of options to merge OM is highest in Panel A and decreasing in industry concentration. Analysis in this Section should be extended to the industries with larger number of firms to obtain clearer picture.

2.5 Conclusion

In this paper I have compared the terms and timing of cash vs. stock mergers for two different settings: in the first one by Lambrecht (2004), the synergy comes from increasing returns to scale and stochastic shock is the same for both bidder and target; the second one, with the synergy linear in pre-merger valuations of the firms, encompasses correlated stochastic processes for the firms and is based on Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008).

I have demonstrated that cash mergers should generally happen at low market valuation, and stock mergers may happen at both low and high market valuations; this conclusion conforms to existing empirical evidence. It partially supports prediction made by Shleifer and Vishny (2003) for the static model.

I have investigated the dynamics of the intra-industry mergers within the first setup. I solved for the optimal order of mergers inside an industry for different initial capital allocations to demonstrate that stock mergers in more concentrated industries occur at higher market valuation (i.e. later) as compared to mergers in less concentrated industries.

Table 2.8: Intra-industry mergers

p_1	p_2	Firm 1	Firm 2	Firm 3	Total	Stage 1	Stage 2
Panel A (HHI=3333)							
		K_1	K_2	K_3	K		
		33	33	33	99		
		OM_1	OM_2	OM_3	OM		
1.928	1.947	17.131	17.131	8.500	42.763	1+2	12+3
1.928	1.947	17.131	8.500	17.131	42.763	1+3	13+2
1.928	1.947	8.500	17.131	17.131	42.763	2+3	23+1
Panel B (HHI=3800)							
		K_1	K_2	K_3	K		
		20	30	50	100		
		OM_1	OM_2	OM_3	OM		
2.053	1.817	10.229	15.945	14.280	40.454	1+2	12+3
2.150	2.009	8.805	7.370	23.935	40.109	1+3	13+2
1.947	2.356	3.872	13.219	22.887	39.978	2+3	23+1
Panel C (HHI=6600)							
		K_1	K_2	K_3	K		
		10	10	80	100		
		OM_1	OM_2	OM_3	OM		
2.286	2.356	3.961	3.961	15.487	23.409	1+2	12+3
3.186	3.319	2.046	1.158	19.450	22.654	1+3	13+2
3.186	3.319	1.158	2.046	19.450	22.654	2+3	23+1

HHI denotes Herfindahl-Hirschman index: $HHI = 10000\frac{K_1^2+K_2^2+K_3^2}{K^2}$.
K is total capital in the industry.
OM is the total value of options to merge.
Parameters are set as follows: merger costs $M_i = 0.05K_i$, $c = 1$, $\gamma = 1.4$, $\theta = 1.2$, $\mu = 0.03$, $\sigma = 0.2$, $r = 0.06$ implying $\beta_2 = 1.5$ so that the condition $\gamma < \beta_2$ holds, $p_0 = 1$.
Value $1 + 2$ in the column Stage 1 means that in the first stage firm 1 merges with firm 2 to create a new firm 12; analogously, value $12 + 3$ in the column Stage 2 means that in the second stage firm 12 merges with firm 3.

Chapter 3

Menu costs in international trade: a dynamic story

3.1 Introduction

The concept of 'menu costs' is the important element of the New Keynesian economics paradigm. It refers to the costs the firms have to incur to change their prices. The seminal study by Mankiw (1985) investigated the behavior of monopolies in the presence of menu costs and the reasons for price stickiness[1].

Clearly, menu costs are also important for international trade since they induce the exporters to adjust their export prices not as often as they would optimally do in in the absence of menu costs.

Imagine an exporter facing the appreciation of the home currency and, consequently, decrease in profits. The easiest way to keep profits stable is to adjust prices constantly (then the exchange rate pass-through equals one), but this would give rise to menu costs and may result in worsened relationships with customers. The other extreme is not to adjust prices at all (then the exchange rate pass-through equals zero) and to exercise the option to exit as soon as the exchange rate jumps out of the certain corridor as in Dixit (1989).

I would like to investigate an intermediate case: an exporter has a number of opportunities to adjust prices paying the sunk menu costs each time; in this case, an average exchange rate pass-through lies between zero and one. The main goal of the study is to solve for the optimal timing of export price adjustments and estimate respective option values.

This paper aims to investigate the influence of menu costs on the behavior of export prices in the dynamic, real options setting. The related real options literature that studies the influence of exchange rates movements on such strategic decisions of multinational firms as extry/exit and market segmentation provides necessary background and benchmarks.

Thus, Friberg (2001) studies the implications of the exchange rate volatility for market segmentation decision by an exporting firm. Exporter operates on two markets; it can either charge the same price everywhere (so that the 'law of one price' would hold), or segment markets at sunk cost. Friberg (2001) argues that the value of the option to segment increases in the exchange rate volatility. His study is related to Dixit (1989) that investigates the entry and exit decisions by exporters and demonstrates that there exists a band of exchange rates where no entry-exit happens;

[1] In his blog `http://gregmankiw.blogspot.com/2007/04/menu-costs.html`, Pr Mankiw explains the origins of the term 'menu costs' as follows:

> I did not make up the name. The earliest reference I can find to the term in J-stor journals is in 'Relative Shocks, Relative Price Variability, and Inflation' by Stanley Fischer, published in 1981, the exact year I was a first-year grad student at MIT. I undoubtedly picked up the name in Stan's course.

the exchange rate pass-through is close to zero within this band and close to one otherwise.

Broll and Eckwert (1999) develops a model where a risk-averse firm supplies either to the home market, or to the foreign market (depending on the realization of the exchange rate at the time when decision is made); markets are segmented, and exchange rate pass-through is set to zero. Broll and Eckwert (1999) demonstrate that export volume increases in exchange rate volatility for moderate degrees of risk aversion.

Botteron, Chesney, and Gibson-Asner (2003) rely on real options framework to analyze the decision of the firm to become partially or fully multinational; they derive a corridor of exchange rates inside which the firm remains fully local, and outside of which it delocalizes its production and/or sales. They also introduce competition to show how it may trigger a preemptive delocalization. Though the present paper deals with a different type of strategic decision of the firm (namely, export price renegotiation), the general approach is similar to the one in Botteron, Chesney, and Gibson-Asner (2003).

In this paper I solve for the timing of optimal export price adjustments for both infinite and finite horizon case. The former relies on traditional real options techniques providing closed form solutions for the export price adjustment thresholds and option values as, for example, in Dixit and Pindyck (1994) or Botteron, Chesney, and Gibson-Asner (2003), whereas the finite horizon problem employs binomial method for option pricing first proposed by Cox, Ross, and Rubinstein (1979). For the finite horizon case I also derive the optimal exercise boundary of the first price adjustment and explain how to obtain the respective boundaries for the second and further price adjustments. In both cases I demonstrate how menu costs make export prices sticky.

According to Broll and Eckwert (1999), exchange rate movements do not have any systematic effect on the prices in the country of destination; they also argue that the degree of the exchange rate pass-through differs very much across countries and industries and is normally much less than 100% according to the empirical literature. This observation together with the fact that export prices should be sticky (because of menu costs) provided another research question to be dealt with in this paper: how volatile are export prices relative to the exchange rates? I show that aggregate export prices are approximately four to five times less volatile than the exchange rates; I also show how volatility changes with the correlations between the pairs of exchange rates in case of multiple exporters.

Thus, this paper's contribution is twofold. First, it provides a simple yet powerful partial equilibrium model of international trade in the presence of menu costs; it demonstrates that even relatively small menu costs may cause export price stickiness. Second, this paper contributes to the literature on real options developing a model of sequential options for both infinite and finite horizon.

The paper is organized as follows: Section 3.2 gives the overview of the model, Section 3.3 investigates the properties of the model under the assumption of the infinite horizon, Section 3.4 provides the numerical solution for the finite horizon, Section 3.5 presents the results of numerical simulation, Section 3.6 concludes.

3.2 Model description

Consider a firm in the Home country that produces both for domestic consumption and for exports to the Foreign country. Constant risk-free rates of interest are δ in the Home country and r in the Foreign country; following the literature, I assume that the dynamics of the exchange rate process[2] $X(t)$ under the risk-neutral probability measure is described by a geometric Brownian motion:

$$dX_t = (\delta - r) X_t dt + \sigma X_t dW_t, \tag{3.1}$$

where W_t is a standard Browninan motion and σ^2 is the constant volatility of the exchange rate.

Consider a very simple setup:

- Home and Foreign markets are uncorrelated to ensure that exchange rate movements do not affect profits in the Home market;
- firm exports one unit of the product per unit time and Foreign demand is inelastic (the firm is really small as compared to the size of the Foreign market);
- optimal price as measured in Home currency is constant and equals P_H;
- initial export price as measured in Foreign currency is set to $P_0 = \frac{P_H}{X_0}$ with X_0 being the value of the exchange rate at time zero;
- menu costs associated with price renegotiation are constant and equal to M (measured in Home currency);
- production costs and profits in the Home market are normalized to zero to simplify the exposition;

[2]The exchange rate is the price of the unit of the Foreign currency measured in the Home currency.

- the instantaneous profit flow of the firm is given by:

$$\pi\left(X_t,\, P_t\right) = P_t X_t, \tag{3.2}$$

where P_t is the export price at time t (initially set to P_0 as above);

- the the value of the firm measured as the sum of future discounted profits is:

$$V\left(X_t,\, P_t\right) = \mathbb{E}_Q\left[\int_t^{\infty} P_t X_s e^{-\delta s} ds\right] = \frac{P_t X_t}{\delta - (\delta - r)} = \frac{P_t X_t}{r}, \tag{3.3}$$

where $\mathbb{E}_Q$ is the expectation under the risk-neutral measure Q;
$V_0 = V\left(X_0,\, P_0\right) = \frac{P_H}{r}$.

It is clear that an appreciation in the Home currency results in the foreign sales decrease, and, consequently, in the decrease in the value of the firm V. To compensate for the exchange rate appreciation at some time τ, the exporter needs to renegotiate the export price and to increase it to the new level $P_\tau = \frac{P_H}{X_\tau} = \frac{P_0 X_0}{X_\tau}$. If this renegotiation were not costly, it would be optimal for the exporter to continuously adjust its export price to compensate for the exchange rate appreciation. Unfortunately, one has to pay menu costs to adjust prices, therefore, prices are generally sticky and are adjusted in a discrete way.

Thus, the exporter waits until the exchange rate process hits from above some endogenous threshold X_τ to pay fixed menu costs M and to switch to the new, optimal level of export price P_τ. What happens next? As time passes and exchange rates continues to appreciate, exporter faces exactly the same problem as before: export profits are decreasing. The remedy is the same: to wait until X_t goes down to hit another threshold X_{τ_2}, to pay menu costs M and to adjust the export price to the level $P_{\tau_2} = \frac{P_H}{X_{\tau_2}}$. The adjustment procedure must be repeated as often as needed.

Then the optimal dynamic price adjustment strategy of the exporter can be summarized as follows:

1. Start at X_0 with export price equal to $P_0 = \frac{P_H}{X_0}$;
2. As X_t first hits the threshold X_1 from above, pay M to switch from P_0 to $P_1 = \frac{P_0 X_0}{X_1} = \frac{P_H}{X_1}$ (Naturally, P_1 will be higher than P_0.);
3. As X_t first hits the next threshold X_2 from above, pay M to switch from P_1 to $P_2 = \frac{P_H}{X_2}$;
4. Repeat previous step switching sequentially from X_2 to X_3, then from X_3 to X_4 and so on.

It is clear that menu costs M is the crucial element that is responsible for discreteness of export price changes and for the frequency of export price adjustments: increasing menu costs not only diminish the option value accruing to the exporter, but also result in less frequent price adjustment; in particular, first adjustment occurs later.

One can restate this problem in terms of options: the exporter has a put-type real option O_N that allows for a number of export price adjustments under the condition of paying fixed menu costs M at each adjustment. Under the assumption of the infinite horizon, the number of price adjustments is infinite (this problem is discussed in Section 3.3), whereas under the assumption of the finite horizon the number of price adjustments is finite (this problem is presented in Section 3.4).

3.3 Options with infinite horizon

This section uses the well-known techniques developed in the literature on real options; for the main reference source, one can refer to Dixit and Pindyck (1994).

First I solve for the value O and exercise threshold X^* of a perpetual option that allows for one price adjustment from P_0 to P^*; this solution will serve us as a benchmark. The payoff to this option equals:

$$PF^*(X_t) = \max\left(V_0 - \frac{X_t P_0}{r} - M,\, 0\right) = \max\left(V_0 - V_0 \frac{X_t}{X_0} - M,\, 0\right) \tag{3.4}$$

as $P_0 = \frac{P_H}{X_0}$ and $V_0 = \frac{P_H}{r}$.

In the continuation region (for $X_t > X^*$) this option satisfies the following ODE:

$$\delta O = (\delta - r) X O' + \frac{\sigma^2}{2} X^2 O'' \tag{3.5}$$

with the general solution being:

$$O = B_1 X^{\beta_1} + B_2 X^{\beta_2}, \tag{3.6}$$

where β_1 and β_2 are negative and positive roots of the characteristic equation $\frac{1}{2}\sigma^2\beta(\beta - 1) + (\delta - r)\beta - \delta = 0$ and $B_2 = 0$ since $\lim_{X \to \infty} O = 0$.

At the exercise threshold X^* the option O should satisfy the following value-matching and smooth-pasting conditions:

$$O\left(X^*\right) = PF^*\left(X^*\right) = V_0 - V_0 \frac{X^*}{X_0} - M \tag{3.7}$$

$$O'\left(X^*\right) = -\frac{V_0}{X_0}. \tag{3.8}$$

Solving for X^* yields a familiar expression for an exercise threshold of a perpetual put option:

$$X^* = \frac{\beta_1}{\beta_1 - 1} X_0 \frac{V_0 - M}{V_0}. \tag{3.9}$$

The option value is then:

$$O\left(X_t\right) = \left(V_0 - M - V_0 \frac{X^*}{X_0}\right) \left(\frac{X_t}{X^*}\right)^{\beta_1} = \frac{V_0 - M}{1 - \beta_1} \left(\frac{\beta_1 - 1}{\beta_1} \frac{V_0}{V_0 - M}\right)^{\beta_1} \left(\frac{X_t}{X_0}\right)^{\beta_1} \tag{3.10}$$

In particular, $O\left(X_0\right)$ which is the option value at time $t = 0$ equals:

$$O\left(X_0\right) = \frac{V_0 - M}{1 - \beta_1} \left(\frac{\beta_1 - 1}{\beta_1} \frac{V_0}{V_0 - M}\right)^{\beta_1}. \tag{3.11}$$

Equations 3.9-3.11 provide a starting point for the analysis. First, the option value $O\left(X_0\right)$ does not depend on the initial level of the exchange rate X_0; this is due to the infinite-horizon nature of the problem in addition to the fact that export price adjustments fully compensate for the exchange rate movements, so that the problem is stationary in terms of the $X_t P_t$.

Second, the coefficient $\frac{\beta_1}{\beta_1 - 1}$ is less than one since $\beta_1 < 0$. It implies that the threshold X^* is smaller than the NPV threshold $X_{NPV} = X_0 \frac{V_0 - M}{V_0}$ that is solved for as the root of the equation $O_1\left(X_{NPV}\right) = 0$. Thus, price adjustment in the real options setting occurs later than it would occur under the traditional NPV rule. It is also possible to compute the 'prohibitive menu costs' level as the solution to the equation $X^*\left(M_p\right) = 0$ to obtain $M_p = V_0$. The results is not striking at all: when the menu costs are equal to the value of the firm itself, it certainly makes no sense to do a price adjustment.

To conclude, one can see that the optimal threshold X^* is a fraction of the initial exchange rate level X_0; it means that the price renegotiation is a reaction to a certain percentage of the appreciation of the Home currency.

With these results in hand, I proceed to solve the general infinite-stage problem. I need to find the value of the option to perform infinite number of price adjustments O^*_{inf} paying a fixed cost

M at each adjustment.

The solution follows the same lines as the one presented above (equations 3.4-3.11). Due to the stationary nature of the problem, it is sufficient to consider the situation between the two price adjustments; initial exchange rate level is X_0 with initial export price being $P_0 = \frac{P_H}{X_0}$. At the moment of price adjustment (when X_t first hits the threshold X_{inf} from above), the exporter benefits from optimizing the export price and from obtaining the option to perform an infinite series of price adjustments again. Thus, he payoff to the option equals:

$$PF_{inf}\left(X_t\right) = \max\left(\frac{P_H}{r} - \frac{X_t P_H}{X_0 r} + O^*_{inf} - M,\, 0\right), \tag{3.12}$$

while the value-matching condition is given by:

$$\begin{aligned} O_{inf}\left(X_{inf}\right) &= PF_{inf}\left(X_{inf}\right) = \frac{P_H}{r} - \frac{X_{inf} P_H}{X_0 r} + O^*_{inf} - M = \\ &= V_0 + O^*_{inf} - M - V_0 \frac{X_{inf}}{X_0}. \end{aligned} \tag{3.13}$$

Therefore, the expressions for the threshold X_{inf} and the option value O_{inf} are as follows:

$$X_{inf} = \frac{\beta_1}{\beta_1 - 1} X_0 \frac{V_0 + O^*_{inf} - M}{V_0} \tag{3.14}$$

$$\begin{aligned} O_{inf}\left(X_t\right) &= \left(V_0 + O^*_{inf} - M - V_0 \frac{X_{inf}}{X_0}\right)\left(\frac{X_t}{X_{inf}}\right)^{\beta_1} = \\ &= \frac{V_0 + O^*_{inf} - M}{1 - \beta_1}\left(\frac{X_t}{X_{inf}}\right)^{\beta_1} = \\ &= \frac{V_0 + O^*_{inf} - M}{1 - \beta_1}\left(\frac{\beta_1 - 1}{\beta_1}\frac{V_0}{V_0 + O^*_{inf} - M}\right)^{\beta_1}\left(\frac{X_t}{X_0}\right)^{\beta_1}. \end{aligned} \tag{3.15}$$

Then $O_{inf}\left(X_0\right)$ which is at the same time equal to O^*_{inf} is as follows:

$$\begin{aligned} O_{inf}\left(X_0\right) &= O^*_{inf} = \frac{V_0 + O^*_{inf} - M}{1 - \beta_1}\left(\frac{\beta_1 - 1}{\beta_1}\frac{V_0}{V_0 + O^*_{inf} - M}\right)^{\beta_1} = \\ &= \frac{1}{1 - \beta_1}\left(\frac{\beta_1 - 1}{\beta_1} V_0\right)^{\beta_1}\left(V_0 + O^*_{inf} - M\right)^{1-\beta_1}. \end{aligned} \tag{3.16}$$

The value of the option to perform infinite number of price adjustments O^*_{inf} can be found from (3.16); generally, the closed-form solution does not exist, therefore I will use the numerical example to demonstrates how the model works.

The values of the input parameters are as follows: $\delta = 0.05$, $r = 0.09$, $\sigma = 0.2$, $P_H = 12$, $X_0 = 1.2$ implying $P_0 = 10$ and $V_0 = 133.33$; menu costs M are set to equal 1% of the initial annual profits P_H so that $M = 0.12$.

Given these values of parameters, there exist two values of O^*_{inf} that satisfy (3.16): $o_1 = 185.5868$ and $o_2 = 207.1656$. To understand which of o_1 and o_2 is relevant to the solution, it is useful to notice that the sequence of options O_n ($n = 1, 2, 3, ...$ is the number of adjustments made under this option) is generated by the iterated function in the following way: $O_1 = f(0)$, $O_2 = f(O_1)$, $O_3 = f(f(O_1))$ etc. where $f(x)$ is the following function:

$$f(x) = \frac{1}{1-\beta_1}\left(\frac{\beta_1 - 1}{\beta_1}V_0\right)^{\beta_1}(V_0 - M + x)^{1-\beta_1}. \tag{3.17}$$

Then it follows that o_1 and o_2 are the fixed points of the sequence of options, with o_1 being an attractive fixed point (for any value of x around this fixed point the sequence converges to o_1) and o_2 not being an attractive fixed point. Therefore, I conclude that $o_1 = 185.5868$ is the solution to the option problem.

Thus, the value of the option to perform infinite number of price adjustments is $O^*_{inf} = 185.5868$ and the price adjustment threshold equals $X_{inf} = \frac{\beta_1}{\beta_1 - 1}X_0\frac{V_0 + O^*_{inf} - M}{V_0} = 0.9673X_0 = 1.1608$.

Figure 3.1 illustrates the behavior of the option sequence O_n for $1 \leq n \leq 600$. It shows that the sequence converges to $O^*_{inf} = 185.5868$ quite fast; the option with as many as 100 price adjustments captures most of the O^*_{inf} option value.

Summarizing, the option value at $t = 0$ and at the time of any price adjustment equals $O^*_{inf} = 185.5868$, whereas the option value in between the price adjustments is given by:

$$O_{inf}(X_t) = \frac{V_0 + O^*_{inf} - M}{1-\beta_1}\left(\frac{\beta_1 - 1}{\beta_1}\frac{V_0}{V_0 + O^*_{inf} - M}\right)^{\beta_1}\left(\frac{X_t}{X_0}\right)^{\beta_1} = 185.5868\left(\frac{X_t}{X_0}\right)^{\beta_1}, \tag{3.18}$$

where X_0 is either the starting value or the exchange rate threshold of the previous price adjustment.

The price adjustment thresholds are computed as follows: $X_n = 0.9673^n X_0$ with $X_1 = X_{inf}$ as in the computations above.

The remaining question is: how much time does it take to reach the threshold? The expected

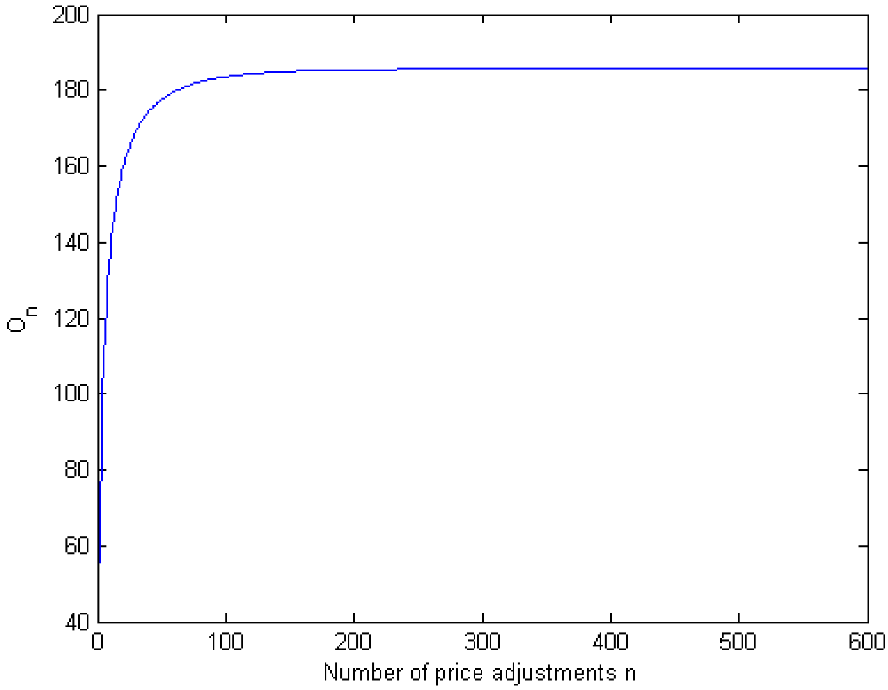


Figure 3.1: Option values for different number of price adjustments

first passage time to hit the barrier X_{inf} starting from X_0 for $\delta - r - \frac{\sigma^2}{2} < 0$ is given by[3]:

$$t_{inf} = \frac{\ln\left(\frac{X_0}{X_{inf}}\right)}{|\alpha|} = 0.5538 \tag{3.19}$$

where $\alpha = \delta - r - \frac{\sigma^2}{2}$ and time is measured in years.

Thus, export price adjustment occurs on average once in 6.6 months as the Foreign currency depreciates by $(1 - 0.9673)100 = 3.27\%$ as compared to the moment of the previous price adjustment.

The infinite horizon model developed in this section outlines the structure of the solution, but it has one main drawback: it hinges on the quite unrealistic assumption of the infinite horizon; therefor, in the next section I ameliorate the model introducing a finite expiration of the option to perform N price adjustments.

[3]See Ingersoll (1987), p. 353-354 for reference.

3.4 Options with finite horizon

To solve for the value of the option that allows to perform N price adjustments over some finite horizon T, I am using the traditional binomial method for the American put option valuation (first proposed by Cox, Ross, and Rubinstein (1979)). I have preferred it to the finite difference method originally proposed by Schwartz (1977) or to the quadratic approximation method by Barone-Adesi and Whaley (1987) due its extreme simplicity and ready availability of the exercise boundary.

I set the horizon to be $T = 3$ years as it seems reasonable that the drift and volatility of the exchange rate process will not dramatically change over this horizon. It should also be emphasized that T is the maturity of the compounded option to perform the whole series of N price adjustments.

It is natural to ask oneself: what is the optimal number N of switches? Strictly speaking, the number of switches is infinite since an option with another option embedded inside has more value than the one without it; that is, the sequence of options O_N $(1 \leq N)$ should be monotonically increasing in N until the fixed point is reached. The problem with the finite horizon model is the computational time needed to compute the value of the option with other options embedded; thus, one has to trade off between the quality of the solution and the price one needs to pay to obtain this solution. In this section I limit myself to the option to perform 5 export price adjustments; as I will demonstrate later, the results are quite satisfactory.

The logic of the solution is the same as in Section 3.3 for the infinite horizon: first I am computing the value of the option to perform exactly one price adjustment F_1. The payoff to this option is given by (3.4) that I repeat here:

$$PF^* \left(X_t\right) = \max\left(V_0 - V_0 \frac{X_t}{X_0} - M,\, 0\right), \tag{3.20}$$

where X^* is the exercise boundary. The payoff at the boundary is as follows:

$$PF^* \left(X^*\right) = \max\left(V_0 - X^* \frac{V_0}{X_0} - M,\, 0\right). \tag{3.21}$$

Due to the specific form of the payoff different from the usual option payoff $\max\left(K - X,\, 0\right)$, initial value for the option calculation is equal to $X_0 \frac{V_0}{X_0} = V_0$, and the strike price is $V_0 - M$. Thus, the value of the option to perform one price adjustment is the function of the initial price V_0, strike price $V_0 - K$, risk-free rates δ and r and the horizon $T = 3$; the total number of time-steps n is set to $n = \max\left(50T,\, 50\right)$.

Table 3.1: Option values under finite horizon of 3 years

F_1	F_2	F_3	F_4	F_5
22.2191	29.4739	33.2950	35.6949	37.3426
	$100\frac{F_2-F_1}{F_1}$	$100\frac{F_3-F_2}{F_2}$	$100\frac{F_4-F_3}{F_3}$	$100\frac{F_5-F_4}{F_4}$
	32.65	12.96	7.21	4.62

The values of the input parameters are as follows: $\delta = 0.05$, $r = 0.09$, $\sigma = 0.2$, $P_H = 12$, $X_0 = 1.2$ implying $P_0 = 10$ and $V_0 = 133.33$; menu costs M are set to equal 1% of the initial annual profits (P_H): $M = 0.12$. F_i denotes the value of the option to perform i price adjustments for $1 \leq i \leq 5$.

Then, given the same set of parameters[4] as in Section 3.3, the value of the option to perform one price adjustments is $F_1 = 22.2191$ over a horizon of $T = 3$ years.

To compute the value of the option to perform exactly two price adjustments over the three year horizon, one needs to do the following: at each time step starting from the pen-ultimate one $T - j\frac{T}{n}$ $(1 \leq j \leq \frac{n}{T})$ increase the strike price by the option value F_1 computed as above but on the horizon of $T - \left(T - j\frac{T}{n}\right) = j\frac{T}{n}$ years. It means, in fact, that at each node of the binomial tree another binomial tree for the option value $F_1\left(j\frac{T}{n}\right)$ is constructed; naturally, it requires more time that a simple F_1 computation. The option value $F_2 = 29.4739$.

The value of the option to perform three price adjustments F_3 follows the same lines: at each time step $T - j\frac{T}{n}$ (except for those at time T), increase the strike price by the value of the option to perform two price adjustments F_2 on the horizon $j\frac{T}{n}$. The value of the option $F_3 = 33.2950$.

Table 3.1 presents the results of computations up to the option value F_5. It shows that F_N is an increasing concave function of N repeating the pattern for the infinite horizon model (see Figure 3.1 for comparison).

The exercise boundary of the first price adjustment for F_1 and F_2 is presented in Figure 3.2. These graphs are based on 1000 time steps for F_1 and 1000 time steps for both the the main computation and the ‘embedded’ option for F_2.

Figure 3.2 demonstrates that in case of the option allowing for two price adjustments F_2, exercise boundary of the first adjustments is reached earlier than in case of F_1; it means that if the firm has only one opportunity to adjust its export price then it waits longer.

The exercise boundary of the second price adjustment in case of the option F_2 can be drawn only after the coordinates of the first price adjustment - exchange rate X_t and hitting time t - are

[4] The values of the input parameters are as follows: $\delta = 0.05$, $r = 0.09$, $\sigma = 0.2$, $P_H = 12$, $X_0 = 1.2$ implying $P_0 = 10$ and $V_0 = 133.33$; menu costs M are set to equal 1% of the expected initial annual profits (P_H): $M = 0.12$.

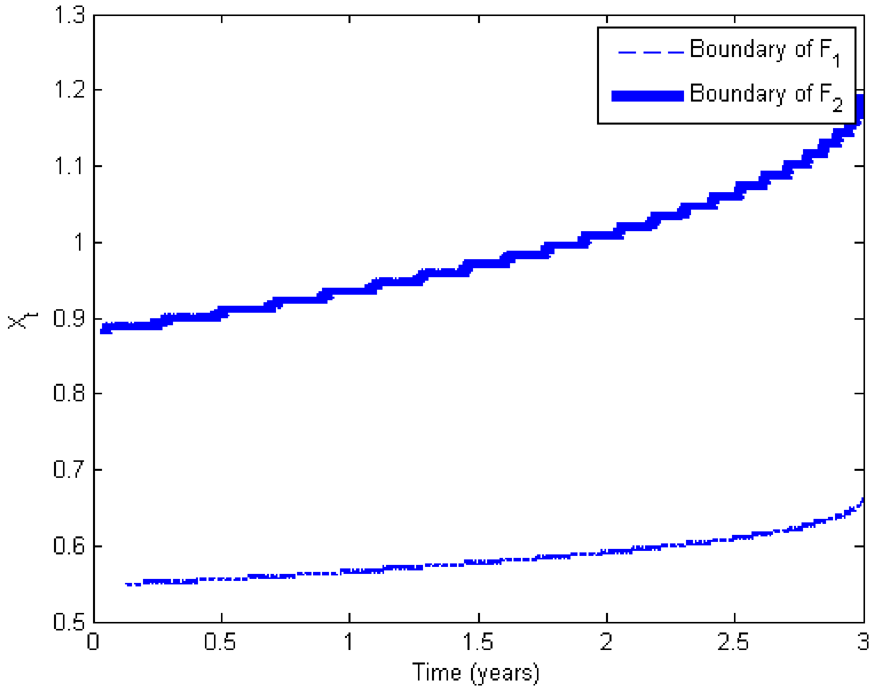


Figure 3.2: Exercise boundary of the first price adjustment for F_1 and F_2

determined, because this boundary is different for each combination of X_t and t.

The same is true for the option to perform three price adjustments F_3: first the exercise boundary for the first price adjustment should be drawn; then, after the position of the first hit is determined, one can construct the exercise boundary of the second price adjustment. Last, after the position of the second hit is identified, the exercise boundary of the third price adjustment can be drawn.

As Table 3.1 demonstrates, the increase in option value from F_4 to F_5 is less than 5%, whereas the computational time needed for the option F_5 is around 5 hours. That is why I decide to limit the number of price adjustments to 5.

To summarize the findings of Sections 3.3 and 3.4, I have demonstrated that even small menu costs (equal to 1% of annual profits in this model) of export price adjustment may result in significant export price stickiness; even under the assumption of infinite horizon of the option to renegotiate the prices (see Section 3.3), the price adjustment occurs on average once in 6.6 months.

This could help explain why the exchange rate movements do not significantly affect the aggregate price level; in the next section I am demonstrating this mechanism.

3.5 Simulations

In this section I simulate the behavior of three exporters in the Foreign market under the assumption of correlated exchange rates. I use the infinite-horizon setup from Section 3.3 because of the availability of the closed-form solution. The values of the parameters are set as follows: risk-free rates for the three exporting countries are $r_1 = 0.04$, $r_2 = 0.07$ and $r_3 = 0.09$ with the risk-free rate in the Foreign country being $r = 0.09$ as in Sections 3.3 and 3.4, volatilities of the increments of the exchange rate processes are as follows: $\sigma_1 = 0.25$, $\sigma_2 = 0.2$ and $\sigma_3 = 0.3$, correlation between the first and the second exchange rate is $\rho_{12} = 0.3$, correlation between the first and the third exchange rate is $\rho_{13} = -0.4$. The starting value of the exchange rate process and the optimal price in the Home market are identical for all the exporting countries and equal $X_0^1 = X_0^2 = X_0^3 = X_0 = 1.2$ and $P_H^1 = P_H^2 = P_H^3 = P_H = 12$ implying the initial export price of $P_0^1 = P_0^2 = P_0^3 = 10$ as in Sections 3.3 and 3.4. Menu cost are set to 1% of P_H as before. I assume that market shares of the exporters are equal; thus, the average price $\bar{P}_t$ in the Foreign market is computed as the simple average of exporters' prices with the initial price being $\bar{P}_0 = 10$

Assuming as much similarity between the exporters as possible allows us to concentrate on the effects of exchange rates movements on the export price in the Foreign market; in order to do so I compute the exchange rate thresholds for each of the exporters using the algorithm from Section 3.3 to obtain the following expressions for the exchange rate thresholds:

$$\begin{aligned} X_n^1 &= 0.9646^n X_0 \\ X_n^2 &= 0.9709^n X_0 \quad , \\ X_n^3 &= 0.9700^n X_0 \end{aligned} \tag{3.22}$$

where X_n^1, X_n^2 and X_n^3 are the exchange rate thresholds for the n^{th} export price adjustment for the first, second and third exporters respectively.

Next I simulate $N = 10000$ realizations of the exchange rates using 50 exercise points per year; then I obtain the time series of the export price for each exporter separately according to the rule: as soon as the first threshold is reached (or, to be precise, overshoot because of the discrete nature of the simulations), adjust the export price, then wait for the next threshold to be reached to adjust the price again and so on. Having obtained the time series of export prices for each of the exporters and for all random paths, I compute the time series of the average export price $\bar{P}_t$ as the simple average of the exporters' prices.

Figure 3.3 presents the dynamics of the exchange rates together with the export prices for three randomly chosen paths. Though I use the infinite-horizon solution based on Section 3.3, I restrict the time period over which the behavior of exporters is observed to three years; thus, I assume that the exporters continue their activity after three-year horizon is reached. Figure 3.3 demonstrates that the average export price is less volatile than the export prices of individual exporters, and that all the export prices are less volatile than the corresponding exchange rates for each of the three random paths conforming to the empirical evidence. It also shows that average export price stays constant for several months (in particular, the average period between the price adjustments equals to 1.54 months in the whole sample).

Next I estimate annualized volatility of the log returns $\hat{\sigma}_{\bar{P}}$ for the average export price series $\bar{P}_t$ to find that $\hat{\sigma}_{\bar{P}} = 0.0516$. Comparing this value with the volatilities of the increments of the exchange rate processes $\sigma_1 = 0.25$, $\sigma_2 = 0.2$ and $\sigma_3 = 0.3$, one clearly sees that average export price is much less volatile than the exchange rates in this market.

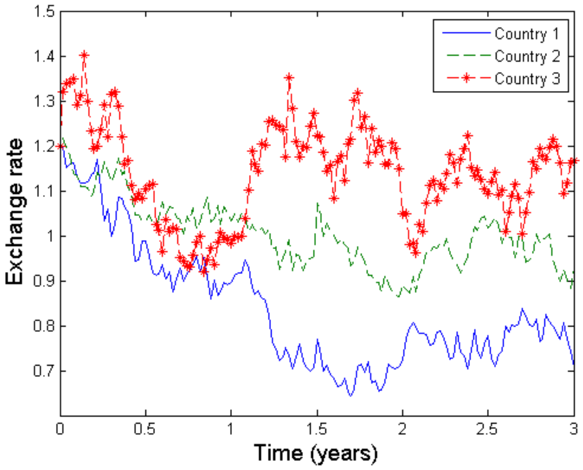
Figure 3.4 exhibits $\hat{\sigma}_{\bar{P}}$ as a function of the correlations ρ_{12} and ρ_{13}. It demonstrates that $\hat{\sigma}_{\bar{P}}$ is generally lower when both correlations are negative - the effect that should be attributed to diversification of imports (these are imports from the point of view of the Foreign country); besides, $\hat{\sigma}_{\bar{P}}$ stays below the level of 10% which is about two times lower than the lowest of σ's which is $\sigma_2 = 0.2$ in this case.

To summarize, the present model generates sticky export prices (especially over relatively short periods of time) and incomplete pass-through that conforms quite well to the existing empirical evidence.

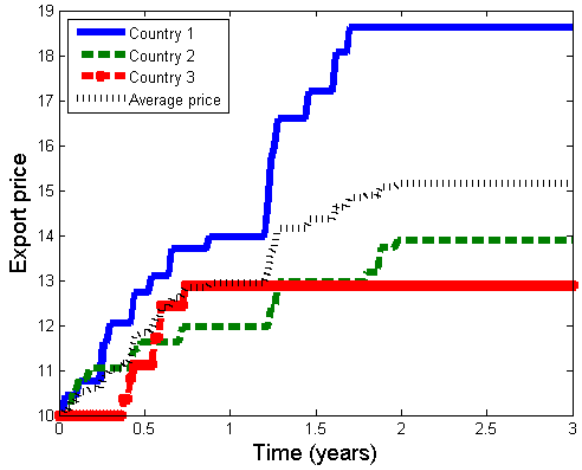
3.6 Conclusion

In this paper I have investigated the behavior of export prices under the assumptions of menu costs of price adjustment and stochastic exchange rates. I have solved for the timing of optimal export price adjustments for both infinite and finite horizon case.

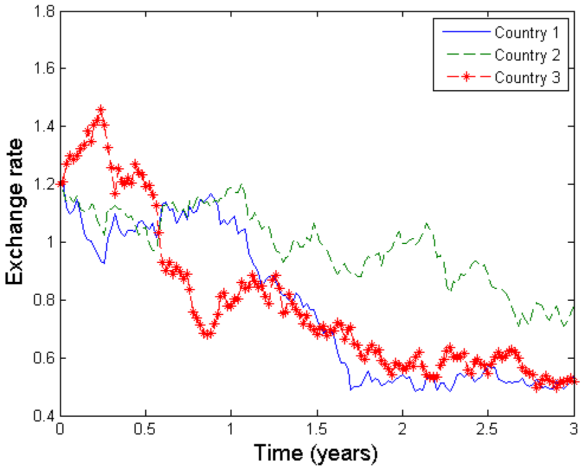
In the former case I have provided closed form solutions for the export price adjustment thresholds and option values as, for example, in Dixit and Pindyck (1994), whereas in the latter case I have used the binomial method for option pricing first proposed by Cox, Ross, and Rubinstein (1979). For the finite horizon case I have also derived the optimal exercise boundary of the first price adjustment and explained how to obtain the respective boundaries for the second and further price adjustments. In both cases I demonstrated that even small menu costs could make export prices sticky; I have also provided the numerical simulations results in order to show how the present model could generate results conforming to the empirical evidence on incomplete (and



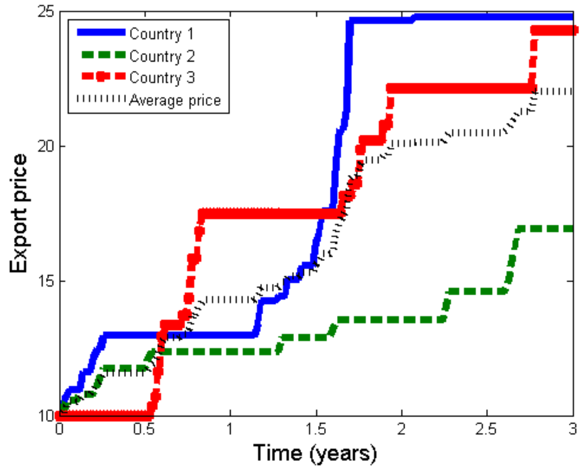
(a) Exchange rates for three exporting countries (path 1)



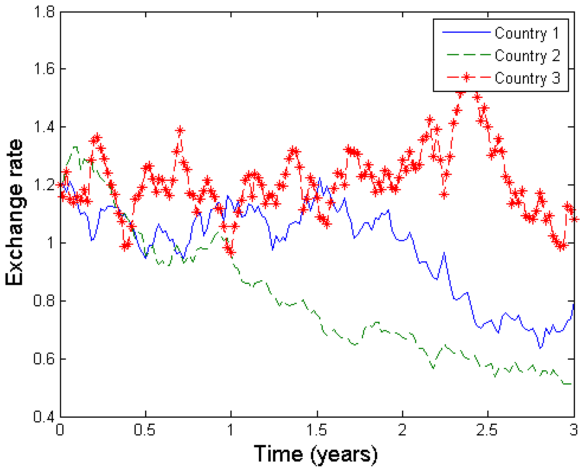
(b) Average price in the foreign market (path 1)



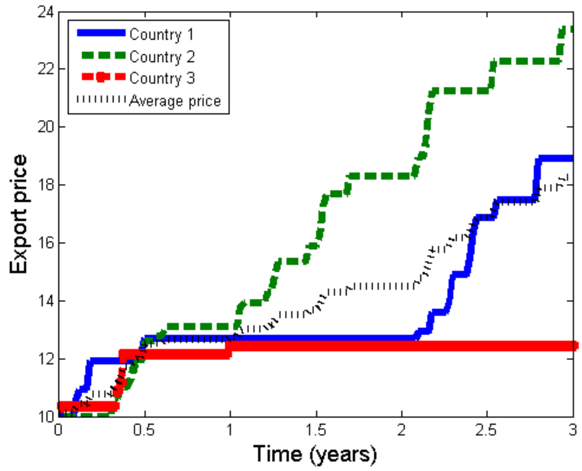
(c) Exchange rates for three exporting countries (path 2)



(d) Average price in the foreign market (path 2)



(e) Exchange rates for three exporting countries (path 3)



(f) Average price in the foreign market (path 3)

Figure 3.3: Exchange rates and export price dynamics over the three-year period for 3 randomly chosen paths

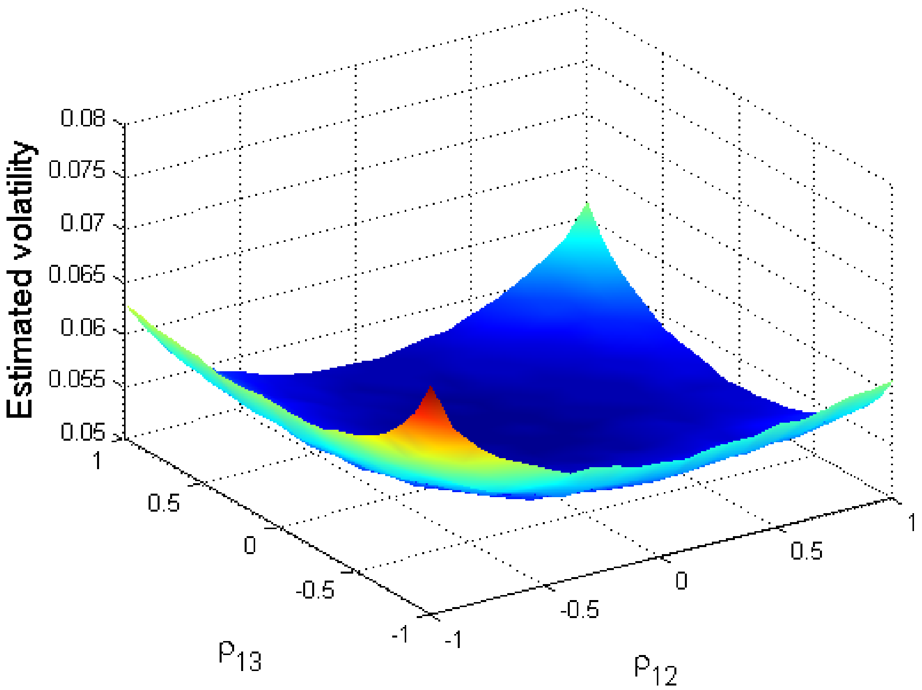


Figure 3.4: Estimated volatility of log returns for the average export price

different over the countries and industries) exchange rate pass-through.

The model can certainly be extended to incorporate the downward sloping demand curve for the exported product (instead of the 'small firm' assumption and inelastic demand that are being used now).

Besides, since the exporters normally sell on their home markets too, the model may also take into account the sales in the home market that can be correlated with the export market. It could then be interesting to explore the effect of the correlation between the Home and the Foreign market on the price adjustment decision by the exporter, and, in particular, to investigate whether there exists any diversification effect in case of negative correlation.

Appendix A

Derivations for Chapter 1

A.1 Derivations for Section 1.2.1

The Incumbent decides on (p_M, q_M) subject to the constraint:

$$V_M = \theta_M q_M - p_M \geq 0. \tag{A.1}$$

Setting $V_M = 0$ yields:

$$p_M = \theta_M q_M. \tag{A.2}$$

The Incumbent maximizes its profits:

$$\pi_M = \frac{\theta_H - \theta_M}{\theta_H - \theta_L}(p_M - \frac{1}{2}c_I q_M^2) = \frac{\theta_H - \theta_M}{\theta_H - \theta_L}(\theta_M q_M - \frac{1}{2}c_I q_M^2) \rightarrow \max \tag{A.3}$$

choosing (θ_M, q_M).

First order conditions are:

$$\frac{\partial \pi_M}{\partial q_M}(\theta_M^*, q_M^*) = \frac{\theta_H - \theta_M^*}{\theta_H - \theta_L}(\theta_M^* - c_I q_M^*) = 0 \tag{A.4}$$

and

$$\frac{\partial \pi_M}{\partial \theta_M}(\theta_M^*, q_M^*) = \frac{q_M^*}{\theta_H - \theta_L}(-2\theta_M^* + \frac{1}{2}c_I q_M^* + \theta_H) = 0. \tag{A.5}$$

From (A.4) and (A.5) the solution (θ_M^*, q_M^*) is as follows:

$$\theta_M^* = \frac{2\theta_H}{3} \tag{A.6}$$

$$q_M^* = \frac{2\theta_H}{3c_I}. \tag{A.7}$$

Then

$$p_M^* = \frac{4\theta_H^2}{9c_I} \tag{A.8}$$

and the profits of the Incumbent are:

$$\pi_M^* = \frac{2\theta_H^3}{27c_I\,(\theta_H - \theta_L)}. \tag{A.9}$$

To ensure that π_M has a maximum at (θ_M^*, q_M^*), check second order conditions at this point:

$$\left.\frac{\partial \pi_M^2}{\partial q_M^2}\right|_{(\theta_M^*, q_M^*)} = -\frac{\theta_H c_I}{3\left(\theta_H - \theta_L\right)} < 0 \tag{A.10}$$

$$\left.\frac{\partial \pi_M^2}{\partial q_M \partial \theta_M}\right|_{(\theta_M^*, q_M^*)} = \left.\frac{\partial \pi_M^2}{\partial \theta_M \partial q_M}\right|_{(\theta_M^*, q_M^*)} = \frac{\theta_H}{3\left(\theta_H - \theta_L\right)} \tag{A.11}$$

$$\left.\frac{\partial \pi_M^2}{\partial \theta_M^2}\right|_{(\theta_M^*, q_M^*)} = -\frac{4\theta_H}{3c_I\left(\theta_H - \theta_L\right)} < 0. \tag{A.12}$$

Since the determinant of the Hessian matrix is positive, the Hessian matrix is negative definite and π_M is maximized at (θ_M^*, q_M^*).

A.2 Derivations for Section 1.2.2

Firms decide on (p_E, q_E) and (p_I, q_I) (such that $q_E < q_I$ and $p_E < p_I$) subject to the following constraints:

$$V_E = \theta_E q_E - p_E \geq 0 \tag{A.13}$$

$$V_I = \theta_I q_I - p_I \geq 0 \tag{A.14}$$

$$\theta_E q_E - p_E \geq \theta_E q_I - p_I \tag{A.15}$$

$$\theta_I q_I - p_I \geq \theta_I q_E - p_E \tag{A.16}$$

with $\theta_I > \theta_E$ by construction.

Setting $V_E = \theta_E q_E - p_E = 0$ yields:

$$p_E = \theta_E q_E. \tag{A.17}$$

Then set $\theta_I q_I - p_I = \theta_I q_E - p_E$ to obtain:

$$p_I = \theta_I(q_I - q_E) + p_E = \theta_I(q_I - q_E) + \theta_E q_E. \tag{A.18}$$

Check whether (A.14) is satisfied:

$$V_I = \theta_I q_I - p_I = \theta_I q_I - (\theta_I(q_I - q_E) + \theta_E q_E) = q_E(\theta_I - \theta_E) > 0. \tag{A.19}$$

Check whether (A.15) is satisfied:

$$\theta_E q_E - p_E = 0 \tag{A.20}$$

and then:

$$\theta_E q_I - p_I = \theta_E q_I - (\theta_I(q_I - q_E) + \theta_E q_E) = (\theta_E - \theta_I)(q_I - q_E) < 0. \tag{A.21}$$

Firms maximize their profits:

$$\pi_E = \frac{\theta_I - \theta_E}{\theta_H - \theta_L}(p_E - \frac{1}{2}c_E q_E^2) = \frac{\theta_I - \theta_E}{\theta_H - \theta_L}(\theta_E q_E - \frac{1}{2}c_E q_E^2) \to \max \tag{A.22}$$

$$\begin{aligned} \pi_I &= \frac{\theta_H - \theta_I}{\theta_H - \theta_L}(p_I - \frac{1}{2}c_I q_I^2) = \\ &= \frac{\theta_H - \theta_I}{\theta_H - \theta_L}(\theta_I(q_I - q_E) + \theta_E q_E - \frac{1}{2}c_I q_I^2) \to \max \end{aligned} \tag{A.23}$$

choosing (θ_E, q_E) and (θ_I, q_I) correspondingly.

First order conditions for the Entrant are:

$$\frac{\partial \pi_E}{\partial q_E}(\theta_E^*, q_E^*) = \frac{\theta_I - \theta_E^*}{\theta_H - \theta_L}(\theta_E^* - c_E q_E^*) = 0 \tag{A.24}$$

$$\frac{\partial \pi_E}{\partial \theta_E}(\theta_E^*, q_E^*) = \frac{q_E^*}{\theta_H - \theta_L}(-2\theta_E^* + \frac{1}{2}c_E q_E^* + \theta_I) = 0. \tag{A.25}$$

From (A.24) and (A.25):

$$\begin{cases} \theta_E^* = \frac{2}{3}\theta_I \\ q_E^* = \frac{2}{3c_E}\theta_I. \end{cases} \tag{A.26}$$

First order conditions for the Incumbent are:

$$\frac{\partial \pi_I}{\partial q_I}(\theta_I^*, q_I^*) = \frac{\theta_H - \theta_I^*}{\theta_H - \theta_L}(\theta_I^* - c_I q_I^*) = 0 \tag{A.27}$$

$$\frac{\partial \pi_I}{\partial \theta_I}(\theta_I^*, q_I^*) = \frac{\frac{1}{2}c_I(q_I^*)^2 - 2\theta_I^*(q_I^* - q_E) + \theta_H(q_I^* - q_E) - \theta_E q_E}{\theta_H - \theta_L} = 0. \tag{A.28}$$

From (A.27) and (A.28):

$$\begin{cases} \theta_I^* = c_I q_I^* \\ \frac{1}{2}c_I(q_I^*)^2 - 2\theta_I^*(q_I^* - q_E) + \theta_H(q_I^* - q_E) - \theta_E q_E = 0. \end{cases} \tag{A.29}$$

At the Nash equilibrium, (A.26) and (A.29) combine into:

$$\begin{cases} \theta_E^* = \frac{2}{3}\theta_I^* \\ q_E^* = \frac{2}{3c_E}\theta_I^* \\ \theta_I^* = c_I q_I^* \\ \frac{1}{2}c_I(q_I^*)^2 - 2\theta_I^*(q_I^* - q_E^*) + \theta_H(q_I^* - q_E^*) - \theta_E^* q_E^* = 0. \end{cases} \tag{A.30}$$

Substitute θ_E^*, q_E^* and $q_I^* = \frac{\theta_I^*}{c_I}$ into the last equation of (A.30):

$$\frac{(\theta_I^*)^2}{2c_I} - 2\theta_I^*\left(\frac{\theta_I^*}{c_I} - \frac{2\theta_I^*}{3c_E}\right) + \theta_H\left(\frac{\theta_I^*}{c_I} - \frac{2\theta_I^*}{3c_E}\right) - \frac{4(\theta_I^*)^2}{9c_E} = 0; \tag{A.31}$$

$$\frac{\theta_I^*}{2c_I} - 2\theta_I^*\left(\frac{3c_E - 2c_I}{3c_E c_I}\right) + \theta_H\left(\frac{3c_E - 2c_I}{3c_E c_I}\right) - \frac{4\theta_I^*}{9c_E} = 0; \tag{A.32}$$

$$\theta_H\left(\frac{3c_E - 2c_I}{3c_E c_I}\right) = \theta_I^*\left(2\left(\frac{3c_E - 2c_I}{3c_E c_I}\right) + \frac{4}{9c_E} - \frac{1}{2c_I}\right). \tag{A.33}$$

The solution to (A.33) is as follows:

$$\theta_I^* = \frac{6\theta_H(3c_E - 2c_I)}{27c_E - 16c_I} \tag{A.34}$$

and then

$$\theta_E^* = \frac{4\theta_H(3c_E - 2c_I)}{27c_E - 16c_I} \tag{A.35}$$

$$q_I^* = \frac{6\theta_H(3c_E - 2c_I)}{(27c_E - 16c_I)c_I} \tag{A.36}$$

$$q_E^* = \frac{4\theta_H(3c_E - 2c_I)}{(27c_E - 16c_I)c_E}. \tag{A.37}$$

Substituting the above solution into the profit functions of the firms (A.22) and (A.23) yields:

$$\pi_E^* = \frac{16\theta_H^3(3c_E - 2c_I)^3}{(27c_E - 16c_I)^3 c_E(\theta_H - \theta_L)} \tag{A.38}$$

$$\pi_I^* = \frac{2\theta_H^3(9c_E - 4c_I)^2(3c_E - 2c_I)^2}{c_E(27c_E - 16c_I)^3 c_I(\theta_H - \theta_L)}. \tag{A.39}$$

Second order conditions for the Entrant at q_E^*, q_I^*, θ_E^*, θ_I^* are:

$$\frac{\partial \pi_E^2}{\partial q_E^2} = -\frac{2c_E\left(3c_E - 2c_I\right)\theta_H}{\left(27c_E - 16c_I\right)\left(\theta_H - \theta_L\right)} < 0 \tag{A.40}$$

$$\frac{\partial \pi_E^2}{\partial q_E \partial \theta_E} = \frac{\partial \pi_E^2}{\partial \theta_E \partial q_E} = \frac{2\left(3c_E - 2c_I\right)\theta_H}{\left(27c_E - 16c_I\right)\left(\theta_H - \theta_L\right)} \tag{A.41}$$

$$\frac{\partial \pi_E^2}{\partial \theta_E^2} = -\frac{8\left(3c_E - 2c_I\right)\theta_H}{c_E\left(27c_E - 16c_I\right)\left(\theta_H - \theta_L\right)} < 0 \tag{A.42}$$

The determinant of the Hessian matrix is $\frac{48(1.5c_E - c_I)^2\theta_H^2}{(27c_E - 16c_I)(\theta_H - \theta_L)^2} > 0$; thus, Hessian matrix is negative-definite and π_E is maximized at q_E^*, q_I^*, θ_E^*, θ_I^* for $\frac{c_I}{c_E} \leq 1$.

Second order conditions for the Incumbent at q_E^*, q_I^*, θ_E^*, θ_I^* are:

$$\frac{\partial \pi_I^2}{\partial q_I^2} = -\frac{c_I\left(9c_E - 4c_I\right)\theta_H}{\left(27c_E - 16c_I\right)\left(\theta_H - \theta_L\right)} < 0 \tag{A.43}$$

$$\frac{\partial \pi_I^2}{\partial q_I \partial \theta_I} = \frac{\left(9c_E - 4c_I\right)\theta_H}{\left(27c_E - 16c_I\right)\left(\theta_H - \theta_L\right)} < 0 \tag{A.44}$$

$$\frac{\partial \pi_I^2}{\partial \theta_I^2} = -\frac{4(3c_E - 2c_I)^2\theta_H}{c_I c_E\left(27c_E - 16c_I\right)\left(\theta_H - \theta_L\right)} < 0 \tag{A.45}$$

The determinant of the Hessian matrix is positive for $\frac{c_I}{c_E} \in (0;\, 0.924)$; this is a technical problem that may be ameliorated by the proper calibration of parameters of the model or by checking that the duopolistic trigger is smaller than 0.924 (which is highly probable).
Still, π_M is maximized at q_E^*, q_I^*, θ_E^*, θ_I^* for $\frac{c_I}{c_E} \in (0;\, 0.924)$.

A.3 Derivations for Section 1.2.3

The firm decides on (p_1, q_1) and (p_2, q_2) (such that $q_1 < q_2$ and $p_1 < p_2$) subject to the following constraints:

$$V_1 = \theta_1 q_1 - p_1 \geq 0 \tag{A.46}$$

$$V_2 = \theta_2 q_2 - p_2 \geq 0 \tag{A.47}$$

$$\theta_1 q_1 - p_1 \geq \theta_1 q_2 - p_2 \tag{A.48}$$

$$\theta_2 q_2 - p_2 \geq \theta_2 q_1 - p_1 \tag{A.49}$$

with $\theta_2 > \theta_1$ by construction.

Setting $V_1 = \theta_1 q_1 - p_1 = 0$ yields:

$$p_1 = \theta_1 q_1. \tag{A.50}$$

Then set $\theta_2 q_2 - p_2 = \theta_2 q_1 - p_1$ to obtain:

$$p_2 = \theta_2(q_2 - q_1) + p_1 = \theta_2(q_2 - q_1) + \theta_1 q_1. \tag{A.51}$$

Check whether (A.47) is satisfied:

$$V_2 = \theta_2 q_2 - p_2 = \theta_2 q_2 - (\theta_2(q_2 - q_1) + \theta_1 q_1) = q_1(\theta_2 - \theta_1) > 0. \tag{A.52}$$

Check whether (A.48) is satisfied:

$$\theta_1 q_1 - p_1 = 0 \tag{A.53}$$

and then:

$$\theta_1 q_2 - p_2 = \theta_1 q_2 - (\theta_2(q_2 - q_1) + \theta_1 q_1) = (\theta_1 - \theta_2)(q_2 - q_1) < 0. \tag{A.54}$$

The firm maximizes its profits:

$$\begin{aligned}\pi_{MG} &= \frac{\theta_2 - \theta_1}{\theta_H - \theta_L}(p_1 - \frac{1}{2}c_I q_1^2) + \frac{\theta_H - \theta_2}{\theta_H - \theta_L}(p_2 - \frac{1}{2}c_I q_2^2) = \\ &= \frac{\theta_2 - \theta_1}{\theta_H - \theta_L}(\theta_1 q_1 - \frac{1}{2}c_I q_1^2) + \frac{\theta_H - \theta_2}{\theta_H - \theta_L}(\theta_2(q_2 - q_1) + \\ &+ \theta_1 q_1 - \frac{1}{2}c_I q_2^2) \rightarrow \max\end{aligned} \tag{A.55}$$

choosing (θ_1, q_1) and (θ_2, q_2).

The first order conditions are:

$$\frac{\partial \pi_{MG}}{\partial q_1} = \frac{(\theta_1^* - \theta_2^*)\left(-\theta_1^* - \theta_2^* + c_I q_1^* + \theta_H\right)}{\theta_H - \theta_L} = 0 \tag{A.56}$$

$$\frac{\partial \pi_{MG}}{\partial \theta_1} = \frac{q_1^*\left(0.5 c_I q_1^* - 2\theta_1^* + \theta_H\right)}{\theta_H - \theta_L} = 0 \tag{A.57}$$

$$\frac{\partial \pi_{MG}}{\partial q_2} = \frac{(\theta_H - \theta_2)\left(-c_I q_2^* + \theta_2^*\right)}{\theta_H - \theta_L} = 0 \tag{A.58}$$

$$\frac{\partial \pi_{MG}}{\partial \theta_2} = \frac{-0.5 c_I q_1^{*2} + 0.5 c_I q_2^{*2} + 2q_1^*\theta_2^* - 2q_2^*\theta_2^* - q_1^*\theta_H + q_2^*\theta_H}{\theta_H - \theta_L} = 0. \tag{A.59}$$

Solving the system of first order conditions (A.56)-(A.59) yields:

$$\theta_1^* = 0.6\theta_H \tag{A.60}$$

$$\theta_2^* = 0.8\theta_H \tag{A.61}$$

$$q_1^* = \frac{0.4\theta_H}{c_I} \tag{A.62}$$

$$q_2^* = \frac{0.8\theta_H}{c_I} \tag{A.63}$$

and the profits are:

$$\pi_{MG}^* = \frac{2\theta_H^3}{25c_I\left(\theta_H - \theta_L\right)}. \tag{A.64}$$

To ensure that π_{MG} has a maximum at $\theta_1^*, \theta_2^*, q_1^*, q_2^*$, check second order conditions at this point:

$$\frac{\partial\pi_{MG}^2}{\partial q_1^2} = \frac{c_I\left(\theta_1 - \theta_2\right)}{\left(\theta_H - \theta_L\right)} < 0 \tag{A.65}$$

$$\frac{\partial\pi_{MG}^2}{\partial q_1\partial q_2} = \frac{\partial\pi_{MG}^2}{\partial q_2\partial q_1} = 0 \tag{A.66}$$

$$\frac{\partial\pi_M^2}{\partial q_2^2} = \frac{c_I\left(\theta_2 - \theta_H\right)}{\left(\theta_H - \theta_L\right)} < 0. \tag{A.67}$$

Since by construction $\theta_1 < \theta_2 < \theta_H$, the Hessian matrix is negative-definite for any $\theta_1, \theta_2, q_1, q_2$, and it follows that π_{MG} is maximized at $\theta_1^*, \theta_2^*, q_1^*, q_2^*$.

A.4 Derivations for Section 1.3.1

The Bellman equation is:

$$F(x) = \pi_0\left(x_0\right)dt + \frac{E\left(F(x+dx)\right)}{1+rdt}. \tag{A.68}$$

Multiplying both sides of (A.68) by $(1+rdt)$ and keeping the terms of order dt only yields:

$$F(x)\left(1+rdt\right) = \pi_0(x_0)dt + F(x) + \lambda dt\left(F\left(x(1-g)\right) - F(x)\right), \tag{A.69}$$

then

$$F(x)rdt = \pi_0(x_0)dt + \lambda dtF\left(x(1-g)\right) - F(x)\lambda dt, \tag{A.70}$$

and

$$F(x)\,(r+\lambda)\,dt = \pi_0(x_0)dt + \lambda dt F\left(x(1-g)\right). \tag{A.71}$$

Dividing both sides of (A.71) by $(r+\lambda)\,dt$ yields:

$$F(x) = \frac{\pi_0\left(x_0\right)}{r+\lambda} + \frac{\lambda}{r+\lambda}F\left(x(1-g)\right). \tag{A.72}$$

Since $x_k = x_0(1-g)^k$ with $x_0 = 1$ and k being the number of jumps needed to reach x_k, one can rewrite (A.71) as:

$$\begin{aligned}
F(x_0) &= \frac{\pi_0\left(x_0\right)}{r+\lambda} + \frac{\lambda}{r+\lambda}F\left(1-g\right) = \\
&= \frac{\pi_0\left(x_0\right)}{r+\lambda} + \frac{\lambda}{r+\lambda}\left(\frac{\pi_0\left(x_0\right)}{r+\lambda} + \frac{\lambda}{r+\lambda}F\left((1-g)^2\right)\right) = \\
&= \frac{\pi_0\left(x_0\right)}{r+\lambda} + \frac{\lambda}{r+\lambda}\left(\frac{\pi_0\left(x_0\right)}{r+\lambda} + \frac{\lambda}{r+\lambda}\left(\frac{\pi_0\left(x_0\right)}{r+\lambda} + \frac{\lambda}{r+\lambda}\left(...\right)\right)\right) = \\
&= \frac{\pi_0\left(x_0\right)}{r+\lambda} \times \frac{1-\left(\frac{\lambda}{r+\lambda}\right)^k}{1-\frac{\lambda}{r+\lambda}} + \left(\frac{\lambda}{r+\lambda}\right)^k F\left((1-g)^k\right) = \\
&= \frac{\pi_0\left(x_0\right)}{r}\left(1-\left(\frac{\lambda}{r+\lambda}\right)^k\right) + \left(\frac{\lambda}{r+\lambda}\right)^k F\left((1-g)^k\right) = \\
&= \frac{\pi_0\left(x_0\right)}{r} + \left(\frac{\lambda}{r+\lambda}\right)^k\left(F\left((1-g)^k\right) - \frac{\pi_0\left(x_0\right)}{r}\right)
\end{aligned} \tag{A.73}$$

A.5 Derivations for Section 1.3.2

The problem of the Incumbent in the general case of non-constant investment cost RD is as follows:

$$\max_k F_M^g = \frac{\pi_{M0}^*}{r} + \frac{1}{r}\left(\frac{\lambda}{r+\lambda}\right)^k\left(\pi_M^*\left((1-g)^k\right) - \pi_{M0} - rRD\left((1-g)^k\right)\right). \tag{A.74}$$

The first order condition is:

$$\begin{aligned}
\frac{\partial F_M^g}{\partial k}\Big|_{k=k_M^{cg}} = \frac{1}{r}\left(\frac{\lambda}{r+\lambda}\right)^{k_M^{cg}}\Bigg(\ln\frac{\lambda}{r+\lambda}\Bigg(\pi_M^*\left((1-g)^{k_M^{cg}}\right) - \\
- \pi_{M0} - rRD\left((1-g)^{k_M^{cg}}\right)\Bigg) + \frac{\partial \pi_M^*}{\partial k} - r\frac{\partial RD}{\partial k}\Bigg) = 0,
\end{aligned} \tag{A.75}$$

where $(1-g)^{k_M^{cg}} = x_M^{cg}$ is the continuous monopolistic trigger for the general solution; k_M^{cg} is the (continuous) number of jumps needed to reach x_M^{cg}.

Since $\pi_M^*\left((1-g)^k\right) = \frac{1}{27(1-g)^k}$, then $\frac{\partial \pi_M^*}{\partial k} = -\frac{\ln(1-g)}{27(1-g)^k}$ and the general solution (for non-constant investment cost RD) to (A.75) is as follows:

$$\frac{\ln\frac{\lambda}{r+\lambda} - \ln(1-g)}{(1-g)^{k_M^{cg}}} = \left(1 + r\frac{RD\left((1-g)^{k_M^{cg}}\right)}{\pi_{M0}^*}\right)\ln\frac{\lambda}{r+\lambda} + r\frac{\frac{\partial RD}{\partial k}}{\pi_{M0}^*} \tag{A.76}$$

where $x_M^{cg} = (1-g)^{k_M^{cg}}$.

In particular, in the case of constant investment cost RD the general solution above reduces to:

$$x_M^c = (1-g)^{k_M^c} = \frac{\ln\frac{\lambda}{r+\lambda} - \ln(1-g)}{\left(1 + r\frac{RD}{\pi_{M0}^*}\right)\ln\frac{\lambda}{r+\lambda}}. \tag{A.77}$$

Consider again the general case $RD = RD_0 + \beta\left(1 - (1-g)^k\right)$ as in (1.46). Then $\frac{\partial RD}{\partial k} = -\beta(1-g)^k \ln(1-g)$ and (A.76) transforms into:

$$\begin{aligned}\frac{\ln\frac{\lambda}{r+\lambda} - \ln(1-g)}{(1-g)^{k_M^{cg}}} &= \left(1 + r\frac{RD_0}{\pi_{M0}^*}\right)\ln\frac{\lambda}{r+\lambda} + \\ &+ \frac{r\beta\left(\ln\frac{\lambda}{r+\lambda} + \ln(1-g)\right)}{\pi_{M0}^*}\left(\frac{\ln\frac{\lambda}{r+\lambda}}{\ln\frac{\lambda}{r+\lambda} + \ln(1-g)} - (1-g)^{k_M^{cg}}\right),\end{aligned} \tag{A.78}$$

where RD_0 is equal to the constant investment cost RD.

Define:

$$A = \ln\frac{\lambda}{r+\lambda} - \ln(1-g) < 0 \tag{A.79}$$

$$B = \left(1 + r\frac{RD_0}{\pi_{M0}^*}\right)\ln\frac{\lambda}{r+\lambda} < 0 \tag{A.80}$$

$$C = \frac{r\beta\left(\ln\frac{\lambda}{r+\lambda} + \ln(1-g)\right)}{\pi_{M0}^*} < 0 \tag{A.81}$$

$$D = \frac{\ln\frac{\lambda}{r+\lambda}}{\ln\frac{\lambda}{r+\lambda} + \ln(1-g)} > 0. \tag{A.82}$$

It is straightforward to show that $\frac{A}{B} < D$:

$$\begin{aligned} D - \frac{A}{B} &= \frac{\ln\frac{\lambda}{r+\lambda}}{\ln\frac{\lambda}{r+\lambda} + \ln(1-g)} - \frac{\ln\frac{\lambda}{r+\lambda} - \ln(1-g)}{\left(1 + r\frac{RD_0}{\pi^*_{M0}}\right)\ln\frac{\lambda}{r+\lambda}} = \\ &= \frac{r\frac{RD_0}{\pi^*_{M0}}\ln^2\frac{\lambda}{r+\lambda} + \ln^2(1-g)}{\left(\ln\frac{\lambda}{r+\lambda} + \ln(1-g)\right)\left(1 + r\frac{RD_0}{\pi^*_{M0}}\right)\ln\frac{\lambda}{r+\lambda}} > 0. \end{aligned} \tag{A.83}$$

Then x^c_M (A.77) can be rewritten as:

$$x^c_M = \frac{A}{B}, \tag{A.84}$$

and x^{cg}_M is the solution to the following quadratic equation based on (A.78):

$$A = x^{cg}_M \left(B + C\left(D - x^{cg}_M\right)\right). \tag{A.85}$$

The roots of (A.85) are:

$$x^{cg}_{M1,2} = \frac{\frac{B}{C} + D \pm \sqrt{\left(\frac{B}{C} + D\right)^2 - 4\frac{A}{C}}}{2}, \tag{A.86}$$

with the first (smaller) root being greater than zero and less than $\frac{A}{B}$:

$$x^{cg}_{M1} = \frac{\frac{B}{C} + D - \sqrt{\left(\frac{B}{C} + D\right)^2 - 4\frac{A}{C}}}{2} < \frac{A}{B} \Leftrightarrow \frac{A}{B} < D, \tag{A.87}$$

and the second root being greater than $\frac{A}{C}$:

$$x^{cg}_{M2} = \frac{A}{C}\frac{1}{x^{cg}_{M1}} > \frac{A}{C} \times \frac{B}{A} > \frac{A}{C} \tag{A.88}$$

since $\frac{B}{A} > 1$.

The second order condition for the general case of non-constant RD is as follows:

$$\frac{\partial^2 F_M^g}{\partial k^2}\Big|_{k=k_M^{cg}} = \frac{1}{r}\left(\frac{\lambda}{r+\lambda}\right)^{k_M^{cg}}\left(\ln\frac{\lambda}{r+\lambda}\left(\frac{\partial \pi_M^*}{\partial k} - r\frac{\partial RD}{\partial k}\right) + \right. \\ \left. + \frac{\partial^2 \pi_M^*}{\partial k^2} - r\frac{\partial^2 RD}{\partial k^2}\right). \tag{A.89}$$

In the case of constant RD (A.89) reduces to:

$$\frac{\partial^2 F_M}{\partial k^2}\Big|_{k=k_M^c} = \frac{1}{r}\left(\frac{\lambda}{r+\lambda}\right)^{k_M^c}\left(\frac{\partial \pi_M^*}{\partial k}\ln\frac{\lambda}{r+\lambda} + \frac{\partial^2 \pi_M^*}{\partial k^2}\right) = \\ = \frac{\ln(1-g)}{27r\,(1-g)^{k_M^c}}\left(\frac{\lambda}{r+\lambda}\right)^{k_M^c}\left(-\ln\frac{\lambda}{r+\lambda} + \ln(1-g)\right) < 0, \tag{A.90}$$

ensuring that k_M^c is the point of maximum.

Return to the general case of non-constant RD. After all necessary substitutions (A.89) becomes:

$$\frac{\partial^2 F_M^g}{\partial k^2}\Big|_{k=k_M^{cg}} = \frac{\beta\ln(1-g)\left(\ln\frac{\lambda}{r+\lambda} + \ln(1-g)\right)}{(1-g)^{k_M^{cg}}}\left(\frac{\lambda}{r+\lambda}\right)^{k_M^{cg}} \times \\ \times\left(\left((1-g)^{k_M^{cg}}\right)^2 - \frac{\pi_{M0}\left(\ln\frac{\lambda}{r+\lambda} - \ln(1-g)\right)}{\beta r\left(\ln\frac{\lambda}{r+\lambda} + \ln(1-g)\right)}\right). \tag{A.91}$$

To ensure the negativity of the second order condition (A.91) for $x_M^{cg} = (1-g)^{k_M^{cg}} \in (0;1]$ it is enough to choose β such that:

$$\frac{\pi_{M0}\left(\ln\frac{\lambda}{r+\lambda} - \ln(1-g)\right)}{\beta r\left(\ln\frac{\lambda}{r+\lambda} + \ln(1-g)\right)} > 1, \tag{A.92}$$

or, in terms of β:

$$\beta < \frac{\pi_{M0}\left(\ln\frac{\lambda}{r+\lambda} - \ln(1-g)\right)}{r\left(\ln\frac{\lambda}{r+\lambda} + \ln(1-g)\right)}. \tag{A.93}$$

Define β as:

$$\beta = K_\beta\frac{\pi_{M0}\left(\ln\frac{\lambda}{r+\lambda} - \ln(1-g)\right)}{r\left(\ln\frac{\lambda}{r+\lambda} + \ln(1-g)\right)}, \tag{A.94}$$

where $K_\beta \in (0;1)$.

Notice that one can rewrite the condition (A.92) as:

$$\frac{A}{C} > 1 \tag{A.95}$$

using the definitions (A.79)-(A.82).

Combining (A.95) with (A.88) yields:

$$x_{M2}^{cg} > \frac{A}{C} > 1 \tag{A.96}$$

provided that β satisfies (A.93).

Thus, it follows from (A.84), (A.87), (A.88) and (A.96) that only the smaller root x_{M1}^{cg} is relevant to the solution in the general case of non-constant investment cost RD (the larger root x_{M2}^{cg} is greater than one) and:

$$x_{M1}^{cg} < x_M^c \tag{A.97}$$

provided β satisfies (A.94).

Substituting (A.79)-(A.82) into the expression for x_{M1}^{cg} (A.87) yields:

$$x_{M1}^{cg} = \frac{E - \sqrt{E^2 - \frac{4}{K_\beta}}}{2}, \tag{A.98}$$

where

$$E = \frac{\left(1 + r\frac{RD_0}{\pi_{M0}^*}\right)\ln\frac{\lambda}{r+\lambda}}{K_\beta\left(\ln\frac{\lambda}{r+\lambda} - \ln(1-g)\right)} + \frac{\ln\frac{\lambda}{r+\lambda}}{\ln\frac{\lambda}{r+\lambda} + \ln(1-g)}. \tag{A.99}$$

A.6 Derivations for Section 1.3.3

The second order condition of the Incumbent is as follows:

$$\begin{aligned}
\frac{\partial^2 F_D}{\partial k^2}\Big|_{k=k_D^c} &= \frac{1}{r}\left(\frac{\lambda}{r+\lambda}\right)^{k_D^c}\left(\frac{\partial \pi_I^*}{\partial k}\ln\frac{\lambda}{r+\lambda} + \frac{\partial^2 \pi_I^*}{\partial k^2}\right) = \\
&= \frac{1}{r\frac{\partial \pi_I^*}{\partial k}}\left(\frac{\lambda}{r+\lambda}\right)^{k_D^c}\left(\ln\frac{\lambda}{r+\lambda} + \frac{\frac{\partial^2 \pi_I^*}{\partial k^2}}{\frac{\partial \pi_I^*}{\partial k}}\right) = \\
&= \frac{1}{r\frac{\partial \pi_I^*}{\partial k}}\left(\frac{\lambda}{r+\lambda}\right)^{k_D^c}\left(\ln\frac{\lambda}{r+\lambda} + t\ln(1-g)\right)
\end{aligned} \tag{A.100}$$

where $t \in (-1; -\frac{839}{2255}]$ for $(1-g)^{k^c_D} \in (0; 1]$ and $\frac{\partial \pi^*_I}{\partial k} > 0$.

Since $\ln \frac{\lambda}{r+\lambda} < \ln(1-g)$ by the properties of the model, then $\ln \frac{\lambda}{r+\lambda} < \ln(1-g) < (-t)\ln(1-g)$ and (A.100) is negative for $(1-g)^{k^c_D} \in (0; 1]$.

Appendix B

Derivations for Chapter 2

B.1 Derivations for Section 2.3.2

Recall from (2.33) that payoffs to the bidder P_B^s and to the target P_T^s at the stock merger are:

$$P_B^s(X,Y) = \max\left(s_B V(X,Y) - KX, 0\right) \tag{B.1}$$

$$P_T^s(X,Y) = \max\left((1-s_B)V(X,Y) - QY, 0\right). \tag{B.2}$$

Then, the state variables for the bidder and the target for LSM would be:

$$\begin{aligned} &S_B^s(X,Y) = s_B V(X,Y) - KX = \\ &= s_B\left(KX + QY + \alpha(K+Q)(X-Y)\right) - KX = \\ &= X\left(s_B\left(K + \alpha(K+Q)\right) - K\right) + Y s_B\left(Q - \alpha(K+Q)\right) = \\ &= Xk_1 + Yk_2 \end{aligned} \tag{B.3}$$

$$\begin{aligned} &S_B^s(X,Y) = (1-s_B)V(X,Y) - QY = \\ &= (1-s_B)\left(KX + QY + \alpha(K+Q)(X-Y)\right) - QY = \\ &= X(1-s_B)\left(K + \alpha(K+Q)\right) + \\ &+ Y\left((1-s_B)\left(Q - \alpha(K+Q)\right) - Q\right) = \\ &= Xq_1 + Yq_2. \end{aligned} \tag{B.4}$$

Solving for the share s_B such that $\frac{k_1}{k_2} = \frac{q_1}{q_2}$ yields:

$$s_B = \frac{K}{K+Q}. \tag{B.5}$$

Substituting $s_B = \frac{K}{K+Q}$ into the expressions for the state variables (B.3) and (B.4) yields:

$$S_B^s(X,Y) = \frac{(X-Y)\left(\alpha K(K+Q) - KQ\right)}{K+Q} = (X-Y)\,a_B \tag{B.6}$$

$$S_T^s(X,Y) = \frac{(X-Y)\left(\alpha Q(K+Q) + KQ\right)}{K+Q} = (X-Y)\,a_T \tag{B.7}$$

summing up to $S_B^s + S_T^s = (X-Y)\,\alpha(K+Q) = (X-Y)(a_T + a_B) = S_{CP}$.

Thus, at $s_B = \frac{K}{K+Q}$ all state variables (S_B^s, S_T^s and S_{CP}) depend on exactly the same stochastic process $X-Y$; it means that the same matrix $P_R = R\left(R'R\right)^{-1}R'$ will be used to compute fitted values in regressions based on either of these state variables[1].

[1] R denotes the matrix of regressors.

It is clear that for $s_B = \frac{K}{K+Q}$ the payoffs to the players are as follows:

$$P_B^s(X,Y) = \max\left(\frac{(X-Y)(\alpha K(K+Q) - KQ)}{K+Q}, 0\right) \tag{B.8}$$

$$P_T^s(X,Y) = \max\left(\frac{(X-Y)(\alpha Q(K+Q) + KQ)}{K+Q}, 0\right) \tag{B.9}$$

$$P_{CP} = \max(\alpha(K+Q)(X-Y), 0). \tag{B.10}$$

Payoffs (B.8)-(B.10) have the same sign if the condition $\alpha(K+Q) > Q$ holds; they are all positive for $X > Y$ (which is the necessary condition for the synergy to be positive ans for the merger to be economically meaningful) and they are all negative for $X < Y$. Thus, positiveness of the payoff to the central planner P_{CP} implies positiveness of P_B^s and P_T^s given $\alpha(K+Q) > Q$. Besides, the ratios $\frac{P_B^s}{P_{CP}}$ and $\frac{P_T^s}{P_{CP}}$ are constant over time for $X > Y$ (and not defined otherwise).

This means that one can solve then stock merger problem from the point of view of central planner (instead of solving it for the bidder and the target) setting $s_B = \frac{K}{K+Q}$ and provided that condition $\alpha(K+Q) > Q$ holds.

It is not surprising that the same conclusion along with the same share s_B and the same necessary condition $\alpha(K+Q) > Q$ appear in the original infinite horizon model by Morellec and Zhdanov (2005).

Thus, one can choose $s_B = \frac{K}{K+Q}$ provided $\alpha(K+Q) > Q$ (that can be rewritten as $\frac{K}{K+Q} > 1-\alpha$) is satisfied; in this paper $\frac{K}{K+Q} = 0.8929 > 1-\alpha = 0.6$.

B.2 LSM sample simulation

Table B.1: LSM sample simulation: results

n	stock			cash			$MARKET_{LSM}$		$PREM_{LSM}$		R_{LSM}	
	O^{Bs}_{LSM}	O^{Ts}_{LSM}	total	O^{Bc}_{LSM}	O^{Tc}_{LSM}	total	cash	stock	cash	stock	cash	stock
1	11.43	5.24	16.67	12.55	1.04	13.59	1.31	1.49	0.27	1.26	1.79	1.94
2	20.19	9.47	29.66	16.30	2.77	19.07	2.72	3.52	0.30	0.86	1.44	1.66
3	19.40	6.29	25.69	8.40	1.43	9.83	2.46	3.83	0.19	0.55	1.23	1.42
4	20.39	7.27	27.66	12.84	2.59	15.42	2.50	3.26	0.31	0.66	1.32	1.49
5	6.13	2.41	8.53	5.44	0.75	6.19	0.75	0.96	0.41	1.30	1.67	2.01
6	4.34	3.64	7.98	3.86	0.80	4.66	1.02	1.39	0.21	0.85	1.41	1.69
7	6.23	3.22	9.45	3.13	1.04	4.17	1.33	1.75	0.19	0.38	1.19	1.29
8	10.46	6.46	16.93	12.31	2.08	14.39	1.94	2.17	0.45	1.36	1.83	2.08
9	12.95	8.00	20.95	15.93	1.96	17.89	1.93	2.18	0.36	1.60	1.91	2.27
10	9.32	7.69	17.00	9.74	2.81	12.55	1.92	2.32	0.33	0.87	1.50	1.70
11	12.36	8.64	21.00	15.60	1.97	17.56	2.02	2.29	0.31	1.45	1.84	2.16
12	12.39	8.59	20.99	10.18	1.58	11.76	2.42	3.33	0.17	0.80	1.39	1.65
13	16.54	9.84	26.38	23.17	1.74	24.91	3.13	3.17	0.32	1.72	2.36	2.36
14	8.91	2.96	11.87	4.35	1.98	6.33	1.45	2.09	0.39	0.34	1.22	1.25
15	12.88	3.55	16.44	13.28	0.63	13.91	1.20	1.35	0.28	1.54	1.95	2.11

Table B.2: LSM sample simulation: input parameters

n	r	δ_X	δ_Y	σ_X	σ_X	ρ	K	Q	α	P	X_0	Y_0
1	0.065	0.0074	0.0338	0.193	0.167	-0.535	113.538	13.075	0.438	12.648	0.967	0.967
2	0.054	0.0089	0.0332	0.246	0.225	0.480	109.290	11.913	0.401	29.277	2.458	2.458
3	0.080	0.0024	0.0399	0.222	0.198	0.778	118.383	9.208	0.388	21.971	2.386	2.386
4	0.049	0.0013	0.0372	0.208	0.215	0.720	104.025	9.687	0.438	23.850	2.462	2.462
5	0.066	0.0023	0.0341	0.204	0.247	0.194	106.305	9.460	0.375	5.997	0.634	0.634
6	0.064	0.0035	0.0310	0.221	0.181	0.310	80.557	12.647	0.366	11.262	0.891	0.891
7	0.055	0.0029	0.0373	0.151	0.158	0.830	85.685	10.401	0.416	14.008	1.347	1.347
8	0.046	0.0093	0.0364	0.228	0.229	-0.134	76.818	9.852	0.376	15.509	1.574	1.574
9	0.041	0.0005	0.0307	0.243	0.187	-0.420	97.227	12.181	0.366	17.377	1.427	1.427
10	0.057	0.0059	0.0312	0.151	0.209	0.264	91.329	14.582	0.378	25.545	1.752	1.752
11	0.047	0.0016	0.0398	0.232	0.168	-0.409	89.365	12.701	0.380	19.117	1.505	1.505
12	0.069	0.0084	0.0350	0.227	0.158	0.244	99.823	13.139	0.351	27.377	2.084	2.084
13	0.055	0.0017	0.0302	0.250	0.165	-0.905	84.085	10.407	0.373	19.988	1.921	1.921
14	0.074	0.0050	0.0305	0.173	0.233	0.989	121.690	10.241	0.417	16.783	1.639	1.639
15	0.069	0.0100	0.0314	0.242	0.180	-0.586	122.009	9.224	0.448	7.510	0.814	0.814

Bibliography

Acharyya, Rajat, 1998, Monopoly and product quality: Separating or pooling menu?, *Economics Letters* 61, 187–194.

Amram, Martha, and Nalin Kulatilaka, 1998, *Real options: managing strategic investment in an uncertain world* (Harvard Business School Press: Boston, Massachusetts).

Andrade, Gregor, Mark Mitchell, and Erik Stafford, 2001, New evidence and perspectives on mergers, *Journal of Economic Perspectives* 15, 103–120.

Barone-Adesi, Giovanni, and Robert Whaley, 1987, Efficient analytic approximation of American option values, *The Journal of Finance* 42, 301–320.

Black, Fischer, and Myron Scholes, 1973, The pricing of options and corporate liabilities, *Journal of Political Economy* 81, 637–654.

Botteron, Pascal, Marc Chesney, and Rajna Gibson-Asner, 2003, Analyzing firms' strategic investment decisions in a real options' framework, *International Financial Markets, Institutions and Money* 13, 451–479.

Boyer, Marcel, Éric Gravel, and Pierre Lasserre, 2004, Real options and strategic competition: a survey, CIRANO working paper Montréal, Canada.

Boyle, Glenn, and Graeme Guthrie, 2003, Investment, uncertainty, and liquidity, *The Journal of Finance* 58, 2143–2165.

Brennan, Michael, and Eduardo Schwartz, 1985, Evaluating natural resource investments, *The Journal of Business* 58, 135–157.

Broll, Udo, and Bernhard Eckwert, 1999, Exchange rate volatility and international trade, *Southern Economic Journal* 66, 178–185.

Cortazar, Gonzalo, Miguel Gravet, and Jorge Urzua, 2008, The valuation of multidimensional American real options using the LSM simulation method, *Computers and Operations Research* 35, 113–129.

Cox, John, Stephen Ross, and Mark Rubinstein, 1979, Option pricing: A simplified approach, *Journal of Financial Economics* 7, 229–263.

Dixit, Avinash, 1989, Hysteresis, import penetration, and exchange rate pass-through, *The Quarterly Journal of Economics* 104, 205–228.

——— , and Robert Pindyck, 1994, *Investment under uncertainty* (Princeton University Press: Princeton, NJ).

Doraszelski, Ulrich, 2001, The net present value method versus the option value of waiting: A note on Farzin, Huisman and Kort (1998), *Journal of Economic Dynamics and Control* 25, 1109–1115.

Evans, Jonathan, Vicky Henderson, and David Hobson, 2008, Optimal timing for an indivisible asset sale, *Mathematical Finance* 18, 545–567.

Farzin, Y., Kuno Huisman, and Peter Kort, 1998, Optimal timing of technology adoption, *Journal of Economic Dynamics and Control* 22, 779–799.

Friberg, Richard, 2001, Two monies, two markets? Variability and the option to segment, *Journal of International Economics* 55, 317–327.

Fudenberg, Drew, and Jean Tirole, 1985, Preemption and rent equalization in the adoption of new technology, *The Review of Economic Studies* 52, 383–401.

Fujita, Yasunori, 2007, Toward a new modeling of international economics: An attempt to reformulate an international trade model based on real option theory, *Physica A* 383, 507–512.

Grenadier, Steven, 1999, The strategic exercise of options: development cascades and overbuilding in real estate markets, *The Review of Financial Studies* 12, 95–129.

——— , 2002, Option exercise games: An application to the equilibrium investment strategies of firms, *The Review of Financial Studies* 15, 691–721.

——— , and Allen Weiss, 1997, Investment in technological innovations: An option pricing approach, *Journal of Financial Economics* 44, 397–416.

Hackbarth, Dirk, and Erwan Morellec, 2008, Stock returns in mergers and acquisitions, *The Journal of Finance* 63, 1213–1252.

Huisman, Kuno, and Peter Kort, 1999, Effects of strategic interactions on the option value of waiting, No. 9992 in CentER DP Tilburg, The Netherlands.

——— , Grzegorz Pawlina, and Jacco Thijssen, 2003, Strategic investment under uncertainty: Merging real options with game theory, No. 2003-06 in CentER DP Tilburg, The Netherlands.

Ingersoll, Jr, Jonathan, 1987, *Theory of financial decision making* (Rowman & Litterfield Publishers: Savage, MD).

Kulatilaka, Nalin, and Enrico Perotti, 1998, Strategic growth options, *Management Science* 44, 1021–1031.

Lambrecht, Bart, 2001, The impact of debt financing on entry and exit in a duopoly, *The Review of Financial Studies* 14, 765–804.

——— , 2004, The timing and terms of mergers motivated by economies of scale, *Journal of Financial Economics* 72, 41–62.

——— , and William Perraudin, 2003, Real options and preemption under incomplete information, *Journal of Economic Dynamics and Control* 27, 619–643.

Leahy, John, 1993, Investment in competitive equilibrium: The optimality of myopic behavior, *The Quarterly Journal of Economics* 108, 1105–1133.

Longstaff, Francis, and Eduardo Schwartz, 2001, Valuing American options by simulation: a simple least-squares approach, *The Review of Financial Studies* 14, 113–147.

Mankiw, Gregory, 1985, Small menu costs and large business cycles: A macroeconomic model of monopoly, *The Quarterly Journal of Economics* 100, 529–537.

Margrabe, William, 1978, The value of an option to exchange one asset for another, *The Journal of Finance* 33, 177–186.

Martynova, Marina, and Luc Renneboog, 2008, A century of corporate takeovers: What have we learned and where do we stand?, *Journal of Banking and Finance* 32, 2148–2177.

McDonald, Robert, and Daniel Siegel, 1986, The value of waiting to invest, *The Quarterly Journal of Economics* 101, 707–728.

Merton, Robert, 1973, Theory of rational option pricing, *Bell Journal of Economics and Management Science* 4, 141–183.

Morellec, Erwan, and Alexei Zhdanov, 2005, The dynamics of mergers and acquisitions, *Journal of Financial Economics* 77, 649–672.

Motta, Massimo, 1993, Endogenous quality choice: price vs. quantity competition, *The Journal of Industrial Economics* 41, 113–131.

——— , 2004, *Competition policy. Theory and practice* (Cambridge University Press: New York, NY).

Mussa, Michael, and Sherwin Rosen, 1978, Monopoly and product quality, *Journal of Economic Theory* 18, 301–317.

Myers, Stewart, 1977, Determinants of the corporate borrowings, *Journal of Financial Economics* 5, 147–175.

Noh, Yong-Hwan, and Giancarlo Moschini, 2006, Vertical product differentiation, entry-deterrence strategies, and entry qualities, *Review of Industrial Organization.*

Pindyck, Robert, 1988, Irreversible investment, capacity choice, and the value of the firm, *The American Economic Review* 78, 969–985.

Schwartz, Eduardo, 1977, The valuation of warrants: implementing a new approach, *Journal of Financial Economics* 4, 79–94.

Shaked, Avner, and John Sutton, 1982, Relaxing price competition through product differentiation, *The Review of Economic Studies* 49, 3–13.

——— , 1983, Natural oligopolies, *Econometrica* 51, 1469–1483.

Shleifer, Andrei, and Robert Vishny, 2003, Stock market driven aquisitions, *Journal of Financial Economics* 70, 295–311.

Shy, Oz, 1996, *Industrial Organization: Theory and Applications* (The MIT Press: Cambridge, MA).

Tirole, Jean, 1988, *The Theory of Industrial Organization* (MIT Press: Cambridge, MA).

Trigeorgis, Lenos, 1996, *Real options: managerial flexibility and strategy in resource allocation* (The MIT Press: Cambridge, Massachusetts).

Vollert, Alexander, 2003, *A stochastic control framework for real options in strategic evaluation* (Birkhäuser: Boston Basel Berlin).

Weeds, Helen, 2002, Strategic delay in a real options model of R&D competition, *The Review of Economic Studies* 69, 729–747.