Int J Earth Sci (Geol Rundsch) (2005) 94: 751–757
DOI 10.1007/s00531-004-0452-1

ORIGINAL PAPER

Tristan G. M. Cornu · Guillaume Bertrand

Backward modeling of the rifting kinematics in the Upper Rhine Graben: insights from an elastic-perfect contact law on the restoration of a multi-bloc domain

Received: 28 October 2003 / Accepted: 25 October 2004 / Published online: 13 January 2005

Abstract This paper addresses the problem of volume restoration for 3-D sedimentary basin kinematic deformation. The primary purpose is methodological and concerns the use of contact mechanics with the finite element method, in order to deform a geological multi-bloc domain. This approach is applied to backward model the later stage of rifting of a segment of the southern Upper Rhine Graben (France–Germany border). Preliminary results from our modeling demonstrate the ability of the method not only to handle complex geometries, but also to successfully perform retro-deformation of a complex geological domain. In addition, they provide or confirm crucial information on the rifting evolution and tectonic features of this segment of the Upper Rhine Graben, such as the distribution of deformation, the asymmetry of the graben and a significant left-lateral strike-slip component of displacement.

Introduction

The work presented here is a retro-deformation of the later extension stage of the rifting of the Upper Rhine Graben (URG). It has been completed within the framework of the European Union funded ENTEC Research and Training network. This project aims at developing and strengthening collaborations between modelers and geologists on regional geological studies, as well as proposing new solutions and methodologies for 3-D approaches of geological problems. When sharing data from many different disciplines (seismicity, 3-D seismic interpretation, sedimentation process, lithosphere flexure, geodesy, etc...) a major issue is the need to build a link between them. From the modeling point of view, three main relevant scales of geological processes can be defined: (1) the surface processes scale, (2) the basin scale, and (3) the lithospheric scale. This paper focuses on the basin scale.

The modeling of basin deformation can be carried out using either a mechanical or a kinematical approach. Until now, in order to avoid the huge complexity of mechanical problems, most of the models developed to balance cross sections favored 2-D kinematical approaches (e.g. Suppe 1983; Contreras and Sutter 1990; Waltham 1989, 1990). 3-D models mostly deal with surface unfolding processes (Gratier and Guillier 1993; Rouby 2000). Only a few addresses the problem of volume restoration, using a pseudo 3-D approach with cylindrical domains composed by a succession of cross sections (Wilkerson and Medwedeff (1993; Egan 1998). Recently, a model was developed that proposes both a valid geological path for 3-D deformation and strain location (Cornu et al. 2003). Nevertheless, this model is limited in its geometrical assumptions to constrain only one boundary of the domain (in addition to the displacement boundary condition). What would happen if one has to constrain two or more boundaries is a problem that still needs to be solved.

To overcome this limitation and properly address the problem of a fully constrained domain, one needs to use a mechanical approach (as opposed to one based upon kinematics, where only one border of the geological domain could be constrained). Thus, a 3-D finite element model is presented here, which was developed at the Vrije Universiteit (Amsterdam) and focuses essentially on the definition of contacts along the faults. Modeling the rifting kinematics in the URG is the

T. G. M. Cornu (✉)
Institut for Aardwetenschapen, Vrije Universiteit,
de Boelelaan, 1085, 1081 HV Amsterdam, The Netherlands
E-mail: tristan.cornu@falw.vu.nl
URL: http://www.geo.vu.nl/users/entec/

G. Bertrand
Geologisches Institut der Universität Freiburg,
Albertstr. 23-B, 79104 Freiburg, Germany

Present address: G. Bertrand
BRGM, BP 6009, 3 av. Claude Guillemin,
45060 Orléans cedex 2, France
E-mail: g.bertrand@brgm.fr

objective of this paper. This case study illustrates that elasticity with perfect contact could be used as a tool to perform retro-deformation of a complex multi-bloc geological domain.

The model

When attempting a backward or forward modeling of basin deformation, one has to deal with the major problem of contact between each geological sub-domain, which takes place on fault surfaces. Therefore, one of the main objectives is to propose an adequate numerical method to properly address this problem of inter-domain contacts along fault planes.

A previous study from Melosh and Williams (1989) focuses on graben formation and fault description. Their "slippery node" method consists in adding 1 or 2 degree(s) of freedom to each node on the predefined fault of the domain. The new numerical system is still solved with a linear resolution. Unfortunately, this very powerful method has a major drawback for the geological time scale as the predefined fault remains fixed, and no deformation occurs to it. More recent works from Laursen and Simo (1993), and Pietrzak and Curnier (1999) address the contact evolution for multi-body domains and seem more appropriate for the geological problem of retro-deformation.

From a purely mathematical point of view, the contact problem is an example of a physical system subject to a governing variational inequality (Duvaut and Lions 1980; Kikuchi and Oden 1988). The use of a linear elastic solid in frictional contact has previously been studied extensively (e.g. Francavilla and Zienkiewics 1975; Hughes 1976). On the other hand, the behavior of an inelastic material, or the replacement of a rigid obstacle by a second deformable body takes the complexity one step further. Their effects on mathematical well-posedness of the problem are not completely understood yet. Previous works on contact problems (frictionless or not) usually suffer at least one of the following: restriction to a particular discretisation, limitation on the admissible motions, or restriction to a rigid obstacle problem. This situation results from the lack of an underlying continuum framework that can be formalized by the following equations (according to Bittencourt and Creus 1998).

The following equation addresses the problem of quasi-static equilibrium and perfect contact for multi-body domain. If we consider a set of bodies $\Omega_{I,I=1,N}$, the variational equality of the equilibrium equations leads to the following formulation:

$$\int_{\Omega_I} \underline{\underline{\sigma}} : \left(\frac{\partial \delta \underline{u}}{\partial \underline{X}}\right) \mathrm{d}V = \int_{\Omega_I} \underline{f}.\underline{\delta u}\,\mathrm{d}V + \int_{\partial\Omega_i^F} \underline{\underline{\sigma}}.\underline{n}.\underline{\delta u}\,\mathrm{d}S + \int_{\partial\Omega_i^C} \underline{T}.\underline{\delta g}\,\mathrm{d}S$$

where $\underline{\underline{\sigma}}.\underline{n}$ and $\underline{f}$ are the prescribed traction and body forces, respectively, $\underline{T}$ is a penalty parameter that represents the contact traction on the contact border, and $\underline{\underline{\sigma}}.\underline{n}$ and $\underline{\delta g}$ are virtual variations of the displacement and gap onto contact borders vectors.

In addition, the gap function and the contact forces must verify the following conditions:

$$g(\underline{X},t) \leq 0$$
$$t_n(\underline{X},t) \geq 0$$
$$t_n(\underline{X},t)g(\underline{X},t) = 0$$

where t_n is the normal component of the contact force (parallel to the normal vector of the contact surface).

Definition of the contact problem

The referential in which the contact problem is solved has to be defined first. The contact elements are extracted from the faces of the 3-D elements (Fig. 1a), where contact occurred, and define a linear contact surface. In case of brick elements, the four-nodes faces are split in two triangles, with a normal vector n. This definition of the contact surface has been tested for previous kinematics studies (Cornu et al. 2003) and proved to be precise enough. On each face a local rotation matrix is defined as

$$\eta = \begin{bmatrix} q_x & q_y & q_z \\ r_x & r_y & r_z \\ n_x & n_y & n_z \end{bmatrix}$$

where q and r are the tangential vectors of the contact surface, normal to n.

The gap function is the normal projection of the slave node onto the target surface (Fig. 1b):

$$g(\underline{X},t) = (\underline{X}^t - \underline{X}^s) \cdot \underline{n}$$

and the contact forces are defined, onto the contact element, according to the penalty matrix and the gap vector:

$$\underline{t} = \underline{\underline{k}} \cdot \underline{g}$$

with the penalty matrix defined as: $\underline{\underline{k}} = \begin{bmatrix} k_t & 0 & 0 \\ 0 & k_t & 0 \\ 0 & 0 & k_n \end{bmatrix}$

The penalty coefficient is null for the tangential component. For the normal component a mean used value can be the Young's modulus of the material

The normal penalty parameter k_n represents the stiffness of the material to penetration, and the tangential penalty parameter k_t the stiffness of the surface to tangential displacement.

The forces computed onto the contact surface have to be expressed into the global referential according to the following relation:

$$\begin{Bmatrix} T_x \\ T_y \\ T_z \end{Bmatrix} = \begin{bmatrix} q_x & r_x & n_x \\ q_y & r_y & n_y \\ q_z & r_z & n_z \end{bmatrix} \begin{Bmatrix} t_q \\ t_r \\ t_n \end{Bmatrix}$$

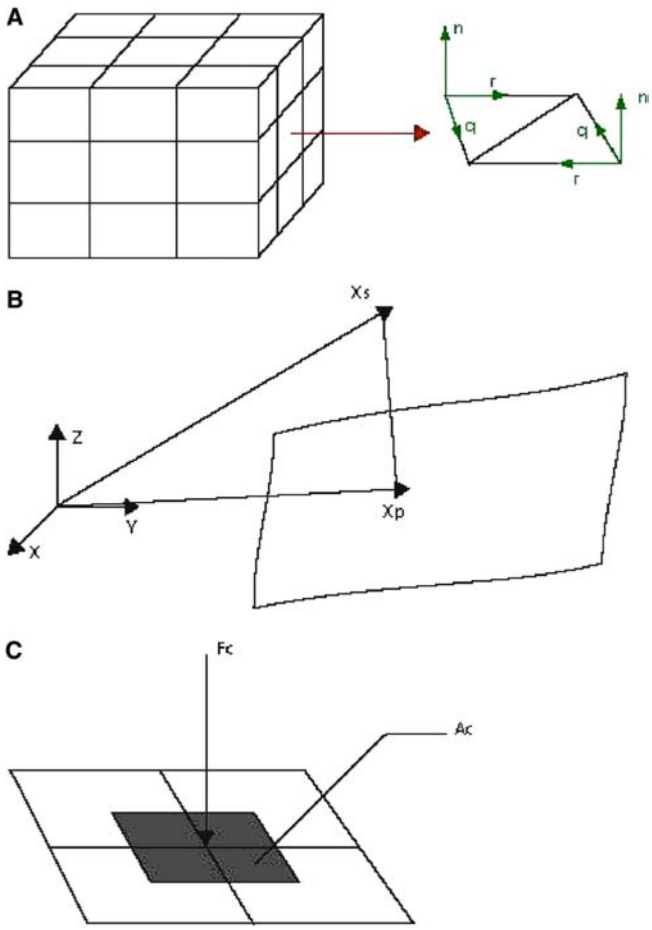

Fig. 1 **a** Extraction of the side element to define the contact element. **b** Projection *Xp* of the slave node *Xs* onto the target surface. **c** Definition of the mean area *Ac* associated to the contact force *Fc*

The traction T must then be integrated over the contact element. Due to convergence requirement, it may be useful to consider the area constant. In this case, the considered area of contact (i.e. A^c) is one fourth of the area of each element that contains the contact node (Fig. 1c), and the forces applied onto A^c are expressed by

$$\underline{F}^c = \underline{T}A^c$$

In order to increase the convergence of the Newton–Raphson algorithm used to solve the non-linear problem, one has to use a tangent stiffness matrix, defined as

$$K^c_{ij} = \eta_{ki}\eta_{lj}k_{kl}A^c$$

The full description of the calculus needed to obtain the tangent matrix can be found in Bittencourt and Creus (1998). Other approaches are fully described in Parisch 1989; Laursen and Simo 1993; Piertzak and Curnier 2000.

Application to the rifting of the Rhine Graben

The case study selected here is a segment of the southern part of the URG. The URG is a NNE-SSW trending tectonic structure that extends over 300 km with boundaries between France, Germany and Switzerland, and an average width of approximately 40 km. It is basically a crustal-scale, small-displacement continental rift (Groshong 1996) and is a segment of the European Cenozoic rift system, which developed in the north-western forelands of the Alps (Ziegler 1992) in response to a 5–7 km extension of the upper crust (Brun et al. 1992). Subsidence and syn-rift sedimentation started in the late Eocene and continue today in parts. The main phase of subsidence, however, was during Oligocene and Miocene (Pflug 1982). The model covers a segment of the URG and its shoulders (Vosges and Black Forest mountains), in the Colmar (northeastern France) and Freiburg–Offenburg (southwestern Germany) areas (Fig. 2). It includes three geological horizons that are, from the youngest to the oldest, the base Tertiary, Top Muschelkalk (middle Triassic) and base Triassic (Fig. 3a). To properly complete our retro-deformation modeling, two tasks of equal importance are required: (1) definition of the geological and numerical domains, and (2) running and interpretation of the numerical computation.

Creation of the domain geometry

The geometry of the model is based upon a digital database created from all available data. The major data source was found in the geothermal synthesis from BRGM Service Géologique Régional Alsace and Geologisches Landesamt Baden-Württemberg (1979) for isobath maps and in Kämpfe (1984) for boreholes. Fault surface geometries were defined from previously published data (e.g. Sittler 1969; Brun et al. 1992). Geological horizons were extrapolated on the rift shoulders from stratigraphic data of Geyer and Gwinner (1991) as well as French and German geological maps.

The construction of the numerical domain is a crucial part of the modeling (Fig. 3). The geological data were digitized from isobath and structural maps, and then exported to geomodelers (GOCAD and 3DMOVE) to build the structural domain (Fig. 3a). This domain was then discretized with a mesh tool (HYPERMESH) into linear brick elements. Finally, it was simplified to four blocks delimitated by three major faults (Fig. 3b). Most of the inner faults, which have been suppressed, are the results of the extension and have been created out of an irreversible process, i.e. the process cannot be reproduced with a backward modeling. The faulting contemporaneous to the extension is addressed in another paper (Cornu and Bertrand, 2005). On the other hand, the central fault is

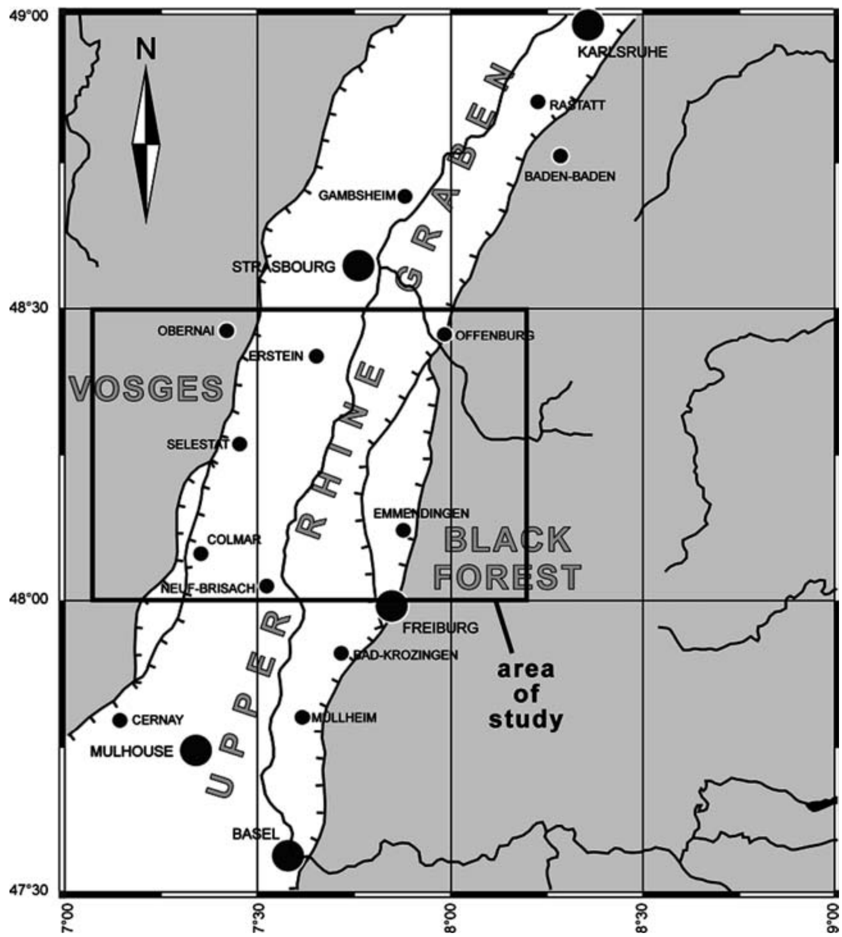

Fig. 2 Schematic map of the southern part of the Upper Rhine Graben, showing the western and eastern main border faults, and the area covered by the numerical model presented in this paper

preserved because it is supposed to have an active role in the lateral motion of the graben. The two "shoulder" blocks have 480 elements, and the two "graben" blocks have 880 elements.

The reduction of the model was selected to (1) test the validity of the algorithm, and (2) prevent a complex and ambiguous analysis of the results as would happen with too many blocks. Although simplifications were made, the domain remains complex from a geometrical point of view, i.e. true geometry of the geological objects was considered (e.g. variations of strike and dip of fault planes).

Results

Apart from the contact condition, the other boundary conditions are (1) shortening of 20 m per iteration on each face of the two shoulders, (2) no lateral motion of the borders in the y (north-south) direction, and (3) an uplift of 5.3 m and 3.7 m per iteration in the French (western) and German (eastern) sides respectively. The shortening and uplift boundary conditions deduced from the possible total amount of extension proposed by Brun (Brun et al. 1992). Also, gravity is removed from the modeling because of the backward settings.

Kinematics

The results displayed in Fig. 4 are obtained after 4,000 m of shortening, i.e. 100 iterations. The kinematics of the deformation is evidenced by the displacement values through the domain along the three directions (Fig. 4). The u (E–W) component (Fig. 4a), in the direction of shortening, shows that compression is accommodated mostly along the two main border faults. The remaining part is distributed inside the graben, and no or very little compression is accommodated within the region of the graben shoulders. Moreover, the western part of the graben seems to have suffered more compression than the eastern side, which in terms of geology fits well the asymmetry of the URG at this latitude. This is highly compatible with a rifting scenario where much of the extension would concentrate along the main border faults, while the remaining part would be accommodated along secondary faults within the graben interior.

The v (N–S) component (Fig. 4b) (i.e. lateral displacement) shows the strike-slip motion along the major faults and within the blocks. The western (Vosgian) main border fault indicates a clear backward right-lateral displacement. On the other hand, the eastern (Black Forest) main border fault shows

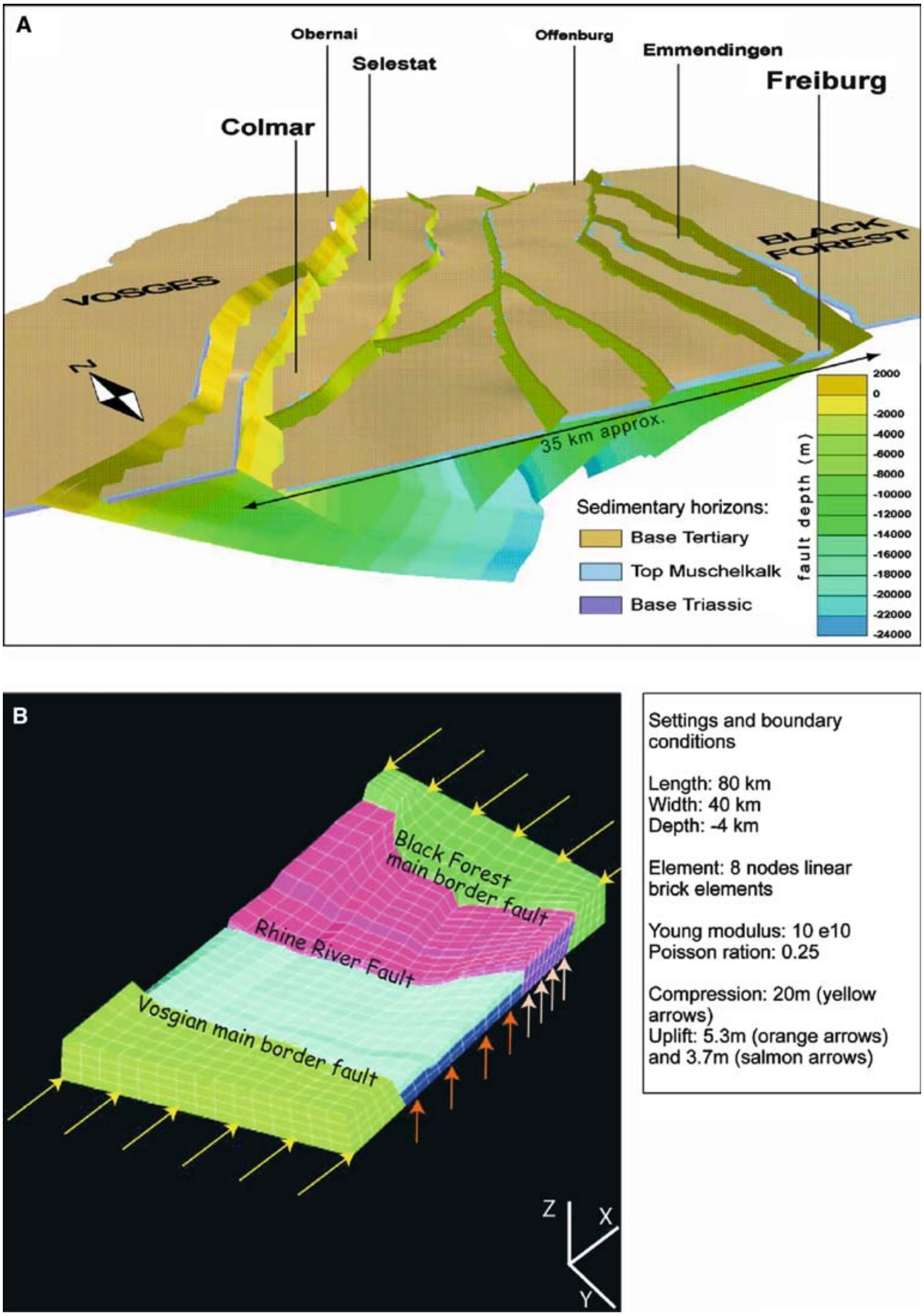

Fig. 3 Description of the geological and numerical domain: **a** geometry of the geological domain in its present-day configuration, looking toward northeast. Geological horizons and faults (color coded for depth, in meters) only are represented. Approximate positions of major cities are given for geographic situation. **b** Numerical model, finite element mesh and the associated boundary conditions

opposite senses of motion that seems to be locally controlled by the strike of the fault that is left-lateral where the fault is striking NW–SE and right-lateral where it is striking NE–SW. These are the strike-slip components one would expect from oblique-slip motion on segments that are not striking perpendicular to the direction of motion. Finally, the Rhine River Fault, in the center of the graben, indicates very little motion, with changing sense (sinistral to the north and dextral to the south), suggesting this structure accommodates no or very little strike-slip motion. These observations indicate a strike-slip component that is inhomogeneous through the graben and concentrated on its western border, which could be an additional argument for the asymmetry of the rift.

The *w* component (Fig. 4 c) shows the vertical motion. It clearly locates the minimum of uplift in the graben shoulders, and the maximum in the western part of the graben, where the depot centers are located in agreement with geological observations.

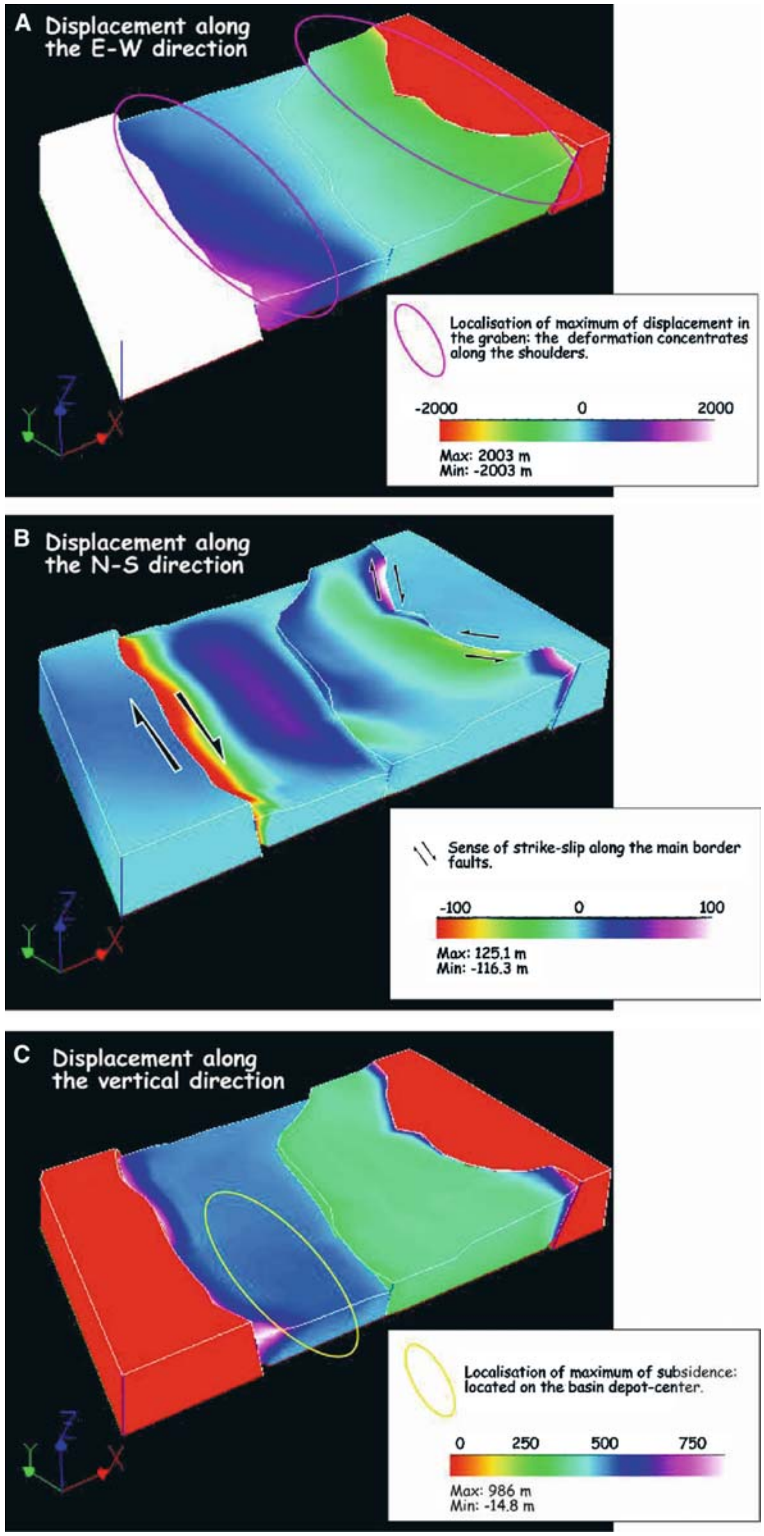

Fig. 4 Map of the displacement component after 4,000 m (60–80% of the extension according to Brun et al. 1992) of retro-deformation: **a** displacement along the *x* (eastward for positive values) direction. **b** displacement along the *y* (northward for positive values) direction. **c** Displacement along the *z* (upward for positive values) direction

To summarize, despite the simplifications assumed within the model (orthogonal - i.e. E–W-extension for instance, while several authors suggest oblique rifting; e.g. Larroque and Laurent 1988; Schumacher 2002; Behrmann et al. 2003), this numerical model provides interesting results that are geologically significant and highly compatible with regional field observations, at least from a qualitative point of view.

From these preliminary results, the chosen modeling approach appears relevant to qualitatively predict aspects of the geological behavior.

Conclusion

A major objective of this paper was to test the modeling methodology. The modeling results demonstrate the adequacy of using finite element methods, with good description of contact mechanics, to retro-deform a complex multi-block geological domain. Results from the numerical model closely fit the first order tectonic observations in the URG.

In addition, the model yields information on the rifting history and tectonic features of this segment of the URG. It first shows that a major part of the displacement is accommodated along the two main border faults while only a minor part is distributed within the graben itself. Secondly, the significantly larger amount of extension, strike-slip and subsidence in the western part of the region is a strong evidence of the asymmetry of the graben, and is in good agreement with geological and geophysical observations. Finally, a significant left-lateral (forward) strike-slip component is an important result with respect to discussion about the direction of the extension axis during rifting. It proves that strike-slip displacement can be obtained not only with oblique, but also, to a certain amount, with purely orthogonal extension (basic hypothesis of our model). It then suggests that kinematical observations alone may not be sufficient to assert the true direction of extension during rifting.

Acknowledgements We wish to thank the European Scientific Community who funded this research through the fifth European Research and Training Network: ENTEC HPRN-CT-2000-00053 (http://www.geo.vu.nl/users/entec).

References

Behrmann J, Hermann O, Horstmann M, Tanner D, Bertrand G (2003) Anatomy and kinematics of oblique continental rifting revealed: a 3D case study of the SE Upper Rhine Graben (Germany). AAPG Bull 87(7):1105–1121

BRGM Service Géologique Régional Alsace and Geologisches Landesamt Baden-Württemberg (1979) Synthèse Géothermique du Fossé Rhénan Supérieur - Geothermische Synthese des Oberrheingrabens. Strasbourg, Freiburg, 109 pages, (23 figures)

Bittencourt E , Creus GJ (1998) Finite element analysis of three-dimensional contact and impact in large deformation problems. Comput Struct 69:219–234

Brun JP, Gutscher MA and Dekorp-Ecors team (1992) Deep crustal structure of the Rhine Graben from DEKORP-ECORS seismic reflection data: a summary. Tectonophysics 208:139–147

Contreras J, Suter M (1990) Kinematic modelling of cross-sectional deformation sequences by computer simulation. J Geophys Res 95:21913–21929

Cornu T, Schneider F, Gratier JP (2003) 3D discrete kinematic modelling applied to extensional and compressional tectonics. In: Niewland D (eds), New insight into structural interpretation and Modelling. Geological Society Special Publication, London, 212:285–294

Cornu T, Bertrand G (2005) Numerical backward and forward modelling of the southern Upper Rhine Graben (France-Germany border): new insights on tectonics evolution of intracontinental rifts. Quat. Sci. Rev., in press, doi: 10.1016

Duvaut G, Lions JL (1980) Inequalities in mechanics and physics. Springer, Berlin

Egan SS, et al. (1998) Computer modelling and visualisation of the structural deformation caused by movement along geological faults. Comput Geosci 25:283–297

Francavilla A, Zienkiewics OC (1975) A note on numerical computation of elastic contact problems. Int J Numer Methods Eng 9:913–924

Garcia-Castellanos D, Fernàndez M, Torné M (1997) Numerical modeling of foreland basin formation: a program relating thrusting, flexure. sediment geometry and lithosphere rheology. Comput Geosci 23(9):993–1003

Geyer OF, Gwinner MP (1991) Geologie von Baden-Württemberg. Institut für Geologie und Paläontologie der Universität Stuttgart. Stuttgart

Gratier JP, Guillier B (1993) Compatibility constraints on folded and faulted strata and calculation of total displacement using computational restoration (UNFOLD program). J Struct Geol 15:391–402

Groshong RH Jr (1996) Construction and validation of extensional cross sections using lost area an strain, with application to the Rhine graben. In: Buchanan PG and Nieuwland DA (eds), Modern developments in structural interpretation, validation and modelling. Geological Society Special Publication, London 99:79–87

Hughes TJR, Taylor RL, Sackman JL, Curnier A, Kanoknukulchai W (1976) A finite element method for a class of impact-contact problems. Comp Methods Appl Mech Eng 8:249–276

Kämpfe C (1984) Tiefbohrungen in Baden-Württemberg und umgebenden Läandern. Arbeiten aus dem Institut für Geologie und Paläontologie an der Universität Stuttgart. n. 80.

Kikuchi N, Oden JT (1988) Contact problem in elasticity: a study of variational inequalities and finite element methods. SIAM, Philadelphia

Larroque JM and Laurent P (1988) Evolution of the stress field pattern in the south of the Rhine Graben from the Eocene to the present. Tectonophysics 148:41–58

Laursen TA, Simo JC (1993) A continuum-based finite element formulation for the implicit solution of multibody, large deformation frictional contact problems. Int J Numer Methods Eng 36:3451–3485

Melosh HJ, Williams CA (1989) Mechanics of graben formation in crustal work: a finite element analysis. J Geophys Res 94(B10):13961–13973

Parisch H (1989) A consistent tangent stiffness matrix for three dimensional non-linear contact analysis. Int J Numer Methods Eng 28:1803–1812

Pflug R (1982) Bau und Entwicklung des Oberrheingrabens: Darmstadt. Wissenschaftliche Buchgesellschaft, Darmstadt, pp 145

Pietrzak G and Curnier A (1999) Large deformation frictional contact mechanics: continuum formulation and augmented Lagrangian treatment. Comput Methods Appl Mech Eng 177:351–381

Rouby D, Xiao H, Suppe J (2000) 3D restoration of complexly folded and faulted surfaces using multiple unfolding mechanisms. AAPG Bull 84(6):805–829

Schumacher ME (2002) Upper Rhine Graben: role of preexisting structures during rift evolution. Tectonics 21(1). DOI 10.1029/2001TC900022

Sittler C (1969) Le fosse Rhenan en Alsace: aspect structural et histoire geologique. Revue de Geographie Physique et de Geologie Dynamique 2, XI(fasc. 5):465–494

Suppe J (1983) Geometry and kinematic of fault-bend folding. Am J Sci 283:684–721

Waltham D (1989) Finite difference modelling of hanging wall deformation. J Struct Geol 11:433–437

Waltham D (1990) Finite difference modelling of sandbox analogues, compaction and detachment free deformation. J Struct Geol 12:375–381

Wilkerson MS, Medwedeff DA (1991) Geometrical modelling of fault-related folds: a pseudo-three-dimensional approach. J Struct Geol 13:801–812

Ziegler PA (1992) European Cenozoic rift system. Tectonophysics 208:91–111