POISSON PROCESSES OF CARBONATE ACCUMULATION ON PALEOZOIC AND HOLOCENE PLATFORMS

BRUCE H. WILKINSON[1], CARL N. DRUMMOND[2], NATHANIEL W. DIEDRICH[1], AND EDWARD D. ROTHMAN[3]
[1] *Department of Geological Sciences, University of Michigan, Ann Arbor, Michigan 48109–1063, U.S.A.*
[2] *Department of Geosciences, Indiana–Purdue University Fort Wayne, Fort Wayne, Indiana 46805–1499, U.S.A.*
[3] *Department of Statistics, University of Michigan, Ann Arbor, Michigan 48109–1027, U.S.A.*

ABSTRACT: **Cambro-Ordovician carbonate lithofacies units in the Elbrook and Conococheague formations exposed at Wytheville, Virginia, as well as those in many other Phanerozoic peritidal sequences, exhibit exponential thickness frequency distributions. That is, occurrence frequency decreases exponentially with linear increase in unit thickness. Such distributions are characteristic of waiting times between independent Poisson events. This relative frequency of spaces of different size between horizons of lithologic change is what one would expect if the horizons were distributed randomly throughout carbonate successions. Abundances of different lithologies, both as net stratigraphic thickness and as number of occurrences, also decreases exponentially among successively rare sediment types, with each lithology being about 60% as plentiful as the next more abundant rock type. Relations between net thickness and number of occurrences for each facies define a linear trend coincident with a mean thickness for all Wytheville units of 0.48 m, a relation indicating that thickness distribution is independent of facies type.**

Similar relations are apparent for the horizontal extent of carbonate sediment bodies from the Holocene Florida–Bahama platform. Areal extents of individual facies units (lithotopes) are described by a frequency distribution in agreement with that anticipated for a population of equidimensional facies elements whose diameter distribution follows an exponential frequency distribution. Although regional gradients in sediment texture and composition are also apparent along most transects from platform margin to interior, such frequency distributions indicate that lateral extents of individual sediment reflect a largely stochastic distribution of facies boundaries across this Holocene surface. Lithotope abundances also yield trends of exponentially decreasing dominance among successively subordinate facies, with each being about 70% as extensive the next more abundant sediment type. Relations between areas and abundances for all lithotopes define a covariant trend corresponding to a mean area of 2.2×10^3 km^2 for all Florida–Bahamas lithotopes.

We consider several numerical models of stochastic carbonate accumulation; although not demonstrably unique, scenarios incorporating the sequential superposition of randomly placed coniform lithotopes result in thickness and area frequency distributions that are the same as those observed in ancient and modern platform deposits. Such simulations of Poisson processes of sediment accumulation are in general agreement with stochastic models of lithologic heterogeneity that have been more widely applied to petroleum reservoirs and groundwater aquifers.

To the as yet unknown degree that peritidal lithofacies area and thickness are correlated, data from Paleozoic and Holocene platforms suggest that carbonate units should exhibit length/height ratios of approximately 10^5. Given the decimeter scale over which facies are designated in most Paleozoic peritidal successions, these relations predict mean lateral extents on the order of several tens of kilometers, a value in general agreement with the few data that exist on spatial continuities of peritidal lithotopes in Paleozoic carbonate sequences.

INTRODUCTION

One of the primary objectives of sedimentologic research is to develop links among processes of accumulation in modern settings and the textural, scalar, and/or compositional attributes of rock units in ancient sequences. Interest in relations between different types of sediment encountered across surfaces of accumulation and resultant lithofacies associations in vertical successions has a long history. In a context of lithologic attributes, comparison of modern environments and ancient sequences extends at least to the time of Walther (1894), who observed that ''only those facies and facies areas can be superimposed primarily which can be observed beside each other at the present time'' (e.g., Middleton 1973). Such genetic kinship between modern depositional surfaces and ancient sedimentary sequences has long served as an important philosophical tool in environmental reconstruction (e.g., Visher 1965).

In a context of quantitative assessment of size attributes, however, analogy among areal distributions of various sediment types in modern settings and their resultant thicknesses in ancient successions has received little attention. One of the difficulties in the reconstruction of depositional history is that the areal extent of some sediment type at a depositional surface may not necessarily relate to the temporal persistence of its accumulation. However, such relations might exist. At one extreme, we might presume that sedimentation is slow and continuous and that cessation of accumulation of one type of material is immediately followed by initiation of deposition of another. In this case, areal extents almost certainly would be reflected in the vertical proportion of that type of sediment in resultant sequences. Conversely, we might presume that sedimentation rates were high, that deposition took place during short episodes of intense physical reworking, and that intervals of accumulation were followed by hiatuses of considerable length. In this case, even though fair-weather sediment compositions would be unrelated to the sedimentologic composition of intermingled tempestite units, thicknesses of individual rock bodies or lithosomes might still correlate with lateral extent. The reason for this is that episodes of intense wave and current activity might persist over larger geographic areas and influence surficial sediment to greater depths below the sediment/water interface.

In spite of these rather obvious relations that might exist among the areal dominance of different sediment types at depositional surfaces and the vertical lithologic extent of resultant sedimentary sequences, data on horizontal persistence are difficult to obtain from outcrops and (especially) in the subsurface (e.g., Kerans and Tinker 1996). The broad field of geostatistical modeling (e.g., Yarus and Chambers 1994), and the use of stochastic models for assessing fluid migration, reservoir heterogeneity, and risk analysis, is at least partly derived from uncertainty about spatial distributions of rock properties between wells and/or outcrops (e.g., Srivastava 1994).

It is in this broader context of vertical and lateral continuity of sedimentary rock bodies that we have undertaken a comparison of vertical magnitude/frequency relations among several North American Paleozoic carbonate sequences with analogous areal relations among mappable facies units on modern shallow-water carbonate platforms. In the following we examine thickness-frequency and facies-frequency distributions from a typical lower Paleozoic carbonate succession, and compare these with analogous distributions from the Florida–Bahama platform. Here we are interested in both the number and extent of different sediment types, and in similarities or differences between these parameters in ancient and modern settings.

For ancient settings, abundant data now exist with respect to facies dom-

JOURNAL OF SEDIMENTARY RESEARCH, VOL. 69, NO. 2, MARCH, 1999, P. 338–350
1073-130X/99/069-338/$03.00

inance and recurrence in many Phanerozoic peritidal sequences, and any one of several well-studied successions would serve to illustrate these relations. As one example, we describe thickness–frequency relations from peritidal units in the Elbrook and Conococheague formations exposed at Wytheville, Virginia (e.g., Wilkinson et al. 1998). However, almost identical relations are apparent in data from the Neoproterozoic Rocknest Formation, Northwest Territories, Canada (Grotzinger 1986), in Middle to Upper Cambrian platform carbonates of the Bonanza King Formation in the southern Great Basin of California (Montañez and Osleger 1993), in the Lower Ordovician El Paso Group of West Texas (Goldhammer et al. 1993), in Lower Ordovician carbonates of the Arbuckle Group near Ardmore, Oklahoma (Wilkinson et al. 1997b), in Upper Ordovician storm-dominated Cincinnatian units of central Ohio (Holland et al. 1997), in the Middle and Upper Devonian Lost Burro Formation of Death Valley, California (Yang et al. 1995), in Upper Devonian successions of southern Canadian Rocky Mountains (McLean and Mountjoy 1994), in Middle Pennsylvanian carbonate–siliciclastic deposits of the southwestern Paradox Basin, Utah (Gianniny and Simo 1996), and in Upper Pennsylvanian sequences of the Pedregosa and Orogrande Basins, Ancestral Rocky Mountains (Soreghan 1994).

ORDOVICIAN CARBONATES

Extensive study of Cambrian and Ordovician sequences exposed throughout the south central Appalachians (Koerschner and Read 1989; Osleger and Read 1991, 1993) provides abundant background data on the general nature of early Phanerozoic carbonate accumulation. Among the many outcrops in this region, those of the Elbrook and Conococheague formations at Wytheville, Virginia, are unsurpassed with respect to quality and completeness of exposure.

Lithologically, the Wytheville succession is typical of lower Paleozoic peritidal carbonates and, within this 303.7-meter-thick sequence, we distinguish 630 discrete units consisting of predominantly ''particulate'' rock types. These range from coarse intraclastic and skeletal sand to mud-size carbonate detritus and their dolomitized equivalents. Most units consist of silt- to mud-size material, and many others are typical ''ribbon rock'' lithologies consisting of flaser- to wavy-bedded alternations of suspended load (carbonate mud) and bed load (silt and fine carbonate sand) material (e.g., Demicco 1983). In addition to particulate carbonate, algal boundstone lithologies as thrombolite, stromatolite, and cryptalgal laminite also are present, but less commonly. From our own observations, and as emphasized by Koerschner and Read (1989), the sequence is notable in that paleosols, caliche, karst features, and other evidence of prolonged subaerial exposure are lacking.

Results of embedded Markov-chain analysis (Wilkinson et al. 1997a) indicate that, over lag distances of less than about 100 stratal elements, there is a general absence of high-frequency order among successive lithofacies. Similarly, tabulation of numbers of deep-to-shallow and shallow-to-deep lithologic transitions indicates little tendency for peritidal units to comprise ''upward-shallowing'' associations. Over stratigraphic intervals spanning up to several dozen stratal elements, units within the Wytheville sequence exhibit no more lithologic order than would be present in a succession in which one lithosome was deposited independently of those deposited before and after. Beyond these observations, two aspects of Wytheville units merit more detailed discussion; these relate to the frequencies with which different thicknesses and lithologic abundances recur.

LITHOLOGIC THICKNESSES

One of the more striking aspects of peritidal carbonate sequences in general, and of the Wytheville succession in particular, is that thicknesses of units (delimited at horizons of lithologic change) exhibit frequency distributions where linear increase in unit thickness is accompanied by an exponential decrease in unit abundance (Fig. 1). Such distributions are a necessary (but not necessarily sufficient) condition for intervals between events that occur randomly in space and/or time, such that the occurrence of any unit top is independent of the stratigraphic recency of the preceding top. Durations separating emission of alpha particles from radioactive isotopes, time intervals between major earthquakes, and other distributions of waiting times between independent events are all random or Poisson processes. In these cases, the number of events that occur over a finite interval of time or space has a Poisson distribution, and sizes of intervals between each event exhibit an exponential distribution of waiting-time frequency. The other requirement for the Poisson model to hold is that there be little or no ordering among unit thicknesses. This matter is discussed in somewhat greater detail below.

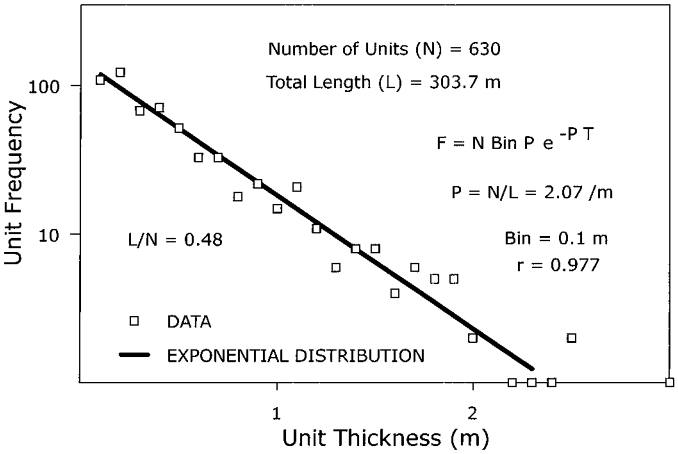

FIG. 1.—Distribution of thickness frequencies of Cambro-Ordovician carbonate lithofacies units in the Elbrook and Conococheague formation exposed at Wytheville, Virginia. The solid line is not a regression; it is the theoretical distribution of intervals between Poisson successes, and is dependent only on number of unit tops ($n = 630$) and the total thickness (303.7 m). Agreement between empirical and theoretical trends suggests that lithologic unit tops occur more or less randomly in this sequence.

In a stratigraphic context, horizons of lithologic change are said to be random if the probability for a depositional event to occur (for a change in lithology to occur) per unit length of section is independent of other depositional events (is independent of the nearness of other horizons of lithologic change). In such cases, lengths of interval (t) between transition horizons give rise to frequency distributions (the exponential distribution) that are dependent only on mean rate of occurrence (e.g., Davis 1986; Swan and Sandilands 1995). In other words, thicknesses of stratal elements in a sequence of length (L) that has been randomly divided into N lithologic units are a function only of L and N. The exponential distribution of lithologic thicknesses resulting from Poisson processes is expressed as

$$F(t) = \text{Bin Size}\frac{N^2}{L}e^{-t(N/L)}$$

Here, the slope of the distribution (N/L) is rate of occurrence of lithologic change per unit length of section, the y intercept (N^2/L) is the product of transition probability and number of transitions, and the reciprocal of the slope (L/N) is mean unit thickness.

This theoretical distribution of independent stratigraphic duration magnitudes is in excellent agreement with that observed for unit thicknesses in the Wytheville section (Fig. 1). Frequencies of occurrence of peritidal thicknesses are dependent only on the number of stratal elements (630) that were identified over this interval (303.7 m). The number of stratal elements in the sequence relative to its total thickness can be thought of as the probability of encountering some lithologic transition per meter of section

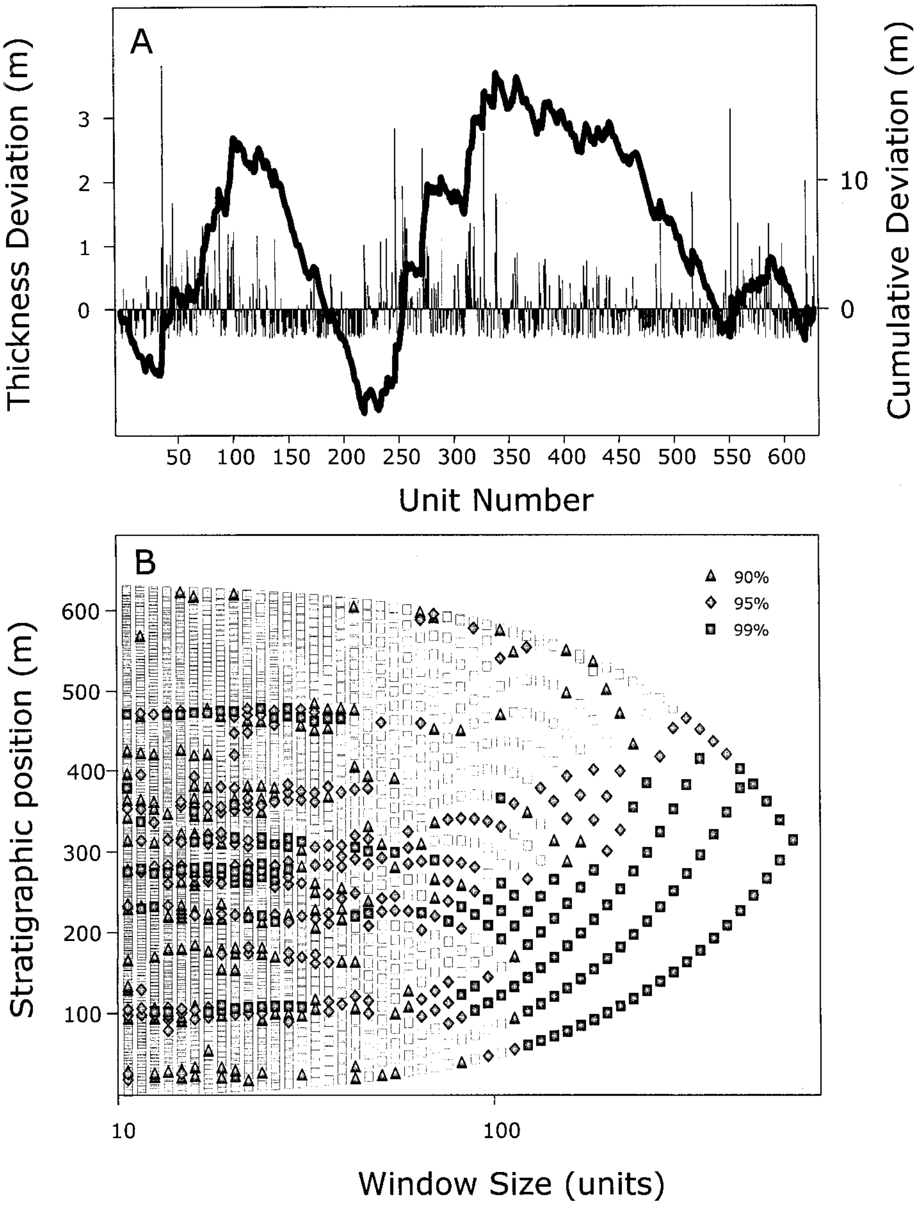

FIG. 2.—Scales of thickness ordering among lithologic units at Wytheville, Virginia. **A)** Thickness deviations (from mean thickness, 48 cm) for 630 lithologic units (vertical bars—left axis) and cumulative deviation (heavy line—right axis) relative to stratigraphic position (horizontal axis; section base to the left). Note several ''cycles'' of change in cumulative deviation, with ascending limbs corresponding to higher frequency of thicker elements. **B)** Comparison of observed (*Oa*) amplitudes of cumulative deviation versus mean amplitudes from 1000 shuffled sequences (*Ra*) relative to stratigraphic interval (horizontal axis). Stippled squares are those intervals where *Oa–Ra* exceeds 3 standard deviations (~ 99%) of randomized amplitudes; diamonds and triangles are intervals where observed amplitudes exceed 95% and 90% of randomized amplitudes, respectively. Note that over intervals smaller than several dozen stratal elements, amplitudes of cumulative deviation fall within the range of amplitudes from randomized successions (open squares).

(P = 2.07/m). It is this parameter that determines the slope of the thickness–frequency relation, as well as the relative difference in numbers of stratal elements contained in adjacent size classes (Fig. 1).

STRATAL ORDER

Purely random segmentation in a stratigraphic sequence requires that the occurrence of any horizon of lithologic change is independent of the recency of any preceding event. Statistically, such accumulation would occur via homogeneous Poisson processes, where similar values of P persist over the entire sequence. Agreement among observed and expected thickness frequencies alone, however, does not preclude the possibility that low-frequency (and perhaps extrabasinal) processes have also had some influence on stratal thicknesses. Long-term variation in rate of sediment generation, subsidence, and/or change in sealevel might well modulate lithologic transition probabilities to the degree that sizes of stratal elements in fact do bear some relation to those of previously deposited units.

Such inhomogeneous Poisson attributes have often been discerned in peritidal sequences through the use of Fischer plots (e.g., Read and Goldhammer 1988), a construction that relates cumulative deviation from mean thickness to stratal element number (Fig. 2A). Regardless of questions concerning their utility for eustatic interpretation (e.g., Drummond and Wilkinson 1992; Sadler et al. 1993; Boss and Rasmussen 1995; Day 1997), such plots are ideal tools for assessing size ordering in peritidal carbonate successions. Because ascending and descending trends of cumulative deviation reflect preferential association of units that are thicker or thinner than average, respectively, amplitudes of Fischer plots afford a useful means to determine scales of statistical nonstationarity among unit thicknesses in peritidal sequences.

To determine sizes of stratigraphic intervals over which unit thicknesses in the Wytheville sequence might have been influenced by longer-term variation in depositional process, it is necessary to compare observed amplitudes of cumulative deviation with amplitudes that would be expected in randomized sequences containing the same lithologic units. Inhomogeneous Poisson accumulation, discernible as some manifestation of thickness ordering, should exhibit amplitudes of cumulative deviation that exceed

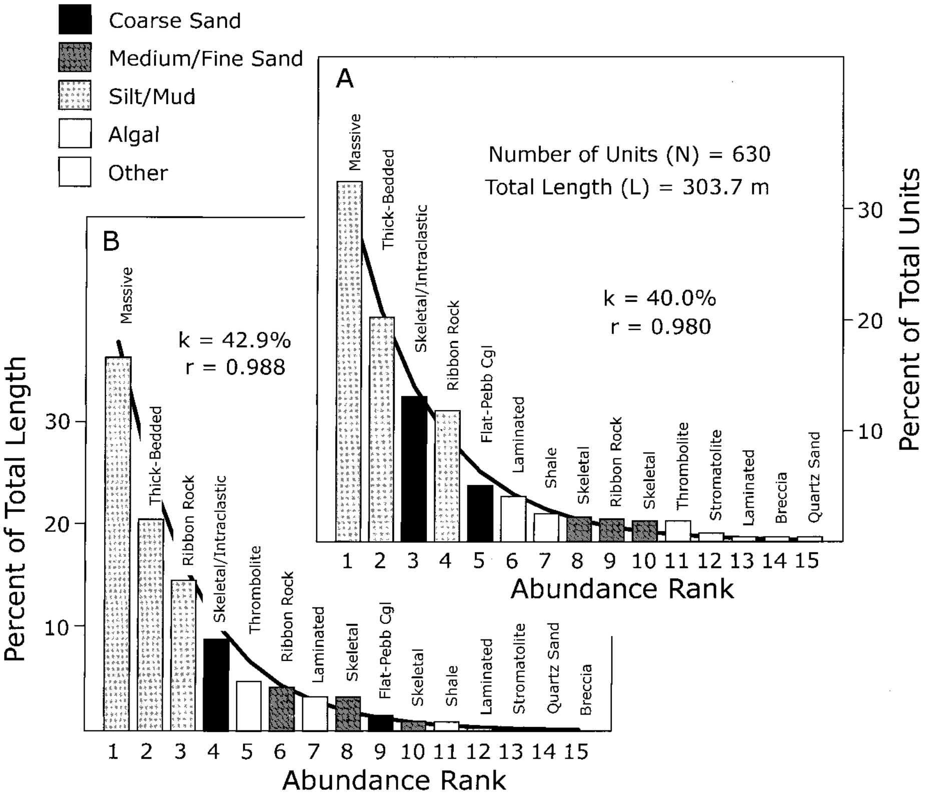

FIG. 3.—Lithologic composition of the Cambro-Ordovician Elbrook and Conococheague Formations at Wytheville, Virginia, expressed as ranked abundance of 15 contained lithologies (*x* axes) relative to: **A)** percent of total units (*y* axis) and **B)** percent of total sequence thickness (*y* axis). Note that massive to thick-bedded carbonate mudstone and siltstone predominates in this succession, and is generally followed in abundance by flasered silt/mudstone and coarser grainstone. Also note that relative to the rank amount (*x* axes), both metrics of lithologic dominance embody distributions of exponentially decreasing abundance with integer decrease in size rank. Relative to immediately larger class sizes, stratal element numbers and section lengths decrease at mean rates of about 40%.

those of randomly shuffled sequences. In addition to determining if size ordering is present, it is also necessary to identify lengths of stratigraphic intervals over which this order might occur. To answer these questions, it is necessary to compare observed amplitudes of cumulative deviation with ranges of amplitudes derived from repeated shuffling of the same sequence, and to do this over a wide range of stratigraphic intervals.

We therefore compare observed and randomized amplitudes of cumulative deviation at window sizes (number of units over which cumulative deviation is determined) from a maximum interval equal to the number of stratal elements in the entire section (630) down to a minimum window of 10 lithologic transitions. For each analysis, we calculate the relative difference (*Rd*) between the amplitude of cumulative deviation observed over some interval (*Oa*) and the mean amplitude of 1000 shuffled sequences (*Ra*) as

$$Rd = \frac{(Oa - Ra)}{Std}$$

where *Std* is the standard deviation of amplitudes from the 1000 randomized intervals. In essence, this metric represents the number of standard deviations separating observed and randomized amplitudes of cumulative deviation. Values of *Rd* can then be plotted as points in space, where the *x* axis is the size of the window being evaluated and the *y* axis is the stratigraphic midpoint of that particular interval under consideration (Fig. 2B).

Moreover, the succession at Wytheville contains 630 lithologic elements (*N*). By considering all possible window sizes down to a minimum window length of say 10 elements (*M*), and shifting each window by one element per calculation, it is theoretically possible to determine $(N - M)(N - M + 1)/2$, or 192,510 values of *Rd*. Because each evaluation also requires repeated shuffling, we have used a logarithmic scale of window sizes and lags or offsets to expedite analysis. This scaling was adjusted to arrive at 2155 values of *Rd* (Fig. 2B).

Using this methodology it becomes apparent that Wytheville thicknesses indeed do exhibit some degree of statistical nonstationarity. Amplitudes of the several low-frequency ''cycles'' of thickness variation apparent in the Fischer plot for the entire sequence (Fig. 2A) exceed the range of amplitudes from randomized intervals at 99% confidence limits. It is reasonable to conclude that these trends may indeed reflect some long-term variation in sedimentation, subsidence, and/or sealevel. Conversely, it is also apparent that, over intervals smaller than about 100 stratal elements, amplitudes of cumulative deviation largely fall within the range of amplitudes anticipated for reshuffled successions. At these scales of consideration, and as suggested by the unit thickness-frequency distribution, unit tops in the Wytheville sequence are indistinguishable from those that would occur randomly.

LITHOLOGIC ABUNDANCES

In addition to unit thicknesses, relative abundances of different carbonate rock types at Wytheville also exhibit a predictable pattern. Each of the 15 lithologies designated in the Wytheville sequence can be rank ordered either with respect to relative numbers of occurrence (Fig. 3A) or with respect to the relative thickness represented by each rock type (Fig. 3B). When either measure of stratigraphic abundance is represented in this way, the abundance of any particular lithology (*A*) relative to its rank (*r*) is approximated as

$$A(r) = e^{-kr}$$

where *k* is the slope of the abundance relation. As with thickness frequencies, this trend represents the proportional difference in numbers of stratal elements contained in adjacent classes, but here defined on the basis of rock texture and composition. With respect to lithologic abundances in this Elbrook/Conococheague sequence, each group contains 40.0% fewer elements (Fig. 3A) and is 42.9% less thick (Fig. 3B) than the next larger class of lithologic units. This proportional difference in lithologic abundance is the same as the relative difference observed for lithologic sizes (Fig. 1).

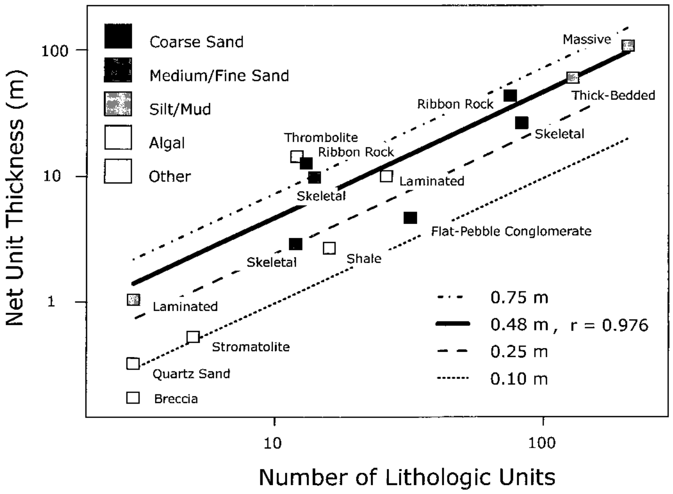

FIG. 4.—Distribution of numbers of lithologic units (x axis) in the Wytheville section relative to the net thickness of each rock type (y axis). Diagonal lines represent several values of unit thickness. Good agreement exists between measured and anticipated thicknesses along the solid line at 0.48 m ($r = 0.98$). This is the mean thickness of all stratal elements, and coincides almost exactly with the least-squares regression through the data.

Comparison of measures of net lithologic abundance (number of occurrences) with net size (total thickness) discloses little relation among composition and unit thickness (Fig. 4). With the exception of a few quartz sand units and rare thin stratiform breccias (which represent secondary modification of preexisting muddy lithologies), relations between net numbers of units and net thicknesses of each rock type follow those anticipated from thickness/abundance relations for the entire sequence. Net thickness for most specific rock types is in agreement with products of lithologic abundance and 48 cm, the average thickness of all units in the succession (Fig. 4). Coarse skeletal sand, for example, is about 28 times more abundant than laminated calcisiltite and mudstone, and comprises a net thickness that is about 26 times as great. Although these lithologies represent opposite ends of the grain-size continuum at Wytheville, and even though they record carbonate accumulation under quite dissimilar current regimes, mean thickness of coarse skeletal sand (0.33 m) is about that of calcisiltite and mudstone (0.36 m). In other words, subsamples drawn from the Virginia succession by lithofacies composition could equally well be random samples of interval thickness. Even though thicknesses of peritidal units at Wytheville reflect the influence of Poisson processes of sediment accumulation, these operated with similar variation in intensity and/or duration on a wide spectrum of sediment grain sizes across the Cambro-Ordovician platform.

HOLOCENE PLATFORMS

Because patterns of lithologic thickness and recurrence frequency apparent in the Wytheville sequence likely bear some relation to the sedimentologic make-up of the two-dimensional Cambro-Ordovician platform surface on which accumulation occurred, we became curious about number and extent of areas of differing sediment composition across modern platform settings. Following Adams and Grotzinger (1996), we envisage peritidal depositional surfaces as perhaps comprising a complex ''mosaic of facies'', with numbers and areal extents of individual elements or lithotopes being analogous to abundances and thicknesses of lithologic units in ancient successions. Like rock sequences, which exhibit lithologic variation in a vertical one-dimensional linear/temporal sense, modern surfaces of carbonate accumulation exhibit two-dimensional compositional heterogeneity in a lateral or spatial regard. Despite this difference in geometry, areas of modern carbonate accumulation are equally amenable to analysis of sizes and recurrences of different sediment types.

We therefore selected the Map of Surface Facies of the Florida–Bahama Plateau (Enos 1974) to tabulate the areal distribution and abundance of various facies types. Compiled from 18 original data sources, the map shows surface sediment compositions over an area of some 723 × 10^3 km^2. Map units are classified on the basis of depositional texture and dominant grain type following the scheme of Dunham (1962). Numbers and areas of each designated region of sediment homogeneity were measured from a scanned image. Of the entire region, some 32.7% consists of unmapped areas and/or areas of subaerially exposed Pleistocene and older bedrock. In addition, about 38.4% lies beyond ''shelf'' limits at water depths in excess of several hundred meters. Although mapped as ''skeletal wackestone'', it is clear that sediment variability at much of these depths is poorly documented. Both regions (71.1% of the map) were therefore excluded. An additional 1.7% of the region, primarily in south Florida, is covered by nonmarine deposits of freshwater marl and/or mangrove peat, and were also excluded from the tabulation. The remaining mapped region (27.2%) of shelf carbonate accumulation includes most of the south Florida shelf, as well as equivalent regions on the Great Bahama Bank, the Little Bahama Bank, and the Gulf of Batabano.

It should be noted that sediment over some of this area represents a thin Holocene veneer on the lithified surface of Pleistocene and older units. Moreover, other areas are covered by what are undoubtedly relict deposits that formed and accumulated during earlier stages of the Holocene evolution of the Florida–Bahama Plateau (e.g., Boss 1996). In spite of these caveats, Holocene platform carbonates still encompass a region of some 197 × 10^3 km^2. This is an area about the size of Nebraska, Minnesota, or South Dakota, and thus affords a large-scale opportunity to examine several aspects of a facies mosaic in a classic area of shallow-water carbonate accumulation.

Unlike predominantly muddy units at Wytheville, most Holocene surface sediment in the south Florida–Bahamas region is composed of coarser particulate carbonate debris, with skeletal/peloidal/oolitic grainstones and packstones making up 66% of total facies occurrences and 78% of the total area. Because most platform-margin and platform-interior coralgal reefs are located on this map with an asterisk (*), areas of reefal boundstone do not appear as discrete sediment types in this tabulation. Among various units in this facies mosaic, skeletal and oolitic grainstones predominate along platform margins, and give way to skeletal and peloidal packstones and wackestones toward and across platform interiors.

FACIES AREAS

One of the most interesting aspects of sediment distributions on Florida–Bahama platform is that size frequencies of individual mosaic elements closely approximate those that would result from the random segmentation of the platform surface into a number of subregions, each with a homogeneous sediment cover (Fig. 5). Specifically, if we assume a generally equidimensional geometry for each of the 90 lithotopes that occur across this platform surface, then frequency of occurrence (F) of a facies element of area (a) is approximated by the relation

$$F(a) = N\,\mathrm{Bin}\frac{2P}{\pi\sqrt{\dfrac{4A}{\pi}}}e^{-P\sqrt{4A/\pi}}$$

where N (90) is the total number of elements and P is the rate of lithotope occurrence per unit area of platform surface. With respect to such a mosaic, P is expressed as

$$P = \sqrt{\frac{N^{\pi}}{2T}}$$

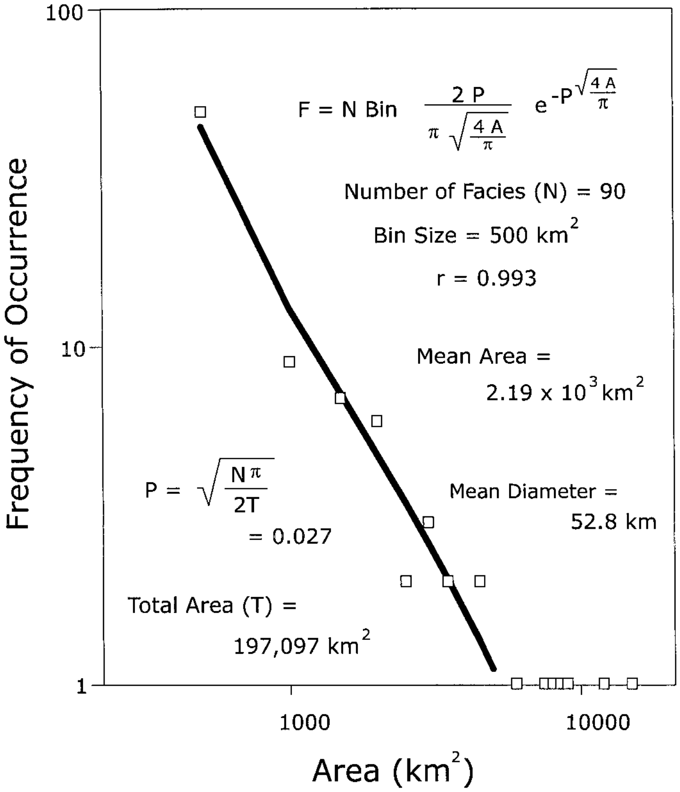

FIG. 5.—Size frequencies of lithotopes in the Florida–Bahamas region. The solid line is not a regression; it is the ideal distribution of Poisson size frequencies where size is calculated assuming each facies element is generally equidimensional; it is the distribution that would result from a population of facies elements with randomly delimited diameters. Note that diameters (and areas) of such elements are dependent only on number of designated units (90) and total mapped area of the Florida–Bahama platform.

where T is total platform area (197 × 10^3 km^2). Here, P is conceptually equivalent to the slope (N/L) of the exponential distribution of lithologic thicknesses (e.g., Fig. 1).

Importantly, this distribution of areas fits that anticipated for a population of equidimensional (e.g., square, circular) shapes, lengths or diameters are characterized by an exponential distribution of element sizes. In other words, frequencies of occurrence of peritidal facies areas across the Florida–Bahama platform (Fig. 5) approximate the density function for a mosaic of equidimensional elements whose linear sizes describe an exponential distribution (e.g., Fig. 1). Area frequency of facies elements on the Florida–Bahama platform is primarily dependent on the number of lithotopes (90) that were designated over this area (197,097 km^2). By analogy with the Wytheville sequence, such predictability of area abundances suggests that instances of lateral sediment change (which ultimately define lithotope area) occur independently, such that the position of one lithotope boundary is independent of the proximity of others. Perhaps more importantly, data from the Florida–Bahamas region suggest that, for any total area and specified number of facies elements across some surface of accumulation, we can predict frequencies of occurrence and lateral extents of mosaic elements.

It is also apparent from the Enos (1974) map that the general shape and sedimentological makeup of individual lithotopes on the Florida–Bahama platform also bears some relation to regional-scale variation in proximity to shelf margins. At a scale of entire platform surfaces, it therefore seems reasonable to conclude that patterns of sediment accumulation may indeed reflect regional variation in water depths and/or current intensities during sediment accumulation. Such scales of lateral nonstationarity are analogous to section-scale changes in lithofacies thicknesses observed in the Wytheville section. Conversely, predictability of lithotope size–frequency trends from total number and total area of sediment bodies on the platform also attests to an important component of stochastic carbonate accumulation. At more localized scales of consideration, and as suggested by the lithotope area-frequency distribution, lithotope boundaries across the Florida–Bahama platform surface are equivalent to those that would occur randomly.

FACIES ABUNDANCES

In addition to area frequency distributions, different carbonate sediment types on the Florida–Bahama platform exhibit predictable differences in relative abundance. It is, of course, a straightforward exercise to rank order each of the 12 facies types designated by Enos (1974) either with respect to relative number of occurrences (Fig. 6A), or with respect to the relative area represented by each sediment type (Fig. 6B). When so ordered, abundance-versus-rank relations among these lithotopes are the same as observed with respect to lithologic elements at Wytheville, Virginia (e.g., Fig. 3); change in abundance is accompanied by a relatively invariant proportional difference in numbers and areas of facies elements in adjacent classes. With respect to abundances of different mosaic elements of Florida–Bahamas peritidal sediment, each subjacent facies group contains 28.1% fewer elements (Fig. 6A) and covers 38.5% less area (Fig. 6B) than units in the next larger class.

Also like lithologies at Wytheville, comparison of sediment dominance as reflected by number of lithotope occurrences with total platform area occupied by that type of facies yields a linear relation for the two (Fig. 7). Except for the rare occurrence of quartz sand and gypsum units, net area of each facies type is in general agreement with products of facies abundance and a mean areal extent of 2.19 × 10^3 km^2.

DISCUSSION

Lithologic data from Cambro-Ordovician peritidal exposures at Wytheville, Virginia (and many other Paleozoic epicratonic sequences) are similar to lithotope data from the Holocene Florida–Bahama platform. Some manifestation of sectional/regional scale order among facies size and/or composition relations is present in both systems. However, frequency distributions from both indicate that dominant depositional processes have served to randomly segment ancient successions and modern platform surfaces with respect to the recurrence of various sizes and types of sedimentary units. Both reflect a dominance of inhomogeneous Poisson processes in time and space during peritidal carbonate accumulation.

Size-Frequency Distributions

Frequency distributions of stratal elements at Wytheville and mosaic elements in the Florida–Bahamas region describe trends where recurrence frequency is dependent on number of designated units and the total size (length or area) of the system. In the case of Paleozoic sequences, frequencies of vertical lithologic extent are predictable from number of designated units and total sequence length (Fig. 1) while, in the case of Holocene platforms, lateral lithotope extents are predictable from number of designated units and mapped platform area (Fig. 5). In the latter case, areal extent in two dimensions is equivalent to linear extent in a one-dimensional transect. Size frequencies are the same as those anticipated from the random distribution of unit tops throughout vertical successions or the independent distribution of lithotope boundaries across surfaces of accumulation. The nature of the genetic link between the two depends on persistence of sedimentation and completeness of sedimentary successions. If deposition rate is invariant, cessation of accumulation of one type of sediment and initiation of accumulation of another would correspond to that instant when some lithotope boundary would migrate across some sectional locale of

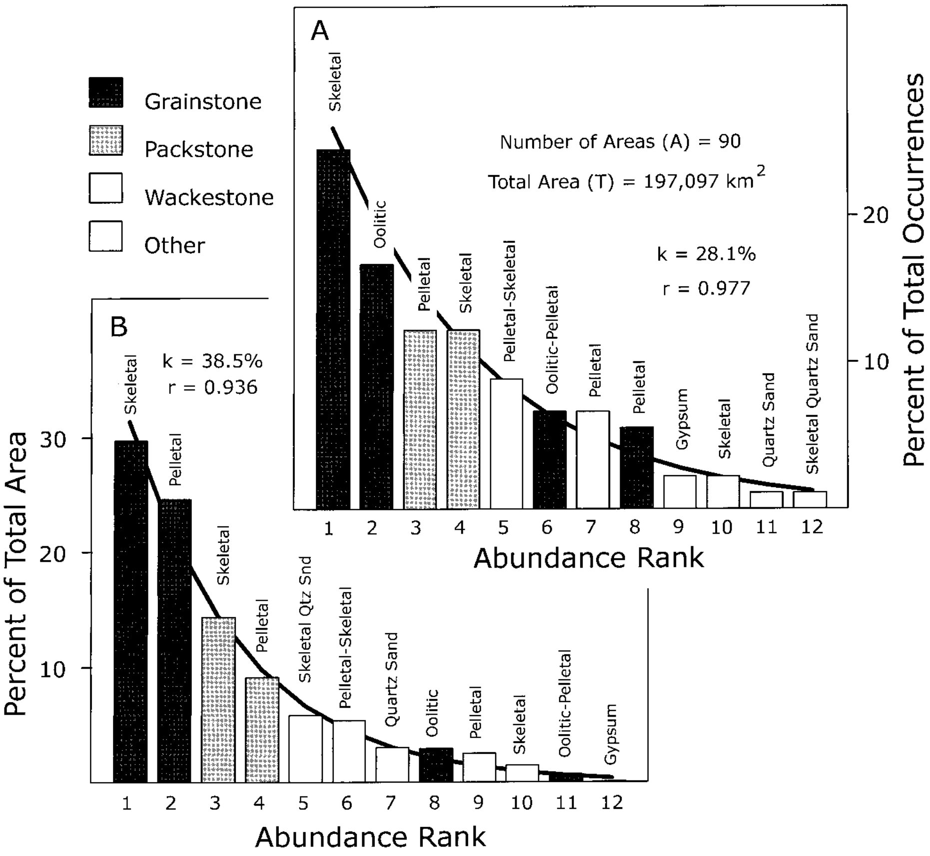

FIG. 6.—Composition of shallow-water sediment across the south Florida–Bahama platform. Ranked abundances of the 12 dominant facies (*x* axes) relative to **A)** percent of total number of facies occurrences (*y* axis) and, **B)** percent of total platform area (*y* axis). Skeletal, oolitic, and peloidal grainstone predominates in this region, and is followed in abundance by more mud-rich packstone and wackestone. Abundance rank (*x* axes) for both measures yield exponential distributions of sedimentologic dominance where, relative to superjacent class sizes, facies element numbers and areas decrease at mean rates of about 33%.

accumulation. Larger lithotopes would therefore give rise to thicker lithofacies. Conversely, if sedimentation is more catastrophic, intervals of more intense reworking and sediment transport would operate over larger geographic areas and to greater depths below the sediment/water interface. Larger lithotopes would still correspond to thicker lithofacies.

However, it is one thing to determine that occurrences of vertical and lateral facies boundaries are Poisson, and yet another to relate patterns of thickness/area frequency to the exact countenance of depositional surfaces. Nevertheless, a reasonable approach to the problem is to devise conceptual models of stochastic accumulation, and then to evaluate these with respect to their similarity to patterns of carbonate accumulation anticipated from examination of Phanerozoic sequences and Holocene platforms. Acceptable depositional models must yield area frequencies similar to that observed in the Florida–Bahamas region. Moreover, because of lithotope area and lithofacies thickness relations noted above, transects across model platform surfaces should yield exponential distributions of chords across each facies type and, by inference, exponential distributions of lithologic thicknesses.

Numerical conceptualizations of facies mosaics have a sparse but interesting history. Tipper (1990), for example, considered the relation between several types of facies mosaics and their anticipated impact on the lateral persistence of lithologic units. Lin and Harbaugh (1984) discussed ramifications of short-memory random processes in two dimensions, and present several numerical simulations of map surfaces derived assuming that patterns of lateral change were largely Markovian. In a similar vein, Moss (1990) considered vertical sequences of lithologic transitions in upper Mesozoic terrigenous successions of the North Sea to reflect the dominant influence of Markov processes, and proceeded to derive complete numerical representations of lithologic distributions in three dimensions. Carle et al. (1998) used a comparable approach to model three-dimensional transitions among Neogene fluvial-fan systems in California. Although analogous techniques are gaining wider appreciation as tools for reservoir modeling (e.g., Tyler et al. 1994; Hatloy 1994), such simulations have not yet been specifically applied to, or calibrated with, size-frequency distributions from peritidal carbonate platforms.

What should scenarios of stochastic carbonate accumulation look like? Conceptually, we might start by thinking of distributions of sediment types on two-dimensional depositional surfaces as being somewhat analogous to the distribution of darts on a board when tossed by very poor players. The specific location of each dart would be random in space because its location would be unrelated to the proximity of its neighbors. Such a distribution would be intermediate between that of a statistically uniform distribution where points are evenly distributed and a distribution anticipated for better players where dart clustering might occur relative to some specific area on the board. Is the distribution of facies elements on the Holocene Florida–Bahama platform similar to that anticipated for such a distribution, and do transects across such a surface yield exponential distributions of length frequency such as those observed for lithologic thicknesses in Phanerozoic successions?

Randomly Distributed Points.—Although several scenarios for the random partitioning of a depositional surface are geologically reasonable, one might presume that the distribution of facies is analogous to that of randomly scattered points, with each point being surrounded by an irregular polygon containing an area of homogeneous sediment composition. Sedimentologically, such a surface might represent the two-dimensional distribution of regions of different depositional processes. These might include laterally discontinuous carbonate mud accumulation under sea grasses, carbonate sand deposition in shallower wave-influenced regions, reef growth along platform margins, patchy bioherm development across the platform, and/or cyanobacterially mediated deposition along restricted parts of the platform interior. Regardless of the presence or absence of platform-scale

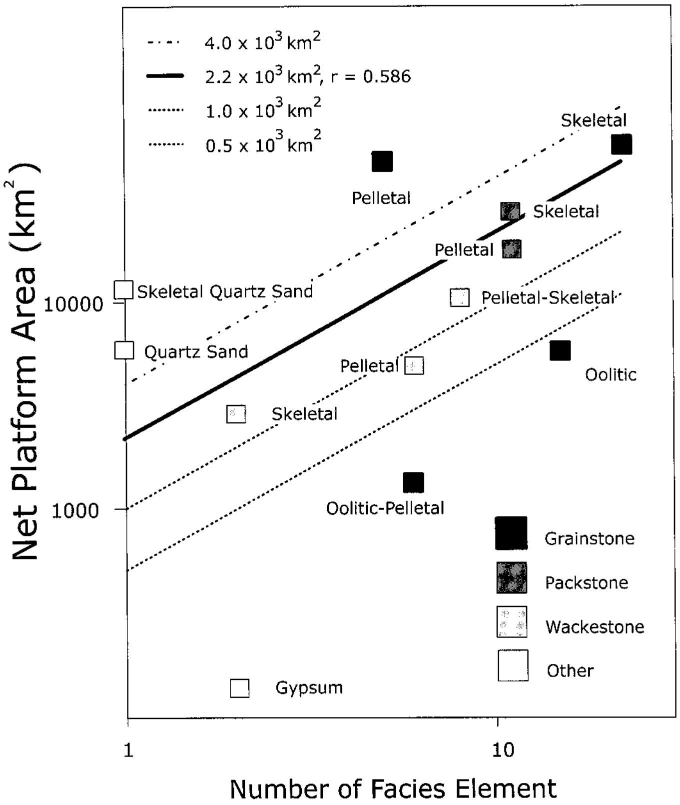

FIG. 7.—Distribution of numbers of facies elements (x axis) in the Florida–Bahamas region relative to net platform area occupied by each sediment type (y axis). Diagonal lines represent several values of lithotope area. Note general agreement between measured areas and those anticipated from a mean element area of 2.2×10^3 km² (heavy solid line). This is the mean area of all lithotopes, and coincides with a least-squares regression through the data ($r = 0.586$).

gradients in lithotope size and/or composition, such distributions might comprise a facies mosaic where the center of each element (and hence its areal extent) is independent of the proximity of adjacent element centers.

An array of points surrounded by irregular polygons (such that all area in each polygon is closer to the enclosed point than to any other point in the array) represents the most compact possible division of space. Such constructions are referred to as Thiessen polygons (e.g., Thiessen and Alter 1911), and are derived from an array of Delaunay triangles, which are triangles that connect three points on a surface such that no other point falls within that circle. Thiessen polygons are formed from the intersection of perpendicular bisectors of Delaunay triangle sides, and a Thiessen polygon surrounding some point therefore bisects all Delaunay lines radiating from that point (Fig. 8). Networks of such polygons, commonly referred to as Voronoi diagrams, have found wide application in many fields (e.g., Okabe et al. 1992), including the Earth sciences (e.g., Fohlmeister 1994; Tipper 1995; Tsai 1993; Gold 1989). Intersections of soap bubbles form an observable network of Voronoi polyhedra (e.g., Davis 1986).

The important questions with respect to such a distribution are: (1) do transects across such a surface (e.g., Fig. 8) yield exponential distributions of facies chord lengths like that observed at Wytheville (e.g., Fig. 1)?, and (2) do area/frequency relations for such a surface resemble that observed on the Florida–Bahama platform (e.g., Fig. 5)? They do not. Calculation of scalar attributes of Thiessen polygons enclosing randomly distributed points yields size frequencies that are dissimilar from those observed at Wytheville and on the Florida–Bahama platform. Lengths of chords to polyhedra from vertical and lateral transects yield skewed frequency distributions (e.g., Fig. 9A), and calculation of polygon areas yields a similarly skewed distribution of size frequencies (Fig. 9B). On the basis of these comparisons, Voronoi diagrams derived from randomly located points appear to be rather imperfect representations of facies mosaics in shallow water carbonate environments.

In retrospect, this may not be surprising. If points are distributed such that they occur randomly along an x–x' or y–y' continuum (where the number of points lying along any unit length of latitudinal or longitudinal transect have a Poisson distribution), then interval frequencies are exponential. However, length spans across, and areas within, Thiessen polygons

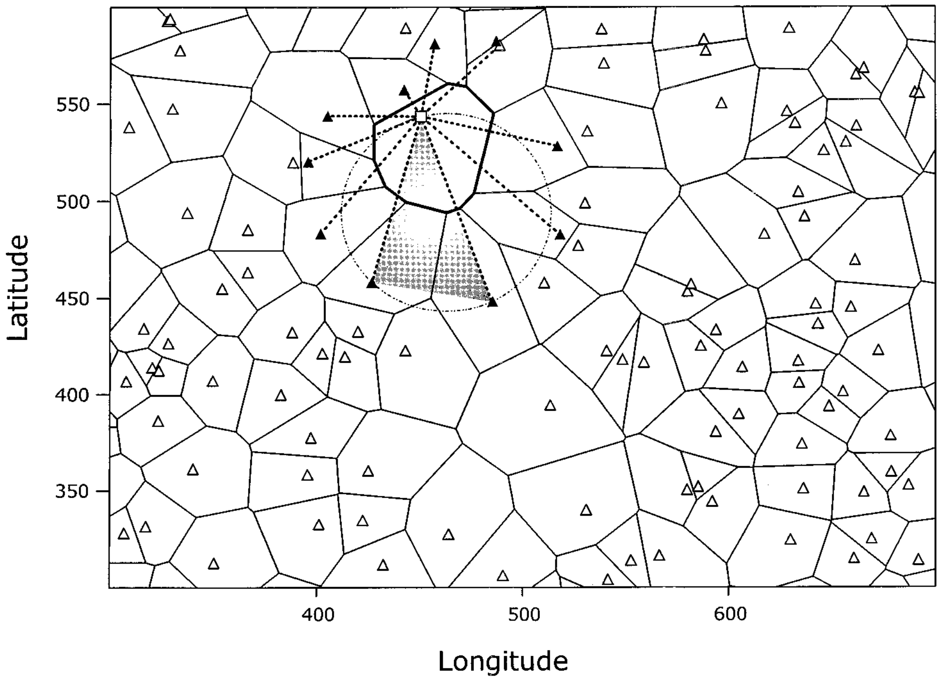

FIG. 8.—Voronoi diagram of Thiessen polygons surrounding 108 randomly distributed points (triangles). As illustration of construction, the ten-sided Thiessen polyhedron (thick line) around the open square was derived from perpendicular bisectors of heavy dashed lines that connect to Thiessen neighbors. The circle encompasses a Delaunay triangle connecting two (of ten) lines to Thiessen neighbors. Note regions of apparent (but spurious) order across this region manifest as domains of higher and lower polygon density.

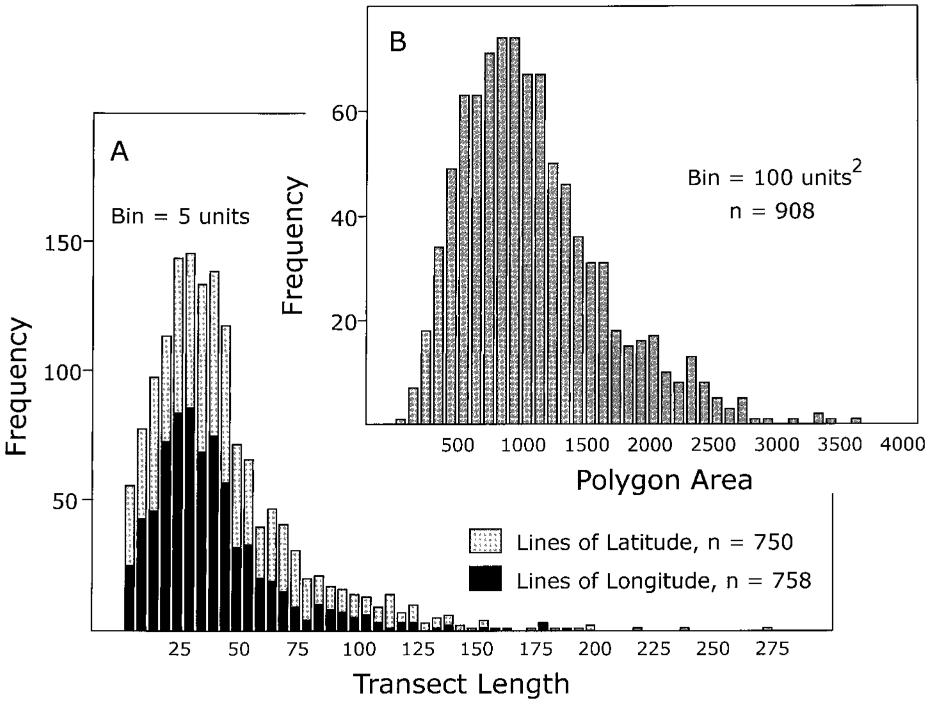

FIG. 9.—Frequency distribution of transect lengths and areas for Thiessen polygons around 908 points that were randomly distributed in *x–y* space (e.g., Fig. 8). **A)** Lengths of polygon transects determined at every 50 units of latitude (lightly stippled) and longitude (densely stippled). **B)** Areas of Thiessen polygons. Note in A and B that both measures of population ''size'' comprise somewhat skewed distributions that are quite dissimilar from thicknesses distributions apparent in the Wytheville sequence (Fig. 1), and from area distributions on the Florida–Bahama platform (Fig. 5).

are dependent not only on the occurrence of the next point in a transect, but also on the *x–y* positions of several nearest-neighbor points. The net result of this dependence is that polyhedral transect lengths and areas are determined by not one but an average of distances between some point and several reference neighbors and, with increasing number of neighbors, distance and/or area distributions become increasingly symmetrical about some mean value.

Randomly Distributed Lithotopes.—If areas surrounding unordered points are inadequate descriptors of peritidal thickness/area frequencies, what other sorts of stochastic models might replicate these thickness/area distributions? We have arrived at one scenario of carbonate deposition that, although almost certainly not unique, incorporates both spatial and temporal aspects of stochastic accumulation in peritidal settings. It constitutes a series of randomly placed and successively accumulated carbonate sediment bodies.

In a spatial context, lithotopes are randomly selected from a theoretical population of units that are equidimensional in map view, and exhibit an exponential distribution of diameter frequencies. Our rationale for using such a population of sizes was of course influenced by the exponential distribution of unit thicknesses at Wytheville, and on conjecture that analogous vertical dimensions may arise from an equivalently scaled suite of lateral dimensions on shelf surfaces of accumulation. Moreover, available data on Holocene peritidal carbonate sedimentation rates (e.g., Sadler 1981) are on the order of a few hundred meters per million years, while rates from most Phanerozoic peritidal platform successions is on the order of a few tens of meters per million years (e.g., Bosscher and Schlager 1993). This difference among short-term and long-term carbonate accumulation rates suggests that interstratal hiatuses may account for perhaps 80–90% of net time (e.g., Sadler 1994). It logically follows that most peritidal successions should therefore reflect relatively abbreviated time intervals of carbonate accumulation and relatively protracted durations at hiatal horizons of nondeposition. If this is indeed the case, it also seems likely that such vertical heterogeneity of time distribution might also exist in lateral dimensions as well. In other words, in a vertical stratigraphic sense we know that time is variably partitioned between lithologic units and interunit lacunae. This model of episodic platformal accumulation merely presumes an equivalent distribution of time and sediment in lateral dimensions such that only a small fraction of the depositional surface is underlain by recently deposited material, and most is surfaced by relict material from some earlier episode of accumulation.

A similar spatial discontinuity of deposition is evident across some Holocene platform surfaces. For example, even though some parts of the ''modern'' Bahamian surface are underlain by Holocene units in excess of 8 meters in thickness, other large regions are covered by relict sediment, mainly as early Holocene palimpsest sand (e.g., Hine 1983). Still other areas consist of older bedrock and/or Pleistocene paleokarst/drainage patterns that are only thinly veneered with more recent carbonate material (e.g., Boss 1996). In spite of this variability in sediment thickness, mean Holocene accumulation rates across most of the Florida–Bahama platform are still several orders of magnitude higher than epoch-duration rates reported for most older peritidal sequences (e.g., Sadler 1981). It is therefore likely that many (if not most) surfaces of epicratonic carbonate accumulation were only in part veneered with recently generated material, and in part (if not largely) covered with palimpsest sediment residual from some earlier interval of sedimentation. Therefore, and in the absence of compelling data to the contrary, we distribute this population of facies units randomly (in *x–y* space) over the depositional surface, such that carbonate material at the sediment/water interface always comprises a range of recently deposited and relict material (Fig. 10).

Do transects across such a surface yield exponential distributions of lithotope length frequency such as that observed at Wytheville, and to areas of different sediment type exposed at the sediment–water interface result in area/frequency relations such as that observed across the Florida–Bahama platform? This seems to be the case. Calculation of distances separating intercepts of lithotope boundaries in long transects across such surfaces of accumulation yield exponential distributions of intercept separations (Fig. 11A), and lithotope area frequencies (Fig. 11B) that are almost identical to that exhibited by surficial units on the Florida–Bahama platform (Fig. 5).

Although other scenarios of peritidal carbonate accumulation might result in equally attractive models, superposition of randomly distributed elements drawn from an exponential distribution of lithotope sizes yields

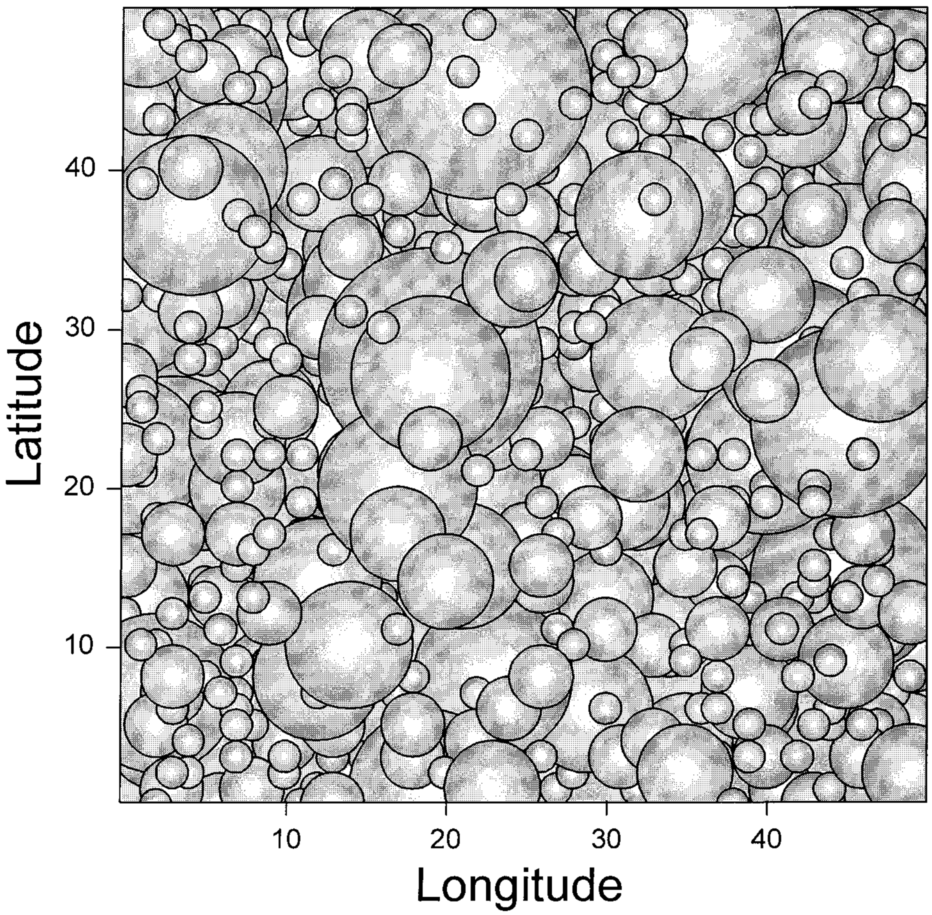

FIG. 10.—Spatial distribution of 394 circular lithotopes drawn from a population exhibiting an exponential distribution of size frequencies. Frequency (*F*) of lithotope radius (*r*) is 333 e-0.666*r*. Area centers are distributed randomly in *x*–*y* space. Mean radius is 1.86 units.

both length and area frequency distributions that are in good agreement with those inferred from Paleozoic sequences and Holocene platforms, respectively. On the basis of this agreement, we suggest that many surfaces of Phanerozoic carbonate accumulation may have comprised a suite of equidimensional facies mosaic elements that also exhibited a consequential degree of heterogeneity with respect to both age and spatial position.

Lithologic Frequency Distributions

Origins of exponential distributions of lithologic abundances at Wytheville (Fig. 3) and on the Florida–Bahama platform (Fig. 6) are more problematic, and analogy with exponential thickness frequencies from Paleozoic sequences (e.g., Fig. 1) provides limited insight as to the origin(s) of this pattern. Recall that the frequency distribution of interval thicknesses for randomly occurring lithologic transitions is dependent on rate of transition occurrence; that is, on the number of unit tops occurring in some length of section. By analogy, exponential distributions of lithologic abundances might (at least theoretically) also be perceived as an artifact of randomly dividing a lithologic continuum of grain sizes and/or rock textures into a series of discrete lithologic types. Imagine a continuum of equally represented carbonate grain sizes, ranging from gravel intraclasts to the finest micrite. If we arbitrarily partitioned this continuum into about a dozen facies, would the relative abundance in each division be similar to that of thickness intervals in a randomly partitioned stratigraphic sequence?

By definition, it would, but it does not then follow that exponential frequencies of lithologic abundance must therefore reflect the influence of some haphazard methodology in our collective discrimination of different types of carbonate rock. Because exponential distributions of lithologic abundance are apparent in all the Paleozoic sequences that we have examined, and because these represent categorization of unrelated rock sequences by dozens of individuals, we feel it is more likely that these patterns indeed do reflect real patterns of lithologic abundances in peritidal settings.

If so, why is it that this pattern of sediment abundance predominates in data from modern and ancient carbonate sequences? It should be noted that, unlike thickness frequencies, which exhibit exponentially decreasing occurrence with linearly increasing size, lithologic frequencies reflect a trend of exponentially decreasing occurrence with increasing abundance rank. Whereas the former represents abundance as a function of stratal thickness, the latter does not. Moreover, it is also evident in data from both Wytheville and the Florida–Bahamas region that the modal sediment size in each system probably lies somewhere in the middle of the range of textural classes represented in each population of facies units. At Wytheville, for example (Fig. 3), massive to thick-bedded silt and mud (and their dolomitized equivalents) predominate both with respect to numbers and with respect to net thickness of identified units. However, it is apparent that several categories of both finer and coarser material, representing a range of lower- and higher-energy settings, occur less abundantly than massive mud, and therefore suggest some environmental control on the dominance of this particular grade of carbonate sediment. Also recall that the Wytheville succession exhibits little evidence that different sediment types also represent different element thicknesses. Abundance/length relations (Fig. 4) reflect similar thickness distributions among all lithologic categories.

Conversely, sedimentary units on the Florida–Bahama platform (Fig. 6) are dominated numerically and areally by skeletal, pelletal, and oolitic grainstone. In south Florida, these pass landward into muddier and more

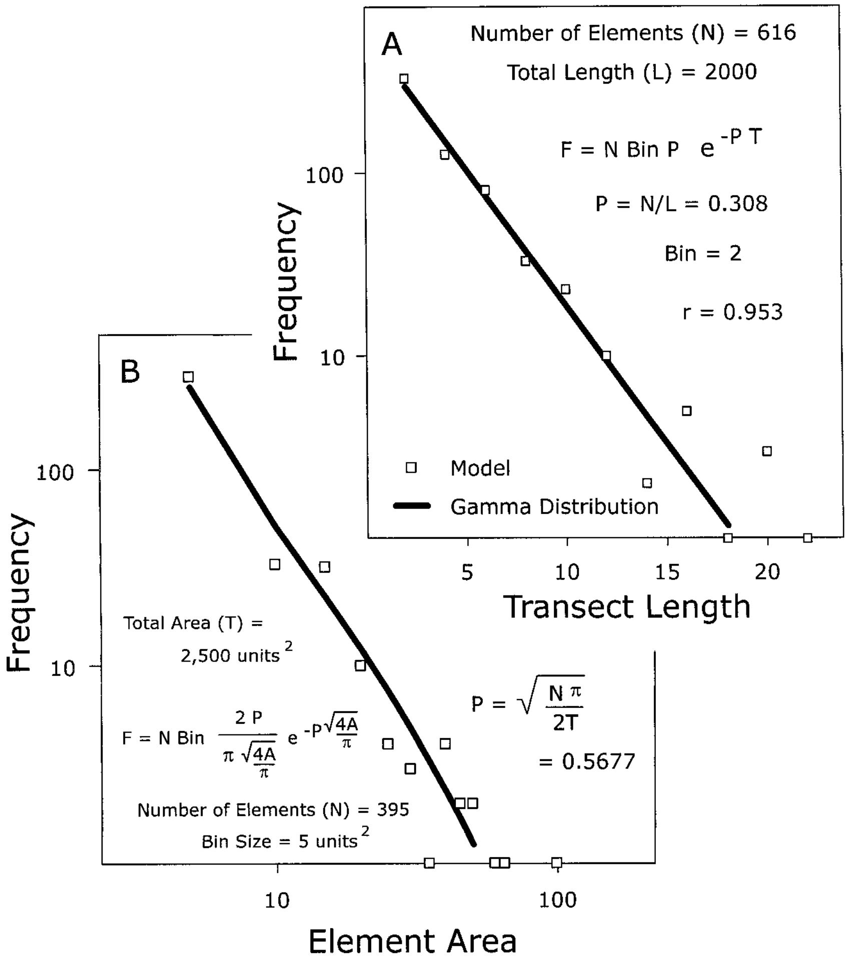

FIG. 11.—Transect lengths and areas from superposition of randomly distributed equidimensional facies elements. **A)** Lengths of 616 transects encountered along a line of 2000 units in total length across a mosaic of lithotopes (e.g., Fig. 10). These exhibit an exponential frequency distribution in excellent agreement with that anticipated for distances between 616 points occurring randomly along a transect of 2000 linear units (solid line), and with element thicknesses observed at Wytheville, VA (Fig. 1). **B)** Areas of the ''exposed'' elements in Figure 10. Note that these exhibit a frequency distribution in excellent agreement with that anticipated for a population of lithotopes whose dimensions comprise an exponential frequency of element diameters (solid line), and with facies areas observed on the Florida–Bahama platform (Fig. 5).

terrigenous lithotopes, while in the Bahamas, packstone and wackestone predominates toward the platform interior. In this case, environmental setting is predicating a dominance of coarser grainstone along platform margins. Size/abundance attributes among these lithotopes also exhibit little indication that different sediment types (e.g., Fig. 7) exhibit different occurrence/area relations across the Florida–Bahamas region.

If it is reasonable to suggest that environmental parameters determine the types of sediment that accumulated in ancient and modern settings, how then might exponential distributions of lithologic abundance be interpreted? Qualitatively, it is possible to reorder data from each setting into symmetrical distributions of sediment abundance, with each centered about some modal textural class. Conceptually, mean particle size should reflect the relative intensities of physical (and other) processes of sediment generation and transport across respective platform surfaces. In the case of the Florida–Bahamas data (Fig. 12B), modal grainstone undoubtedly reflects the dominance of stronger currents along platform margins, processes that perhaps decrease exponentially in intensity toward platform interiors. With respect to Wytheville, however, modal massive mud and silt more probably reflect the dominance of perhaps intermediate energy conditions across much of the region (Fig. 12A). Presumably, these depositional processes exponentially increased and decreased in abundance and/or intensity across different parts of the Cambro-Ordovician bank surface.

CONCLUSIONS

Thicknesses of lithologic units in many peritidal carbonate sequences are approximated by exponential distributions of thickness frequency. Relative abundances of different unit thicknesses are dependent on sequence length and numbers of designated stratal elements, and are those that would result from the random segmentation of a stratigraphic succession. Similarly, areas of carbonate lithotopes on the south Florida–Bahama platform yield frequency distributions that are approximated by those for populations of generally equidimensional facies elements that exhibit an exponential distribution of lithotope diameters. Relative abundance of different facies areas is dependent on total platform area and number of designated elements, and suggests that the spatial boundaries between adjacent lithotopes occur randomly. Size/abundance data from both Paleozoic and Holocene settings reveal no systematic relation among sizes of different facies and their sedimentological makeup. Even though units from either setting exhibit exponential distributions of size and abundance, specific lithologies are not dissimilar in thickness or areal extent.

Results from several numerical models of stochastic carbonate deposition suggest that superposition of randomly distributed equidimensional facies elements, which comprise exponential distributions of lithotope diameter, approximate size/abundance data from Paleozoic sequences and Holocene surfaces. Such models suggest an important component of unpredictability of vertical and lateral variation in the size and composition of peritidal units that make up shallow-water carbonate sequences.

Within a spectrum of possible interpretations, different individuals have variably embraced the importance of deterministic versus stochastic processes in interpreting the nature of global sedimentary sequences. Much of the interpretational justification of ''sequence stratigraphy'', for example, relies on assumptions about relations between sequence size and shape, and

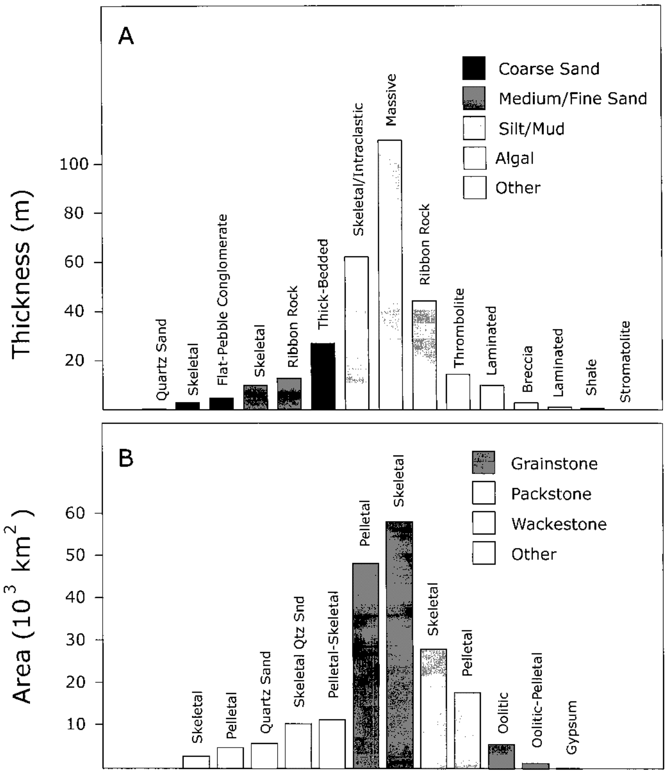

FIG. 12.—Abundance frequencies for lithologic elements exposed **A**) at Wytheville, Virginia, and **B**) on the Florida–Bahama platform. These are derived from the same data as in Figures 3 and 6 but here are reordered as two hypothetical quasi-symmetrical distributions reflecting potentially decreasing thickness/area abundances relative to a modal abundance as massive silt/mud and skeletal grainstone in the Phanerozoic sequences and on the Holocene platforms, respectively.

the Phanerozoic history of sealevel change. In and of itself, the reality of this linkage is perhaps less important than its reliance on deterministic conjectures, that large-scale geometries of associated stratal elements are correlated with some simple variable such as eustatic change. Regardless of the validity of these inferences, and at a considerably finer scale of consideration, numbers, sizes, and compositions of carbonate units in Paleozoic and Holocene systems imply a less deterministic relation between environment of accumulation and the lateral/vertical distribution of different stratal elements in cratonic peritidal sequences.

ACKNOWLEDGMENTS

Many individuals have both directly and indirectly contributed to the ideas presented above. Bob Demicco, Linda Hinnov, Bob Goldhammer, John Grotzinger, Michelle Kominz, David Osleger, Jon Pelletier, Peter Sadler, and Scott Tinker have served as valuable sources of information and motivation during these investigations. Early versions of this paper benefited from critical review by Linda Albertzart, Julie Kupecz, Alberto Malinverno, Jon Pelletier, Peter Sadler, and Alex Woronow, and from truly outstanding editing by John Southard. Support of research on patterns of sediment accumulation at the University of Michigan is supported by the national Science Foundation, NSF Grant EAR-94–05551, and at Indiana University Purdue University Fort Wayne by the American Chemical Society, PRF grant 29737-B8.

REFERENCES

ADAMS, R.D., AND GROTZINGER, J.P., 1996, Lateral continuity of facies and parasequences in Middle Cambrian platform carbonates, Carrara Formation, southeastern California, U.S.A.: Journal of Sedimentary Research, v. 66, p. 1079–1090.

BOSS, S.K., 1996, Digital shaded relief image of a carbonate platform (northern Great Bahama Bank)—scenery seen and unseen: Geology, v. 24, p. 985–988.

BOSS, S.K., AND RASMUSSEN, K.A., 1995, Misuse of Fischer plots as sea-level curves: Geology, v. 23, p. 221–224.

BOSSCHER, H., AND SCHLAGER, W., 1993, Accumulation rates of carbonate platforms: Journal of Geology, v. 101, p. 345–355.

CARLE, S.F., LABOLLE, E.M., WEISSMANN, G.S., VAN BROCKLIN, D., AND FOGG, G.E., 1998, Conditional simulation of hydrofacies architecture—a transition probability/Markov approach, *in* Fraser, G.S., and Davis, J.M., eds., Hydrologic Models of Sedimentary Aquifers: SEPM, Concepts in Hydrology and Environmental Geology no. 1, p. 147–170.

DAVIS, J.C., 1986, Statistics and Data Analysis in Geology, 2nd Edition: New York, Wiley, 646 p.

DAY, P.I., 1997, The Fischer diagram in the depth domain: a tool for sequence stratigraphy: Journal of Sedimentary Research, v. 67, p. 982–984.

DEMICCO, R.V., 1983, Wavy and lenticular-bedded carbonate ribbon rocks of the upper Cambrian Conococheague limestone, central Appalachians: Journal of Sedimentary Petrology, v. 53, p. 1121–1132.

DRUMMOND, C.N., AND WILKINSON, B.H., 1992, On the use of cycle thickness diagrams as records of long-term sealevel change during accumulation of carbonate sequences: Journal of Geology, v. 101, p. 687–702.

DUNHAM, R.J., 1962, Classification of carbonate rocks according to depositional texture: American Association of Petroleum Geologists, Memoir 1, p. 108–121.

ENOS, P., 1974, Surface sediment facies of the Florida–Bahamas Platform: Map and Chart Series, Geological Society of America, MC-5.

FOHLMEISTER, J.F., 1994, Hotspots, mantle convection, and plate tectonics—a synthetic calculation: Pure and Applied Geophysics, v. 143, p. 673–695.

GIANNINY, G.L., AND SIMO, J.A., 1996, Implications of unfilled accommodation space for sequence stratigraphy on mixed carbonate–siliciclastic platforms—an example from the Lower Desmoinesian (Middle Pennsylvanian), southwestern Paradox Basin, Utah, *in* Longman, M.W., and Sonnenfeld, M.D., Paleozoic Systems of the Rocky Mountain Region: SEPM, Rocky Mountain Section, p. 213–234.

GOLD, C.M., 1989, Surface interpolation, spatial adjacency, and GIS, *in* Raper, J., ed., Three Dimensional Applications in Geographical Information Systems: London, Taylor & Francis, Ltd., p. 21–35.

GOLDHAMMER, R.K., LEHMAN, P.J., AND DUNN, P.A., 1993, The origin of high-frequency platform carbonate cycles and third-order sequences (Lower Ordovician El Paso Gp, West Texas)—constraints from outcrop data and stratigraphic modeling: Journal of Sedimentary Petrology, v. 63, p. 318–359.

GROTZINGER, J.P., 1986, Cyclicity and paleoenvironmental dynamics, Rocknest platform, northwest Canada: Geological Society of America, Bulletin, v. 97, p. 1208–1231.

HATLOY, A.S., 1994, Numerical facies modeling combining deterministic and stochastic methods, *in* Jarus, J.M., and Chambers, R.L., eds., Stochastic Modeling and Geostatistics: American Association of Petroleum Geologists, Computer Applications in Geology, no. 3, p. 109–120.

HINE, A.C., 1983, Relict sand bodies and bedforms of the northern Bahamas—evidence of extensive early Holocene sand transport, *in* Peryt, T.M., ed., Coated Grains: Heidelberg, Springer-Verlag, p. 116–131.

HOLLAND, S.M., MILLER, A.I., DATTILO, B.F., MEYER, D.L., AND DIEKMEYER, S.L., 1997, Cycle anatomy and variability in the storm-dominated type Cincinnatian (Upper Ordovician): coming to grips with cycle delineation and genesis: Journal of Geology, v. 105, p. 135–152.

KERANS, C., AND TINKER, S.W., 1997, Sequence Stratigraphy and Characterization of Carbonate Reservoirs: SEPM, Short Course Notes 40, 140 p.

KOERSCHNER, W.F., AND READ, J.F., 1989, Field and modeling studies of Cambrian carbonate cycles, Virginia Appalachians: Journal of Sedimentary Petrology, v. 59, p. 654–687.

LIN, C., AND HARBAUGH, J.W., 1984, Graphic Display of Two- and Three-dimensional Markov Computer Models in Geology: New York, Van Nostrand Reinhold, 157 p.

MCLEAN, D.J., AND MOUNTJOY, E.W., 1994, Allocyclic control on Late Devonian buildup development, Southern Canadian Rocky Mountains: Journal of Sedimentary Research, v. B64, p. 326–340.

MIDDLETON, G.V., 1983, Johannes Walther's law of the correlation of facies: Geological Society of America, Bulletin, v. 84, p. 979–988.

MONTAÑEZ, I.P., AND OSLEGER, D., 1993, Parasequence stacking patterns, third-order accommodation events, and sequence stratigraphy of Middle to Upper Cambrian platform carbonates, Bonanza King Formation, southern Great Basin, *in* Loucks, R.G., and Sarg, J.F., eds., Recent Advances and Applications of Carbonate Sequence Stratigraphy: American Association of Petroleum Geologists, Memoir 57, p. 305–325.

MOSS, B.P., 1990, Stochastic reservoir description—a methodology, *in* Hurst, A., Lovell, M.A., and Morton, A.C., eds., Geological Applications of Wireline Logs: Geological Society of London, Special Publication 48, p. 57–75.

OKABE, A., BOOTS, B., AND SUGIHARA, K., 1992, Spatial Tessellations—Concepts and Applications of Voronoi Diagrams: Wiley Series in Probability and Mathematical Statistics: Chichester, U.K., Wiley, 532 p.

OSLEGER, D.A., AND READ, J.F., 1991, Relation of eustasy to stacking patterns of meter-scale carbonate cycles, late Cambrian, U.S.A.: Journal of Sedimentary Petrology, v. 61, p. 1225–1252.

OSLEGER, D.A. AND READ, J.F., 1993, Comparative analysis of methods used to define eustatic variation in outcrop: Late Cambrian interbasinal sequence development: American Journal of Science, v. 293, p. 157–216.

READ, J.F., AND GOLDHAMMER, R.K., 1988, use of Fischer plots to define third-order sealevel curves in Ordovician peritidal cyclic carbonates, Appalachians: Geology, v. 16, p. 895–899.

SADLER, P.M., 1981, Sediment accumulation rates and the completeness of stratigraphic sections: Journal of Geology, v. 89, p. 569–584.

SADLER, P.M., 1994, The expected duration of upward-shallowing peritidal carbonate cycles and their terminal hiatuses: Geological Society of America, Bulletin, v. 106, p. 791–802.

Sadler, P.M., Osleger, D.A., and Montañez, I.P., 1993. On labeling, length, and objective basis of Fischer plots: Journal of Sedimentary Petrology, v. 63, p. 360–368.

Soreghan, G.S., 1994, Stratigraphic responses to geologic processes: Late Pennsylvanian eustasy and tectonics in the Pedregosa and Orogrande Basins, Ancestral Rocky Mountains: Geological Society of America, Bulletin, v. 106, p. 1195–1211.

Srivastava, R.M., 1994, An overview of stochastic models for reservoir characterization, *in* Yarus, J.M., and Chambers, R.L., eds., Stochastic Modeling and Geostatistics; Principles, Methods, and Case Studies: American Association of Petroleum Geologists, Computer Applications in Geology no. 3, p. 3–16.

Swan, A.R.H., and Sandilands, M., 1995, Introduction to Geological Data Analysis: London, Blackwell Science, 446 p.

Thiessen, A.H., and Alter, J.C., 1911, Climatological data for July, 1911; District Number 10, Great Basin: Monthly Weather Review, p. 1082–1089.

Tipper, J.C., 1990, Correlation in mosaic environments: Geological Society of London, Journal, v. 147, p. 487–494.

Tipper, J.C., 1995, Assessing geometric algorithms: some comments arising from the paper ''Fast topological construction of Delaunay triangulations and Voronoi diagrams'' by Victor J.D. Tsai: Computers and Geosciences, v. 21, p. 433–436.

Tsai, V.T.D., 1993, Fast topological construction of Delaunay triangulations and Voronoi diagrams: Computers and Geosciences, v. 19, p. 1463–1474.

Tyler, K., Henriquez, A., and Svanes, T., 1994, Modeling heterogeneities in fluvial domain: a review of the influence of production profiles, *in* Jarus, J.M., and Chambers, R.L., eds., Stochastic Modeling and Geostatistics; Principles, Methods, and Case Studies: American Association of Petroleum Geologists, Computer Applications in Geology, no. 3, p. 77–90.

Visher, G.S., 1965, Use of vertical profiles in environmental reconstruction: American Association of Petroleum Geologists, Bulletin, v. 49, p. 41–61.

Walther, J., 1894, Einleitung in die Geologie als historische Wissenschaft: Jena, Verlag von Gustav Fischer, 1055 p.

Wilkinson, B.H., Diedrich, N.W., Drummond, C.N., and Rothman, E.D., 1998, Michigan hockey, meteoric precipitation, and rhythmicity of accumulation of peritidal carbonate platforms: Geological Society of America, Bulletin, v. 110, p. 1075–1093.

Wilkinson, B.H., Drummond, C.N., Diedrich, N.W., and Rothman, E.D., 1997b, Biological mediation of stochastic peritidal carbonate accumulation: Geology, v. 25, p. 847–850.

Wilkinson, B.H., Drummond, C.N., Rothman, E.D., and Diedrich, N.W., 1997a, Stratal order peritidal carbonate sequences: Journal of Sedimentary Research, v. 67, p. 1068–1082.

Yang, W., Harmsen, F., and Kominz, M.A., 1995, Quantitative analysis of a cyclic peritidal sequence, the Middle and Upper Devonian Lost Burro Formation, Death Valley, California—a possible record of Milankovitch climatic cycles: Journal of Sedimentary Research, v. 65, p. 306–322.

Yarus, J.M., and Chambers, R.L., 1994, Stochastic Modeling and Geostatistics; Principles, Methods, and Case Studies: American Association of Petroleum Geologists, Computer Applications in Geology no. 3, 379 p.

Received 21 January 1998; accepted 26 October 1998.