Pricing and Informational Efficiency of the MIB30 Index Options Market.

An Analysis with High Frequency Data*

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Abstract

We analyze the pricing and informational efficiency of the Italian market for options written on the most important stock index, the MIB30. We find several indications inconsistent with the hypothesis that the Italian MIBO is an efficient market. We report that a striking percentage of the data consists of option prices violating basic no-arbitrage conditions. This percentage declines but never becomes negligible when we relax the no-arbitrage restrictions to accommodate for the presence of bid/ask spreads and other frictions. The result holds in general for all levels of moneyness and time to maturity. We also document abrupt changes of the implied volatility surface that can hardly be explained by changes in market beliefs. Finally we investigate the informational efficiency of the MIBO and conclude that option prices are poor predictors of the volatility of MIB30 returns.

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1. **Introduction**

In this paper we investigate the efficiency properties of the market for options on the Italian MIB30 index, the so-called MIBO market, one of the most important segments of the Italian Derivatives Market (IDEM). Following the creation of the index futures market (FIB30) by approximately one year, trading on the MIBO started in November 1995. The birth of the MIBO has been a crucial step towards the completion of the Italian stock market. MIB30 index options have soon become an important financial instrument, especially for institutional investors (like mutual funds) whose portfolios have typically a large share allocated to securities with a high degree of correlation with the MIB30 index.[1] More recently, new financial derivatives have been introduced so that the IDEM is bound to undergo further, important developments.

The importance of the Italian MIBO market parallels the prominent role of index options markets in the structure of modern financial markets. In the first place, these markets are essential instruments for risk sharing. On one hand, options enable portfolio managers to substantially improve their ability to hedge the risk of unpredictable changes of financial prices. On the other hand, investors may also easily and swiftly take speculative positions consistent with their views on future asset price movements. In the absence of well functioning option markets both these activities — hedging and speculation — would either be too costly or simply not feasible. Second, index options markets represent the best available instrument for aggregating investors' opinions concerning the future volatility of asset returns. Therefore an efficient options market should: (i) foster the implementation of hedging and speculative activities at affordable costs (*risk-sharing and pricing efficiency*), and (ii) accurately aggregate market beliefs concerning asset returns volatility (*informational efficiency*). The main objective of our paper is to assess the efficiency of the Italian MIBO market using a high frequency data set that spans a 9 months interval, April 1999 - January 2000. The systematic investigation of the efficiency properties of the MIBO30 market gives us an opportunity to contribute to at least three distinct literatures.

First, the analysis of the risk-sharing efficiency of a financial market always raises issues on the existence of arbitrage opportunities, the starkest antithesis to risk sharing. The assessment of the presence of arbitrage opportunities has a long tradition in empirical finance.[2] In fact one may derive from abstract models of frictionless markets well defined constraints on options prices that, whenever not respected, provide evidence of arbitrage opportunities that may be exploited by explicit portfolio strategies. These constraints have the advantage of being free of any assumption

[1] During 1999, the year to which our analysis refers, the volume of trades on stocks belonging to the MIB30 basket accounted for approximately 76.4% of the total trading volume on the Italian stock exchange. The ratio of the volume of trades on the MIBO with respect to that of the MIB30 basket stocks accounted for approximately 53%.

[2] Seminal papers are Stoll (1969), Gould and Galai (1974), and Klemkosky and Resnik (1979). More recently this issue has been addressed by Ackert and Tian (1999, 2000), George and Longstaff (1993), Kamara and Miller (1995), Nisbet (1992), Ronn and Ronn (1989), and Yadav and Pope (1994). Needless to say, most of these papers focus on North American markets, especially on S&P 100 and S&P 500 options.

on the stochastic process of the price of the underlying asset, and provide sensible predictions based solely on first principles. The best example of this approach is the test of the put/call parity, which has been repeatedly performed over the years for different markets and samples (see Stoll (1969) for pioneering work). We will perform this analysis in Section 3. On this point we improve on the previous literature by entertaining and testing a wider and more complete set of no arbitrage restrictions. In particular, we develop and test a new condition, the *maturity spread*, that is particularly useful in establishing a link between the level of efficiency of options and futures markets. Also, we carefully distinguish between tests that jointly rely on the efficiency of the markets for options and the underlying from more restrictive tests exclusively measuring MIBO efficiency (see Ackert and Tian (2000)). While in some papers (see Nisbet (1992)) it is simply shown that most arbitrage opportunities vanish after the introduction of frictions,[3] in Section 4 we take the view that the amount of market imperfections needed in order to restore absence of arbitrage opportunities should itself be considered an indicator of market efficiency, more precisely of the efficiency of the market microstructure. Therefore, differently from previous contributions, we avoid conducting a simple test of the hypothesis of market efficiency given transaction costs but instead treat frictions as a parameter and represent efficiency as a curve on a graph with axes representing the frequency and entity of arbitrage violations vs. the size of transaction costs.

Second, recent years have witnessed many attempts at modeling the structure of the pricing mechanism at work in modern options markets by looking at the structure and dynamics of the implied volatility surface. Since the world market crash of October 1987, it is widely known that many option markets (certainly the North American markets) are characterized by systematic deviations from the constant volatility benchmark of Black and Scholes (1973). These anomalies have been described either in terms of a volatility smile vs. moneyness (or a smirk, see Rubinstein (1994) and Dumas et al. (1998)) or as the presence of a term structure (Campa and Chang (1995)). Similar patterns have been documented for other markets as well (see Peña et al. (1999), Cavallo and Mammola (2000)). In Section 5 we take on the task of modeling and estimating the implied volatility surface characterizing the MIBO market. Furthermore, we try to characterize and measure the instability of the implied volatility surface using a Markov chain, hidden state approach. We consider the efforts of Sections 5.1 and 5.2 as directed at gaining further insights in the efficiency of the MIBO since abrupt changes in the implied volatility surface in a high frequency context are possibly indicative of a market dominated by "rumors" rather than fundamentals.

Third, starting from Canina and Fliglewski (1993) several authors have measured the informational efficiency of options markets by testing the unbiasedness of implied volatility as a predictor of ex post, realized volatility of stock returns. The common finding has been that implied volatilities are poor forecasts of future volatility and that a prediction based on option prices can easily be improved upon by using variables commonly included in the agents' information

[3] Cavallo and Mammola is an example directly pertaining to the MIBO30. However other Authors (see Ackert and Tian (2000)) establish that arbitrage violations persist despite the frictions, although to a lesser extent.

sets.[4] In Section 5.3 we couple traditional methods of investigation (such as GMM) with novel, panel-oriented econometric tools. Furthermore, we explore the relationship between pricing and informational efficiency by probing the robustness of our results to the use of data sets exposed in different degrees to the presence of arbitrage opportunities.

Our results for the Italian options market are only partially consistent with previous findings concerning North American markets. The no arbitrage restrictions are not satisfied for a high percentage of the data and this suggests that market frictions should be incorporated when fictitiously implementing arbitrage strategies.[5] We then compute the level of frictions which would be consistent with a reasonably low ratio of arbitrage opportunities. Implied frictions are quite substantial and this casts doubts on the overall risk-sharing efficiency of the MIBO market. Our assessment of the degree of efficiency of the market is even weakened by the investigation of the implied volatility surface. The surface is subject to dramatic changes often taking place in the span of a few hours only. It is at least doubtful that on a regular basis the IDEM might be affected by news of such an impact to completely unravel the structure of market beliefs with such a high frequency. Finally, and this time in full accordance with results concerning other markets, we assess a very low degree of MIBO informational efficiency: MIBO implied volatilities are lousy predictors of future volatility.

Our work has antecedents in a limited number of papers that have already examined the Italian index options market (see Barone and Cuoco (1989)). In particular, in a recent paper Cavallo and Mammola (2000) also investigate the efficiency properties of the MIBO market. Similarly to us, they use high-frequency observations.[6] However they only test the put-call parity relation and apply their tests to at-the-money, short-term contracts only, while we extend our investigations to all categories of contracts. Cavallo and Mammola partly use available data on the bid-ask spread, and partly guess the relevant transaction costs, while (as stressed above) we treat frictions endogenously and consider them as a crucial component of the notion of market efficiency.

The paper is organized as follows. In Section 2 we describe our data set and some of the institutional characteristics of the MIBO30 market. In Section 3 we run the no-arbitrage tests, showing in detail which conditions are more often violated, in which segment of the market this happens more frequently, and what is the level of the average profits associated to each type of violation. In addition to the usual no arbitrage conditions, we introduce a new one, not yet tested in the literature, named maturity spread, and discuss its features. Given our finding of pervasive violations of the most basic no-arbitrage restrictions, in Section 4 we proceed to imply

[4] Other references are Ahmed and Swidler (1998) and Christensen and Prabhala (1998).

[5] Although in Shleifer and Vishny (1997) it is correctly noticed that also the amount of capital necessary to exploit arbitrage opportunities should be considered, we follow the literature and focus on operational costs only, such as the bid/ask spread or fixed transaction costs when short-selling of the underlying is required.

[6] Their data set is shorter and refers to an earlier period (July 29, 1996 - February, 1997). Markets rule differ slightly.

out of the data the level of frictions compatible with pricing efficiency. Section 5 documents the intrinsic informational inefficiency of the MIBO market, both by applying direct tests and by modeling the dynamics of the implied volatility surface. We obtain indications of instability and of the importance of arbitrage violations in estimating the law of motion of the surface over time. Section 6 concludes by offering some hints to policy issues and directions for future research.

2. The Data

We analyze a high-frequency data set on European-style options written on the most important Italian stock index, the MIB30. Our sample contains data collected at a frequency of 30 minutes from 9 a.m. to 6 p.m. each day starting on April 6, 1999 and ending on January 31, 2000, for a total of 300 calendar days and approximately 15 different infra-daily observations a day. Each observation reports the value of the MIB30 index, the risk-free interest rate, the cross-section of MIBO30 option prices (over alternative strikes and maturities) and the bid and ask volumes. The interest rate is computed as an average of the bid and ask three months LIBOR rates. Although far from constant over the whole period, the risk-free rate shows only two major breaks in its mean, May 5, 1999 and September 29, 1999 when the mean switches respectively from about 2.95% to 2.69% and then from 2.69% to approximately 3.37%. Figure 1 plots the time series of the LIBOR interest rate. To give a clear picture of the market dynamics during the period to which our analysis refers to, Table I also reports summary statistics for the MIB30 index level and the (annualized, continuously compounded) MIB30 returns. Figure 1 also plots the high-frequency time series of the MIB30: between the Spring and the Fall of 1999 the market was bearish, to become strongly bullish instead during the last 3 months of the year.

The vector of option prices reports the transaction price for contracts with different strike prices and maturities. According to IDEM market rules, prices are quoted for the option with strike price nearest to the index, two strikes above and two below it. Strike prices differ from one another by 500 "index points", each of the value of 2.5 euro. Furthermore, prices are quoted for options with the three shortest maturities in the calendar month and the three shortest quarterly maturities. We therefore have a vector of approximately 25 prices for call and for put contracts at each point in time (day/time of the day) which sum up to a total of 116,772 prices. From this sample we preliminary discard 40,872 prices for which either the price itself or the underlying where not correctly reported or for which we do not have a positive bid or ask volume. We are therefore left with a total 75,900 prices, of which 37,920 refer to calls and 37,980 to puts.[7]

Call S_t the value of the MIB30 index at time t, K the strike price of a given option contract, and τ the number of days to the maturity of the contract. A major characteristic of an option is moneyness (z_t), i.e. either the ratio of the spot index over strike (S_t/K, for a call) or its inverse (K/S_t, for a put). By distinguishing contracts on the basis of moneyness and the length

[7]This makes the dimension of our data set considerable with respect to other works in this field. For instance, Cavallo and Mammola (2000) effectively use only 3,642 observations.

of their residual life we can obtain a detailed description of the composition of the sample. In Table II we have adopted the following definitions: an option is considered being at the money (ATM) if the strike price is within 2% from the index; if it is within 5% (but apart for more than 2%) the option will be considered in the money (ITM) or out of the money (OTM) respectively, depending on the sign of its intrinsic value; an option is said to be deep in-the-money (DITM) or deep out-of-the-money (DOTM) if its strike price differs from the value of the underlying by more than 5%. We also adopt the following maturity classes: a contract has *very short* time to expiration if $\tau \in (0, 7]$, *short* if $\tau \in (7, 25]$, *medium* if $\tau \in (25, 50]$, *long* if $\tau \in (50, 90]$, and *very long* when $\tau \in (90, \infty)$. It is evident from Table II that more than half of the sample is composed of options which expire within a month, while almost no option appears with residual life exceeding three months. This last remark entitles us to discard this group from the sample and gives the main justification for choosing a three months reference for the risk-free interest rate. The most important class in the sample is that of ATM options with short residual life (16%). More generally, ATM options represent more than one third of the data set, while short- and medium-term contract account for almost 80%.[8]

Finally, it is worthwhile to stress that the high-frequency structure of our data set allows us to avoid two common problems plaguing empirical studies of option markets: the potential non-synchronicity between option and stock index prices, and liquidity issues, potentially reflecting the fact that some of the prices could be of little economic significance because of the limited transaction volume. As far as synchronicity is concerned, the high frequency structure of the data set excludes that the non simultaneous recording of the option price and of the index may be troublesome for options that are liquid enough. Furthermore, to restrict our attention to liquid option prices only, we check for the bid and ask volumes recorded simultaneously with the option price, and discard prices for which either of these values were nil.

3. **Arbitrage Tests**

In abstract terms, for frictionless options markets absence of arbitrage is equivalent to the pricing rules

$$c_t(K,\tau) = E_{Q,t}\left[e^{-r\tau}\max(S_T - K, 0)\right] \tag{1a}$$

$$p_t(K,\tau) = E_{Q,t}\left[e^{-r\tau}\max(K - S_T, 0)\right], \tag{1b}$$

where c_t and p_t indicate the time t price of a call and a put with strike K and time to maturity $\tau \equiv T - t$. Q is the risk neutral probability measure. For the sake of simplicity, we will assume throughout that the interest rate r is constant. It is then possible to derive from (1) some explicit relationships that have to be satisfied by option prices if arbitrage is absent. We consider the following conditions:[9]

[8] These simple figures stress the arbitrariness of restricting the analysis to either ATM or short-term contracts only.

[9] Dependence of c_t and p_t on K and τ is omitted when no ambiguity arises.

Lower bound:

$$c_t \geq S_t - Ke^{-r\tau} \tag{2a}$$

$$p_t \geq Ke^{-r\tau} - S_t \tag{2b}$$

Strike Monotonicity ($K' > K$):

$$c_t(K) \geq c_t(K') \tag{3a}$$

$$p_t(K) \leq p_t(K') \tag{3b}$$

Maturity Monotonicity ($\tau_2 > \tau_2$):

$$c_t(\tau_2) \quad \geq \quad c_t(\tau_1) \tag{4a}$$

$$p_t(\tau_2) \quad \geq \quad p_t(\tau_1) + K\left[e^{-r\tau_1} - e^{-r\tau_2}\right] \tag{4b}$$

Butterfly ($K'' > K > K'$):

$$\frac{K-K'}{K''-K'}c_t(K'') + \frac{K''-K}{K''-K'}c_t(K') \quad > \quad c_t(K) \tag{5a}$$

$$\frac{K-K'}{K''-K'}p_t(K'') + \frac{K''-K}{K''-K'}p_t(K') \quad > \quad p_t(K) \tag{5b}$$

Put/call parity:[10]

$$p_t(K) - c_t(K) + S_t \quad \geq \quad Ke^{-r\tau} \tag{6a}$$

$$p_t(K) - c_t(K) + S_t \quad \leq \quad Ke^{-r\tau} \text{ or} \tag{6b}$$

$$p_t(K) - c_t(K) + S_t \quad = \quad Ke^{-r\tau}.$$

All of these restrictions are well known and have been repeatedly tested on data from many different markets (see, among others, Klemkosky and Resnik (1979), Phillips and Smith (1980), Figlewski (1989), Nisbet (1992), George and Longstaff (1993), and Kamara and Miller (1995)). For this reason we will not make explicit the trading strategies that ensure arbitrage profits whenever any of these conditions is not satisfied. In all cases, buying and selling option contracts, the underlying and the riskless asset according to the sign with which the corresponding prices appear in the above conditions (expressed as non negativity constraints) leaves the investor with a portfolio giving a positive payoff at maturity. Hence the current price of such a portfolio ought to be positive. If not, an arbitrage opportunity exists. For example, if (4a) is violated, an investor could buy a call with long maturity, sell one with short maturity and if the latter is in the money at maturity, sell the index short while lending the amount K on the money market. This would give the investor $c_t(\tau_1) - c_t(\tau_2) > 0$ at the initial date, 0 at $t + \tau_1$ and

$$\max(S_{t+\tau_2} - K, 0) + Ke^{r(\tau_2-\tau_1)} - S_{t+\tau_2} \geq \max(K - S_{t+\tau_2}, 0) > 0$$

[10]The trading strategies needed to exploit violations of (6) are quite different, depending on which inequality prevails: this motivates our choice to write that condition as a double inequality.

at $t+\tau_2$ (and analogously for the put condition (4b)).[11]

The informational content of conditions (2)-(6) differs significantly because of the role played by the underlying. The outcome of a market efficiency test using (2) - (6) does very much depend on the existence of arbitrage profits on the underlying market as well as on the derivatives market. Thus (2)-(6) are truly *joint* conditions and it is hard to disentangle the contributions of the two markets to possible violations. The actual implementation of the corresponding trading strategies is also rather delicate, because in order to exploit violations some of the conditions in (2)-(6) (for instance, (2a) or (6a)) the investor is required to take a short position in the underlying and this is going to be very complex and costly (if possible at all). In order to partially overcome these problems and gain more detailed information on the actual degree of efficiency of the MIBO market *in isolation*, we also consider the following conditions:

Reverse Strike Monotonicity ($K_1 > K_2$):

$$c_t(K_2) - c_t(K_1) \leq (K_1 - K_2)\, e^{-r\tau} \tag{7a}$$

$$p_t(K_1) - p_t(K_2) \geq (K_1 - K_2)\, e^{-r\tau} \tag{7b}$$

Box Spreads:

$$[p_t(K_1) - c_t(K_1)] - [p_t(K_2) - c_t(K_2)] \geq (K_1 - K_2)\, e^{-r\tau} \tag{8a}$$

$$[p_t(K_1) - c_t(K_1)] - [p_t(K_2) - c_t(K_2)] \leq (K_1 - K_2)\, e^{-r\tau} \tag{8b}$$

i.e.

$$[p_t(K_1) - c_t(K_1)] - [p_t(K_2) - c_t(K_2)] = (K_1 - K_2)\, e^{-r\tau}.$$

Maturity Spreads:

$$[p_t(\tau_1) - c_t(\tau_1)] - [p_t(\tau_2) - c_t(\tau_2)] \geq K\left[e^{-r\tau_1} - e^{-r\tau_2}\right] \tag{9a}$$

$$[p_t(\tau_1) - c_t(\tau_1)] - [p_t(\tau_2) - c_t(\tau_2)] \leq K\left[e^{-r\tau_1} - e^{-r\tau_2}\right] \tag{9b}$$

i.e.

$$[p_t(\tau_1) - c_t(\tau_1)] - [p_t(\tau_2) - c_t(\tau_2)] = K\left[e^{-r\tau_1} - e^{-r\tau_2}\right]$$

[11] It is often believed that the value of a call option increases with time to maturity only in the Black and Scholes model. To show that this is not true one derives from (1a) the inequality

$$\begin{aligned} c_t(\tau_1) &= E_{Q,t}\left\{\max\left(S_{t+\tau_1} - K, 0\right) e^{-r\tau_1}\right\} \\ &\leq E_{Q,t}\left\{\max\left(S_{t+\tau_1} e^{-r\tau_1} - K e^{-r\tau_2}, 0\right)\right\} \\ &= E_{Q,t}\left\{\max\left[E_{Q,t+\tau_1}\left(S_{t+\tau_2} e^{-r\tau_2}\right) - K e^{-r\tau_2}, 0\right]\right\} \\ &\leq E_{Q,t}\left\{E_{Q,t+\tau_1}\left\{\max\left(S_{t+\tau_2} - K, 0\right) e^{-r\tau_2}\right\}\right\} \\ &= E_{Q,t}\left\{\max\left(S_{t+\tau_2} - K, 0\right) e^{-r\tau_2}\right\} \\ &= c_t(\tau_2) \end{aligned}$$

and similarly for a put contract. As we will make clear shortly, condition (4) will mainly be used as a control on the quality of the data set.

(8) and (9) directly follow from (6) evaluated over a pair of different strike prices and maturities, respectively and taking the difference between the values so obtained. (7) follows from combining (8) with (3). The advantage of (7)-(9) over (6) is that they do not depend on the underlying and therefore the corresponding portfolios do not imply any position but in the options market. (7) and (8) have been introduced recently by Ronn and Ronn (1989). Only (9) is, to our knowledge, entirely new, and this justifies a more detailed analysis.

Suppose that (9a) is violated and consider an investor that buys a put and sells a call maturing in τ_1 days with the same strike price K. The final payoff of her position and that of an investor who sells a future with identical maturity differ by the deterministic quantity $K - F(t, t+\tau_1)$ only. Given that the initial value of a future is 0 by construction, it must be that $p_t(K, \tau_1) - c_t(K, \tau_1) - e^{-r\tau_1}K = -e^{-r\tau_1}F(t, t+\tau_1)$. Taking the reverse position for options with time to maturity $\tau_2 > \tau_1$ may be compared to buying a futures maturing at $t + \tau_2$. Summing the two positions and subtracting the amount $K\left[e^{-r\tau_1} - e^{-r\tau_2}\right]$ raised issuing debt is therefore entirely analogous to buying a futures maturing at $t + \tau_2$ and selling one maturing at $t + \tau_1$. Then the value of such a portfolio must be:

$$\begin{aligned} 0 &= e^{-r\tau_2}F(t, t+\tau_2) - e^{-r\tau_1}F(t, t+\tau_1) \\ &= \left[p_t(K, \tau_2) - p_t(K, \tau_1)\right] - \left[c_t(K, \tau_2) - c_t(K, \tau_1)\right] + \left[e^{-r\tau_1} - e^{-r\tau_2}\right]K \end{aligned}$$

If this quantity is positive, then $F(t, t+\tau_1) \leq e^{-r(\tau_2 - \tau_2)}F(t, t+\tau_2)$ and it would be strictly convenient to buy the synthetic futures with short maturity while selling the one with long maturity at $t + \tau_1$. Condition (9) is then to be considered more as a test on the absence of arbitrage opportunities on the futures market, as mimicked by the market for options: for this reason we will not include it when computing the total number of arbitrage opportunities on the MIBO market and simply exploit it as a device to check the robustness and meaning of our results.

Tables III and IV give aggregate and disaggregated results of this battery of no-arbitrage tests.[12] Table III shows first of all that the number of arbitrage violations is indeed outstanding, approximately 50% of the sample. For what concerns the different conditions tested, it clearly emerges that the most often violated are Box spreads (8), the put/call parity (6) and Butterfly spreads (5). The two Box conditions are violated almost equally often, but the put-call parity condition is highly asymmetrical and it is the short hedge (6a) the most relevant one. This finding reflects the higher difficulty and costs to take a short position in the index required to exploit this violation.[13] Also the convexity condition (Butterfly) is often not satisfied. This is due to the very general form in which we test it.[14] For an option appearing in the n^{th} position

[12]When tracking deviating options, one is faced with the problem of how to identify such contracts when the condition tested implies more than one contract at the time, as in the put/call parity. In order to avoid duplications we arbitrarily impute violations of such conditions to put contracts.

[13]Cavallo and Mammola (2000) report the same conclusion.

[14]In most studies the butterfly condition is tested by confronting each contract with just a pair of contracts, those with strike price immediately larger and smaller.

inside a vector of 25 option prices sorted by the strike price, there are in fact $(n-1) \times (25-n)$ possible hedges, $n = 2, \ldots, 24$, for a total of $2,300$. The small number of violations of the vertical spreads (Strike Monotonicity and Maturity Monotonicity) provides some evidence that lack of synchronicity between the MIB30 and option prices is not serious in our data set. In fact, whenever an option contract have not been traded for a while it would easily be the case that the intervening variation in the value of the index results in a vertical misalignment of option prices.[15] The distribution of the number of violations across moneyness, time to maturity and type of condition violated is also interesting (Table IV). It is clear that the number of arbitrage opportunities detected increases the shorter the time to maturity and higher the moneyness. Short-term, ITM and DITM options are normally considered not very liquid: Those investors who own option contracts which are in the money and with short time to maturity have good prospects of receiving a positive final payoff if they hold their contracts rather than trade for arbitrage profits. We thus find some correlation between mispricing and liquidity. Nevertheless, even the more liquid segments of the market tend to display ratios above 30%, i.e. inefficiency is maximum when coupled with low liquidity, but it characterizes in general the MIBO30.

The maturity spread condition (9) is also frequently violated and, as remarked already, this is to be interpreted as a signal of the inefficiency of the futures (FIB30) market. The striking fact here is the deep asymmetry between the short side (9a) — which is negligible — and the long side, which is often violated. Remember that (9b) is equivalent to buying long maturities and selling short ones on the futures market and reflects the expectation of long-term increases of the index, which is consistent with the bull market of the last part of 1999 shown by Figure 1. This finding has an interesting interpretation in terms of market microstructure. Current IDEM rules imply an almost complete lack of maturity overlap between the options and the futures markets. The shortest maturity for which futures contracts are traded is one month, Section 2 shows that 54% of traded option contracts expires within 25 days. Hence the synthetic futures created by trading options with the aim of exploiting arbitrage opportunities arising from the maturity spreads do not find correspondence in any actual futures contract. One possibility is that such synthetic positions are set up for hedging purposes, rather than for arbitrage operations. Especially in this case, a better synchronization of the MIBO30 and the FIB30 cycles might affect the incidence of this type of mispricing and improve the overall efficiency of the IDEM.

4. **The Role of Frictions**

The real issue remains the interpretation and explanation of our findings. Whereas conditions (2)-(9) refer to a model of frictionless markets (such as Black and Scholes (1973)), frictions are obviously important in actual markets. Therefore, a first way to look at the preceding results is to consider them as an indication of the distance between abstract, frictionless finance theory

[15] Problems with the automatic quoting system made such an event extremely frequent in the first months of 1999. For this reason these additional 3 months were not included in our study.

and the real world.

An initial assessment of the role of frictions on the MIBO may be simply obtained by computing the exact amount of the arbitrage profits corresponding to Tables III-IV. Obviously, it is not only interesting to quantify the number and incidence of arbitrage opportunities but also to measure the level of the implied profits. This information is contained in Table V, in which we report average profits for each arbitrage condition and type of contract, along with other descriptive statistics of their empirical distribution. Taking into account that the average price of contracts in the sample is about $1,250$ index points, it turns out that while some conditions, such as the butterfly spread, provide (loosely speaking) an average profit of about 5%, other conditions such as the Lower Bound for call contracts (2a) and the long put-call parity (9) offer substantial arbitrage returns, ranging from 30 to 40%. The whole distribution of arbitrage profits is interesting and plotted in Figure 2. It turns out that conditions (3) through (7) are rather concentrated around modest profit levels and that more than 60% of the sample of arbitrage profits are below 200 index points. Nevertheless, conditions (8) and (9) display much higher profits so that the maximum available arbitrage profits (reported in panel C of Figure 2) have a distribution that is right-skewed. As expected, the short side of the put/call parity (6a) is more profitable than the long one. Figure 3 gives details on the empirical distribution of profits as a ratio of the price of the corresponding contract(s). To make the plots readable, in Panel A we only report the distributions of some conditions (namely (5), (8a) and (9b)) as the remaining ones would anyway lie in between. Panel B shows the distribution of the overall rate of profit. Although for some conditions the rate of arbitrage profits seems to be rather low (for instance, for the butterfly spread), among those contracts that allow for an arbitrage operation, about half guarantee a rate of at least 10% and about one fifth of the contracts provide a rate of at least 30%.

In a market with no "real" arbitrage opportunities, arbitrage profits deduced from theory should indicate nothing but the amount paid by investors as commissions, margins, taxes, etc. Table V may then be simply interpreted as reporting the average amount of transaction costs of trading on the MIB30 index and/or the IDEM. When trying to understand the nature of such costs, it is reasonable to restrict attention to the following aspects:

1. *Microstructural issues.* Inserting more than one order in the market maker's book does not guarantee that all of them will be instantaneously executed. In volatile asset markets, possible delays tend to make the final payoff of the trading strategy uncertain. Although it may be objected that — at least for certain groups of investors, such as market makers (their orders take priority over the others) — automated market circuits reduce this evenience, it is not clear how to take this aspect in due account. Similarly to previous literature (see Gould and Galai (1974), Nisbet (1992), and Ackert and Tian (2000)) we check whether arbitrage opportunities are still available in the observation that immediately follows the one in which it was originally detected, i.e. whether arbitrage opportunities persist for at

least half-an-hour.[16] We obtain results very similar to those reported in Tables III and IV and therefore omit them.

2. *Taxation*. When constant tax rates apply to capital gains, the existence of taxes does not alter the *existence* of arbitrage profits (but clearly affects their amount). However the main problem when assessing the impact of taxes is that they it depends on the overall fiscal position of investors so that identical transactions could in principle undergo very different tax regimes, depending, for example, on whether the investor is home-based or foreign. As most papers, we prefer to omit this element rather than take a totally arbitrary approach.

3. *Dividends*. Known future dividends can be easily accommodated into option pricing models, although in practice (especially for a stock index) dividends are rarely known in advance. Although it may be observed that historically dividends on the Italian stock market tend to be small and therefore not a major factor (especially for short- and medium-term option contracts), a simple test of the relative importance of dividends for our purposes can be derived by noticing that dividends are most often paid out in the months of June and July. Figure 4 plots the monthly distribution of the percentage incidence of mispricing. Arbitrage violations are not particularly concentrated in any particular month, with the exception of April 1999. No special concentration of mispricing is noted in June and July. Nevertheless we follow the literature and try to imply a high frequency series of the dividend yield out of the available data set in three alternative ways: (i) from the spot-future parity, following Aït-Sahalia and Lo (1998);[17] (ii) from the put-call parity;[18] (iii) from the maturity spread condition. The results obtained are very similar in all three cases: Approximately 70% of the computed dividend yields turns out to be negative, which is clearly meaningless while the average yield is approximately 0.02%. Incorporating these values into our exercise has almost no effect.

4. *Transaction costs*, in particular the presence of bid/ask spreads. Given our conclusions concerning the frictions under 1. - 3., we devote the remaining part of this section to transaction costs properly said.

[16] For end-of-the-day misspricings, the test checks whether the opporunity persists until the market opening, the following business day. Notice that allowing for such delays without conflicting with the idea that arbitrage is a sure profit would in pricinciple require a data set with much higher frequency than ours. However the literature reports examples of ex-post arbitrage profits analysis based on daily data sets (for instance, Ackert and Tian (2000)) in which the implementation of the tests has little significance since the free lunch nature of arbitrage profits is obviously not preserved.

[17] Call $\delta_{t,\tau}$ the continuous dividend yield between time t and $t+\tau$, and $F_{t,\tau}$ the time t price of FIB30 futures contracts with maturity in $t+\tau$. Then $\delta_{t,\tau} = \frac{1}{\tau}\ln\left(\frac{S_t}{F_{t,\tau}}\right) + r_{t,\tau}$. In our case calculation of $\delta_{t,\tau}$ for all maturities requires some interpolation as the FIBO30 market only trades quarterly maturities, while the MIBO30 trades many more maturities.

[18] $\delta_{t,\tau}(K) = \frac{1}{\tau}\ln\left(\frac{p_t(K)-c_t(K)+S_t}{K}\right) + r_{t,\tau}$. Of course, given the high ratio of put-call parity violations even for ATM contracts, the method is at least doubtful.

Let T_t^S represents the *fixed* cost of a given date t sale of the underlying, the MIB30 index, the superscript a denotes ask prices, and b bid prices. In Table VI we rewrite the arbitrage profits arising from violations of the conditions introduced in Section 3 and taking into account transaction costs and bid/ask spreads. Apart from T_t^S, we adopt the choice of treating all *proportional* costs related to buying or selling assets — for example brokers' commissions — together with the bid/ask spread. As for T_t^S, it represents the differential impact of transaction costs when the arbitrage strategy requires the investor to sell the underlying index short, and therefore it appears in conditions (2a) and (6a). Selling a stock index short can be accomplished in different ways. The exact replication of an index sale can only be obtained by selling the future on the MIB30. This is an ordinary sale transaction so that no particular costs apply apart from the corresponding bid/ask spread. Unfortunately, this strategy is not always available in the Italian stock market since the expiration dates of futures and options match only imperfectly. All the alternatives to a short position in the matched futures have the disadvantage to display high but imperfect correlation with the underlying and therefore imply implicit costs (risk). One obvious possibility is of course to sell the index according to the amount already existing in an arbitrageur's portfolio. Apart from correlation issues, this is a cheap alternative in which no true transaction cost appears other than the bid/ask spread on the stocks composing the index.[19] Finally, another possibility is for the investor to participate to a stock loan (*riporto*), a contract that has some importance in the Italian stock market and by which investors gain temporary possession of the stocks upon lending a corresponding amount of money at a prescribed rate, often lower than the current market short interest rate. In this case the transaction cost can be measured in terms of the interest loss implied by the contract.[20] Bid/ask spreads turn out to be the major type of friction in our analysis. Two main difficulties arise. First of all one should in principle reconstruct the bid /ask spread on the index as an average of the corresponding spread on the individual assets composing the index, and this is clearly unfeasible. Therefore the spread must be estimated rather than exactly computed. A second concern refers to the fact that the bid/ask spread is not uniform: it varies considerably across classes of options and categories of investors. A MIBO market maker taking position of her own, rather than playing an intermediary role, would actually pay no such spread when buying or selling options (if not as a shadow cost); even the spread on the index would be considerably smaller relative to other investors. In a sense, for arbitrage strategies purely involving options positions market makers' face equations not very dissimilar to those introduced in Section 3. On the opposite, for other investors the spreads

[19]Of course this strategy is actually available only to investors who already have a long position in the MIB30 and who are not committed to satisfy binding portfolio constraints.

[20]The costs implied by the stock loan contract appear to be of little significance if significant at all. To quantitatively assess this suspicion, we compute the interest rate that makes the put/call parity hold at each point in time and compare it with the actual level of that rate. It turns out that the implied rate is often negative and very rarely its spread with respect to the actual rate is below 100%. It appears that in the presence of the low nominal riskless rates of the period under examination almost any reasonable interest loss incurred by investors because of transaction costs (the use of riporti) would hardly modify Tables III and IV.

do apply but also vary considerably from one contract to the other. According to IDEM rules upper bounds to the spreads for each class of options are imposed as a function of moneyness. Nevertheless both moneyness and the spread are expressed in absolute values, which may result in the corresponding bounds on the ratio of the spread to the option price being extremely wide.

Finally, to make the profit formulas in Table VI more tractable we will assume that both the bid/ask spreads and the transaction cost T_t^S are constant ratios of the corresponding asset value. In particular we call α the spread on the option — for instance, $c_t^b = (1-\alpha)\, c_t$ and $c_t^a = (1+\alpha)\, c_t$ in the case of a call — β the spread on the MIB30 — $S_t^b = (1-\beta)\, S_t$ and $S_t^a = (1+\beta)\, S_t$ — and γ the fixed transaction cost on sales of the underlying, $T_t^S = \gamma S_t$. Then the bid/ask spread is 2α and 2β for options and the MIB30, respectively. Under these assumptions — and assuming $K_1 < K_2$ so that $K^\lambda = \lambda K_1 + (1-\lambda)\, K_2$ — we derive the following lower bounds for arbitrage profits:[21]

Lower bound:

$$K \exp(-r\tau) - \left[\frac{S_t}{1-\beta}\,(1+\beta) + p_t\,(K)\,(1+\alpha)\right] \tag{10a}$$

$$-K \exp(-r\tau) + \left[\frac{S_t}{1+\beta}\,(1-\beta)\,(1-\gamma) - c_t\,(1+\alpha)\right] \tag{10b}$$

Strike Monotonicity:

$$p_t\,(K_1)\,(1-\alpha) - p_t\,(K_2)\,(1+\alpha) \tag{11a}$$

$$c_t\,(K_2)\,(1-\alpha) - c_t\,(K_1)\,(1+\alpha) \tag{11b}$$

Maturity Monotonicity ($\tau_2 > \tau_1$):

$$c_t\,(\tau_2)\,(1-\alpha) - c_t\,(\tau_1)\,(1+\alpha) \tag{12a}$$

$$p_t\,(\tau_2)\,(1-\alpha) - p_t\,(\tau_1)\,(1+\alpha) - K\,[\exp(-r\tau_1) - \exp(-r\tau_2)] \tag{12b}$$

Reverse Strike Monotone ($K_1 > K_2$):

$$c_t\,(K_1)\,(1-\alpha) - c_t\,(K_2)\,(1+\alpha) + (K_1 - K_2)\, e^{-r\tau} \tag{13a}$$

$$p_t\,(K_1)\,(1-\alpha) - p_t\,(K_2)\,(1+\alpha) + (K_1 - K_2)\, e^{-r\tau} \tag{13b}$$

Butterfly ($K_1 > K > K_2$, $\lambda = \frac{K-K_2}{K_1-K_2}$):

$$p_t\,(K)\,(1-\alpha) - [\lambda p_t\,(K_1) + (1-\lambda)\, p_t\,(K_2)]\,(1+\alpha) \tag{14a}$$

$$c_t\,(K)\,(1-\alpha) - [\lambda c_t\,(K_1) + (1-\lambda)\, c_t\,(K_2)]\,(1+\alpha) \tag{14b}$$

[21] In equations (10) and (16) we have slightly modified the implemented strategy by imposing that the amount of the underlying sold (bought) be $\frac{1}{1+\beta}$ ($\frac{1}{1-\beta}$) so that at maturity the investor will pays out (receives) the whole price of the undelying, less of the spreads. In essence we impose that the investor may hedge today against changes in future spreads.

Box spreads:

$$[p_t(K_1)+c_t(K_2)](1-\alpha)-[p_t(K_2)+c_t(K_1)](1+\alpha)-(K_1-\ K_2)e^{-r\tau} \quad (15a)$$

$$[p_t(K_2)+c_t(K_1)](1-\alpha)-[p_t(K_1)+c_t(K_2)](1+\alpha)-(K_2-\ K_1)e^{-r\tau} \quad (15b)$$

Put/call parity:

$$\left[\frac{S_t}{1+\beta}(1-\beta)(1-\gamma)+p_t(1-\alpha)-c_t(K)(1+\alpha)\right]-K\exp(-r\tau) \quad (16a)$$

$$\left[c_t(K)(1-\alpha)-\left(\frac{S_t}{1-\beta}(1+\beta)+p_t(1+\alpha)\right)\right]+K\exp(-r\tau) \quad (16b)$$

Maturity spreads:

$$[p_t(\tau_1)+c_t(\tau_2)](1-\alpha)-[p_t(\tau_2)-c_t(\tau_1)](1+\alpha)-K\left[e^{-r\tau_1}-e^{-r\tau_2}\right] \quad (17a)$$

$$[p_t(\tau_2)+c_t(\tau_1)](1-\alpha)-[p_t(\tau_1)+c_t(\tau_2)](1+\alpha)-K\left[e^{-r\tau_2}-e^{-r\tau_1}\right] \quad (17b)$$

Furthermore, when using these formulas we will set $\alpha=\beta$. When $\alpha=\gamma=0$ we immediately deduce that (10) - (17) are exactly equivalent to (2) - (9). For positive values of the parameters, (2) - (9) are far more restrictive thus explaining how profit opportunities that exist in theory may not exploitable in practice.

This simplified structure for the allowable frictions is similar to others already used and/or estimated in the literature. Nisbet (1992, pp. 392-393) estimates transaction costs on the London Traded Options Market (LTOM) during 1988 finding $1.2\% \leq 2\alpha \leq 6.8\%$ and $0.2\% \leq \gamma \leq 5.7\%$. Yadav and Pope (1994, pp. 925-926) investigate the market for futures on the FTSE 100 index between 1986 and 1990 estimating $2\alpha \leq 0.6\%$ and $\gamma \leq 0.75\%$, i.e. substantially lower values. Longstaff (1995) studies the S&P 100 index options market (CBOE) over 444 days between 1988 and 1989: he notices that α varies considerably across different classes of options and that for options that are deep OTM the bid/ask spread can be extremely high (up to 30% of the average price). Using the same data set, George and Longstaff (1993) study the distribution of the average spreads. Table VII reports their results. The average of the bid/ask spread is 2.5% but it can be two or even three times higher in percentage terms for DOTM (and hence low unit price) contracts.[22]

Our objective is now to imply out of the available MIBO data a structure for transaction costs that makes the data compatible with the presence of varying degrees of violation of the basic no-arbitrage conditions (10) - (16). Since experience with this type of scenario analysis shows that changing γ makes very little difference for the result on the number and size of the existing arbitrage opportunities, we set $\gamma=0$ and focus on α only. Table VIII contains the detailed

[22] Ackert and Tian (2000, p. 45) report that on the S&P 500 for contracts with price above (respectvely, below) \$3 the bid ask spread was about 1/16 (1/32) of a point. Therefore the bid/ask spread should at least 2% for low price (DOTM) options and can be as high as 4% for medium price options.

outcomes of our scenario simulations (for $\alpha = \beta = 5\%$) while Table X describes outcomes as α is increased from 0 to 10%. Correspondingly, Figure 5 plots the number and percentage incidence of violations as a function of α. In panel of A of Figure 5 we plot the ratio of arbitrage violations over the sample size for conditions implying a low number of violations. Percentage ratios in this case rapidly converge to zero as α increases.[23] Panel B refers to the two sides of the put-call parity: although it is still clear that violations of the short end occur more frequently that those of the long side, it is remarkable that the high ratios of these violations quickly drop to zero as soon as α reaches about 2%. This is entirely consistent with the findings of Cavallo and Mammola (2000). However, the put-call parity is not the only arbitrage restriction that option prices should obey: panel C stresses the existence of other conditions that are both frequently violated in the absence of frictions (box and maturity spreads implied about 20% violations in Table III) and that remain important throughout the range of all the plausible values for α we consider. For instance, at $\alpha = 4\%$ the two box spread conditions still originate a 5% each of violations, while the maturity spread condition is still at a very persistent 15%.[24] The plot documents that some kinds of pricing inefficiencies are very persistent and that in particular condition (9b) is scarcely influenced by the value of the transaction costs.[25] Table IX details the findings for $\alpha = 5\%$ by breaking the results down as a functions of moneyness and time to maturity. As expected, most of the violations keep occurring for DOTM (for which we are probably underestimating the size of the bid/ask spread on options) and very short-term contracts (which are less liquid and hence likely to imply frequent mispricing as well as higher transaction costs than allowed for by our framework).

Summarizing, these findings give three indications. First, even modest frictions can significantly reduce the number of violations, implying that most of the violations detected in Section 3 actually corresponded to very thin profit levels. For example, with $\alpha = 2\%$ violations of the monotone spreads (with respect to either strike or maturity) drop to about 1.6% thus confirming that our data set is not affected by systematic misrecordings. Second, as α increases the arbitrage conditions that remain into play are the box and maturity (long) spreads. Third, it is not possible to completely get rid of the mispricing even by raising α beyond reasonable tresholds (8% or more) as mispricing persist in more than 2.5% of the sample. Although this number is not high enough to cast doubts on the meaningfulness of our tests, it witnesses the existence of niches of

[23] Violations of the maturity monotone spread conditions are not plotted altogether because negligeable in number and economic significance.

[24] A strong belief that the MIB30 was bound to rise during 1999 is a reasonable explanation for these misspricings. It would be interesting to consult directly elicited beliefs (polls) from market operators.

[25] The resilience of the violation of the spread conditions is not totally surprising. Observe that in the spread conditions the bid/ask spread is less important than in others, given our assumptions. As a pure matter of scale the spread has high impact especially when applied to the underlying (the MIB30 has a price which is several orders of magnitude higher than option contracts). Since trading in the underlying is not required to exploit misspricings evidenced by the spread conditions, increasing bid/ask spreads will reduce their percentage relevance very slowly.

pricing inefficiencies that cannot be simply explained away by the existence of frictions.[26]

Notice that a vanishing number of arbitrage opportunities does not imply that the surviving mispricing cannot be anyway quite profitable. Figure 6 plots average rates of arbitrage profit vs. the bid/ask spread α. These plots confirm one of our previous conclusions: the profit rate does not converge to zero as α increases. On the contrary, average profit rates deriving from arbitrage strategies exploiting violations of conditions such as the butterfly spread seem to be increasing with α. The reason for this puzzling behavior is that higher bid/ask spreads have two effects: (i) they discard from the sample of arbitrage opportunities those associated with a lower level of profits; (ii) they cut down the profitability of those opportunities that remain in the sample. The combined effect may result in a positive relationship between profits and bid/ask spread whenever the number of low profit strategies is important. Figure 6 also stresses that the short edge of the put-call parity is on average, and independently of α, much more profitable than the short edge.

We conclude that the MIBO market is characterized by remarkable arbitrage opportunities even after considering the role of transaction costs and other frictions. Some inefficiency may actually be caused by the loose bounds on bid/ask spreads (when expressed as ratios) mandated by the IDEM. Also, the current structure of the IDEM appears to be responsible for some of the mispricing we have detected, especially for the most persistent ones that are not much affected by the introduction of frictions. For instance, we suspect that a relevant percentage of the violations detected might be easily removed by improving the feasibility of synchronized trading (on instruments with identical maturity) on the MIBO and FIB30 markets, given that some arbitrage strategies require investors to simultaneously submit orders on the two markets. On the other hand, there is no doubt that the level of arbitrage profits detected after filtering for the bid/ask spread is not at all negligeable, especially if we consider that the spreads do not evenly apply to all investors. Finally, trading strategies triggering short positions in the index tend to exhibit higher profitability, and this is maybe a consequence of the existence of portfolio constraints, particularly for institutional investors. The latter is clearly a kind of market imperfection that cannot be accounted for by any data analysis although we may conjecture that it does have significant role.

5. **Informational Efficiency**

5.1. *The MIBO30 IV surface and its determinants*

Some valuable information on market efficiency as well as on the pricing mechanism is conveyed by the structure and dynamics of the implied volatility (IV) surface, i.e. the behavior of Black-Scholes

[26]Longstaff (1995) reports that violations deriving from conditions such as monotonic or butterfly do not reach 1.5% of his sample, and the associated profits would amount to a few cents. In order to recover the same ratio in our data set we would need to impose a value of α close to 6%, considerably higher than those documented by Longstaff. Moreover, residual profits would still be rather high.

IVs as a function of either moneyness or time-to-maturity.[27] In this sub-section we accurately examine the MIBO IV surface. To this purpose we often distinguish between the original, raw data set and a sub-sample of option prices obtained after filtering arbitrage opportunities existing when setting $\alpha = \beta = 5\%$, a total of 67,962 observations. We will often refer to the latter data set as the arbitrage-free one.

Starting with IVs vs. moneyness, Figure 7 plots full-sample as well sub-periods averages and medians of implied volatility when classified in 21 mutually exclusive moneyness intervals of 1% size, starting at 0.89 and up to 1.10. Overall, IVs describe a very asymmetric smile when plotted against moneyness. Medians and means are not very different, confirming that DITM options command an IV which is 5-8% higher than ATM options; while OTM options have IVs slightly lower than ATM options, DOTM options imply again IVs above the ATM levels. Although the meaning of averaging (or calculating the median of) the IVs of contracts with different time-to-maturity is ambiguous, it is clear that one of the basic assumptions of Black-Scholes — constant volatility independent of the behavior of the underlying spot price — hardly applies to the Italian options market. The bottom panel of Figure 7 plots average IVs vs. moneyness for three subperiods of equal length: 04/06/1999 - 07/15/1999, 07/15/1999 - 10/25/1999, and 10/26/1999 - 01/31/2000. While the first and last periods produce jagged smiling shapes, the second is an asymmetric smile similar to the one obtained for the full sample. The variety of shapes obtained through a simple decomposition into three subsamples makes us suspect the presence of remarkable instability in the MIBO IV surface.[28]

We now examine the IV surface for a few alternative days. For instance consider April 16, 1999. Figure 8 plots four IV curves as a function of moneyness for three consecutive trading times covered by our data set (11:49 am, 12:19 p.m., and 12:49 p.m.), besides the closest moment to market closing in our sample, 5:19 p.m. Reading the plots in a clockwise direction, we have an initial example of stability of the IV surface (between 11:49 am and 12:19 p.m., when it describes an almost perfectly skewed shape, an asymmetric smile) followed by a sudden shift to (an almost equally perfect) smile. However, by the end of the day (5:19 p.m.) the IV surface has once more changed, taking a shape in which DOTM options have much higher IV than all other moneyness classes. Figure 8 contributes to our consciousness that on the MIBO30 market the IV surface can take (even in the space of a few hours) many alternative shapes and be subject to sudden breaks, apparently signalling drastic revisions of market expectations. Figure 9 shows that similar remarks apply to the other dimension of the IV surface, the term structure of implied volatilities (IVs vs. time-to-expiration). Focussing on the afternoon of Sept. 7, 1999,[29] we can

[27] The relationship between the option price and the level of volatility is stricly monotonic increasing and we will therefore interpret implied volatility as a scale change of the original price *independently of the actual applicability of Black-Scholes model.*

[28] Figure 7 refers to the raw data. We have replicated the same type of analysis with reference to the arbitrage-free data obtaining very similar plots that we omit to save space. Therefore asymmetric smiles in the IV surface do not entirely depend on the presence of arbitrage opportunities.

[29] This choice is not totally random, as in order to be able to draw IV term-structure plots we require the trading

see that not only a variety of shapes of the IV surface vs. time-to-maturity are possible — at first hump-shaped, then upward sloping, then 'smiling', and finally downward sloping — but also that dramatic changes can occur in half-an-hour only. For instance, on that day the term structure evolved from hump-shaped to upward sloping between 1:05 p.m. and 2:35 p.m., with two further breaks between 2:35 p.m. and 3:35 p.m.. At market close, the IV surface was decreasing vs. time-to-maturity, another possible structure never appeared while the MIBO market was open during the day. Also in this case, sudden breaks are possible and on the whole the IV shapes are highly unstable.

Since it is difficult to draw conclusions as to what affects the IV surface by merely observing a few plots, we also fit a few alternative structural models describing the relationship between implied volatilities and contract features. We estimate the following nine models:

1. $\ln \sigma^{IV}(z_t, \tau_t) = \beta_0 + \epsilon(z_t, \tau_t)$
2. $\ln \sigma^{IV}(z_t, \tau_t) = \beta_0 + \beta_1 \ln z_t + \epsilon(z_t, \tau_t)$
3. $\ln \sigma^{IV}(z_t, \tau_t) = \beta_0 + \beta_1 \ln z_t + \beta_2 (\ln z_t)^2 + \epsilon(z_t, \tau_t)$
4. $\ln \sigma^{IV}(z_t, \tau_t) = \beta_0 + \beta_1 OTM_t + \beta_2 (ITM_t)^2 + \epsilon(z_t, \tau_t)$
5. $\ln \sigma^{IV}(z_t, \tau_t) = \beta_0 + \beta_1 OTM_t + \beta_2 (\ln z_t)^2 + \epsilon(z_t, \tau_t)$
6. $\ln \sigma^{IV}(z_t, \tau_t) = \beta_0 + \beta_1 OTM_t + \beta_2 (\ln z_t)^2 + \beta_3 ITM_t + \epsilon(z_t, \tau_t)$
7. $\ln \sigma^{IV}(z_t, \tau_t) = \beta_0 + \beta_1 \ln z_t + \beta_2 (\ln z_t)^2 + \gamma_1 \tau_t + \gamma_2 (\ln z_t) \tau_t + \epsilon(z_t, \tau_t)$
8. $\ln \sigma^{IV}(z_t, \tau_t) = \beta_0 + \beta_1 \ln z_t + \beta_2 (\ln z_t)^2 + \gamma_1 \tau_t + \gamma_2 (\ln z_t) \tau_t + \gamma_3 \tau_t^2 + \epsilon(z_t, \tau_t)$
9. $\ln \sigma^{IV}(z_t, \tau_t) = \beta_0 + \beta_1 NS_t + \beta_2 NS_t^2 + \gamma_1 \tau_t + \gamma_2 NS_t \tau_t + \epsilon(z_t, \tau_t)$

Models 1-6 follow the specification search in Peña et al. (1999, pp. 1159-1160), apart for the fact that the regressand is specified as the logarithm of the IV. Since by construction $\sigma^{IV}(z_t, \tau_t) > 0$, the right-hand side is always well defined.[30] The advantage of this specification is to make the random regressands consistent with the errors $\epsilon(z_t, \tau_t)$, commonly interpreted as (possibly normal) random draws from a distribution symmetric around zero. Model 1 corresponds to the assumption of constant volatility (e^{β_0}) independent of contract features of Black-Scholes' model. It is a useful benchmark as it allows to measure which is the percentage of variability captured by additional regressors. Models 2 and 3 correspond to the case of an IV surface which is either a linear or a quadratic function of moneyness, although the IV surface does not depend on time-to-expiration. As for models 4-6, define the following piecewise functions:

$$OTM_t = \begin{cases} \ln z_t & \text{if } z_t < 1 \\ 0 & \text{if } z_t \geq 1 \end{cases}, \qquad ITM_t = \begin{cases} 0 & \text{if } z_t < 1 \\ \ln z_t & \text{if } z_t \geq 1 \end{cases}. \tag{18}$$

The former indicator measures moneyness when the contract is OTM and zero otherwise, while the latter measures moneyness when the contract is ITM. Clearly, $OTM_t + ITM_t = \ln z_t \; \forall z_t$.

days to have at least three different maturities simultaneously traded.

[30] Ncube (1996, p. 74) makes the same point.

Consequently, model 4 captures an asymmetric smile, linear for $z_t < 1$ and quadratic for $z_t \geq 1$. Model 5 still represents an asymmetric smile, as for $z_t < 1$ the IV surface is described by a polynomial of second degree, while for $z_t \geq 1$ the IV surface reduces to the upward sloping branch of a quadratic function. Model 6 is yet another variation, in which for $z_t < 1$ the IV surface is a polynomial of second degree with coefficients β_1 and β_2, while for $z_t \geq 1$ a different polynomial of second degree is fitted, this time with coefficients β_2 and β_3. Models 7 and 8 are inspired by Dumas et al. (1998, p. 2068). Model 7 allows the IV surface to change as a function of time-to-expiration τ_t. τ_t also appears in an interaction term, $\tau_t z_t$. Model 8 differs from 7 as also a quadratic term in τ_t is used as a regressor. Finally, model 9 follows Gross and Waltner (1995) in using an alternative to the variable z_t, the (so-called) normalized strike, NS_t.[31] Model 9 is otherwise identical to model 7.

Although it is well known that traditional OLS estimates are not efficient and produce a biased estimate of the covariance matrix, in the following we take the non-sphericality of the pricing errors into account by running OLS regressions of models 1-9 but calculating heteroskedasticy-autocorrelation consistent estimates of the covariance matrix of the estimated regression coefficients as in Newey and West (1987).[32]

Panel A of Table XI reports the estimation output for the original data set purged of the observations violating the lower bound condition only. In general all models and estimated coefficients are highly statistically significant, even when the possible non-sphericalities are taken into account in the estimation of the covariance matrix of the OLS estimates.[33] In spite of this, there are substantial differences in the explanatory power of the alternative structural models. In particular, models 7 and 8 — those reflecting a possible term-structure in the IV surface and the interactions between time-to-maturity and moneyness — provide a $\bar{R}^2$ (0.13-0.14) which is much higher than all other models.[34] For instance, focussing on model 8, it turns out that the ATM volatility for a typical medium-term option 50 days to maturity ought to be 23%. However this value grows to 24.2% for a contract DOTM ($z_t = 0.9$) and to 48% for a contract DITM ($z_t = 1.1$), holding time-to-maturity constant. These numbers are perfectly consistent with a strongly asymmetric smile (smirk). However, even taking into account that this data set contains a certain percentage of observations violating basic arbitrage conditions as well as the high frequency with which the option prices are sampled, an adjusted R-square of 0.14 is quite modest in the light of the literature. For instance, Ncube (1996) obtains a 0.48 estimating a model with similar structure. Panel B of Table XI estimates the same models on the arbitrage-free data set. Although also in this case most of the estimated coefficients are highly significant, there is

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[31] $NS_t = \frac{\ln\left(\frac{S_t e^{(r_t - \delta_t)\tau_t}}{K}\right)}{\sqrt{\tau_t}}$,where τ_t is expressed as a fraction of a 365 days' year.

[32] In a companion paper we explore other econometric methodologies able to account for heteroskedasticity and serial correlation of the random shocks affecting the IV surface, see Cassese and Guidolin (2001).

[33] Following the suggestion in Newey and West (1987), the truncation lag is set equal to the integer part of $4\left(\frac{T}{100}\right)^{\frac{2}{9}}$ where T is the sample size.

[34] Model 9 and the alternative definition of standardized moneyness it employs perform quite poorly.

now an important change as model 9 — employing a standardized definition of moneyness — provides a much higher $\bar{R}^2$ than all other models, 0.26. It seems then that the particular shape that best fits the MIBO30 IV surface does depend on the fact that the data might contain some observations out-of-line, i.e. violating basic no-arbitrage conditions (see Cassese and Guidolin (2001)). The better $\bar{R}^2$ achieved is only partially surprising if we think that prices that are not arbitrage-free are likely to be the reflection of either misrecordings or pure noise. On the other hand, the overall explanatory power of our structural models remains rather limited. The implication is that the time variation in the structure of the MIBO30 IV surface is so strong that simple structural models relating IVs to basic contract features hardly explain the sudden shifts in the surface. This result casts further doubts on the fact that MIBO30 prices might be actually changing as a result of information flows affecting market expectations. This motivates our investigation of the informational content of MIBO30 prices.

5.2. *Heterogeneity and instability of the IV surface*

Given the heterogeneous shapes of implied volatility as a function of both moneyness and time-to-maturity documented in 5.1, we classify these shapes in a few alternative categories. This provides another set of descriptive measures that allows us to document in precise ways the instability of the IV surface and to suggest possible explanations for the unusually low explanatory power of the IV surface regressions.[35] At each point in time (day and time of the day) for which at least 3 observations on option prices for alternative moneyness levels are available, we classify the MIBO30 IV curve as a function of moneyness in five distinct categories:[36]

1. *Smiles*, if the average implied volatility DITM and DOTM are above the average near-ATM volatility;
2. *Smirks*, if the average DITM implied volatility exceeds the average near-ATM which in its turn exceeds the average DOTM implied volatility;
3. *Reverse smirks* (downward sloping shapes), if the average DOTM implied volatility exceeds the average near-ATM which in its turn exceeds the average DITM implied volatility;

[35] David and Veronesi (2000) have recently stressed the instability of the implied volatility curves in the US (S&P 500) options market and the possibility that at times implied volatilities would show either 'frowns' or upward sloping shapes when plotted against the ratio K/S_t (put moneyness). Interestingly, they use these observations and the fact that smiles and smirks are anyway deemed to be prevalent as a benchmark for the empirical assessment of their theoretical option pricing model.

[36] The information from OTM and ITM options is ignored and that accomodates for very imperfect smiling or sneering shapes, in which irregularity of OTM and ITM IVs behavior occurs. Also, we drop IV curves in which either *both* DOTM *and* OTM or *both* ITM *and* DITM options are absent. Finally, we avoid mis-classifying any of the shapes when the average IVs are actually not very different for different moneyness classes, by requiring a minimum distance of $\delta = 1\%$ (in annualized percentage terms) when running pairwise comparisons. One implication is that category 5, besides collecting many irregular and odd shapes, will also sweep up all the cases of near-flat (BS-consistent) IV curves.

4. *Frowns* (reverse smiles), if the average near-ATM implied volatility exceeds both the average DITM and the average DOTM volatilities;

5. Other shapes, all the cases that cannot be classified in any of 1.-4. above.

In the original data we find that for the full sample smirks dominate appearing 48% of the time, followed by other shapes (including flat, BS-type shapes, 27%) and by smiles (21%). Frowns and upward sloping smirks are relatively uncommon, representing a combined 4% only. Table XII reports these results along with the unconditional probabilities (sample frequencies) for the three sub-samples defined above. The frequencies are remarkably stable over time, with one exception: in the third subperiod covering November 1999 to January 2000 there are many less cases of smiling IV shapes (11%) and a peak (9%) of upward sloping smirks. Table XII also stresses that the frequencies are quite robust to changes of the parameter δ, at least in the sense that excluding the residual shapes, smirks always dominate over smiles and a part of the smiles seem to become reverse smirks during the third subperiod, which is interesting.[37] The bottom panel of Table XII applies these criteria to the arbitrage-free data set obtained in Section 4. Apart from a small increase in the importance of smiles, the picture is essentially unchanged.

Another feature of MIBO30 option prices is that the corresponding IV surfaces are highly volatile, changing (often in dramatic ways) quite rapidly in a matter of days or even during the same trading day. We try to quantify the speed of change of IV curves vs. moneyness by calculating the frequency with which switches occur from shapes of type i to shapes of type j, $i, j = 1, ..., 5$ and where 1 stands for smiles, 2 for smirks, etc.,[38] Interpreting IV shapes as market states and assuming a first-order homogeneous Markov chain, our calculations yield a sample estimate of the transition matrix describing the stochastic behavior of the market state. Table XIII reports the findings for both the full data set and for the three subsamples. Looking at the full-sample results, it is evident that not all the states 'communicate', meaning that starting from uncommon shapes it is not possible to land in all other states, and viceversa. In general all the sample frequencies (estimated transition probabilities) are low, implying a remarkable degree of dynamics in the IV surface. For instance, interpreting the sample frequencies of the 5 shapes as their stationary distribution, we have that the unconditional probability of remaining in any of the five states for two consecutive periods (often as short as half an hour) is 0.4183 meaning that almost 60% of the time the IV shape vs. moneyness at t will switch to another shape at $t+1$, an

[37]There is an obvious structural effect in the sense that as δ increases the percentage of IV curves that fall into the residual, 'other shapes' category cannot but increase. In other words, if we apply a very restrictive criterion to classify a pair of IVs for different maturities as different, clearly we will get a substantial number of shapes that are classified as flat curves.

[38]Notice that since at each point in time contracts for different maturities are traded, it is important to quantify the dynamics of IV shape switches by comparing 'slices' of the IV surface having (approximately) identical time-to-expiration. We therefore impose a bound of (approximately) 5 hours on how distant in time two trading dates (defined by day and hour of the day) can be for a switch between shapes of identical maturity to be accounted for.

unequivocal sign of instability of the IV surface. The corresponding unconditional probabilities of remaining in a state for two consecutive periods are 0.382, 0.460, and 0.422 for the three sub-samples, a sign that the instability in the IV surface is structural and does not depend on some short, aberrant period of time. Table XIII also documents that — although the estimated transition matrix changes over time — the underlying instability of the IV surface is pervasive.[39] The bottom panel of Table XIII describes the transition frequencies for the arbitrage-free data set. The unconditional probability of a switch in the IV surface is in this case 0.59, confirming that the impression of a latent instability in the pricing function at work in the MIBO market does not entirely derive from situations of obvious mispricing.

We repeat similar experiments with reference to the shape of the implied volatility function vs. time-to-maturity. The multiplicity of traded strike prices at each point in time forces us to 'slice' the IV surface at a precise moneyness level, ATM, and to compare IVs for different maturities. Since at most three maturities are traded at each point in time, we classify the term structure described by IVs in five mutually exclusive categories:

1. *Upward sloping structures*, if the average implied volatility for ATM options with the longest maturity exceeds volatility for intermediate maturity, which in its turn is above the IV for shorter maturities;

2. *Downward sloping structures*, in the opposite situation (long is less than medium which is less than short);

3. *Smiling term structures*, if the IV for medium term ATM contract is above the IV of both short- and long-term ATM contracts;

4. *Hump-shaped structures*, if the IV for medium term ATM contract is below the IV of both short- and long-term contracts.

5. Other shapes, all the cases that cannot be classified in any of 1.-4. above.

We drop observations in cases in which for ATM options less than three maturities are traded at a point in time. Similarly to what done above, we prevent mis-classifications in the presence of minimal differences of the ATM IVs for different maturities by requiring a minimum distance of $\delta = 1\%$. We apply these criteria of classifications to the original data set. Since the MIBO30

[39] As a robustness check on our findings, we recalculate the transition matrix for the full sample ignoring switches that involve different days. Notice that our criterion of a maximum difference of 5 hours between subsequent observations does not rule out that one IV shape be classified (say) on Monday evenining and the subsequent valid IV shape with corresponding (but notice, not identical) maturity be classified on Tuesday morning. Since the two maturities are then different (and this may matter a lot for very-short term options) and the passage of the night might easily bring in new information (for instance, from North-American markets) that should be reflected in the IV surface, we decide to run this experiment. We find that the resulting transition matrix is very similar to the one reported in Table XIII.

market is not very deep as far as the variety of traded maturities, the resulting classification concerns many less days/hours than in the case of moneyness. Unconditionally, we find that the IV surface tends to be flat (61.09%), and for the rest of the time it is either hump-shaped (14.53%) or 'smiling' (14.46%). The typical upward sloping term structures studied in the US options markets (see Das and Sundaram (1999) and Campa and Chang (1995)) are on the opposite quite uncommon in the MIBO30 (5.45%), and anyway as likely as downward sloping ones (4.47%).[40]

5.3. *The Informational Content of Implied Volatilities*

A much debated question in the literature concerns the informational content of implied volatility. The idea is that, given the one-to-one relationship between option prices and Black-Scholes volatility, if the options market efficiently incorporates all available information through efficient pricing mechanisms, then it should not be possible to improve volatility forecasts over and above the implied volatility themselves. Notice that this type of tests is independent of the actual applicability of Black-Scholes model to option pricing in the MIBO market. For instance, the tests might allow the market to revise the IV characterizing a given contract as a function of changes of the underlying, of interest rates, of the passage of time, etc. (see the discussion in Canina and Fliglewski (1993, p. 661)). Also, these tests link two different notions of market efficiency, *informational* and *risk-sharing* efficiency, in the sense that testing and measuring informational efficiency assumes that the market is risk-sharing efficient, so that no arbitrage opportunities exist.

We conjecture that the MIBO market is informational inefficient. Sections 3 and 4 have provided plenty of indications against the pricing efficiency of the MIBO30 market. We suspect that an imperfect pricing mechanism may prove unable to serve as an unbiased aggregator of beliefs. In particular, in the following we will try to sort out the link between informational and pricing efficiency by comparing econometric tests for the original data and the arbitrage-free one.[41] Subsections 5.1 and 5.2 have also shown that the MIBO30 IV surface is highly unstable over time, to the point that the typical structural models proposed in the literature have little explanatory power. This suggests the Italian options market might be dominated by noise and therefore be a poor information aggregator.

A first issue in implementing this type of tests is defining the *realized* volatility of returns on the underlying asset. Given a series of high-frequency (sampled at half an hour intervals) MIB30 prices $\{S_j\}_{j=t}^{\tau}$ we follow Canina and Fliglewski (1993) (CF for short) and define realized

[40] The dates/hours we are able to classify are not enough to allow a meaningful disaggregation based on the three subperiods used above. In practice, for long intervals of time only two maturities were traded. Also, we check the robustness of our finding by using either $\delta = 0.5\%$ or $\delta = 1.5\%$, and by using arbitrage-free data only. Results are quite robust.

[41] Most of the previous literature does not pay any attention to the effects of arbitrage violations on the tests of informational efficiency, apart from the preventive elimination of all prices violating the lower bound conditions (i.e. of all negative IVs). Therefore, pricing efficiency is simply assumed. Cf. Canina and Fliglewski (1993, p. 664).

volatility between t and τ as the annualized standard deviation of the continuously compounded MIB30 returns $\{R_j\}_{j=t+1}^{\tau}$. Since in general we have observations on 15 half-an-hour periods per trading day, the annualized volatility of the MIB30 can be simply obtained by multiplying the high-frequency standard deviations by the square root of $15\times 252 = 3,780$. We call this measure $\sigma^*(t, \tau)$.

Assuming that at time t the IV on an option with maturity in $\tau > t$ represents the MIBO market's prediction of the future volatility over the interval $[t, \tau]$,[42] it is well known that a fundamental test of rationality of this forecast can be obtained from the regression

$$\sigma^*(t, \tau) = \alpha + \beta IV(t, z_t, \tau) + u(t, z_t, \tau), \tag{19}$$

in the sense that rationality implies $\alpha = 0$ and $\beta = 1$ so that $E[\sigma^*(t, \tau) | \mathcal{F}_t] = IV(t, \tau)$ (unbiasedness, from $E[u(t, \tau)|\mathcal{F}_t] = 0$, where $\mathcal{F}_t$ is the information set available to the market at time t). Deviations of α and/or β from the values 0 and 1 illustrate the presence of biases and hence the irrationality of markets' forecasts. Notice that the notation $IV(t, z_t, \tau)$ stresses that at time t several IVs corresponding to different moneyness levels z_t are available with maturity $t + \tau$. Analogously, since $IV(t, z_t, \tau)$ should be formed by an efficient market able to incorporate all available information into beliefs and hence prices, if $x_t \in \mathcal{F}_t$ is any piece of public information, then the (encompassing) regression

$$\sigma^*(t, \tau) = \alpha + \beta IV(t, z_t, \tau) + \gamma x_t + u(t, z_t, \tau) \tag{20}$$

should still give $\alpha = 0$, $\beta = 1$, and also $\gamma = 0$. In the following we use two alternative definitions of x_t. First, following CF (p. 671) we employ a 30 (trading) days moving average of realized daily volatility σ^*, appropriately annualized.[43] Second, we use $IV(t-1, \tau)$, the IV of the trading period (normally half-an-hour) immediately before the one under consideration. Obviously, both these variables belong to $\mathcal{F}_t$.

Unfortunately, it is not possible to simply estimate regressions (19)-(20) using all implied volatilities corresponding to different days/time of the day, moneyness, and time-to maturity levels by OLS. The problem is that since the observations come from a panel data set the random disturbances $u(t, z_t, \tau)$ are unlikely to be *spherical*, i.e. to have identical variance and to be uncorrelated. For instance, it is plausible that because of the lower liquidity, certain categories of contracts (for instance, DITM and DOTM) be characterized by more volatile random shocks to their forecasting power, a source of heteroskedasticity. Similarly, it is likely that in a high frequency data set certain times of the day (like opening, lunch time, etc.) be characterized by more volatile random influences than others. Finally, depending on the dynamics of the market's

[42] This assumption is quite common in the older literature, see Poterba and Summers (1986) and Day and Lewis (1988) among the others.

[43] The moving window refers to the 30 days preceding time t. CF used a 60-days average, but given our shorter sample size (in terms of days, not of overall number of observations) we opt for 30. However CF expressly point out that similar results were obtained using 30 instead of 60.

(risk neutral) beliefs underlying the pricing of derivative securities, it is plausible that disturbances to the informational efficiency of the options market be correlated across moneyness classes and/or across maturities. Shocks affecting beliefs and thus prices might also be long-lived thus creating serial correlation across random disturbances for all option contracts. It is well known that in these circumstances OLS estimates are inefficient and would produce a biased estimate of the covariance matrix. In the following we follow three strategies to take the non-sphericality of the pricing errors into account: (i) we estimate the regression coefficients using Parks' (1967) method after applying suitable procedures of reduction and transformation of the original data sets; (ii) using the same transformed data set, we apply feasible GLS estimation to each separate class of moneyness/maturity; (iii) as in CF, apply GMM estimation.

First, we take two steps:

a. We subject our data set to a reduction process by which, for each recorded trading time, we extract only 20 observations, corresponding to all the possible combinations (the order does not matter) of 5 categories of moneyness (DOTM, OTM, ATM, ITM, DITM) and 4 categories of time-to-maturity (very short, short, medium, long). The classes of moneyness and time-to-expiration are defined as in Section 2. It often happens that a given moneyness class contains multiple observations. In these cases we extract the observation with the lowest (highest) moneyness in the case of DOTM (DITM) options, and simply use the mid-point observation based on a moneyness ranking for the remaining three classes. Since our high frequency sample consists of 3,434 observations over time, the resulting panel data set is in principle composed of 68,680 observations, thus implying a minimal loss of information. In practice, it happens that a few classes of moneyness may not be represented; especially in the case of time-to-maturity, at most two classes are simultaneously present throughout the sample. It turns out that the 'reduced' data set consists of 21,240 observations, between 1/3 and 1/4 of the original number. On the other hand, the resulting data set has the structure of a balanced panel in which the cross-sectional identifier are now the 20 moneyness/time-to-maturity classes.

b. We allow the covariance matrix of the random errors affecting (19)-(20) to have arbitrary patterns of heteroskedasticity, as well as *serial* and *cross-sectional* correlation, as synthesized by a full rank covariance matrix Ω.

Write the generic model for time t (defined by day/hour of the day) as

$$\begin{aligned} y_{it} &= \beta_0 + \mathbf{x}'_{it}\boldsymbol{\beta} + u_{it} \qquad i = 1, ..., 20 \quad \text{or} \\ \mathbf{y}_t &= \beta_0 \boldsymbol{\iota}_{20} + X_t\boldsymbol{\beta} + \mathbf{u}_t \end{aligned}$$

where $E[\mathbf{u}_t] = 0$ and $E[\mathbf{u}_t\mathbf{u}'_t] = \Sigma_t$. y_{it} collects the ex-post realized volatility between time t and $t+\tau$, while the row vector $\mathbf{x}'_{it}$ contains the regressors, $IV(t, z_t, \tau)$ in (19) and $[IV(t, z_t, \tau)\ x_t]'$ in

(20). Let's now stack the 3,434 observations on the different times in the sample and write the model in compact fashion as:

$$Y = \boldsymbol{\beta}_0 + X\boldsymbol{\beta} + \mathbf{u}$$

where $\boldsymbol{\beta}_0$, $\boldsymbol{\beta}$ and X appropriately incorporate any restrictions on the parameters between cross-sectional units. Following Parks (1967), we initially assume Σ_t is constant over time and that no *serial* correlation patterns be present, so that the overall covariance matrix of the IV errors can be effectively described by $\Omega = \Sigma \otimes I_T$, where $T = 3,434$.[44] It is well known that the GLS estimator

$$\hat{\boldsymbol{\beta}}^{GLS} = (X'\Omega X)^{-1}X'\Omega Y$$

is consistent and efficient, and also yields consistent estimates of the covariance matrix of the estimated coefficients, $(X'\Omega X)^{-1}$. Unfortunately, Ω is unknown and must be first replaced by a consistent estimate, such as

$$\widehat{\Omega}^{OLS} = \hat{\Sigma}^{OLS} \otimes I_T = \left[T^{-1}\sum_{t=1}^{T}(\hat{\mathbf{u}}_t^{OLS})(\hat{\mathbf{u}}_t^{OLS})'\right] \otimes I_T$$

where $\hat{\mathbf{u}}_t^{OLS} = \hat{\mathbf{y}}_t - \hat{\beta}_0^{OLS}\boldsymbol{\iota}_{20} - \mathbf{X}_t'\hat{\boldsymbol{\beta}}^{OLS}$. The resulting estimator

$$\hat{\boldsymbol{\beta}}^{FGLS} = \left(X'\widehat{\Omega}^{OLS}X\right)^{-1} X'\widehat{\Omega}^{OLS}Y$$

is called the feasible GLS.[45] Under a variety of conditions (see Parks (1967)) it has been shown to be consistent and unbiased. Asymptotically, it is also equivalent to the MLE and therefore is fully efficient. Even in the absence of normality, it can be interpreted as a pseudo-maximum likelihood estimation that retains all the asymptotic properties of MLE estimators (see Gouriéroux and Monfort (1984)). Notice however that the assumption of Σ_t constant over time is easily rejected by most data sets. In our case, it is likely (at least within a given class of contracts) that forecasting errors might be long-lived and hence serially correlated. Therefore we resort to a further step. We regress (by OLS) the panel residuals on their lagged values and estimate the matrix R in the multivariate model

$$\hat{\mathbf{u}}_t^{FGLS} = R\hat{\mathbf{u}}_{t-1}^{FGLS} + \boldsymbol{\nu}_t \qquad (21)$$

where $\boldsymbol{\nu}_t$ is spherical. Finally, we apply OLS to the (so called Prais-Winsten) transformed model

$$y_t - \hat{R}y_{t-1} = \boldsymbol{\beta}_0(I - \hat{R}) + (X_t - X_t\hat{R})\boldsymbol{\beta} + \boldsymbol{\nu}_t,$$

[44] Since in our case the time dimension of the panel (3,434) is much larger than the cross-sectional dimension, some Authors have argued that this is often the natural approach (see Davidson and MacKinnon (1993, p. 321)), no other corrections being required.

[45] In practice we iterate over the two steps of finding a consistent estimator for Ω based on the residuals obtained in step $i-1$, estimating $\hat{\boldsymbol{\beta}}_{(i)}^{FGLS}(\hat{\Omega}_{(i-1)})$ and then calculating the corresponding residuals for step i until convergence of the estimates of $\boldsymbol{\beta}$ is obtained. Although our data set is relatively large, convergence is always obtained quickly.

which yields consistent and efficient estimates of $\hat{\boldsymbol{\beta}}_0^{Parks}$ and $\hat{\boldsymbol{\beta}}^{Parks}$, along with an unbiased estimate of their covariance matrix.

Table XIV reports descriptive statistics (mean, median, and standard deviation) for each of the 20 classes defined above. Most of the contract classes are represented in the sample in a well balanced way, although (as it is to be expected) long-term, deep ITM and OTM contracts are under-represented (less than 1,000 observations each). As for the estimation result, in the case of the original data set we find that the first-stage, FGLS residual do indeed display a high degree of persistency ($\hat{\rho} = 0.999$). Therefore we apply the second stage of Parks's method. We estimate by OLS the model:

$$\hat{\mathbf{u}}_t^{FGLS} = \rho I_{20} \hat{\mathbf{u}}_{t-1}^{FGLS} + \boldsymbol{\nu}_t,$$

a simplification of (21) to the case in which serial correlation is common in intensity to all classes of option contracts. Since we do not have any theoretical reason to assume that random disturbances to informational efficiency have a different persistence as a function of moneyness and/or time-to maturity, and this assumption remarkably simplifies the task, we proceed to derive our final (Parks) estimates from the Pras-Winsten modified regression:

$$y_t - \widehat{\rho} y_{t-1} = \boldsymbol{\alpha}(1 - \widehat{\rho}) + (X_t - \widehat{\rho} X_t)\boldsymbol{\beta} + \boldsymbol{\nu}_t.$$

We thus find (p-values in parenthesis under the estimates):

$$\sigma^*(t,\tau) = \underset{(0.0000)}{0.213} - \underset{(0.0000)}{0.0027}\, IV(t, z_t, \tau) + u(t, z_t, \tau).$$

The R^2 is of 0.8% only. Using the same econometric techniques we obtain a much higher explanatory power by estimating the encompassing regressions

$$\sigma^*(t,\tau) = \underset{(0.0000)}{0.36} - \underset{(0.0000)}{0.0021}\, IV(t, z_t, \tau) - \underset{(0.0000)}{0.8754}\sigma^*_{MA}(t-30, t) + u(t, z_t, \tau) \quad (R^2 = 0.2183)$$

$$\sigma^*(t,\tau) = \underset{(0.0000)}{0.214} - \underset{(0.0000)}{0.0037}\, IV(t, z_t, \tau) - \underset{(0.0000)}{0.0025} IV(t-1, z_t, \tau) + u(t, z_t, \tau) \quad (R^2 = 0.0105).$$

Particularly in the first case, the R^2 is quite high, and a simple 30-days rolling window standard deviations forecasts future MIB30 volatility much better than IVs. As expected, the MIBO market seems to couple pricing and informational inefficiencies in large amounts. In the case of the arbitrage-free data, the results are very similar. For instance (and through the same steps above), the forecasting regression is estimated to be:

$$\sigma^*(t,\tau) = \underset{(0.0000)}{0.237} - \underset{(0.1169)}{0.0003} IV(t, z_t, \tau) + u(t, z_t, \tau).$$

Although the panel-FGLS estimate of the common β stops being significantly negative, this is meaningless as IV forecasts remain largely biased and the R^2 is negligible. Also from this econometric exercise we conclude that the clues of informational inefficiency are strong and probably unrelated to pricing efficiency.[46]

[46] Results from encompassing regressions are similar and therefore not reported.

Second, we estimate equations (19)-(20) by FGLS separately for each of the 20 classes of moneyness/time to maturity. This choice remarkably simplifies the econometrics, since with only one observation per day we only have to worry about serial correlation of the disturbances.[47] Furthermore, the exercise is meaningful as it may reveal which segments of the MIBO30 market are more inefficient. Table XV presents the results for the different classes. Panel A refers to the original data: for all subsamples IVs fail the rationality tests as predictors of future MIB30 volatility during the life of the contract. The intercept is significantly positive (with p-values of at most 5%) for *all* classes of contracts. On the other hand, the slope is most of the time (12 out of 20 cases) not statistically different from zero, and when it is different from zero it is often negative (7 cases). In general the coefficient of the IVs are very small, and the maximum R^2 is 0.0245. Overall, also with this method our first data set seems to strongly reject the hypothesis of informational efficiency of the MIBO market. These results are similar — if not stronger — to those of CF. Panel B offers a very similar picture for the arbitrage-free data set. Improvements are minimal, in the sense that only one of the estimated α fails to be statistically significant, while there are still four β coefficients which are significantly negative; the R^2 coefficient generally increases, but the fundamental idea is still that options' IVs have little to do with the ex-post realized volatility.

A third and final strategy mixes the previous two and corresponds to the one adopted by CF in their seminal paper. Define $\boldsymbol{\theta} \equiv [\alpha\ \beta]'$, $\mathbf{x}_i \equiv [1\ IV_i]'$, $X = \left[\mathbf{x}_1\ \mathbf{x}_2\ ...\ \mathbf{x}_{N_{m,\tau}}\right]'$ and let $N_{m,\tau}$ denote the total number of observations (over time) that fall in a given class of moneyness and time-to-maturity (the index m, τ). CF estimate (19)-(20) by GMM using data for each separate class of contracts as defined by moneyness and time-to-maturity.[48] Although the point estimates of the regressors coefficients are identical to OLS, the advantage of this estimation method is that the resulting estimate of the covariance matrix of the estimators,

$$Cov\left(\hat{\boldsymbol{\theta}}_{m,\tau}^{GMM}\right) = (X'X)^{-1}\left[N_{m,\tau}^{-1}\sum_{i=1}^{N_{m,\tau}}\hat{u}_i^2\mathbf{x}_i\mathbf{x}_i' + N_{m,\tau}^{-1}\sum_{i=1}^{N_{m,\tau}}\sum_{j=i+1}^{N_{m,\tau}}I_{i,j}\hat{u}_i\hat{u}_j(\mathbf{x}_i\mathbf{x}_j' + \mathbf{x}_j\mathbf{x}_i')\right](X'X)^{-1}, \tag{22}$$

is robust to arbitrary forms of correlation in the errors $u(t, z_t, \tau)$ arising from the fact that many contracts overlap. In particular, the indicator variable $I_{i,j} = 1$ if and only if two observations are associated with overlapping contracts, and zero otherwise. By running a few Monte Carlo experiments, CF show that for the problem at hand even in small samples the standard errors

[47] It turns out that most of the contract classes are characterized by strongly persistent residuals (the first order autocorrelations range from 0.91 to 1) and hence very low levels of the D-W statistics. Therefore GLS corrections for serial correlations are most needed and result in the estimates of Table XV. In any case it is true that most of the OLS estimates for α were positive, while most of the OLS βs were negligible and sometimes negative.

[48] Notice that no restriction of panel nature is imposed, in the sense that in correspondence to a day/hour of the day it is possible (and highly likely) to have multiple observations belonging to the same category. Therefore $N_{m,\tau}$ can exceed the total number of points in time for which observations are available. From this point view the data used are not obtained by reduction but fully corresponds to each of the two data sets under examination.

extracted from (22) are quite accurate. Table XVI reports the results of this last set of tests for the original data set. Although $\hat{\alpha}^{GMM}$ is never significant, $\hat{\beta}^{GMM}$ is also never close to 1; in 12 cases out of 20 $\hat{\beta}^{GMM}$ is not even significantly different from zero, in one case it is significantly negative. The R^2 coefficients are in general very small, between 0.38% for OTM, very short-term options and 15% for ATM, very short-term contracts. For the 7 classes of contracts for which $\hat{\alpha}^{GMM}$ is statistically nil while $\hat{\beta}^{GMM} > 0$, $\hat{\beta}^{GMM}$ is always statistically less than one and in 6 cases the IVs anyway fail the encompassing tests (20). In general, a moving window volatility index calculated on high frequency MIB30 returns seems to have a good forecasting power (the R^2 in the encompassing regression always doubles), although the sign of $\hat{\gamma}^{GMM}$ is significantly negative for 13 categories of contracts, possibly an indication of mean reversion. All in all, it seems clear that (with the exception of ATM, very-short term contracts) also this battery of tests reveals strong informational inefficiency of the MIBO market. This conclusion is confirmed by GMM estimation and tests applied to all the original data (bottom right cell of Table XVI).

Our findings improve only slightly when GMM methods are applied to arbitrage-free data. This confirms that pricing efficiency is not sufficient for informational efficiency to result. Table XVII reports on the arbitrage-free data set estimates: we obtain higher values for the R^2s (38% for OTM, short-term options). However, with the only exception of ITM short-term contracts (that anyway fail the encompassing tests), it remains true that the estimated slope coefficients are in general quite small, often not even significantly positive (occasionally significantly negative), which is again inconsistent with informational efficiency.

6. **Conclusion**

This paper has analyzed the pricing and informational efficiency of the Italian market for options on the most important stock index, the MIB30. We find several indications inconsistent with previous findings of the literature (see Cavallo and Mammola (2000)) that have led to the conclusion that the Italian MIBO30 is quite an efficient options market favoring risk-sharing activities and unbiased aggregation and dissemination of information. On the opposite, we report that a striking percentage of the data consists of option prices violating some basic no-arbitrage condition. This percentage declines but never becomes negligible when we relax the no-arbitrage restrictions to accommodate for the presence of bid/ask spreads and other frictions. The result holds generally for all levels of moneyness and time-to-expiry. We also tentatively map the presence of niches of resilient arbitrage opportunities to a few microstructural features of the Italian derivatives market, such as the mismatch between the calendar cycle of futures (FIB30) and options markets, and the fact that maximum bid/ask spreads are set in absolute values and therefore can be particularly wide for low-price contracts (DOTM). We also document abrupt changes of the implied volatility surface that — using high-frequency data as we do — can hardly be explained by changes in market beliefs. We suspect that option pricing on the MIBO can at times become erratic and prone to microstructural factors hardly consistent with the roles a developed, efficient options market ought to play. Finally, using a variety of econometric tools and approaches for the treatment of our high-frequency panel, we investigate the informational efficiency of the MIBO and conclude that option prices are indeed very poor aggregator and predictors of future volatility of MIB30 returns.

There are many possible extensions of this work. First, although Section 5 insists on checking the robustness of the results concerning the dynamics of the IV surface and the degree of informational efficiency to the presence of observations potentially violating the basic conditions of Sections 3 and 4, we believe it is still a long way before obtaining a clear understanding of how the pricing efficiency and microstructural aspects of a derivatives market may influence our perceptions of the IV surface as well as of the informational efficiency of option prices.[49] For instance, a more systematic approach mapping frictions into models of the IV surface or forecasting ability of IVs might be fruitful. Second, in many points in the paper we have speculated that apparent mispricing might be due to the objectives and constraints of the actors involved in the MIBO market, such as market makers and institutional investors. To perform analysis in the style of the current paper but with the benefit of better data sets including the characteristics of the trader, etc. (practically, using information from market makers' books) would be an exciting challenge to our speculations.

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Table I

Summary Statistics.

Summary statistics of the financial prices (options, the MIB30 index, and the interest rate) used in the paper. All the values are expressed in MIB30 index points. MIB30 index returns are continuously compounded and annualized.

	Minimum	**Maximum**	**Mean**	**Std Dev.**
Call prices	1	5,260	1,003.99	855.41
Put prices	1	4,300	882.25	667.97
All contracts - price	1	5,260	942.55	768.97
Strike price	31,000	44,000	37,500	3,968.63
Residual Life	1	109	26.07	16.93
Black-Scholes implied volatility	0.0393	1.5474	0.2548	0.0775
ATM – BS implied volatility	0.0515	0.7755	0.2437	0.0477
MIB30 index	31,518	43,476	35,821	2,923.63
MIB30 index returns (%)	-107.15	68.22	0.141	0.178
Risk-free Rate (LIBOR)	2.48	3.54	2.99	0.3605

Table II

Summary Statistics – Percentage Composition of the Data Set By Moneyness and Time to Maturity.

Moneyness is measured by $z = S/K$ for a call option and $z = K/S$ for a put option. In the case of a call, moneyness classes correspond to: $z < 0.95$ (DOTM), $0.95 \le z < 0.98$ (OTM), $0.98 \le z \le 1.02$ (ATM), $1.02 < z \le 1.05$ (ITM) and $1.05 < z$ (DITM). The classes for time to maturity (τ) are defined as follows: $\tau \le 7$ (Very Short), $7 < \tau \le 25$ (Short), $25 < \tau \le 50$ (Medium), $50 < \tau \le 90$ (Long) and $\tau > 90$ (Very Long).

	Very Short	**Short**	**Medium**	**Long**	**Very Long**	**Total**
DOTM	1.29	5.13	5.69	0.69	0.01	12.81
OTM	2.55	9.97	9.02	2.25	0.13	23.92
ATM	4.36	15.76	13.75	3.39	0.22	37.48
ITM	2.14	8.12	5.97	1.29	0.10	17.62
DITM	1.06	3.62	3.03	0.39	0.07	8.16
Total	11.40	42.60	37.46	8.01	0.52	100

Table III

Arbitrage Opportunities: Absolute Number of Violations.

The table reports the number of arbitrage opportunities detected in the sample for each listed condition. The last line reports the number of contracts that violate at least one of the first seven conditions (thus excluding Maturity spreads). Maturity spreads are not included because, as explained in the text, they are considered more of a test of the FIBO30 market. The last column indicates the ratio of the whole sample in which the corresponding condition is violated.

		Violations	Total	% of the sample
Lower Bound	Put	943	2,371	3.12
	Call	1,428		
Strike Monotonicity	Put	628	1,343	1.77
	Call	715		
Maturity Monotonicity	Put	31	31	0.04
	Call	0		
Reverse Monotonicity	Put	938	2,260	2.98
	Call	1,322		
Butterfly	Put	5,448	10,993	14.48
	Call	5,545		
Box spread	Short	17,516	33,957	23.08
	Long	16,441		21.66
Put/call parity	Short	18,699	30,280	24.64
	Long	11,581		15.26
Maturity spread	Short	223	13,652	0.29
	Long	13,429		17.69
Overall		39,277	39,277	51.75

Table IV

Arbitrage opportunities. Sample Composition by Maturity and Moneyness

The table reports the distribution over maturity and moneyness classes of the arbitrage opportunities detected in the sample for each condition. Moneyness and maturity classes are defined in Table II and in the main text.

	Very Short	Short	Medium	Long	Total	
DOTM	521	1,438	1,339	103	3,401	(34.82%)
	(52.95%)	(36.77%)	(32.25%)	(19.29%)		
OTM	1,020	527	2,893	578	7,818	(42.86%)
	(52.47%)	(6.93%)	(42.07%)	(31.85%)		
ATM	1,785	6,543	5,727	1,453	15,508	(54.27%)
	(53.70%)	(54.45%)	(54.63%)	(52.80%)		
ITM	1,286	4,026	1,532	485	8,329	(61.99%)
	(78.80%)	(65.03%)	(33.66%)	(45.75%)		
DITM	642	1,984	1,465	130	4,221	(67.84%)
	(79.46%)	(71.88%)	(63.42%)	(37.04%)		
Total	5,254	17,318	13,956	2,749	39,277	(51.52%)
	(60.45%)	(53.32%)	(48.86%)	(42.27%)		

Table V

Average Arbitrage Profits: Absolute Numbers and Rates of Return

The table reports mean, standard deviation, and fifth and ninety-fifth percentiles of arbitrage profits in Tables III and IV. The last column reports the average ratio of profits over the price of the option(s), a sort of instantaneous rate of return from the arbitrage strategy. Profits are expressed in MIB30 index points.

		Average		Std. Dev.	5% percentile	95% percentile	% ratio
Lower Bound	Call	492.38	349.26	654.22	5.54	2001.08	186.78
	Put	132.54					
Strike Monotonicity	Call	224.21	231.47	361.50	2	730	37.83
	Put	239.75					
Maturity Monotonicity	Call	0	130.37	147.54	12.93	458.37	4.85
	Put	130.37					
Reverse Monotonicity	Call	125.44	114.63	266.64	0.18	360.06	21.21
	Put	114.63					
Butterfly	Call	68.47	61.12	125.98	2.5	200	4.71
	Put	53.62					
Box spread	Short	150.47		477.98	5.62	368.89	75.16
	Long	89.95		164.24	4.38	275.70	70.06
Put/call parity	Short	127.17		346.19	4.74	331.13	48.34
	Long	80.30		159.31	3.44	252.76	51.40
Maturity spread	Short	93.60		107.73	5.50	274.55	70.06
	Long	394.81		177.15	130.32	710.77	387.29
Overall		165.31		402.85	7.85	409.3	74.38

Table VI

Expressions for Arbitrage Profits Inclusive of Transaction Costs

The table reports formulas for the no-arbitrage conditions once transaction costs (fixed costs of buying and selling the underlying index and bid/ask spreads) are taken into account. In the table, T_t^S represents the fixed cost of a given date t transaction on the underlying, the MIB30 index. The superscript *a* denotes ask prices and *b* the bid prices. These formulas also reflect that on the IDEM options contracts are settled in cash.

Condition		Formula
Lower bound	Put	$\left[S_T^b + (K - S_T)^+\right] e^{-r\tau} - [S_t^a + p_t^a(K)]$
	Call	$\left[(S_T - K)^+ - S_T^a\right] e^{-r\tau} + [S_t^b - c_t^a(K) - T_t^S]$
StriKe Monotonicity	Put	$\left[(K_2 - S_T)^+ - (K_1 - S_T)^+\right] e^{-r\tau} + p_t^b(K_1) - p_t^a(K_2)$
	Call	$\left[(S_T - K_1)^+ - (S_T - K_2)^+\right] e^{-r\tau} + c_t^b(K_2) - c_t^a(K_1)$
Maturity Monoton.	Put	$p_t^b(\tau_1) - p_t^a(\tau_2) - K\left[e^{-r\tau_1} - e^{-r\tau_2}\right]$
	Call	$c_t^b(\tau_1) - c_t^a(\tau_2)$
Reverse Monoton.	Put	$p_t^b(K_1) - p_t^a(K_2) - \left[(K_2 - S_T)^+ - (K_1 - S_T)^+\right] e^{-r\tau}$
	Call	$c_t^b(K_1) - c_t^a(K_2) + \left[(S_T - K_2)^+ - (S_T - K_1)^+\right] e^{-r\tau}$
Butterfly	Put	$\left[\alpha(S_T - K_1)^+ + (1-\alpha)(S_T - K_2)^+ - (S_T - K^*)^+\right] e^{-r\tau} + c_t^b(K^*) - c_t^a$
	Call	$\left[\alpha(K_1 - S_T)^+ + (1-\alpha)(K_2 - S_T)^+ - (K^* - S_T)^+\right] e^{-r\tau} + p_t^b(K^*) - p_t^a$
Box	Short	$[p_t^b(K_1) - c_t^a(K_1)] - [p_t^a(K_2) - c_t^b(K_2)] - (K_1 - K_2)e^{-r\tau}$
	Long	$[p_t^b(K_2) - c_t^a(K_2)] - [p_t^a(K_1) - c_t^b(K_1)] + (K_1 - K_2)e^{-r\tau}$
Put/call parity	Short	$[S_t^b + p_t^b(K) - c_t^a(K) - T_t^S] - [K + (S_T^b - S_T)]\, e^{-r\tau}$
	Long	$[c_t^b(K) - S_t^a - p_t^a(K)] + [K + (S_T^b - S_T)]\, e^{-r\tau}$
Maturity spreads	Short	$[p_t^b(\tau_1) - c_t^a(\tau_1)] - [p_t^a(\tau_2) - c_t^b(\tau_2)] - K\left[e^{-r\tau_1} - e^{-r\tau_2}\right]$
	Long	$[p_t^b(\tau_2) - c_t^a(\tau_2)] - [p_t^a(\tau_1) - c_t^b(\tau_1)] + K\left[e^{-r\tau_1} - e^{-r\tau_2}\right]$

Table VII

Average Bid-Ask Spreads As a Function of Moneyness and Time-to-Expiration for S&P 100 Index Options (from George and Longstaff (1993))

The table reports the values in Figures 1 and 2, and in Table 1 of George and Longstaff (1993). Contracts are classified as short-term if they are less than 30 days to maturity, as medium-term if they are between 30 and 60 days to maturity, and as long-term if they are more than 90 days to maturity. In the case of put options, DOTM means that their strike is more than 7.5 points less than the index, OTM between 7.5 and 2.5 points below the index, ATM between 2.5 points below and 2.5 points above the index, ITM between 2.5 and 7.5 points above the index, and DITM if their strike is more than 7.5 points above the index. Switching the strike with the spot index, the relevant definitions for call options are obtained.

	Short-term		Medium-term		Long-term	
	Put	Call	Put	Call	Put	Call
DOTM	7.34	6.82	3.48	3.44	5.38	4.12
OTM	3.99	4.16	3.19	2.79	3.58	3.19
ATM	3.04	2.81	2.54	2.14	2.91	2.82
ITM	2.29	1.92	2.34	2.05	3.15	2.91
DITM	2.46	2.09	2.83	2.31	3.23	2.71

Table VIII

Arbitrage Opportunities with Bid/Ask Spreads: Absolute Numbers and Percentage Incidence ($\alpha = \beta = 5\%$)

Number of arbitrage opportunities for each listed condition, in the presence of bid/ask spreads. The last row indicates the number (absolute numbers and percentage ratios, respectively) of contracts that violate at least one of the first seven conditions (thus excluding Maturity spreads). The scenario simulations set γ=0 and $\alpha = \beta = 5\%$.

		Number of violations	Total	%
Lower Bound	Put	0	0	0
	Call	0		
Strike Monotonicity	Put	503	1,136	1.50
	Call	634		
Maturity Monotonicity	Put	5	5	0.01
	Call	0		
Reverse Monotonicity	Put	129	293	0.39
	Call	164		
Butterfly	Put	483	1,162	1.53
	Call	679		
Box spread	Short	2,573	4,163	3.39
	Long	1,590		2.09
Put/call parity	Short	70	70	0.09
	Long	0		0
Maturity spread	Short	21	11,648	0.03
	Long	11,627		15.32
Overall			6,065	7.99

Table IX

Arbitrage Opportunities with Bid/Ask Spreads: Sample Composition by Maturity and Moneyness

The table reports the number of arbitrage opportunities for each listed condition, in the presence of bid/ask spreads. These scenario simulations set γ=0 and $\alpha = \beta$ = 5%.

			Very Short	Short	Medium	Long	Total	
DOTM	Butterfly	Put	65	51	48	3	167	278
		Call	42	39	30	0	111	
	Box spread	Short	98	135	173	0	406	631
		Long	73	86	65	1	225	
	Mat Spread	Short	0	0	0	0	0	464
		Long	199	260	5	0	464	
	Overall		320	321	338	9	988	988
			(32.68%)	(8.24%)	(7.83%)	(1.69%)	(10.16%)	(10.16%)
OTM	Butterfly	Put	49	8	53	3	113	154
		Call	17	4	18	2	41	
	Box spread	Short	157	297	126	6	586	1,033
		Long	162	180	94	11	447	
	Mat Spread	Short	0	0	0	0	0	3,388
		Long	941	2,389	58	0	3,388	
	Overall		396	524	321	22	1,263	1,263
			(20.46%)	(6.92%)	(4.69%)	(1.22%)	(6.96%)	(6.96%)
ATM	Butterfly	Put	6	25	116	48	196	291
		Call	15	21	54	5	95	
	Box spread	Short	262	574	193	44	1,073	1,760
		Long	227	281	165	14	687	
	Mat Spread	Short	0	0	0	0	0	5,964
		Long	1,471	4,384	109	0	5,964	
	Overall		517	1,025	621	113	2,278	2,278
			(15.62%)	(8.57%)	(5.95%)	(4.12%)	(8.01%)	(8.01%)
ITM	Butterfly	Put	11	97	47	12	167	313
		Call	47	43	56	0	146	
	Box spread	Short	77	230	81	16	404	581
		Long	22	98	42	15	177	
	Mat Spread	Short	1	6	0	0	7	1,637
		Long	389	1,225	16	0	1,630	
	Overall		209	566	296	37	1,098	1.098
			(12.87%)	(9.18%)	(6.53%)	(3.51%)	(8.21%)	(8.21%)
DITM	Butterfly	Put	3	26	6	2	37	127
		Call	9	19	62	0	90	
	Box spread	Short	9	49	44	2	104	158
		Long	4	25	25	0	54	
	Mat Spread	Short	2	12	0	0	14	195
		Long	39	140	2	0	181	
	Overall		43	186	204	5	438	438
			(5.34%)	(6.77%)	(8.87%)	(1.43%)	(7.07%)	(7.07%)
Total	Butterfly	Put	134	207	270	68	679	1,162
		Call	130	126	220	7	483	
	Box spread	Short	603	1,285	617	68	2,573	4,163
		Long	488	670	391	41	1,590	
	Mat Spread	Short	3	18	0	0	21	11,648
		Long	3,039	8,398	190	0	11,627	
	Overall		1,485	2,612	1,785	186	6,065	6,065
			(17.16%)	(8.08%)	(6.28%)	(2.87%)	(7.99%)	(7.99%)

Table X
Arbitrage Opportunities with Bid/Ask Spreads: Absolute Numbers, Percentage Incidence, Average Profits and Profitability

The table reports the absolute number (N), the percentage ratio on the total sample size, the average profit, and the average profitability deriving from violations of the no-arbitrage conditions listed in the first column of the table, and under the structure of transaction costs assumed in Section 4. Each column refers to a different half-size of the bid/ask spreads for options and spot index markets. These scenario simulations set γ=0 and also impose the restriction $\alpha = \beta$.

		Half-size of the bid-ask spread (α)								
		0	1	2	3	4	5	6	7	8
Lower Bound	N	2,371	265	85	12	2	0	0	0	0
	%	3.12	0.35	0.11	0.02	0	0	0	0	0
	π	349	919	600	698	109	0	0	0	0
	π (%)	186	711	371	170	24	0	0	0	0
Strike Monotonicity	N	1,343	1,274	1,240	1,202	1,177	1,136	1,115	1,021	934
	%	1.77	1.68	1.63	1.58	1.55	1.5	1.47	1.35	1.23
	π	231	22	217	209	199	192	181	167	154
	π (%)	38	37	36	35	34	33	32	30	29
Maturity Monotonicity	N	31	20	13	7	6	5	3	2	0
	%	0.04	0.03	0.02	0.01	0.01	0.01	0	0	0
	π	130	142	151	194	167	133	143	43	0
	π (%)	0	5	5	6	5	4	4	1	0
Reverse Monotonicity	N	2,260	1,345	900	626	413	293	205	128	89
	%	2.98	1.77	1.19	0.82	0.54	0.39	0.27	0.17	0.12
	π	114	141	160	179	220	260	325	422	519
	π (%)	21	32	45	62	90	124	172	263	364
Butterfly spread	N	10,993	6,681	4,111	2,683	1,710	1,162	800	463	341
	%	14.48	8.8	5.42	3.53	2.25	1.53	1.05	0.61	0.45
	π	61	65	72	77	88	101	120	162	188
	π (%)	4	5	5	6	8	10	12	17	19
Box spread – Short	N	17,516	11,699	7,436	4,942	3,485	2,573	1,996	1,314	866
	%	23.08	15.41	9.8	6.51	4.59	3.39	2.63	1.73	1.14
	π	150	177	231	301	383	476	574	786	1,109
	π (%)	75	82	97	113	127	139	148	159	180
Box spread – Long	N	16,441	10,451	6,192	3,823	2,466	1,590	1,033	524	305
	%	21.66	13.77	8.16	5.04	3.25	2.09	1.36	0.69	0.4
	π	89	91	102	115	130	153	188	263	357
	π (%)	70	78	95	112	129	149	174	212	254
Put/call parity – Short	N	18,699	260	191	150	121	70	12	0	0
	%	24.64	0.34	0.25	0.2	0.16	0.09	0.02	0	0
	π	127	1,905	1,696	1,287	823	499	65	0	0
	π (%)	48	289	275	222	142	80	25	0	0

Table X (continued)

Arbitrage Opportunities with Bid/Ask Spreads: Absolute Numbers, Percentage Incidence, Average Profits and Profitability

The table reports the absolute number (N), the percentage ratio on the total sample size, the average profit, and the average profitability deriving from violations of the no-arbitrage conditions listed in the first column of the table, and under the structure of transaction costs assumed in Section 4. Each column refers to a different half-size of the bid/ask spreads for options and spot index markets. These scenario simulations set $\gamma=0$ and also impose the restriction $\alpha = \beta$.

		Half-size of the bid-ask spread (α)								
		0	1	2	3	4	5	6	7	8
Put/call parity – Long	N	11,581	97	46	1	0	0	0	0	0
	%	15.26	0.13	0.06	0	0	0	0	0	0
	π	80	677	323	186	0	0	0	0	0
	π (%)	51	238	124	88	0	0	0	0	0
Maturity spread – Short	N	223	125	78	40	27	21	16	6	3
	%	0.29	0.16	0.1	0.05	0.04	0.03	0.02	0.01	0
	π	93	102	99	126	132	120	106	82	34
	π (%)	70	4	4	5	5	4	4	2	1
Maturity spread – Long	N	13,429	13,232	12,994	12,688	12216	11627	11,021	9,270	7,420
	%	17.69	17.43	17.12	16.72	16.09	15.32	14.52	12.21	9.78
	π	394	364	334	306	282	261	241	215	200
	π (%)	387	347	309	275	250	229	211	192	181
Overall	N	39,277	25,336	17,105	11,661	8248	6065	4,623	3,091	2,253
	%	51.75	33.38	22.54	15.36	10.87	7.99	6.09	4.07	2.97
	π	165	153	170	202	243	292	348	451	560
	π (%)	74	78	82	91	101	110	118	125	130

Table XI

OLS Implied Volatility Regressions in Pooled Cross Section Time Series Data - Competing Specifications.

The table reports the output from the OLS estimation of models 1. –9. in the main text. Panel A of the table uses the original data set, purged of deviations of the lower bound condition only (73,529 observations). Panel B refers instead to the arbitrage-free data set obtained in Section 3 (67,962 observations). In both cases the reported p-values (in the parenthesis below the estimates) are calculated using the HAC covariance estimator of Newey and West (1987).

Panel A – Data purged of lower bound violations only

	Regressors											Model stats	
	const	lnz_t	$(lnz_t)^2$	OTM_t	ITM_t^2	$100\tau_t$	$lnz_t\ \tau_t$	$1000\times\tau_t^2$	NS_t	NS_t^2	$100\times NS_t\tau_t$	$\bar{R}^2$	F-stat
Model 1	-1.40											0.000	0.00
	(0.00)												
Model 2	-1.40	1.54										0.066	0.00
	(0.00)	(0.00)											
Model 3	-1.43	1.47	14.92									0.085	0.00
	(0.00)	(0.00)	(0.00)										
Model 4	-1.42			0.68	32.70							0.077	0.00
	(0.00)			(0.00)	(0.00)								
Model 5	-1.40		28.90	2.72								0.079	0.00
	(0.00)		(0.00)	(0.00)									
Model 6	-1.44		4.34	0.42	2.51							0.086	0.00
	(0.00)		(0.00)	(0.00)	(0.00)								
Model 7	-1.35	1.75	14.92			-0.29	-0.01					0.127	0.00
	(0.00)	(0.00)	(0.00)			(0.00)	(0.00)						
Model 8	-1.30	1.70	15.98			-0.79	-0.01	0.09				0.147	0.00
	(0.00)	(0.00)	(0.00)			(0.00)	(0.00)	(0.00)					
Model 9	-1.35					-0.23			-0.04	0.24	0.55	0.070	0.00
	(0.00)					(0.00)			(0.00)	(0.00)	(0.00)		

Panel B – Arbitrage-free data set

	Regressors											Model stats	
	const	lnz_t	$(lnz_t)^2$	OTM_t	ITM_t^2	$100\tau_t$	$lnz_t\ \tau_t$	$1000\times\tau_t^2$	NS_t	NS_t^2	$100\times NS_t\tau_t$	$\bar{R}^2$	F-stat
Model 1	-1.41											0.000	0.00
	(0.00)												
Model 2	-1.42	1.47										0.071	0.00
	(0.00)	(0.00)											
Model 3	-1.44	1.42	14.45									0.092	0.00
	(0.00)	(0.00)	(0.00)										
Model 4	-1.43			0.67	32.34							0.086	0.00
	(0.00)			(0.00)	(0.00)								
Model 5	-1.42		28.20	2.67								0.087	0.00
	(0.00)		(0.00)	(0.00)									
Model 6	-1.42		15.34	1.73	15.72							0.088	0.00
	(0.00)		(0.00)	(0.00)	(0.00)								
Model 7	-1.38	1.54	14.46			-0.23	-0.00					0.124	0.00
	(0.00)	(0.00)	(0.00)			(0.00)	(0.22)						
Model 8	-1.33	1.52	15.42			-0.67	-0.00	0.07				0.142	0.00
	(0.00)	(0.00)	(0.00)			(0.00)	(0.24)	(0.00)					
Model 9	-1.42					-0.09			0.05	0.66	1.11	0.258	0.00
	(0.00)					(0.00)			(0.00)	(0.00)	(0.00)		

Table XII
Classification of Market States According to the Shape of the Implied Volatility Function vs. Moneyness – Sample Frequencies

For each point in time (day and time during the day) for which at least 3 observations on option prices for alternative moneyness levels are available, we classify the MIBO30 IV curve as a function of moneyess in five categories: smiles, smirks, reverse smirks, frowns, and other residual shapes (see the main text for accurate definitions). Classifications are based on the relative magnitude of the MIB30 implied volatility for DOTM, OTM, ATM, ITM, and DITM options, where the definitions are given in Section 2. IVs at different levels of moneyness are considered different if they diverge by more than δ percent. The following table reports the sample frequency (i.e. an estimate for the unconditional probability) of the five states as a function of δ for both the full-sample period and for three sub-period of equal length (in months). The bottom part of the table refers to the full-sample period when frictions are taken into account ($\alpha = \beta = 5\%$).

	Smiles	**Smirks**	**Reverse Smirks**	**Frowns**	**Other shapes**
			$\delta = 1\%$		
Full sample: 04/06/1999 – 01/31/2000	20.71%	48.22%	3.09%	0.95%	27.02%
04/06/1999 – 07/15/1999	17.96%	49.57%	1.17%	1.05%	30.25%
07/15/1999 – 10/25/1999	33.85%	43.76%	0.68%	0.17%	21.54%
10/26/1999 – 01/31/2000	10.66%	51.01%	8.55%	1.65%	28.13%
			$\delta = 0.5\%$		
Full sample: 04/06/1999 – 01/31/2000	27.72%	52.14%	3.95%	1.68%	14.52%
04/06/1999 – 07/15/1999	25.86%	54.20%	1.85%	2.04%	16.05%
07/15/1999 – 10/25/1999	40.94%	46.75%	1.20%	0.34%	10.77%
10/26/1999 – 01/31/2000	16.27%	54.87%	10.02%	2.57%	16.27%
			$\delta = 1.5\%$		
Full sample: 04/06/1999 – 01/31/2000	13.33%	42.99%	2.48%	0.49%	40.72%
04/06/1999 – 07/15/1999	10.80%	43.64%	0.68%	0.43%	44.44%
07/15/1999 – 10/25/1999	23.85%	39.23%	0.34%	0.09%	36.50%
10/26/1999 – 01/31/2000	5.79%	46.05%	7.44%	1.01%	39.71%
			$\delta = 1\%$		
Arbitrage-free data set (67,962 obs.)	21.50%	46.40%	3.48%	0.98%	27.64%

Table XIII

Sample Transition Matrix of Market States According to the Shape of the Implied Volatility Function vs. Moneyness.

For each point in time (day and time during the day) for which at least 3 observations on option prices for alternative moneyness levels are available, we classify the MIBO30 IV curve as a function of moneyess in five categories: 1 - smiles, 2 - smirks, 3 - reverse smirks, 4 - frowns, and 5 - other residual shapes (see the main text for accurate definitions). Classifications are based on the relative magnitude of the MIB30 implied volatility for DOTM, OTM, ATM, ITM, and DITM options, where the definitions are given in Section 2. IVs at different levels of moneyness are considered different if they diverge by more than δ = 1% percent. The following table reports the sample frequency of the switches from shapes of type i to shapes of type j, i,j = 1,...,5 where 1 stands for smiles, 2 for smirks, etc. for both the full-sample period and for three sub-period of equal length (in months). The bottom part of the table refers to the full-sample period when frictions are taken into account ($\alpha = \beta$ = 5%).

	Smiles	Smirks	Reverse Smirks	Frowns	Other shapes
			Full sample: 04/06/1999 – 01/31/2000		
Smiles	34.86%	33.81%	1.44%	1.04%	28.85%
Smirks	17.17%	52.83%	3.88%	1.07%	25.04%
Reverse Smirks	10.17%	54.24%	11.86%	3.39%	20.34%
Frowns	16.67%	55.56%	2.78%	0%	25%
Other shapes	24.74%	39.69%	2.47%	0.62%	32.47%
			04/06/1999 – 07/15/1999		
Smiles	24.25%	37.31%	1.12%	1.87%	35.45%
Smirks	16.17%	49.85%	0.75%	1.50%	31.74%
Reverse Smirks	22.22%	33.33%	0%	5.56%	38.89%
Frowns	25%	75%	0%	0%	0%
Other shapes	21.46%	46.24%	2%	0%	30.31%
			07/15/1999 – 10/25/1999		
Smiles	44.79%	30.21%	0.52%	0.26%	24.22%
Smirks	29.41%	55.99%	0.87%	0%	13.73%
Reverse Smirks	50%	37.5%	0%	12.5%	0%
Frowns	50%	50%	0%	0%	0%
Other shapes	47.06%	23.08%	0.45%	0%	29.41%
			10/26/1999 – 01/31/2000		
Smiles	25.66%	38.05%	5.31%	1.77%	29.20%
Smirks	8.18%	53.82%	10.18%	1.45%	26.36%
Reverse Smirks	4.35%	59.78%	15.22%	2.17%	18.48%
Frowns	5.56%	38.89%	5.56%	0%	50%
Other shapes	13.13%	42.09%	4.71%	2.02%	38.05%
			Arbitrage-free data set (67,962 observations)		
Smiles	36.07%	33.50%	1.54%	1.16%	27.73%
Smirks	18.39%	51.20%	4.55%	0.70%	25.16%
Reverse Smirks	10.00%	56.15%	6.92%	1.54%	25.38%
Frowns	19.44%	50.00%	8.33%	0%	22.22%
Other shapes	24.05%	39.22%	3.20%	1.14%	32.40%

Table XIV

Descriptive Statistics for the Reduced, Balanced Panel Data Set Used in FGLS Estimation

The table reports means, medians, standard deviations (along with the total number of cross-sectional observations) of implied volatility for the two balanced panels built by reduction of the original data sets (lower-bound violations and arbitrage violations-free, respectively) in Section 5. The reduction is applied by extracting information on Black-Scholes IVs and contract features for 20 classes defined along the mutually exclusive dimensions of moneyness — {DOTM, OTM, ATM, ITM, DITM}— and time-to-expiration — {very short, short, medium, long}. The relevant definitions of the categories of option contracts can be found in Section 2.

Panel A — panel derived from lower bound violation-free data (21,589 obs.)				
	Very short	Short	Medium	Long
DOTM	0.4204 0.3683 0.2051 (222)	0.2363 0.2263 0.0502 (1,020)	0.2257 0.2172 0.0438 (822)	0.2125 0.2068 0.0244 (163)
OTM	0.3020 0.2751 0.1291 (615)	0.2380 0.2195 0.0846 (1,890)	0.2283 0.2187 0.0480 (1,993)	0.2156 0.2086 0.0286 (536)
ATM	0.2507 0.2503 0.0554 (805)	0.2390 0.2332 0.0420 (2,102)	0.2421 0.2378 0.0420 (2,373)	0.2285 0.2205 0.0332 (698)
ITM	0.2968 0.2887 0.0753 (675)	0.2660 0.2459 0.1070 (1,802)	0.2542 0.2556 0.0499 (1,999)	0.2469 0.2396 0.0343 (600)
DITM	0.4046 0.3768 0.1372 (489)	0.3096 0.2620 0.2126 (935)	0.2648 0.2566 0.0839 (1,274)	0.2528 0.2437 0.0464 (288)
Panel B — panel derived from lower bound violation-free data (21,165 obs.)				
	Very short	Short	Medium	Long
DOTM	0.4157 0.3659 0.2075 (216)	0.2346 0.2260 0.0473 (1,020)	0.2234 0.2151 0.0244 (822)	0.2131 0.2078 0.0244 (163)
OTM	0.2929 0.2769 0.0971 (615)	0.2265 0.2180 0.0406 (1,843)	0.2251 0.2176 0.0406 (1,986)	0.2162 0.2068 0.0271 (536)
ATM	0.2514 0.2453 0.0585 (804)	0.2386 0.2378 0.0419 (2,068)	0.2405 0.2378 0.0406 (2,372)	0.2328 0.2245 0.0323 (720)
ITM	0.3010 0.2862 0.0873 (623)	0.2489 0.2424 0.0436 (1,764)	0.2533 0.2550 0.0450 (1,999)	0.2467 0.2391 0.0346 (600)
DITM	0.4263 0.3858 0.1662 (447)	0.2717 0.2610 0.0624 (905)	0.2568 0.2587 0.0460 (1,274)	0.2563 0.2466 0.0482 (288)

Table XV

Informational Content of MIBO30 Implied Volatility – Regression Tests for Classes of Moneyness and Time-to-Maturity (FGLS)

The table reports the FGLS estimates of the coefficients α and β in the regression tests of forecast rationality:

$$\sigma^*(t, \tau) = \alpha + \beta\ IV(z_t, t, \tau) + u(z_t, t, \tau),$$

where $\sigma^*(t, \tau)$ is the annualized standard deviation of MIB30 (infra-daily) log- returns, and $u(z_t, t, \tau)$ is a white-noise residual. * indicates that a coefficient is significant at 5%, while ** means significant at 1%.The two panels report results for the two balanced panels built by reduction of the original data sets (lower-bound violations and arbitrage violations-free) in Section 5. When the IVs represent rational forecasts of future volatility, α = 0 and β = 1.

Panel A — panel derived from lower bound violation-free data (21,589 obs.)					
		Very short	Short	Medium	Long
DOTM	$\hat{\alpha}^{FGLS}$	0.2000**	0.1610**	0.1538**	0.1720**
	$\hat{\beta}^{FGLS}$	-0.0060	0.0024	-0.0040**	-0.0002
	R^2	0.0137	0.0007	0.0245	0.0000
	Numb. Obs.	(210)	(976)	(720)	(147)
OTM	$\hat{\alpha}^{FGLS}$	0.3800*	0.1556**	0.2680*	0.7700**
	$\hat{\beta}^{FGLS}$	-0.0039*	-0.0000	-0.0005	0.0008
	R^2	0.0095	0.0000	0.0005	0.0056
	Numb. Obs.	(594)	(1,768)	(1,747)	(445)
ATM	$\hat{\alpha}^{FGLS}$	0.1607**	0.1667**	0.1900**	0.2000**
	$\hat{\beta}^{FGLS}$	-0.0020	0.0100**	-0.0012*	-0.0026*
	R^2	0.0001	0.0059	0.0022	0.0133
	Numb. Obs.	(794)	(1,999)	(2,052)	(491)
ITM	$\hat{\alpha}^{FGLS}$	0.1593**	0.2070**	0.1860**	0.1840**
	$\hat{\beta}^{FGLS}$	-0.0043	-0.0026*	0.0002	0.0015
	R^2	0.0045	0.0030	0.0002	0.0001
	Numb. Obs.	(660)	(1,682)	(1,742)	(472)
DITM	$\hat{\alpha}^{FGLS}$	0.1778**	0.2190**	0.9540**	0.1930**
	$\hat{\beta}^{FGLS}$	-0.0002	-0.0055**	-0.0019**	-0.0035
	R^2	0.0001	0.0139	0.0120	0.0044
	Numb. Obs.	(458)	(865)	(1,109)	(186)
Panel B — panel derived from arbitrage-free data (21,165obs.)					
		Very short	Short	Medium	Long
DOTM	$\hat{\alpha}^{FGLS}$	0.1969**	0.1610**	0.1500**	0.1552**
	$\hat{\beta}^{FGLS}$	-0.0086*	0.0076*	-0.0003	-0.0005
	R^2	0.0297	0.0041	0.0001	0.0001
	Numb. Obs.	(201)♣	(976)	(720)	(147)
OTM	$\hat{\alpha}^{FGLS}$	0.2330	0.1584**	0.2777*	0.7720**
	$\hat{\beta}^{FGLS}$	-0.0010	0.0114*	-0.0000	0.0007
	R^2	0.0005	0.0056	0.0000	0.0043
	Numb. Obs.	(594)	(1,722)	(1,726)	(445)
ATM	$\hat{\alpha}^{FGLS}$	0.1600**	0.1679**	0.3360**	0.1828**
	$\hat{\beta}^{FGLS}$	-0.0034**	-0.0004	-0.0012	-0.0018
	R^2	0.0006	0.0001	0.0022	0.0007
	Numb. Obs.	(792)	(1,950)	(2,017)	(573)
ITM	$\hat{\alpha}^{FGLS}$	0.1647**	0.1711**	0.1917**	0.1843**
	$\hat{\beta}^{FGLS}$	-0.0078**	0.0029	0.0008	0.0015
	R^2	0.0181	0.0008	0.0013	0.0002
	Numb. Obs.	(580)	(1,628)	(1,722)	(467)
DITM	$\hat{\alpha}^{FGLS}$	0.2138**	0.1820**	0.3710**	0.1920**
	$\hat{\beta}^{FGLS}$	-0.0027*	-0.0073	0.0006	-0.0037
	R^2	0.0148	0.0092	0.0021	0.0040
	Numb. Obs.	(405)	(824)	(1,071)	(186)

Table XVI

Informational Content of MIBO30 Implied Volatility – Forecasting and Encompassing Regression Tests for Classes of Moneyness and Time-to-Maturity (GMM). Original Data (Purged of Violations of the Lower Bound Condition)

The table reports the GMM estimates of the coefficients α, β, and γ in the regression tests of forecast rationality:

$$\sigma^*(t, \tau) = \alpha + \beta\, IV(z_t, t, \tau) + u(z_t, t, \tau) \text{ and}$$
$$\sigma^*(t, \tau) = \alpha + \beta\, IV(z_t, t, \tau) + \gamma\, x_t + u(z_t, t, \tau)$$

where $\sigma^*(t, \tau)$ is the annualized standard deviation of MIB30 (infra-daily) log- returns, and $u(z_t, t, \tau)$ is a white-noise residual. x_t corresponds to either the rolling window standard deviation of (infra-daily) MIB30 returns over the 30 days preceding t, or the lagged value if IV. * indicates that a coefficient is significant at 5%, while ** means significant at 1%. In the case of encompassing regressions we only report the estimate of γ for the two definitions of x_t. When the IVs represent rational forecasts of future volatility, α = 0 and β = 1. In encompassing regressions, $\alpha = \gamma = 0$ and β = 1.

		Very short	Short	Medium	Long	All maturities
DOTM	$\hat{\alpha}^{GMM}$	0.1868	0.1050	0.1246	0.1192	0.1395
	$\hat{\beta}^{GMM}$	-0.0143	0.2561*	0.1231	0.2562**	0.0907
	R^2	0.0060	0.1320	0.0781	0.1003	0.0773
	Numb. Obs.	(688)	(2,567)	(2,685)	(367)	(6,307)
	$\hat{\gamma}^{GMM}$ x_t = RW vol.	-1.0621**	-0.1839*	0.2043**	-1.1331**	0.0653
	$\hat{\gamma}^{GMM}$ $x_t = IV_{t-1}$.	-0.0101	0.1755*	0.0891	0.1663**	0.0383
OTM	$\hat{\alpha}^{GMM}$	0.1571	0.1322	0.1693	0.1224	0.1467
	$\hat{\beta}^{GMM}$	0.0183	0.1217	-0.0414	0.2206*	0.0590
	R^2	0.0038	0.0711	0.0064	0.0561	0.0137
	Numb. Obs.	(2,006)	(6,362)	(5,322)	(1,335)	(15,025)
	$\hat{\gamma}^{GMM}$ x_t = RW vol.	-0.3080**	-0.1861	-0.2519*	-1.0194**	-0.1985
	$\hat{\gamma}^{GMM}$ $x_t = IV_{t-1}$.	0.0132	0.0594*	-0.0242	0.1354	0.0410
ATM	$\hat{\alpha}^{GMM}$	0.1005	0.1292	0.2014	0.0971	0.1408
	$\hat{\beta}^{GMM}$	0.2230*	0.1357	-0.1638	0.2992**	0.0846
	R^2	0.1516	0.0467	0.0848	0.0660	0.0104
	Numb. Obs.	(5,210)	(10,399)	(10,419)	(2,590)	(28,618)
	$\hat{\gamma}^{GMM}$ x_t = RW vol.	-0.1307	-0.1433	-0.2845*	-1.1863**	-0.2796*
	$\hat{\gamma}^{GMM}$ $x_t = IV_{t-1}$.	0.1362	0.0894	-0.1072	0.1953	0.0529
ITM	$\hat{\alpha}^{GMM}$	0.1332	0.1541	0.2048	0.1116	0.1740
	$\hat{\beta}^{GMM}$	0.0698	0.0317	-0.1643*	0.2312**	-0.0505
	R^2	0.0589	0.0143	0.1349	0.0470	0.0066
	Numb. Obs.	(2,183)	(5,801)	(5,741)	(1,447)	(15,172)
	$\hat{\gamma}^{GMM}$ x_t = RW vol.	0.2034**	0.0199	-0.3555**	-1.2281**	-0.1569
	$\hat{\gamma}^{GMM}$ $x_t = IV_{t-1}$.	0.0495	0.0140	-0.1097	0.1605*	-0.0167
DITM	$\hat{\alpha}^{GMM}$	0.1457	0.1722	0.1890	0.1336	0.1796
	$\hat{\beta}^{GMM}$	0.0155	0.0165	-0.0742	0.0544*	-0.0340
	R^2	0.0088	0.0068	0.0977	0.0200	0.0138
	Numb. Obs.	(1,041)	(3,119)	(3,689)	(558)	(8,407)
	$\hat{\gamma}^{GMM}$ x_t = RW vol.	0.3547**	-0.3296*	-0.2239**	-1.3585**	-0.1870*
	$\hat{\gamma}^{GMM}$ $x_t = IV_{t-1}$.	0.0119	0.0037	-0.0379	0.0282	-0.0154
All levels of moneyness	$\hat{\alpha}^{GMM}$	0.1423	0.1062	0.1882	0.1438	0.1508
	$\hat{\beta}^{GMM}$	0.0435	0.2394	-0.1113	0.1000	0.0351
	R^2	0.0264	0.0771	0.0298	0.0136	0.0055
	Numb. Obs.	(11,128)	(28,248)	(27,856)	(6,297)	(73,529)
	$\hat{\gamma}^{GMM}$ x_t = RW vol.	-0.0481	-0.1382	-0.2874	-1.1610**	-0.1747
	$\hat{\gamma}^{GMM}$ $x_t = IV_{t-1}$.	0.0348	0.1617	-0.0737	-0.0720	0.0236

Table XVII

Informational Content of MIBO30 Implied Volatility – Forecasting and Encompassing Regression Tests for Classes of Moneyness and Time-to-Maturity (GMM). Arbitrage-Free Data.

The table reports the GMM estimates of the coefficients α, β, and γ in the regression tests of forecast rationality:

$$\sigma^*(t, \tau) = \alpha + \beta\, IV(z_t, t, \tau) + u(z_t, t, \tau) \text{ and}$$

$$\sigma^*(t, \tau) = \alpha + \beta\, IV(z_t, t, \tau) + \gamma\, x_t + u(z_t, t, \tau)$$

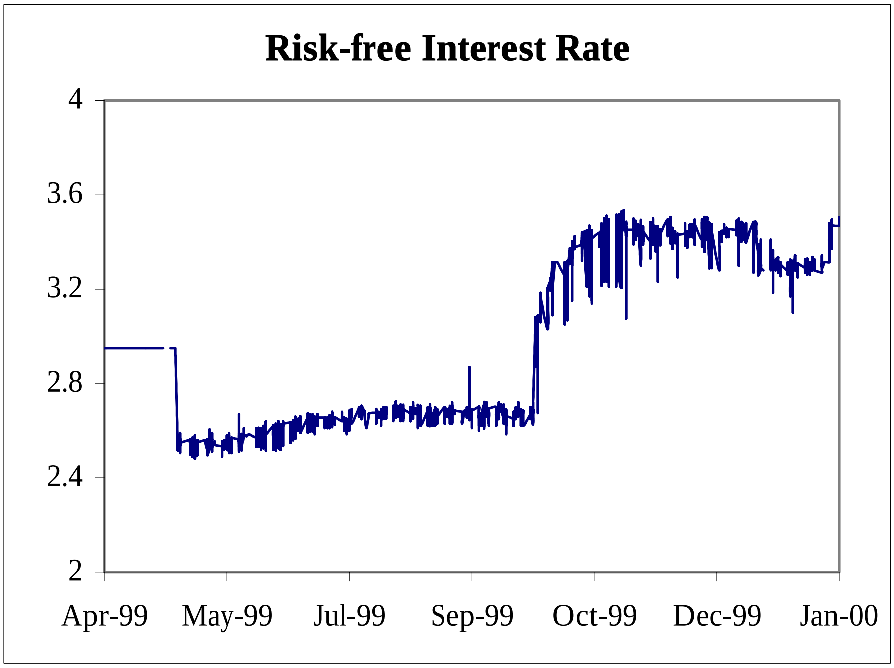
where $\sigma^*(t, \tau)$ is the annualized standard deviation of MIB30 (infra-daily) log- returns, and $u(z_t, t, \tau)$ is a white-noise residual. x_t corresponds to either the rolling window standard deviation of (infra-daily) MIB30 returns over the 30 days preceding t, or the lagged value if IV. * indicates that a coefficient is significant at 5%, while ** means significant at 1%. In the case of encompassing regressions we only report the estimate of γ for the two definitions of x_t. When the IVs represent rational forecasts of future volatility, $\alpha = 0$ and $\beta = 1$. In encompassing regressions, $\alpha = \gamma = 0$ and $\beta = 1$. ♣ indicates that a slope coefficient is not significantly different from 1 at 5%.

		Very short	Short	Medium	Long	All maturities
DOTM	$\hat{\alpha}^{GMM}$	0.1737	0.0709	0.1182	0.1221	0.1326
	$\hat{\beta}^{GMM}$	0.0016	0.4079^{**}	0.1522	0.2485^{**}	0.1249
	R^2	0.0001	0.2967	0.0948	0.0860	0.1067
	Numb. Obs.	(463)	(2,507)	(2,550)	(363)	(5,883)
	$\hat{\gamma}^{GMM}$ x_t = RW vol.	-0.8217^{**}	0.2860^{**}	0.2051^{**}	-1.1238^{**}	0.3685^{**}
	$\hat{\gamma}^{GMM}$ $x_t = IV_{t-1}$.	0.0008	0.2510^{*}	0.1052	0.1531	0.0632
OTM	$\hat{\alpha}^{GMM}$	0.1458	0.0461	0.1761	0.1856	0.1275
	$\hat{\beta}^{GMM}$	0.0474	0.5168	-0.0515	-0.0269	0.1580
	R^2	0.0175	0.3774	0.0054	0.0003	0.0600
	Numb. Obs.	(1,580)	(6,182)	(5,012)	(1,274)	(14,048)
	$\hat{\gamma}^{GMM}$ x_t = RW vol.	-0.0015	0.2344^{*}	-0.2107^{*}	-1.2297^{**}	0.1569
	$\hat{\gamma}^{GMM}$ $x_t = IV_{t-1}$.	0.0359	0.2870^{*}	-0.0344	-0.0204	0.1071
ATM	$\hat{\alpha}^{GMM}$	0.0799	0.0346	0.1962	0.3269	0.1093
	$\hat{\beta}^{GMM}$	0.3076^{**}	0.5587^{*}	-0.1026	-0.6441^{**}	0.2567
	R^2	0.2721	0.2971	0.0108	0.0938	0.0686
	Numb. Obs.	(3,642)	(10,114)	(9,760)	(2,572)	(26,088)
	$\hat{\gamma}^{GMM}$ x_t = RW vol.	0.1620^{*}	0.1912	-0.1293	-1.6943^{**}	0.0370
	$\hat{\gamma}^{GMM}$ $x_t = IV_{t-1}$.	0.1784	0.2899	-0.0635	-0.4245^{*}	0.1552
ITM	$\hat{\alpha}^{GMM}$	0.1288	0.0261	0.2098	0.3565	0.1452
	$\hat{\beta}^{GMM}$	0.0922	$0.5966^{*,♣}$	-0.1058	-0.7103^{**}	0.1342
	R^2	0.0778	0.2745	0.0096	0.1555	0.0232
	Numb. Obs.	(1,746)	(5,505)	(5,526)	(1,385)	(14,162)
	$\hat{\gamma}^{GMM}$ x_t = RW vol.	0.3840^{**}	0.3213^{*}	0.0292	-1.8175^{**}	0.2741
	$\hat{\gamma}^{GMM}$ $x_t = IV_{t-1}$.	0.0646	0.3386	-0.0701	-0.4664^{**}	0.1247
DITM	$\hat{\alpha}^{GMM}$	0.1510	0.0639	0.1648	0.3605	0.2002
	$\hat{\beta}^{GMM}$	0.0163	0.4859	0.1580	-0.6795^{**}	-0.0042
	R^2	0.0090	0.2762	0.0225	0.3107	0.0001
	Numb. Obs.	(892)	(2,872)	(3,471)	(546)	(7,781)
	$\hat{\gamma}^{GMM}$ x_t = RW vol.	0.4820^{**}	0.3881^{*}	0.1979	-3.1775^{**}	0.4971
	$\hat{\gamma}^{GMM}$ $x_t = IV_{t-1}$.	0.0124	0.3156	0.1003	-0.3712^{**}	0.0485
All levels of moneyness	$\hat{\alpha}^{GMM}$	0.1399	0.0388	0.1413	0.2542	0.1374
	$\hat{\beta}^{GMM}$	0.0598	0.5497	0.1539	-0.3101	0.1455
	R^2	0.0476	0.3431	0.0227	0.0404	0.0482
	Numb. Obs.	(8,323)	(27,180)	(26,319)	(6,140)	(67,962)
	$\hat{\gamma}^{GMM}$ x_t = RW vol.	0.0459	0.2146	-0.1039	-1.5882^{**}	0.2308
	$\hat{\gamma}^{GMM}$ $x_t = IV_{t-1}$.	0.0480	0.3275	0.0975	-0.2160	0.1057

Figure 1

Plots of the High-Frequency Time Series of the Interest Rate and of the MIB30 Index

The upper panel plots the (risk-free) interest rate over the period April 6, 1999 – January 31, 2000. The interest rate is calculated as the average of the bid and ask of the three months LIBOR rate. The bottom panel plots the MIB30 index over the same time period (in index points, each valued 2.5 Euros). In both plots financial prices are sampled throughout the day at regular intervals of 30 minutes, between 9 a.m. and 6 p.m..



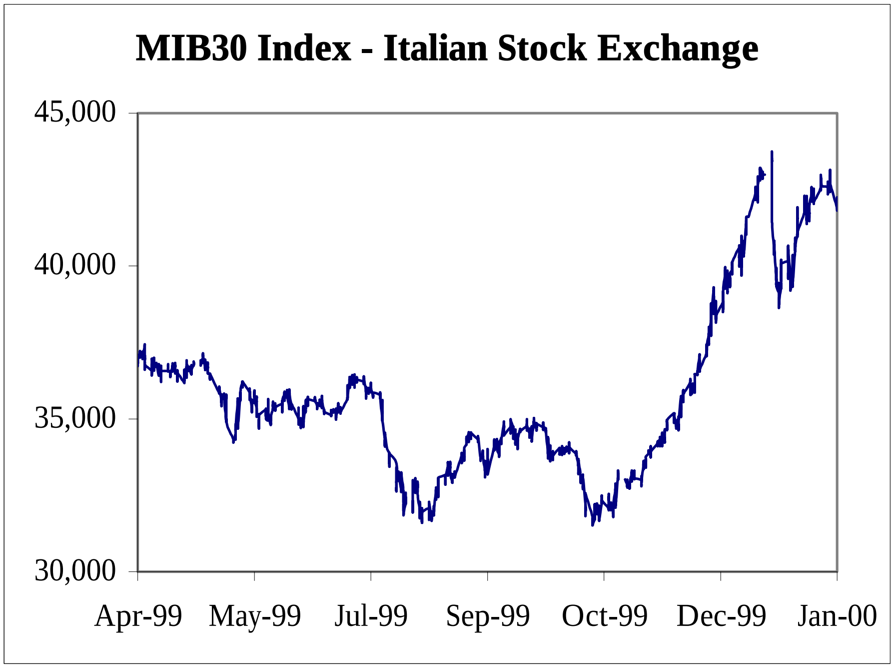
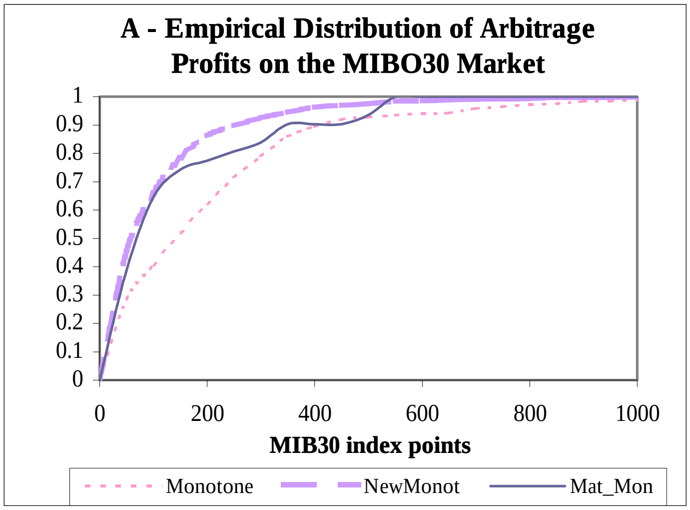
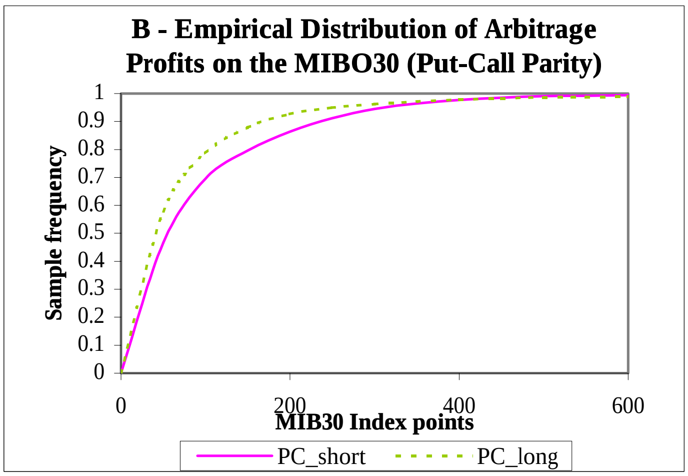


Figure 2

Empirical Distribution of Arbitrage Profits on the MIBO30 Market

The first two graphs plot the cumulative empirical distribution of arbitrage profits resulting from the violation of a few of the no-arbitrage conditions listed in the main text. For clarity, values exceeding 1,500 MIB30 index points have been omitted. The bottom panel (C) reports the empirical density and distribution function of the maximum arbitrage profits across the different conditions. Arbitrage profits are expressed in MIB30 index points.





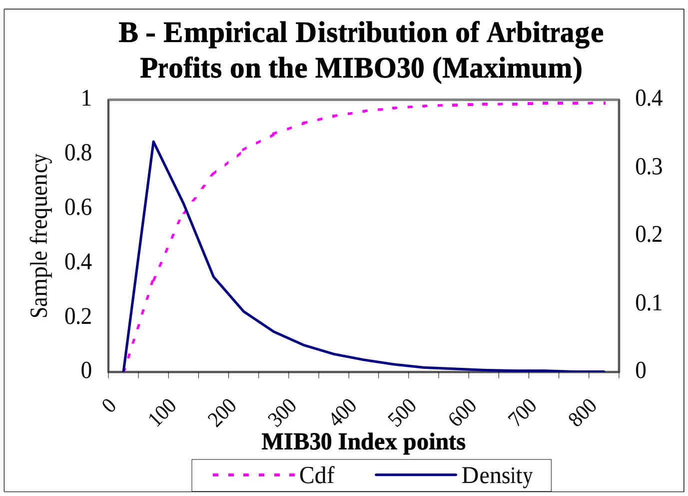
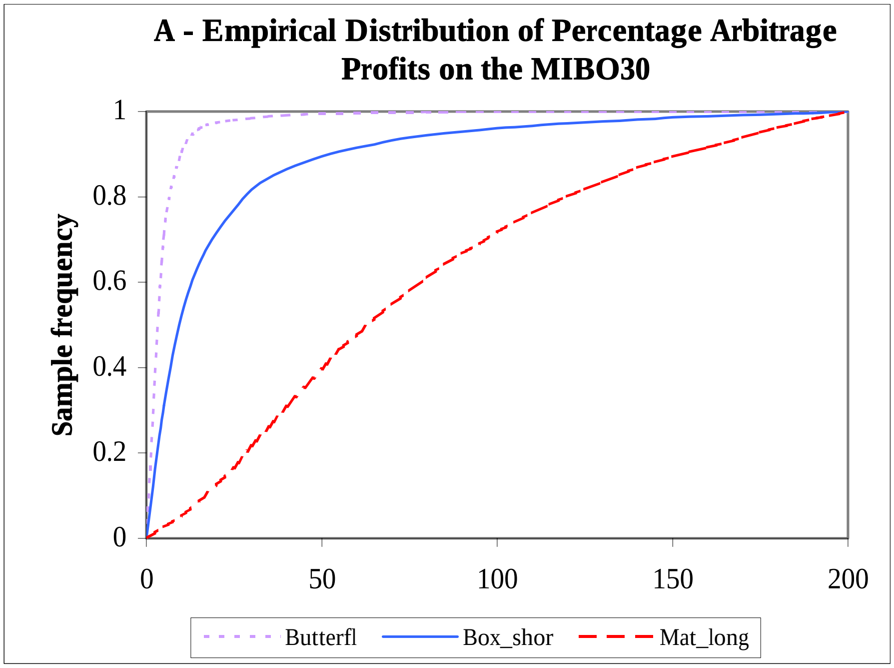


Figure 3

Empirical Distribution of Percentage Arbitrage Profits on the MIBO30 Market

The graphs plot the cumulative, empirical distribution of arbitrage profits resulting from the violation of a few of the no-arbitrage conditions listed in the main text as a ratio of the value of the option contract(s) causing the violation (or to which the violation has been imputed). For clarity, values in excess of 200% have been omitted. Panel B refers to the maximum arbitrage profit across the different conditions.



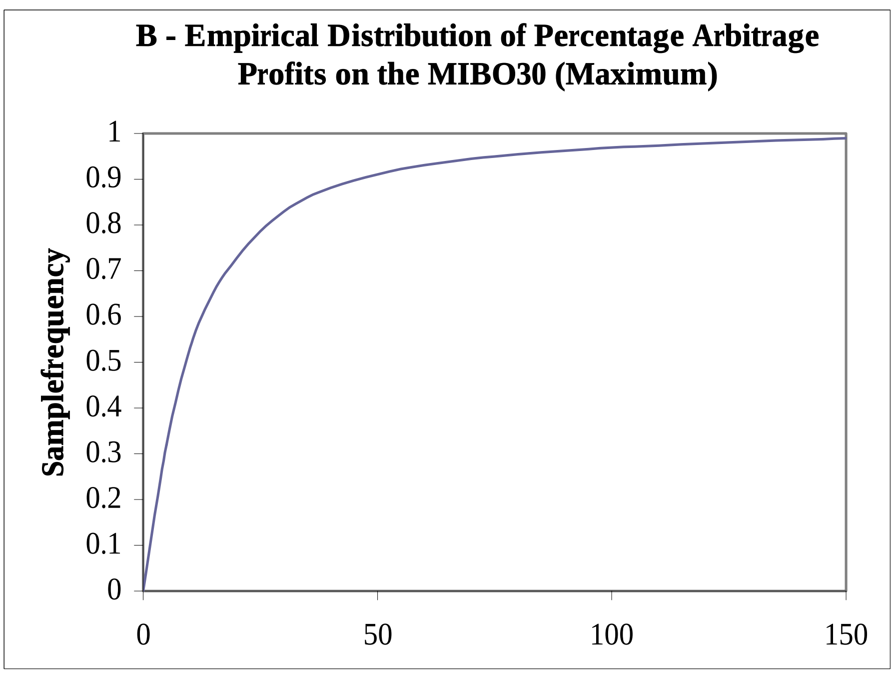


Figure 4

Monthly Variation of the Percentage Incidence of Arbitrage Violations on the MIBO30 Market

The graph plots the percentage of the sample observations on MIBO30 option prices in each month between April 1999 and January 2000 displaying arbitrage violations of any type analyzed in Section 3 of the main text. The dotted, straight line represents the average over the entire sample.

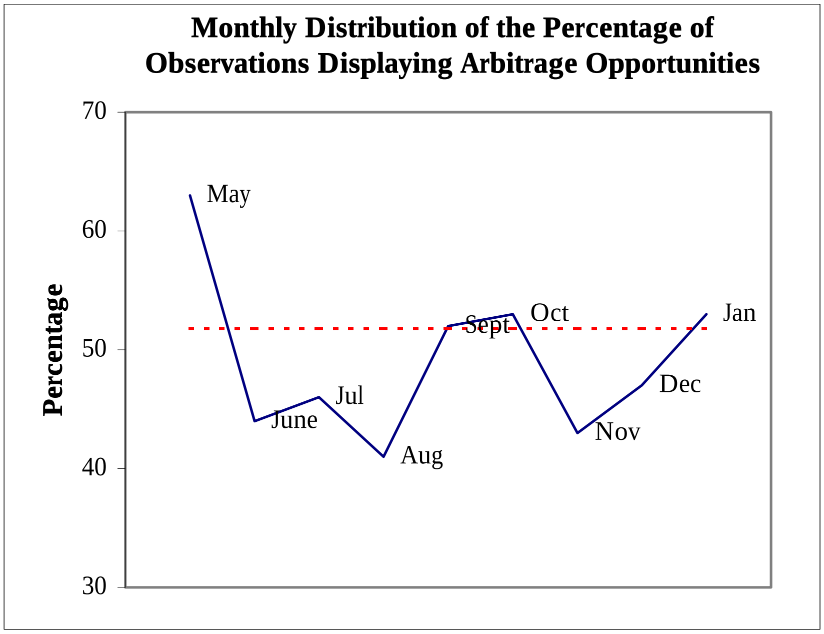
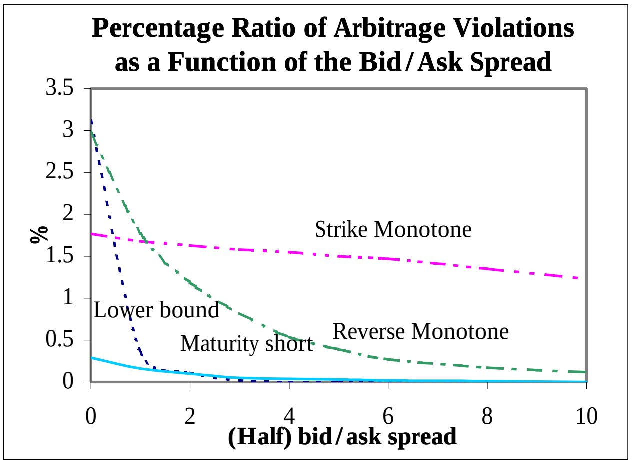
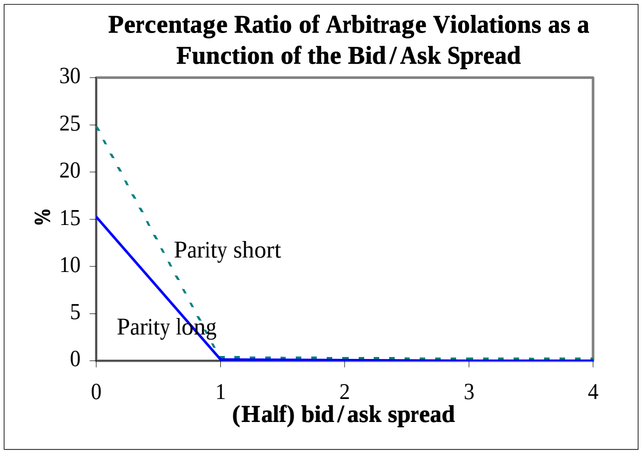


Figure 5

Percentage Incidence of Arbitrage Violations As a Function of Alternative Levels of the Bid/Ask Spread.

The graphs plot the changes in the percentage of the data displaying violations of the basic no-arbitrage conditions derived in Section 4 as a function of the (half-) size of the bid/ask spreads α and β characterizing the MIBO30 (options) and the MIB30 index (the underlying) markets, respectively. These scenario simulations set γ=0 and also impose the restriction $\alpha = \beta$. Different plots report on different no-arbitrage conditions.





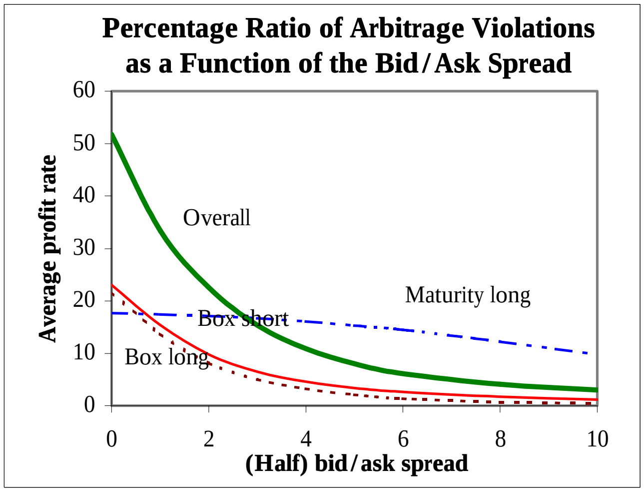


Figure 6

Average Arbitrage Profit Rates as a Function of Alternative Levels of the Bid/Ask Spread.

The graphs plot the changes in the average profit rates obtained by exploiting the presence of violations of the basic no-arbitrage conditions derived in Section 4 as a function of the (half-) size of the bid/ask spreads α and β characterizing the MIBO30 (options) and the MIB30 index (the underlying) markets, respectively. These scenario simulations set γ=0 and also impose the restriction $\alpha = \beta$. Different plots report on different no-arbitrage conditions.

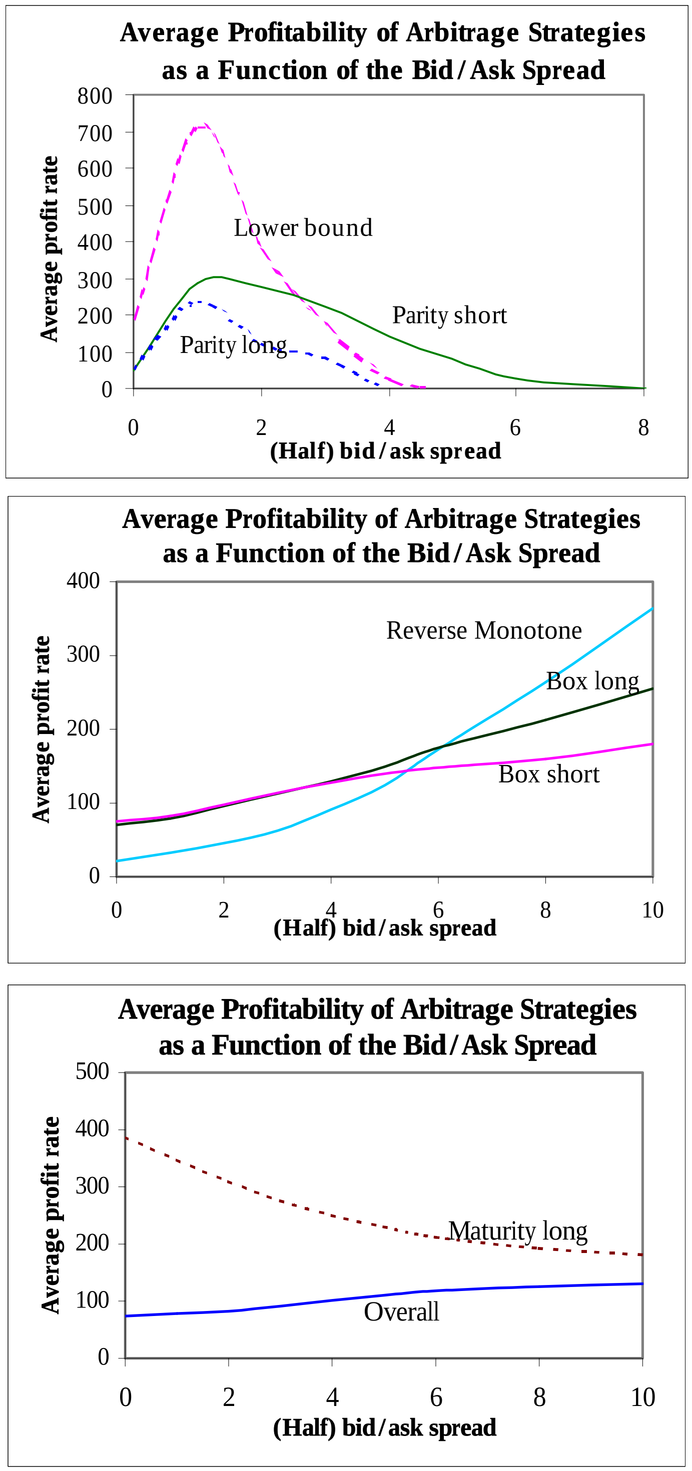
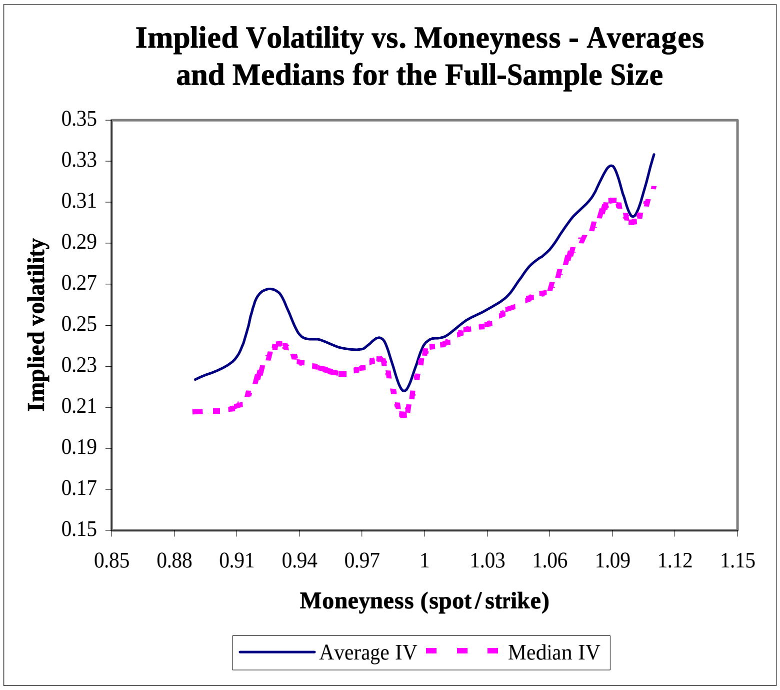


Figure 7

Implied Volatility as a Function of Moneyness

The graphs plot medians and averages of IVs of MIBO30 options for 21 mutually exclusive intervals of moneyness (from 0.89 to 1.1). The reference period is the full sample 04/06/1999 – 01/12/2000 in the top panel and three alternative sub-periods (04/06/1999 - 07/15/1999, 07/15/1999 - 10/25/1999, and 10/26/1999 - 01/31/2000) in the bottom panel (in this case only average IVs are reported).



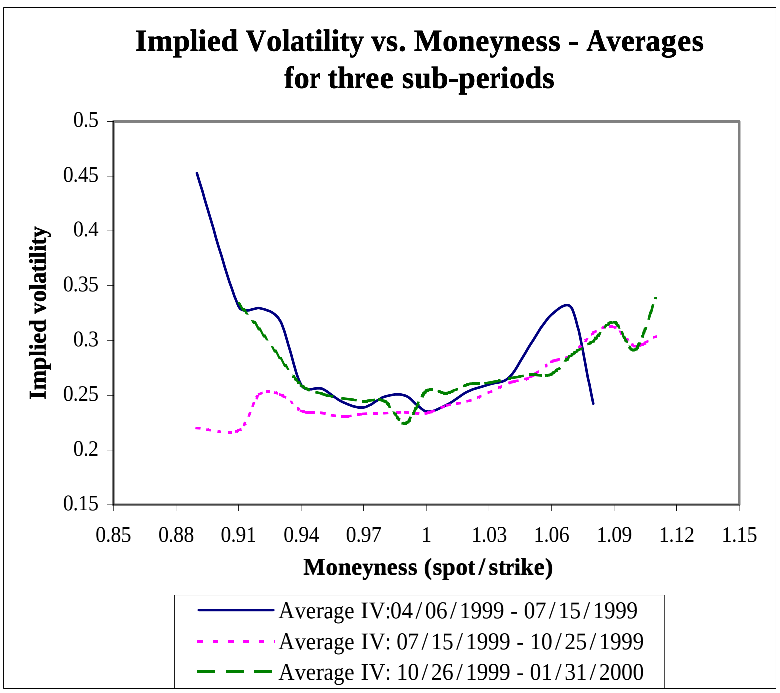


Figure 8

Implied Volatility as a Function of Moneyness on April 16, 1999

The graphs plot implied volatilities as a function of moneyness when sampled at four different times on April 16, 1999. All the contracts considered expired in May 1999 (short-term options). The plots should be read clockwise, illustrating a sudden change of the IV surface (stable between 11:49 am and 12:19 pm) between 12:19 pm and 12:49 pm. The right panel at the bottom shows the IV curve at the end of the day, at market close.

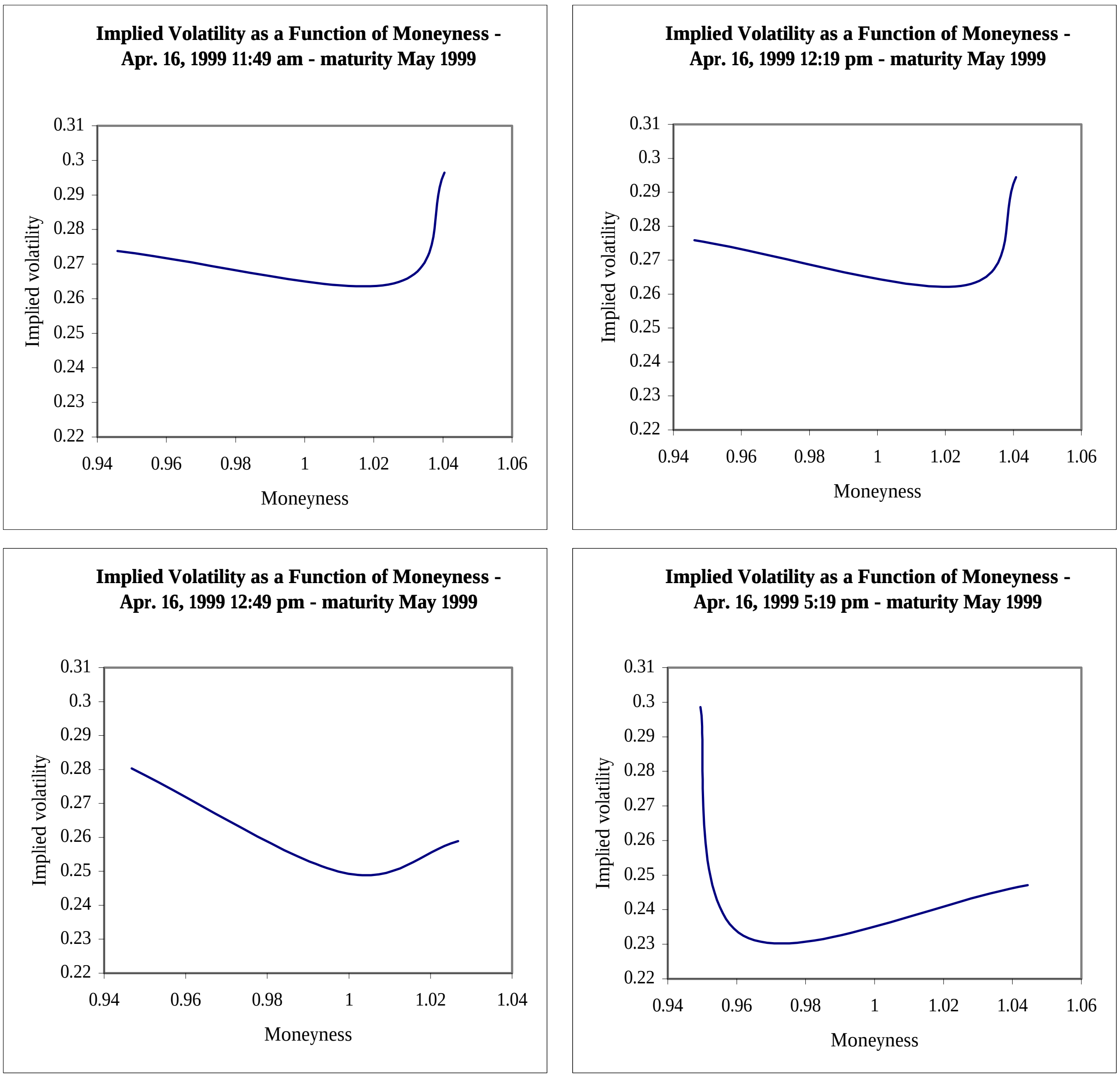


Figure 9
Implied Volatility as a Function of Maturity on September 7, 1999

The graphs plot implied volatilities as a function of time to maturity when sampled at six different times on September 7, 1999. The contracts considered are the closest-at-the-money that were traded at the particular time of the day indicated in the graphs. The plots should be read clockwise, illustrating a sudden change of the IV surface (stable during the morning of the same day) between 1:05 pm and 2:35 pm and again between 2:35 pm and 3:35 pm. The last panel at the right- bottom shows the IV curve at the end of the day, at market close.