Perspectives on the Measurement Problem

Perspectives from the Measurement Problem

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Arne Hansen

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Stefan Wolf

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Dissertation Committee

Antonio Carzaniga   Università della Svizzera italiana, Lugano, Switzerland
Igor Pivkin   Università della Svizzera italiana, Lugano, Switzerland
Časlav Brukner   University Vienna, Vienna, Austria
Christian Wüthrich   University Geneva, Geneva, Switzerland
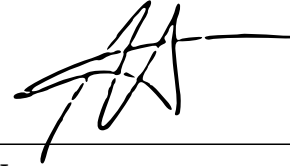
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Research Advisor

Stefan Wolf

PhD Program Director

Walter Binder

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I certify that except where due acknowledgement has been given, the work presented in this thesis is that of the author alone; the work has not been submitted previously, in whole or in part, to qualify for any other academic award; and the content of the thesis is the result of work which has been carried out since the official commencement date of the approved research program.

Arne Hansen
Lugano, 23 June 2020

Abstract

In quantum mechanics, the measurement problem is commonly regarded as reason for deep concern. It seems that, either, the problem can be solved and there is hope for quantum mechanics, or the theory better be left behind in search of another one. We investigate the prospect of finding a solution to the measurement problem—within quantum mechanics, as well as in theoretical frameworks beyond it. As we do so, there emerges a perspective on doing physics drawing from considerations in the philosophy of language and in epistemology. Developing this perspective for further aspects in physics reveals: The notion of a *system* is similarly problematic. Eventually, the question arises whether these are problems of specific physical theories, or rather of specific stances towards physics. In this regard, we examine in particular the role and understanding of *assumptions*.

Acknowledgements

Contents

Prependix

1 Not a prologue

The conceptualization of a measurement in quantum mechanics gives rise to problems—facets of the so-called measurement problem. Is the measurement problem a problem of quantum mechanics, or even a contradiction within the theory? A hint that we have not found the ultimate truth yet? A cue that we do not truly understand quantum mechanics yet, or that we have to search for another truer theory? As subsequently discussed, the measurement problem affects a wider class of theories—namely theories that require empirical evidence for the occurrence of a measurement, empirical evidence for our contact with the world out there. Thus, the problem poses rather as a clash of epistemological[1] stances than as a defect of specifically quantum mechanics. We cannot commensurate the requirement of empirical evidence for our contact with the world with the idea of definite measurement results as the basis of certain knowledge. The measurement problem takes us back to considerations by Dewey and Bohr on the epistemological import of quantum mechanics. Before the wake of quantum mechanics, the doubt uttered by Wittgenstein with the following quote,

> A *picture* held us captive. And we could not get outside it, for it lay in our language and language seemed to repeat it to us inexorably. [129, §115]

could be confronted by emphasizing the special status of the picture. That was to say, the picture that holds us captive *is* the objective reality. While quantum mechanics warrants our contact with the world, it does no longer present us with clear picture that allows for an unquestioned acceptance of this captivity.

In light of the unclarity of the picture, the focus is commonly turned to structures that "shine through the picture." The picture analogy can, however, be

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[1] Epistemology is, roughly speaking, concerned with what we can know and how we get to know what we know.

developed another way by raising the question what pictures—i.e., what ways of *doing physics*—result in regarding the measurement as problematic.

If physics aims for a description of the world, then reflections on the nature of language by Nietzsche, Wittgenstein, Sellars, Putnam, and Rorty apply also to quantum mechanics: Can we hope to fully describe a measurement, or a system? How can a physical theory exhaustively capture an observer's account of experience, while it draws legitimacy from experimental findings? Can we reduce scientific practice to pure description short of any normative elements? The stir that the "problem" creates among physicists, becomes a symptom for the prevalence of the assumption that physical theories, at least possibly, exhaust anything that can be said. We are left with a dilemma: *The measurement problem leads us to a reflection that has no room within the philosophic and epistemological stances that lead to the measurement problem.* In order to see the problem in the measurement as a technical problem of quantum mechanics that asks for a solution, one needs to assume the exhaustiveness of the physical description—not to question it.

The dilemma also reflects in what to make of an assumption: What is an assumption under the assumption of exhaustiveness? Is it a statement whose truth value cannot be decided yet? Or is it, instead, a creative and contingent act of forming agreement—a starting point for merely one among many possible ways of thinking, speaking and describing; neither necessary nor sufficient? The latter idea that language relies on creative elements that entail their own contingencies prohibits itself to be proven by an absolute and finally decisive argument. Eventually, it seems, one cannot evade obstacles before entering into new language games as noted by Wittgenstein: Without having had similar thoughts before, the reader might not easily be convinced. *The ladder is merely useful to those, who have already stepped onto it.*

2   Why not to read on — a disclaimer

There is reason not to read this document: Language games cannot be un-played. If cognition is a creative act of *forming and being formed*[2], then this document

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[2]"Denn Erkennen ist weder passive Kontemplation noch Erwerb einzig möglicher Einsicht im fertig Gegebenen. Es ist ein tätiges, lebendiges Beziehungseingehen, ein Umformen und Umgeformtwerden, kurz ein Schaffen. [...] Erkenntnisse werden von Menschen gebildet, aber auch umgekehrt: sie bilden Menschen." [33, p. 48] — "For cognition is neither passive contemplation nor acquirement of uniquely possible insight in the readily given. It is an active, lively partaking in relationships, a shaping and being reshaped, in short a creative act. [...] Insights are formed by people, but also inversely: they form people." [33, p. 48, own translation]

creates its own contingencies. We hope that the following encourages the reader to dare, at least in some aspects, a re-description [103].[3] Though, if we cannot step outside language, then, we suspect, this is an irreversible move.

3   Outline

In the subsequent Section 4, we anticipate the measurement problem, before we introduce the quantum mechanical formalism in Section 5. In Section 6, we discuss common reading of the measurement problem in greater detail. The following sections widen and alter the perspective on the measurement problem. In Section 7, we examine how the measurement is problematic for a wider class of physical theories. This raises epistemological questions. In Section 8, we investigate what stance on doing physics gives rise to the measurement problem by looking closer at linguistic subtleties surrounding the Wigner's-friend experiment. In light of this critical perspective onto the measurement problem, we conclude: Instead of giving way to the urge to solve the problem, one might rather reflect on our ways of doing aspects. To this end, we examine facets of prominent concepts in physics—the *state*, in Section 9, *information*, in Section 10, and *systems*, in Section 11.[4]

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[3]"Ich möchte nicht mit meiner Schrift Andern das Denken ersparen. Sondern, wenn es möglich wäre, jemand zu eigenen Gedanken anregen." [130, preface]

[4]We suspect that there is less of a linear narrative than this outline might make believe. The lack of a directedness in the figure below supports this suspicion. In a sense, it constitutes a visual argument for holism. The figure shows textual elements with color transitions indicating the tree that emerges from the document structure via the chapters and sections down to the (sub-)*subsections. The drawn edges show references between these elements.

We encourage the reader to try different entry points. To this end, we point out the considerations about Gibbs paradox in Section 10.2 and 11.2, and about the Object/Relation Impedance Mismatch in computer science considered in 11.4.
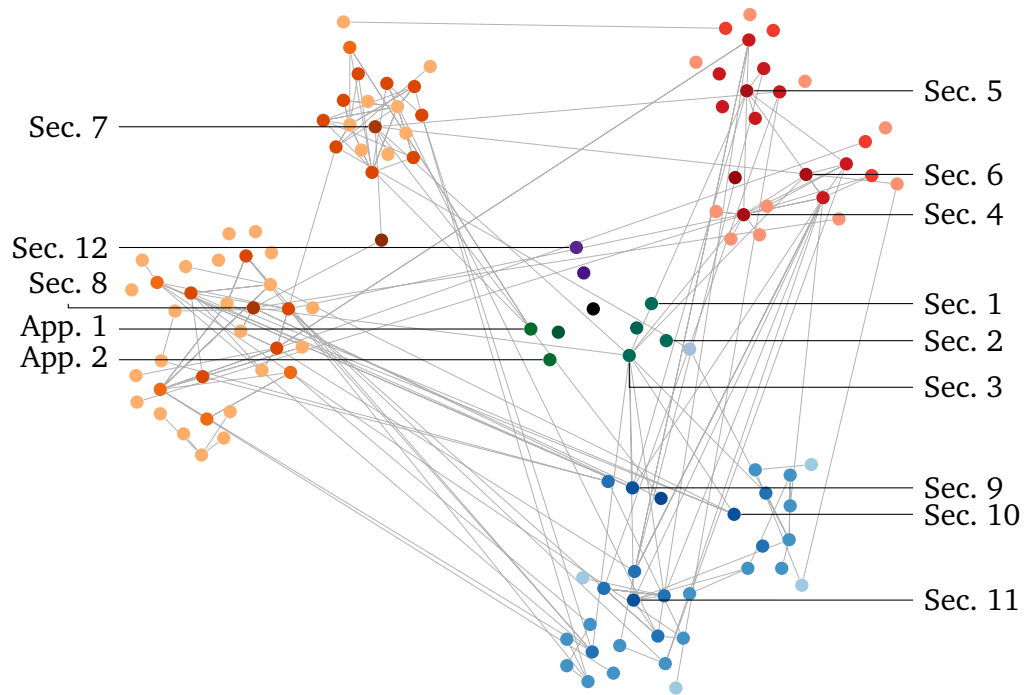
Introduction

4 The Problem

The bone of contention is the *measurement problem* in quantum mechanics. Before we give a detailed introduction to quantum mechanics, we roughly outline the problem in this section.

Linearity in quantum mechanics (Q1) seems at odds with measurements yielding exclusively one of multiple possible results (Q2):

(Q1) In quantum mechanics, a system is assigned a *state*, i.e., a normalized element in a Hilbert space. Any normalized superposition of vectors is again a permissible state. The composition of systems via tensor products preserves this linearity. The time evolution of a state is governed by Schrödinger's equation, which yields a unitary time evolution operator for isolated systems, and, thus, preserves linearity.

(Q2) A measurement yields a definite result. In a Stern-Gerlach experiment, each silver atom *either* goes up *or* down. If we measure the brain activity of a cat, we find it *either* dead *or* alive. We might loose or forget the result. But we do not know, receive, or perceive two results. What allows us to speak of two *different possible* results, is exactly *this distinction*. What allows us to say that we *have obtained a result*, is that we get *one of multiple possible results*. This aspect appears in quantum mechanics in the form of *orthogonality*.

Combining these aspects within quantum mechanics creates problems:

(P1) There is a divide into two different realms, the world of quantum states and the world of measurement results. The latter is termed *classical* by an inflation of the distinction of quantum and classical mechanics. This invites to think, that there is a world of *classical information*, constituted by measurement results and governed by classical mechanics, as opposed to

a world of *quantum information* governed by quantum mechanics. Where is the border between these worlds? Where do we leave one and step into the other?

(P2) Quantum mechanics does not forbid its application to an experimenter observing a quantum mechanical system. How is it, that the observer sees a definite result (see (Q2)) despite the linearity[5] of quantum mechanics?

The two problems are related. The latter problem can be termed as follows: How can an observation be part of the quantum realm? This problem is attenuated in the *Wigner's-friend experiment*. After introducing the formalism of quantum mechanics in Section 5, we elaborate on the separation into two irreconcilable realms (P1), in Section 6.1; and we discuss the Wigner's-friend experiment in detail in Section 6.2.

5 En route to Quantum Mechanics: The Stern/Gerlach experiment

The experiment proposed by Stern in 1921 [115] and carried out by Gerlach in 1922 [42] is an experimental cornerstones of quantum mechanics: Single silver atoms are accelerated and shot through an inhomogeneous magnetic field which deflects the silver atom. The deflecting magnetic field allows to measure how much the magnetic dipole moment—in a sense the inner magnetization of the silver atoms—is oriented up or down along a given axis, i.e., the axis along which the magnetic field is oriented. If we shoot the single silver atoms—one after the other with a sufficiently large time gap in between them to ensure that they do not interact with one another—, then we find that some of the silver atoms are deflected up and some are deflected down. In particular, the silver atoms do not go up and down to different degrees but, instead, bunch around an upper and a lower point.

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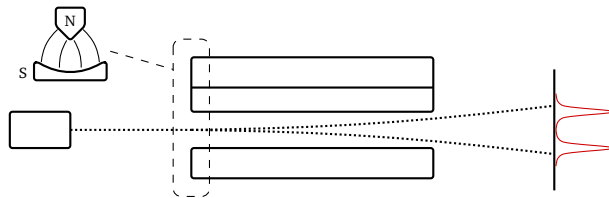
[5]More precisely, unitarity is required here as we will discuss below: The notion of an *isolated system* turns out to be essential to the measurement problem.

This is little surprising, if the silver atoms *from the beginning* come as a mixture of two categories. Experimentally we may *find* equally many in the up and in the down category. If we measure the ones that are deflected up a second time, we find all of them going up; and, vice versa, for the ones that are deflected down.
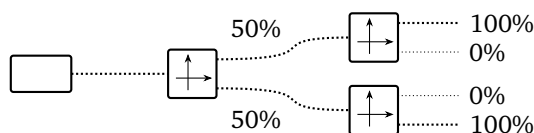
We can rotate the magnetic field, and measure how much the magnetic dipole of the silver atom is aligned with another spatial axis.[6] Surprisingly, we still find the silver atoms bunched around two distinct points, up and down along the axis we measure. In a suitable experimental setup, the distribution of the silver atoms does not change as we rotate the magnetic field.[7]

(1)
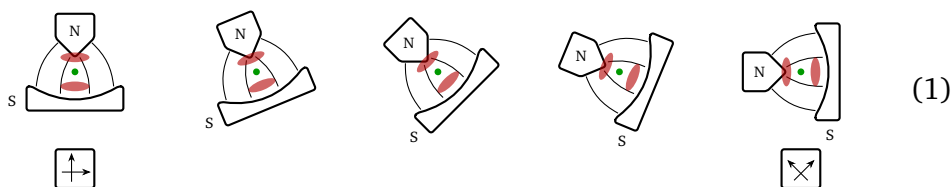
While we can inquire about arbitrarily many different questions, there are, for any of these questions, merely two different answers. This seems at odds with what we initially thought we would measure—namely, *how much* the dipole moment of the silver atoms is aligned with a given axis. This question seems to be a binary question[8] *only if* the particles already come in two categories. Then, however, as we rotate the magnetic field, we expect the gap between the two points on the screen to eventually close as the axis of the magnetic field becomes orthogonal to the dipole moment of the incoming silver atoms. That is, we expect to obtain an answer from a continuous set to the question we inquire about.

How does this affect the stability of the answers? Do we obtain the same answers to subsequent inquiries of the same question? It turns out that, indeed, we do: No matter the orientation of the magnetic field, measuring the particles going up (down), we still find them to go up (down), as along as we measure in

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[6]This is called commonly a "measurement in another basis" because of the mathematical representation of measurements in quantum mechanics. We, therefore, illustrate measurements with a set of basis vectors.

[7]The red ellipses illustrate the bunched particle positions detected on the screen; the green dot illustrates the incoming silver atoms.

[8]That is, a question with two possible answers; a yes-no-question.

the same basis.
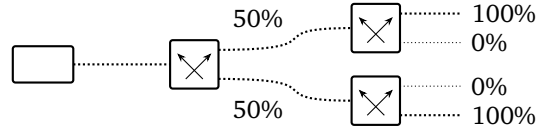
In this sense, we can confirm previous measurements by inquiring about the same question again.

In light of the previous findings, one should maybe let go of the idea that, no matter the orientation, we always measure the magnetic dipole moment of the silver atoms, i.e., the same characteristic, merely from different perspectives. Are then the measurements with different orientations of the magnetic field rather inquiries about *independent questions* corresponding to *different characteristics* of the silver atoms? If we perform different measurements on the *same* silver atom, can we compile the information of measurement results for specific choices of bases and expect later measurements to reproduce the listed results? Can we, thus, *characterize* the silver atom by the results for different measurement bases? If we perform measurements in different bases, then we find that hopes to this end are disappointed:
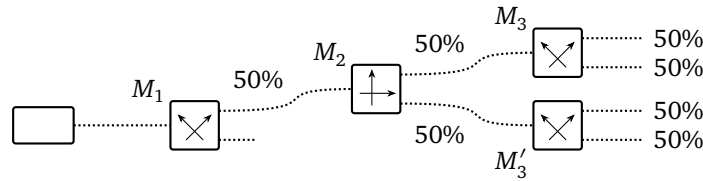
In half of the cases, the measurements M_1 and M_3 (M_3') yield different results despite having used the *same* measurement bases. The intermediate measurement in a *different* basis, M_2, disturbs the result. The silver atom does not seem to inherently carry the answers to the questions we inquire about.

This, however, offers another, complementary insight: If we measure twice the same silver atom in the same bases, and we obtain *different* answers, then we conclude that the silver atom has been interacted with in a way that corresponds to inquiring about a measurement in a different basis. There is an apparent trade-off: The limitation on speaking about "the inherent and antecedently existent properties of the silver atoms" has yielded us the new ability to empirically trace our interactions with the particles under investigation. It enables us to *meaningfully* speak about the interactions necessary for empirical evidence, i.e., it renders our connection to the world out there empirically tangible.

There are two routes from here: On the one hand, the situation above compares to wave mechanics. Along this path we are lead to the formalism of quan-

tum mechanics for single systems in Section 5.1. On the other hand, we can attribute the second measurement M_2 to another observer. Then, the question arises: If we include the other observer into our quantum mechanical consideration, does then the combined system carry the answers to the questions we inquire about? Can we control for the disturbance the other observer causes, by including him into our considerations? In other words, we may wonder: Are we addressing the right questions to the right system? Along this latter route, we introduce composite systems in Section 5.5.

5.1   Superposition

If we modify the picture above slightly, discarding the downwards deflected silver atoms, we obtain:
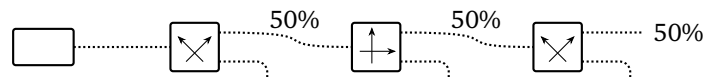
The behavior resembles the one of initially unpolarized light shun onto a sequence of polarizing filters with the depicted orientation:

The percentages refer to the *intensity* after the filter relative to the intensity before it. For light thought of as electromagnetic waves, it is not surprising to find half the intensity after a filter oriented at 45° angle relative to the polarization direction of the light before the filter. The *superposition* of waves allows to think of vertically polarized light as the sum of two vectors:

The interference patterns in a single particle double-slit experiment are another striking example of wave properties of single particles, i.e., superposition in quantum mechanics. Conversely, the electromagnetic effect shows that light has particle properties, and, thus, follows the scheme of single particles with wave properties that show in their statistical behavior. This motivates the introduction of the quantum states as elements in a vector space.

The quantum state

An *isolated* quantum system is associated with a normalized vector in a complex or real Hilbert space,

$$\psi \in \mathscr{H} \qquad \|\psi\|^2 = \langle \psi | \psi \rangle = 1$$

where the norm is derived from the inner product, written here in the Dirac notation. Any linear combination, or superposition of quantum states $\psi_1, \psi_2 \in \mathscr{H}$ yields another valid state, $\alpha \psi_1 + \beta \psi_2$, if we ensure that its norm is preserved, i.e., $|\alpha|^2 + |\beta|^2 = 1$.

A structural comparison with wave mechanics has prompted us to turn to a vector space structure. But, how does one associate a state to a system? As we have seen above: The term "state" should not be understood as codification of answers to questions we might inquire about—answers that, e.g., a silver atom carries with itself. The discussion above also shows: After we have performed a measurement and we know its results, we *can say* about a system that performing a second time the same measurement recovers that result *with certainty*.[9] In other words, after having performed a measurement M and obtained a result x, the state should codify that we expect to retrieve x with probability one, and $\neg x$ ("not x") with probability zero if we perform M again. Thus, the state characterizes a probability distribution for results of a measurement M corresponding to the question "x or $\neg x$?".

How do we characterize a measurement, so that a vector carries the aforementioned meaning? We observe that $\|\Pi_\psi \psi\|^2 = 1$ and $\|\Pi_\psi^\perp \psi\|^2 = 0$ where Π_ψ is the projector onto the subspace spanned by ψ, and $\Pi_\psi^\perp$ the projector onto the orthogonal complement to that subspace. The projectors Π_ψ and its orthogonal complement $\Pi_\psi^\perp$ allow to derive the desired probability distribution, and, thus, provide a representation of the measurement "x or $\neg x$?". To turn this reasoning around, we associate different measurement results with mutually orthogonal subspaces. A measurement is then represented by a complete set of projectors onto an orthogonal subspace of $\mathscr{H}$,

$$M = \left\{ \Pi_\lambda \;\middle|\; \lambda \in R_M \subset \mathbb{R}, \sum_\lambda \Pi_\lambda = \mathbb{1}, \mathrm{Tr}(\Pi_{\lambda_i} \Pi_{\lambda_j}) = \delta_{ij} \right\} \tag{2}$$

where the *range* R_M is the set of possible, *real-valued* results of M. The *measurement basis* of M is an orthonormal basis $\mathscr{B}$ of the Hilbert space that allows to

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[9]We implicitely assume that the system is isolated.

express the projectors as

$$\Pi_{\lambda \in B_\lambda} = \sum_{\phi \in B_\lambda} |\phi\rangle\langle\phi|$$

where $\{B_\lambda\}$ is a partitioning of $\mathscr{B}$. The spectral theorem allows to associate measurements with Hermitian operators via the linear combination

$$A = \sum_{\lambda \in R_M} \lambda \Pi_\lambda.$$

To obtain a (positive) probability measure, we compute the probability to obtain the result $\lambda \in \mathbb{R}$ associated with a subspace with projector Π_λ as the norm squared,

$$\mathrm{P}(\lambda) = \|\Pi_\lambda \psi\|^2$$

of a system being in the state ψ, i.e., after having previously measured x. The normalization of quantum states ensures the normalization of the probability distributions over measurement results.

Measurements

We generalize the notion expressed in (2) for arbitrary Hilbert spaces: A measurement in quantum mechanics is represented by a *projector valued measure*, i.e., a map $P : \mathscr{B}(\mathbb{R}) \to \mathfrak{L}(\mathscr{H})$ from the Borel sets of $\mathbb{R}$ to the ($C^\star$-algebra of) linear, continuous operators on the Hilbert space $\mathscr{H}$ that satisfies

1. $P(B) \geq 0 \qquad \forall B \in \mathscr{B}(\mathbb{R})$,

2. $P(B)P(B') = P(B \cap B') \qquad \forall B, B' \in \mathscr{B}(\mathbb{R})$,

3. $P(\mathbb{R}) = \mathbb{1}$,

4. for any countable family $\{B_n\}_{n \in \mathbb{N}} \subset \mathscr{B}(\mathbb{R})$ of *disjoint sets*, $B_n \cap B_m = \emptyset$ if $n \neq m$:[a]
$$\sum_n P(B_n) = P\Big(\bigcup_n B_n\Big).$$

It follows, that P maps elements from $\mathscr{B}(\mathbb{R})$ to projectors, i.e., $P(B)$ is self-adjoint and idempotent. We denote the set of projectors on $\mathscr{H}$ by $\mathfrak{P}(\mathscr{H})$. The spectral theorem establishes a correspondence between PVMs and Hermitian operators. PVMs generalize further to *positive operator-valued measures* (POVM) if we drop the second requirement.

> The probability to obtain a value in a set $B \in \mathscr{B}(\mathbb{R})$ as a result if one performs the measurement M on a system in state ψ is
>
> $$\mathrm{P}(\lambda) = \|P(B)\psi\|^2.$$
>
> ───────────
> [a]We assume here that this sum exists and is well-defined. For the technical details we refer to [82].

If we perform a measurement and obtain a result associated with a *one-dimensional* subspace of $\mathscr{H}$, we know what state to assign to the measured system. The situation is, however, less clear if one obtains a result associated with a subspace of dimension larger than one. To remedy the issue, we generalize the quantum states from above to *density matrices*.

5.2   Density Matrices

Above, we consider subsequent measurements with bases at a relative angle of 45° degrees, i.e., where the magnetic field is rotated by 90°. Then, the probability to retrieve the same measurement result *after an intermediate measurement in a rotated basis* drops to 1/2. What happens if this angle is varied? Experimentally, we find the following behavior:
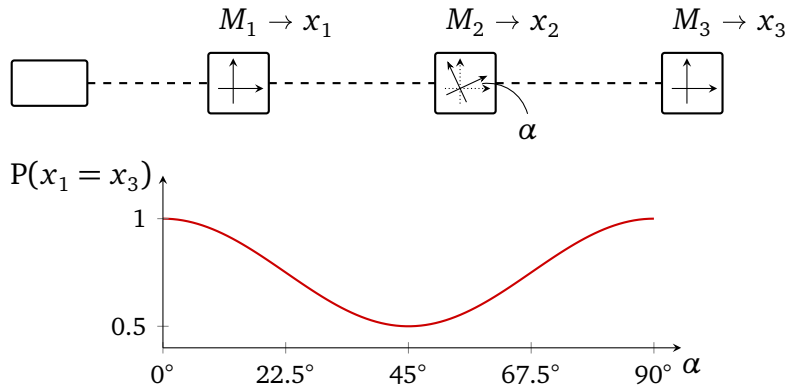
This leads to the following observations: Firstly, the continuity of possible questions does not (necessarily) reflect in the continuity of possible answers, but in the continuity of the possible probability distributions. Secondly, the case we discuss in Section 5 (in particular the scenario in (1)) is *not* covered here: If we could associate a state $\psi \in \mathscr{H}$ to the system leaving the source on the left, then rotating the magnetic field in a first measurement M_1 appropriately should yield one particular result with certainty. In a suitable Stern-Gerlach experiment, however, the probability for both results is 1/2, *no matter the orientation of the*

magnetic field. How to account for *this kind of uncertainty*? What state should one associate to the system leaving the source? A normalized linear combination of vectors is not a solution here: It merely changes for *which* measurement the probability distribution becomes deterministic. Instead, a normalized linear combination *of projectors* yields a way out.

Density matrices

A system is associated with a density matrix, i.e., with a trace-class, positive semi-definite operator ρ on $\mathscr{H}$, i.e.,

$$\rho \in \mathfrak{L}(\mathscr{H}) : \ \rho \geq 0 \ \text{and} \ \text{Tr}(\rho) = 1 \,.$$

We silently drop the requirement for the system to be isolated for reasons that become clear in the next section. The states from before—i.e., the normalized *vectors*—integrate here as the one-dimensional projectors $\rho_\psi = \Pi_\psi = |\psi\rangle\langle\psi|$, and are called the *pure states*. The density matrices require to adapt the computation of measurement probabilities.

Measuring and assigning density matrices

The probability to obtain the result in $B \in \mathscr{B}(\mathbb{R})$ if one performs the measurement M with a PVM P_M on a system with a density matrix ρ is

$$\text{P}(B) = \text{Tr}(P_M(B) \cdot \rho) \,.$$

After having measured a value in B, the system is assigned the density matrix $\rho' = 1/n \, P_M(B) \rho \, P_M(B)$ where $n = \text{Tr}(P_M(B)\rho)$.

What if there is no previously known ρ? If in a measurement M we obtain a value λ and there exists a set $B \in \mathscr{B}(\mathbb{R})$ such that $B \ni \lambda$ with $P_M(B)$ being trace class, i.e., if $\text{Tr}(P_M(B)) < \infty$, then one can assign the state $\rho' = P_M(B)/n$ where $n = \text{Tr}(P_M(B))$. For finite-dimensional Hilbert spaces, with $\dim \mathscr{H} = n$, all projectors are trace class and one can initially assign the maximally mixed state $\rho_{\text{init}} = \mathbb{1}/n$.

5.3   The lattice of projectors

How does the mathematical framework apply to the case of multiple, subsequent measurements? First, we confirm that if we measured the value $\lambda \in R_{M_1}$, then

the density matrix assigned to the system after the measurement

$$\rho' = \frac{1}{n}\Pi_\lambda \rho \Pi_\lambda$$

satisfies $\text{Tr}(\Pi_\lambda \rho') = \text{Tr}(\rho') = 1$ as for projectors, by definition, we have $\Pi_\lambda \Pi_\lambda = \Pi_\lambda$. This means that any subsequent measurement in the same basis yields the same results as we demanded above. Generally, the joint probability to obtain the results λ and ν in two subsequent measurements given an initial density matrix ρ is

$$P(\lambda, \nu) = P(\nu|\lambda)P(\lambda) = \text{Tr}(\Pi_\nu \Pi_\lambda \rho \Pi_\lambda) = \text{Tr}(\Pi_\nu \Pi_\lambda \rho \Pi_\lambda \Pi_\nu).$$

The probability of the same results in the inversely ordered measurements is equal to $P(\lambda, \nu)$ if and only if $\Pi_\lambda \Pi_\nu = \Pi_\nu \Pi_\lambda$. If the latter is the case, we say that the projectors *commute*. The binary measurements associated with commuting projectors do not disturb each other. Note that projectors for different values in the range R_M and their complements $\mathbb{1} - \Pi_\lambda$, i.e., the elements in the set

$$\{\Pi_\lambda \mid \lambda \in R_M\} \cup \{\mathbb{1} - \Pi_\lambda \mid \lambda \in R_M\},$$

commute.[10] Thus, we can think of any measurement M as a series of binary questions, each corresponding to question "λ or $\neg\lambda$?" and we can focus the discussion about measurements on the projectors, i.e., elements in $\mathfrak{P}(\mathcal{H})$.

In this new context of projectors on a Hilbert space, Gleason's theorem [46] strengthens the notion of density matrices as probability measures.

Theorem 1 (Gleason). *For a Hilbert space $\mathcal{H}$ of dimension* $\dim \mathcal{H} \geq 3$ *(or infinite dimensional and separable), any map* $\mu : \mathfrak{P}(\mathcal{H}) \to \mathbb{R}_+$ *with*

$$\mu(\mathbb{1}) = 1 \quad \text{and} \quad \mu\left(\sum_i P_i\right) = \sum_i \mu(P_i)$$

can be expressed with a positive trace-1 operator $\rho \in \mathfrak{L}(\mathcal{H})$ *as*

$$\mu(P) = \text{Tr}(\rho P) \quad \forall P \in \mathfrak{P}(\mathcal{H}).$$

With the exception of two-dimensional Hilbert spaces, where there exist measures on $\mathfrak{P}(\mathbb{C}^2)$ that cannot be expressed by means of a two-dimensional density matrix,[11] the density matrices can be seen as a *consequence* of representing measurement by elements in $\mathfrak{P}(\mathcal{H})$.

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[10]This follows generally for PVM from characteristic (2).

[11]See, e.g., Remark 7.26 (4) in Ref. [82].

A way to understand the structure of $\mathfrak{P}(\mathscr{H})$ is to regard it as lattice, i.e., a partially ordered set for which any two elements have a unique least upper bound and a unique greatest lower bound. Lattices can be seen as a detour towards a Boolean algebra—the standard logical structure that we know, e.g., from classical mechanics[12]: In an orthocomplemented lattice the order relation corresponds to the material conditional, the orthocomplement to the negation, logical conjunction and disjunction to the greatest lower and least upper bounds. In a distributive or Boolean lattice all elements can be consistently assigned truth values. The set $\mathfrak{P}(\mathscr{H})$ together with the order relation *equivalently characterized by any* of the following features[13]

1. $P \leq Q$, defined as $\langle x|Px \rangle \leq \langle x|Qx \rangle \ \forall x \in \mathscr{H}$,

2. $P(\mathscr{H})$ is subspace of $Q(\mathscr{H})$,

3. $PQ = QP = P$,

forms a orthocomplemented lattice—albeit *not a distributive one* if the dimension of the Hilbert space $\mathscr{H}$ is greater or equal than two. In other words, associating measurements with elements in $\mathfrak{P}(\mathscr{H})$ allows to merely go part of the way towards a Boolean algebra: The elements in $\mathfrak{P}(\mathscr{H})$ cannot be assigned values $\mathtt{t}$ and $\mathtt{f}$ consistently.

There are different ways to arrive at this conclusion:

1. From Gleason's Theorem 1, it follows that there is no deterministic probability distribution on $\mathfrak{P}(\mathscr{H})$ if the dimension of the Hilbert space is larger than three.

2. The lattice $\mathfrak{P}(\mathscr{H})$ has a trivial center, i.e., only 0 and $\mathbb{1}$ commute with all other elements in $\mathfrak{P}(\mathscr{H})$. Thus, with the result by Jauch and Piron [66] (see also [50]), there is no dispersion-free state on $\mathfrak{P}(\mathscr{H})$, i.e., no deterministic probability distribution.

3. Kochen and Specker show in Ref. [70] that there is no consistent assignment of values to orthogonal bases (see also [61]).

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[12]In classical mechanics, the binary questions are of the form "Is the state of a system in a given subset of phase space?" The corresponding lattice is the power set of phase space with subset relation as partial order.

[13]See [82, Proposition 7.16].

with trivial center/ without superselection rules
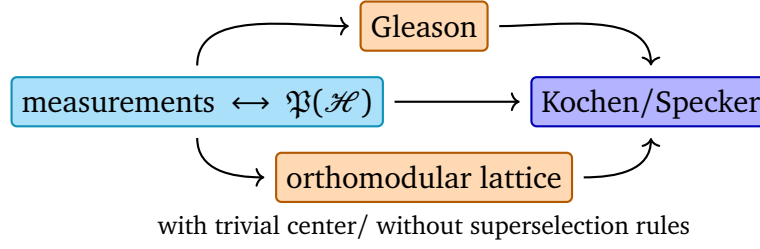
These statements ascertain mathematically what we suspected previously: As long as we characterize measurements with projectors on a Hilbert space, i.e., $\mathfrak{P}(\mathcal{H})$ (and we do not exclude projectors by means of a super-selection rule), we cannot assign consistently truth values to all possible questions. Thus, a quantum system generally cannot carry answers to all possible questions.

5.4   Time evolution: General classes of same questions

What does it mean, to "ask the same question"? Before, when we considered silver atoms in a Stern/Gerlach experiment, we assumed that it is possible to measure twice the same silver atom, once in a measurement M_1 at time t_1 and then M_2 at some later time t_2 and take the two measurements to be the same. In some cases, the measurements M_1 and M_2 might be characterized by the same sets of projectors. But this is not necessarily case. Let $f : \mathfrak{P}(\mathcal{H}) \to \mathfrak{P}(\mathcal{H})$ be the map that associates the projectors at t_1 with the projectors at t_2. We require that the projectors of a measurement $M = \{\Pi_\lambda \mid \lambda \in R_M\}$ are mapped again to a measurement, i.e., that

$$\left\{ f\left(\Pi_\lambda\right) \,\middle|\, \lambda \in R_M \right\}$$

satisfy

$$\sum_\lambda f\left(\Pi_\lambda\right) = \mathbb{1} \qquad \mathrm{Tr}\left(f\left(\Pi_{\lambda_i}\right) f\left(\Pi_{\lambda_j}\right)\right) = \delta_{ij}\,\mathrm{Tr}\left(f(\Pi_{\lambda_i})\right).$$

Further, we ask that $f(0) = 0$ and $f(\mathbb{1}) = \mathbb{1}$.

We generalize the second requirement to the following: *Conditional probabilities are preserved.* The probability of measuring λ_i in a measurement M_1 at time t_1 and λ_j in a measurement M_2 at time t_2 only depends on the equivalence class—provided that the order is preserved.

$$P((\lambda_j, M_2, t_1) \mid (\lambda_i, M_1, t_1)) = P((\lambda_j, M_2, t_2) \mid (\lambda_i, M_1, t_1)) \qquad (3)$$
$$= P((\lambda_j, M_2, t_2) \mid (\lambda_i, M_1, t_2)) \qquad (4)$$

The requirement ensures continuity as it can be read as: The conditional probability is the same in the limit $t_1 \to t_2$, i.e., for cases in which M_1 is performed

infinitesimally before M_2. With the state assignment rules from above we obtain that

$$\frac{\text{Tr}\left(\Pi_{\lambda_i}^{M_1}\Pi_{\lambda_j}^{M_2}\right)}{\text{Tr}\left(\Pi_{\lambda_i}^{M_1}\right)} = \frac{\text{Tr}\left(f\left(\Pi_{\lambda_i}^{M_1}\right)f\left(\Pi_{\lambda_j}^{M_2}\right)\right)}{\text{Tr}\left(f\left(\Pi_{\lambda_i}^{M_1}\right)\right)}.$$

It follows, by setting $\Pi^{M_1} = \mathbb{1}$, that f is trace-preserving, i.e.,

$$\text{Tr}(f(\Pi)) = \text{Tr}(\Pi) \quad \forall \, \Pi,$$

and the requirement for conditional probability distributions reduces to

$$\text{Tr}\left(\Pi_{\lambda_i}^{M_1}\Pi_{\lambda_j}^{M_2}\right) = \text{Tr}\left(f\left(\Pi_{\lambda_i}^{M_1}\right)f\left(\Pi_{\lambda_j}^{M_2}\right)\right).$$

In particular, this requirement has to hold for any two one-dimensional projectors. Thus, using Wigner's Theorem, we conclude that f is of the form

$$f : \Pi \mapsto U\Pi U^\dagger$$

for some unitary (or anti-unitary) map U, i.e., $U^\dagger = U^{-1}$. Note, that the requirement (3) ensures that f preserves the lattice structures, and constitutes thus a lattice isomorphism.

To turn the above static Wigner symmetry into a continuous one, we require a map $\mathbb{R} \ni t \mapsto U_t$, such that[14]

$$U(t+s) = U(t) \cdot U(s) \qquad U(0) = \mathbb{1}.$$

The time-evolution, i.e., the classes of equivalent questions, is characterised by unitary one-parameter groups.

Instead of translating the projection operators in time, we can apply the dual map

$$f^\star : \rho \mapsto U^\dagger \rho U$$

to the density matrix.

The above characterizes inquiries to *isolated systems*. What happens, however, if the system we are dealing with is *not* isolated? One way to approach systems that are not isolated is to include enough of their environment to turn it into an isolated one. For that, we need to introduce the composition of systems, i.e., how to formally consider two systems S_1 and S_2 together as *one combined system*.

---

[14]This answers the question whether we should opt for unitary or for anti-unitary operators: The set of anti-unitary operators is not closed under multiplication. Thus, as $\mathbb{R}$ allows for a square root, we must restrict to unitary operators.

5.5   Composite systems

In order to jointly describe two systems so that the superposition of the subsystems carries over to the joint system we require a binary operation "$\otimes$" such that for all $\phi, \phi_i \in \mathcal{H}_a, \psi, \psi_i \in \mathcal{H}_b, \alpha \in \mathbb{C}$

$$\phi \otimes (\psi_1 + \psi_2) = \phi \otimes \psi_1 + \phi \otimes \psi_2$$
$$(\phi_1 + \phi_2) \otimes \psi = \phi_1 \otimes \psi + \phi_2 \otimes \psi$$
$$(\alpha \phi) \otimes \psi = \alpha(\phi \otimes \psi) = \phi \otimes (\alpha \psi).$$

These are the defining properties of the *tensor product* of vector spaces [124]. The tensor product extends to operators: For an operator S on $\mathcal{H}_a$ and T on $\mathcal{H}_b$, the tensor product is defined by how they act on a tensor product of elements in $\mathcal{H}_a \otimes \mathcal{H}_b$,

$$(S \otimes T)(\phi \otimes \psi) = S\phi \otimes T\psi.$$

Thus, also density matrices $\rho_1 \in \text{End}(\mathcal{H}_a)^{15}$ and $\rho_2 \in \text{End}(\mathcal{H}_b)$ combine to a density matrix $\rho_1 \otimes \rho_2 \in \text{End}(\mathcal{H}_a \otimes \mathcal{H}_b)$.

For a density matrix $\rho \in \text{End}(\mathcal{H}_1 \otimes \mathcal{H}_2)$, the *partial trace* on $\mathcal{H}_a$ defined by the linear extension of the map

$$\text{Tr}_b : \text{End}(\mathcal{H}_a \otimes \mathcal{H}_b) \rightarrow \text{End}(\mathcal{H}_a)$$
$$S \otimes T \mapsto \text{Tr}(T)S$$

allows to retrieve the density matrices of the subsystem $\mathcal{H}_a$ and, vice versa, an analogously defined $\text{Tr}_b : \text{End}(\mathcal{H}_a \otimes \mathcal{H}_b) \rightarrow \text{End}(\mathcal{H}_b)$ the density matrices of the subsystem $\mathcal{H}_b$. On a first glance it might seem unnecessary to introduce a second operation for deriving the density matrices of the subsystems from the joint density matrix: With the normalization of the density matrix, we obtain

$$\text{Tr}_b(\rho_1 \otimes \rho_2) = \rho_1 \quad \text{and} \quad \text{Tr}_a(\rho_1 \otimes \rho_2) = \rho_2.$$

There are, however, vectors on the joint systems that are *not* of the form $\phi \otimes \psi$. For instance, the vector

$$|\psi^+\rangle = \frac{1}{\sqrt{2}} \left(|0\rangle_a \otimes |1\rangle_b + |1\rangle_a \otimes |0\rangle_b\right) = \frac{1}{\sqrt{2}} \left(|0,1\rangle + |1,0\rangle\right) \in \mathcal{H}_a \otimes \mathcal{H}_b$$

is *entangled* or *not separable*, i.e., cannot be written as a tensor product of a vector in $\mathcal{H}_a$, and a vector in $\mathcal{H}_b$. Then, also the corresponding density matrix $\rho = |\psi^+\rangle\langle\psi^+|$ cannot be written as a tensor product of density matrices on the subsystems.

---

[15] With $\text{End}(\mathcal{H}_a)$, we denote the set *endomorphisms*, i.e., of homomorphism of a space $\mathcal{H}_a$ into itself.

One might be tempted to assume that a density matrix is, analogously, entangled, if it *cannot* be written in the form $\rho_1 \otimes \rho_2$. This is sufficient, however not necessary: A density matrix is entangled if it *cannot* be written as a linear combination of such products, i.e., *not* in the form

$$\sum_i \lambda_i \rho_a^{(i)} \otimes \rho_b^{(i)}.$$

If a density matrix ρ on a joint system is separable, then the joint conditional probability distribution for *separate measurements* is

$$
\begin{aligned}
P(a, b \,|\, x, y) &= \operatorname{Tr}\left(\Pi_x^{(a)} \otimes \Pi_y^{(b)} \rho \right) \\
&= \sum_i \lambda_i \operatorname{Tr}\left(\left(\Pi_x^{(a)} \otimes \Pi_y^{(b)} \right) \left(\rho_a^{(i)} \otimes \rho_b^{(i)} \right) \right) \\
&= \sum_i \lambda_i \operatorname{Tr}\left(\Pi_x^{(a)} \rho_a^{(i)} \otimes \Pi_y^{(b)} \rho_b^{(i)} \right) \\
&= \sum_i \lambda_i \operatorname{Tr}\left(\Pi_x^{(a)} \rho_a^{(i)} \right) \operatorname{Tr}\left(\Pi_y^{(b)} \rho_b^{(i)} \right) \\
&= \sum_i \lambda_i \, P(a \,|\, x, i) \cdot P(b \,|\, y, i).
\end{aligned}
$$

A joint conditional probability distribution describes the behavior of a bipartite input-output box, i.e., a black box that produces outputs a and b upon receiving inputs x and y.

$$
\begin{array}{cc}
a & b \\
\uparrow & \uparrow \\
\multicolumn{2}{c}{\boxed{P(a, b \,|\, x, y)}} \\
\uparrow & \uparrow \\
x & y
\end{array}
$$
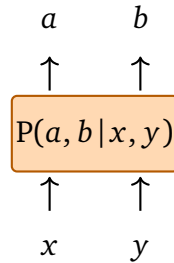
The behavior of a bipartite box is called *local* if it can be written in the above form. As the weights λ_i are positive and normalized to sum to one, we can write it as

$$P(a, b \,|\, x, y) = \sum_i P(i) \, P(a \,|\, x, i) \cdot P(b \,|\, y, i).$$

Thus, a local behavior can be explained by a so-called *hidden variable i* with distribution $P(i)$ previously shared between parties A and B. To produce a local behavior, all A needs to know from B—and, vice versa, B from A—is the shared hidden variable i. Then, each party's output merely depends on their respective

input and the value of the hidden variable. We have discussed above that a quantum state cannot be seen as a collection of definite answers to possible questions due to contextuality. Thus, sharing a quantum state is not the same as sharing a hidden variable. Indeed, Bell shows in Ref. [10] that there exist non-local behaviors deriving from measurements on entangled quantum states. Bell's theoretical results have found experimental confirmation, e.g., in Ref. [3, 78].[16]

The set of local behaviors forms a polytope. This polytope is a proper subset of the polytope of *non-signaling* behaviors that satisfy

$$\sum_b P(a, b \,|\, x, y) = P(a \,|\, x) \quad \text{and}$$
$$\sum_a P(a, b \,|\, x, y) = P(b \,|\, y)$$

and, thus, are in accordance with relativity theory. That is, no messages can be transmitted employing the behavior and, therefore, special relativity theory cannot be falsified by transmitting information faster than light.

Quantum mechanics accounts for a convex set of behaviors—not a polytope—that contains the local behaviors as a proper subset and is itself properly contained in the set of non-signaling behaviors. The latter follows from the observation that the partial trace suffices to compute probabilities for measurements performed on the respective subsystem. If A performs a measurement, then the probability to obtain the result associated with $\Pi^{(a)}$ is

$$P(a) = \text{Tr}_A\big(\text{Tr}_B\big((\Pi^{(a)} \otimes \mathbb{1}_B)\rho\big)\big) = \text{Tr}_A\big(\Pi^{(a)}\,\text{Tr}_B(\rho)\big)\ .$$

Here we used

$$\text{Tr}\,((A \otimes \mathbb{1}_B)\rho) = \text{Tr}_A(\text{Tr}_B\,((A \otimes \mathbb{1}_B)\rho)) = \text{Tr}_A(A\,\text{Tr}_B(\rho))\ ,$$

which follows, e.g., from expanding both sides in a product basis (see also [1, 21]). Consequently, the probabilities for results of measurements performed by A do *not* depend on the input of B, and any behavior derived from measurements on joint quantum states is non-signaling. In other words, the probabilities of

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[16]There has since been a debate about possible loopholes by means of which information could still have been transferred from one remote site to another without violating relativity theory. There has also been a corresponding race for "loophole-free" experimental evidence for Bell non-locality. Whether any experiment can claim being "loophole-free" seems debatable in itself. Usually, these claims refer to closed *known* loopholes. So the "loophole-free" does not mean "free from any loopholes" but rather "free from the loopholes previously discovered." Yet, the connotation of a general absence of loopholes remains implicitly invoked, and spurs the hope of a certainty.

measurements performed on subsystems are fully characterized by the respective partial traces. In this sense, quantum mechanics allows for a separability in the sense that parts of the world around us can be described independent of other such parts. This separability is crucial for the notion of a system, as has been pointed out by Einstein and as we will discuss in Section 8.3. We do, however, not intend to imply that, *inversely*, the joint behavior derives from the behavior of the subsystems and previously shared information. This would take us back to a local description of the world and, thus, be contradicting the findings of Bell. Furthermore, we note that given a system with non-prime dimension it is not clear how to split the sytem into subsystems. The choice how to split the system into subsystems amounts to the choice on a particular basis (see Section 8.3 and 11.3).

The formulation in terms of input-output boxes is lent from information theory and has spurred the search for a characterization of the set of quantum behaviors in terms of information theory and complexity theory. Conversely, research on the foundations of quantum mechanics has employed information theory. Along these lines we have attempted to strengthen results ruling our hidden faster-than-light communication an explanation for non-locality building on the work of Alberto Montina [81, 56].

5.5.1 Stinespring's dilation

The initial idea of including enough of the environment into our description in order to isolate a system manifests itself in Stinesprings's dilation. Above, we have characterized classes of equivalent measurement by a one-parameter group of unitary operators. This is dual to the unitary time evolution of the state of an isolated system. If the system is *not* isolated, then the time evolution of a state is given by a *completely-positive, trace-preserving map* (CPTP map)

$$T : \text{End}(\mathcal{H}) \to \text{End}(\mathcal{H})$$
$$\text{Tr}(T(\rho)) = \text{Tr}(\rho)$$
$$T(\rho) \otimes \tau > 0 \quad \forall \rho, \tau > 0 .$$

A CPTP map does, in general, not preserve inner products. Any such map can, however, be regarded as "part" of a unitary acting on a larger system: By the Stringspring dilation,[17] there exist for every CPTP map T acting on $\mathcal{H}$ a Hilbert space $\mathcal{H}_E$ and a isometry $V \in \mathfrak{L}(\mathcal{H}, \mathcal{H} \otimes \mathcal{H}_E)$ so that

$$T : \rho \mapsto \text{Tr}_E(V\rho V^\dagger) .$$

---

[17]The Stinespring dilation can be derived from Choi-Jamiolkowski map as shown in Ref. [21].

As there exists a unitary embedding U of V in $\mathscr{H} \otimes \mathscr{H}_E$ so that

$$U : w \otimes w_0 \mapsto Vw$$

for some fixed $w_0 \in \mathscr{H}_E$, the CPTP map T can be regarded as the effect of unitary map U on the joint system $\mathscr{H} \otimes \mathscr{H}_E$.

5.5.2   The relative state formalism

A measurement, or rather the effect of a measurement on the measured system, can be described by a CPTPM acting on the measured system. Thus, it follows from the Stinespring dilation: If we include enough of the system's environment into our quantum mechanical description, then we are left with a unitarily evolving isolated system. The measurement can be regarded as an *interaction* of the system that is to be measured, henceforth denoted S, and a part of its environment M. The unitary time evolution U describes this interaction. This perspective of measurement as an interaction between two quantum systems has been introduced by Everett [30] and is commonly referred to as the *relative-state formalism*. The system M is also referred to as the memory. Eventually, an observer's brain acts as such memory. If for the time span in which U acts, other parts of the observer remain isolated, we can take M to correspond to the observer. The interaction corresponding to measuring a PVM with the orthonormal basis $\{\phi_i\}_{i \in I}$[18] is characterized by a unitary that acts on the orthonormal basis as

$$U : \ |\phi_i\rangle_S \otimes |0\rangle_M \mapsto |\phi_i\rangle_S \otimes |i\rangle_M \, .$$

The vectors $\{|i\rangle_M\}_{i \in I}$ form an orthonormal set and can be completed to a basis of M. Thus, the dimension of the memory system is at least as large as the cardinality $|I|$. The system M is at times taken to be in an initial "ready" state orthogonal to span of $\{|i\rangle_M\}_{i \in I}$, and the dimension of M is at least $|I| + 1$. If the system S is initially in a superposition state with respect to the measurement basis $\{|\phi_i\rangle\}$, then the joint system $S \otimes M$ ends up in an entangled state after the action of U. To obtain measurement statistics in the relative-state formalism, we apply the Born rule to the partial trace

$$\mathrm{P}(i) = \mathrm{Tr}(\mathrm{Tr}_S(U(|\psi\rangle_S \otimes |0\rangle_M)(\langle\psi|_S \otimes \langle 0|_M U^\dagger)|i\rangle\langle i|_M)$$

where i indexes the result corresponding to the basis vector $|\phi_i\rangle$.

---

[18]We assume a finite-dimensional Hilbert space. The projectors Π_k correspond to disjoint subsets of $\{|\phi_i\rangle\}_{i \in I}$.

To compare results of different measurement on the system S one introduces different memories. The stability of the measurement results shows in the correlation—with $P(i, j) = 0$ if $i \neq j$—if the two measurements are equivalent and if the system is isolated.[19]

5.6   The perspective onto quantum mechanics

In the previous sections, we not only give an introduction to quantum mechanics, but also put it into a specific perspective. From classical mechanics, one is used to see the state of a system as the definite characteristics sufficient to describe the system's evolution through time and space. This perspective onto the state is shattered in quantum mechanics due to non-locality and contextuality. The notion of a state in classical mechanics is merely possible because the measurements—represented by the subsets of phase space—form a Boolean lattice. The orthomodular lattice of projectors in quantum mechanics is no longer distributive or Boolean—resulting in the above-mentioned contextuality. Thus, the notion of a state needs to be altered, as anticipated by Hermann:[20]

> In contrast to this [states in classical physics], the quantum mechanical formalism requires for the description of a state new symbols that express the mutual dependency of the determinability of different measurements. [63, own translation]

Instead of taking quantum states to be of a different kind than states in classical mechanics, we shift the focus away from states altogether. Thus, we deviate from the common approach maintaining the prominent role of states by featuring them in the "1st Axiom" of quantum theory.

We place the measurements, or questions that can be inquired about, in the focus of our perspective on quantum mechanics. In this regard, the second pillar of our understanding of quantum mechanics—beside the orthomodular lattice of projectors on a Hilbert space—is Gleason's theorem that establishes a one-to-one correspondence between density matrices and probability distributions over the set of projectors.

With the emphasize on measurements, the Heisenberg picture suggests itself. For an isolated system this amounts to classes of equivalent questions generated

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[19]We assume that equivalent memory states for the different measurement have the same index.

[20]"Im Gegensatz dazu [Zustände der klassischen Physik] braucht der quantenmechanische Formalismus zur Zustandsbeschreibung neuartige Symbole, die die gegenseitige Abhängigkeit in der Bestimmbarkeit verschiedener Größen zum Ausdruck bringt." [63]

by the one-parameter group of unitary operators.

This picture of quantum mechanics has been the starting point and inspiration for the more general considerations about the measurement problem in Section 7.

6 The Measurement Problem

Equipped with a formal basis of quantum mechanics, we retrace the common characterization of the problem with measurements arising in quantum mechanics. This extends the description in Section 4. Quantum mechanics has given rise to philosophical discussions and doubts from the very beginning. In Section 6.1, we discuss these early considerations and the older conceptual problem of a dualism between a quantum world and a classical one. Everett with his relative state formalism characterized the measurement as an interaction between two system that are both described, at least in principle, by quantum mechanics. This leads to the description of the measurement problem by means of the so-called Wigner's-friend experiment discussed in Section 6.2.

6.1 Dualism: Quantum vs classical world

Bohr, when recounting discussions with Einstein about fundamental problems arising with quantum mechanics in Ref. [17], turns to the Fifth Physical Conference of the Solvay Institute. Einstein uttered concerns about the abandonment of a "causal account in space and time," and provided to this end the following example:

> To illustrate his attitude, Einstein referred at one of the sessions to the simple example, illustrated by Fig. I, of a particle (electron or photon) penetrating through a hole or a narrow slit in a diaphragm placed at some distance before a photographic plate. [17, p. 212]
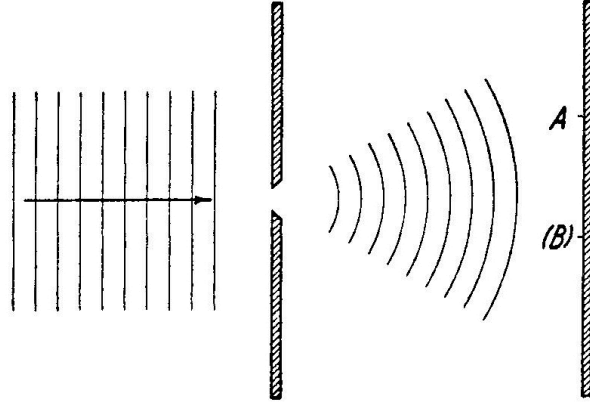
Fig. 1

Bohr describes the problem arising in this example as follows:[21]

> The apparent difficulty, in this description, which Einstein felt so
> acutely, is the fact that, if in the experiment the electron is recorded
> at one point A of the plate, then it is out of the question of ever ob-
> serving an effect of this electron at another point (B), although the
> laws of ordinary wave propagation offer no room for a correlation
> between two such events. [17, p. 212f]

The "correlation between events" is the *exclusivity* of measurement results: We
either measure a particle at position A *or* at a position B. This reflects, more
formally, in the notion of orthogonality: The orthogonality of

$$q_A = \text{``effect observed at } A\text{''}$$

and

$$q_B = \text{``effect observed at } B\text{''}$$

is, analogue to orthogonality on a complemented lattice [91], defined by $\neg q_A$
(not q_A) implying q_B.[22] The orthogonality of propositions about measurement re-
sults is not warranted by the account of waves in electrodynamics or as coupled-
oscillators. Considering the same scenario with electromagnetic waves leads to

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[21] Bohr remarks after this quote: "The discussions [among Einstein, Ehrenfest, and Bohr in
Solvay, 1927], however, centered on the question of whether the quantum-mechanical descrip-
tion exhausted the possibilities of accouting for observable phenomena or, as Enstein maintained,
the analysis could be carried further and, especially, of whether a fuller description of the phe-
nomena could be obtained by bringing into consideration the detailed balance of energy and
momentum in individual processes." [17, p. 213f] We turn to the question about physical theo-
ries offering an exhaustive description in Section 8.

[22]Note the symmetry: $\neg q_A \to q_B$ if and only if $\neg q_B \to q_A$.

statements about intensities at different positions on the screen: The relation
between "the intensity at *A*" and "the intensity at *B*" is generally *not* character-
ized by exclusivity or orthogonality. The relation can also be regarded as the
spatio-temporal integrity of the system: If the detection on the screen is a mea-
surement performed on single particles—i.e., yields a statement about that single
particle—, then orthogonality allows to still refer to a single particle.[23]

We encounter the incommensurability of definite measurement results and
unitary evolution of isolated systems that characterizes the measurement prob-
lem. Landau and Lifshitz, thus, conclude that quantum mechanics necessarily
relies on classical mechanics.[24]

> Thus quantum mechanics occupies a very unusual place among phys-
> ical theories: it contains classical mechanics as a limiting case, yet
> at the same time it requires this limiting case for its own formula-
> tion. [74, §1, p. 3]

quantum states   classical (mechanics|information)
superposition/linearity   definite results
                                          apparatus
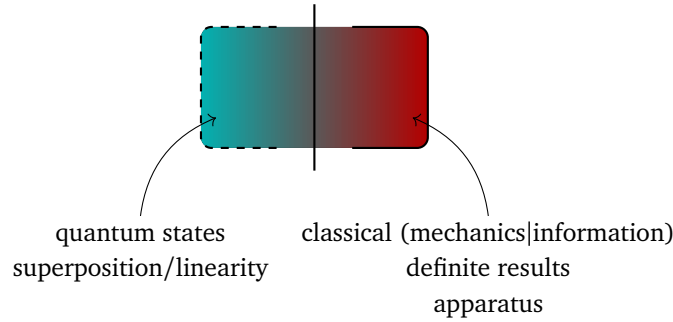
A measurement is characterized in Ref. [74] as the interaction between the quan-
tum system and the "classical object" referred to as the *apparatus*.[25] This classical-
quantum dualism comes with problems:[26] For instance, the question where to

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[23]This is weaker than to require a general criterion for the identity of the particle, by, e.g.,
spatio-temporal continuity. Here, we merely consider speaking meaningfully about *one* particle.
We return to this discussion in Section 11.2.

[24]Also other "classical theories" can be placed here. The term "classical information" has been
used to represent the orthogonality of measurement results. The inverse conclusion, that there
must exist "quantum information," is questionable. Note also that requiring the orthogonality
does not imply the necessity for classical *mechanics* on our side of the border.

[25]"In this connection the 'classical object' is usually called *apparatus*, and its interaction with
the electron is spoken of as *measurement*. However, it must be emphasised that we are here not
discussing a process of measurement in which the physicist-observer takes part. By *measurement*,
in quantum mechanics, we understand any process of interaction between classical and quantum
objects, occuring apart from and independently of any observer. The importance of the concept
of measurement in quantum mechanics was elucidated by N. Bohr." [74, §1, p. 2]

[26]These problems parallel general concerns about dualisms as uttered by, e.g., Quine and
Davidson [47].

draw the line between the two realms remains unclear. If all observers must find themselves on the classical side of the line, then who or what counts as an observer? Bell puts this ironically as follows:[27]

> It would seem that the theory is exclusively concerned about 'results of measurement', and has nothing to say about anything else. What exactly qualifies some physical systems to play the role of 'measurer'? Was the wavefunction of the world waiting to jump for thousands of millions of years until a single-celled living creature appeared? Or did it have to wait a little longer, for some better qualified system . . . with a Ph. D.? If the theory is to apply to anything but highly idealised laboratory operations, are we not obliged to admit that more or less 'measurement-like' processes are going on more or less all the time, more or less everywhere? Do we not have jumping then all the time? [9, p. 216]

The quote highlights another aspect that often goes with the divide into two worlds: the ontologization[28] of the state-assignment rule (see Section 5). Where the quantum and the classical world meet—in the interaction that goes with a measurement—, the unitary evolution of an isolated system is interrupted and the observed system "collapses" or "jumps" into the state associated with the measurement result. This is then turned around in the context of the Wigner's-friend experiment (see Section 6.2): For a definite result to occur, the observed system must collapse into the respective state.

6.2 Wigner's friend

After having introduced concepts of quantum mechanics in Section 5, we now introduce the Wigner's-friend experiment mentioned in (P2). First we recapitulate the experiment in its usual wording—employing references to states. Afterwards, we reframe it in terms of measurements—i.e., more in line with the above developed perspective onto quantum mechanics.

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[27]Despite the ironic tone, note that "applying to idealised laboratory operations" is not some complement of "saying something about the 'wavefunction of the world' " which, then, leads us to expect "jumping everywhere." We return to descriptions of "the state of the world" in Section 9.

[28]With *ontology* we refer to what one holds to be the basic building blocks of reality—the basic furniture of the world. Consequently, with the ontologization of the "collapse," the latter becomes a primitive building block of quantum mechanics.
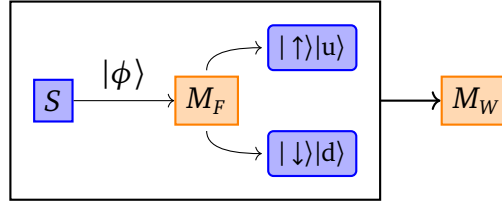
Figure 1. The Wigner's-friend experiment.

6.2.1   The traditional account

Employing references to states, the setup of the Wigner's-friend experiment is described as follows: A source emits a system in a state

$$\vert\phi\rangle = \frac{\vert 0\rangle + \vert 1\rangle}{\sqrt{2}} \in \mathscr{H}_S \, .$$

This state is then measured by an observer, called *Wigner's friend*, and modelled by a quantum system with Hilbert space $\mathscr{H}_F$, in the basis $\{\vert 0\rangle, \vert 1\rangle\}$. Finally, Wigner himself measures the joint system $\mathscr{H}_S \otimes \mathscr{H}_F$ in a basis

$$\left\{ \frac{\vert 0\rangle_S \vert 0\rangle_F + \vert 1\rangle_S \vert 1\rangle_F}{\sqrt{2}} , \frac{\vert 0\rangle_S \vert 0\rangle_F - \vert 1\rangle_S \vert 1\rangle_F}{\sqrt{2}} , \ \ldots \ \right\} .$$

Within Everett's *relative-state formalism Q* of quantum mechanics [30][29], the joint system $\mathscr{H}_S \otimes \mathscr{H}_F$ after the friend's measurement is in a state

$$\begin{aligned} V(\vert\phi\rangle_S) &= \frac{1}{\sqrt{2}}\big(V(\vert 0\rangle_S) + V(\vert 1\rangle_S)\big) \\ &= \frac{1}{\sqrt{2}}\big(\vert 0\rangle_S \otimes \vert 0\rangle_F + \vert 1\rangle_S \otimes \vert 1\rangle_F\big), \end{aligned}$$

where V is the isometry modelling the measurement of the friend [7]. Thus, Wigner's final measurement yields the eigenvalue, corresponding to the first basis vector with probability one.

Within Q it seems, however, unclear, how we end up observing "definite results." How are we to make sense of the superposition state

$$\frac{1}{\sqrt{2}}\big(\vert 0\rangle_S \otimes \vert 0\rangle_F + \vert 1\rangle_S \otimes \vert 1\rangle_F\big)$$

---

[29]Here, we do not refer to the *many-worlds interpretation* of quantum mechanics. We want to emphasize not to confuse the formalism with an interpretation [8].

if it corresponds to a measurement that yields *either* the result "0" *or* the result "1"? Recall the state assignment rule: Assuming that in a measurement we obtain a result corresponding to a one-dimensional projector $|\psi\rangle\langle\psi|$ on the Hilbert space associated with the observed system, we assign the normalized vector $|\psi\rangle$ as the state of that system. This is usually put as "the state of the system collapses to $|\psi\rangle$." In this regard, Wigner's friend can apply a formalism Q_C assigning the state $|0\rangle$ or $|1\rangle$ to the system S depending on the result of his measurement. Note, however, that Q_C introduces a collapse merely for *the system under consideration*. Thus, if Wigner applies Q_C, he assigns a state to the *joint* system $S \otimes F$ following *his* measurement. For Wigner, the friend's measurement does *not* induce a collapse. In other words: A measurement inducing a collapse in Q_C is an interaction of the system under consideration and its environment. Wigner and his friend consider different systems. For Wigner, the friend's interaction with S is not a measurement in this sense. Therefore, Q_C yields predictions about the outcome of Wigner's final measurement identical to those of Q. But we still lack an "explanation" for the friend obtaining a definite result in his measurement performed on S.

To account for the idea that measurements (generally) yield definite results, let us consider a third formalism with an *objective collapse*: In Q_{OC}, the joint system $\mathcal{H}_S \otimes \mathcal{H}_F$ collapses after the interaction modelled by the isometry V above, conditioned on the obtained result. Importantly, this collapse is happening independently of the observer, not merely subjectively—that is, it has to be considered also by Wigner (similarly to "GRW" [43]). For Wigner, who does not know the result of the friend's measurement, the entangled state $V(|\phi\rangle)$ collapses to a mixture

$$\rho = \frac{1}{2}\left(|0,0\rangle\langle0,0| + |1,1\rangle\langle1,1|\right), \tag{5}$$

and Wigner measures either of the two first vectors in the basis above with equal probability.

So, the question is: Does the interaction between S and F induce a collapse as modelled in Q_{OC} despite $S \otimes F$ being isolated—thus, evolving unitarily according to Q and Q_C? Combining idea of "S interacting with F" and "the friend obtaining a definite result about S" in the term measurement leaves us with an unclarity, or even a contradiction:[30] Following the former notion of a measurement, we

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[30]Wigner already remarked on the imprecise use of terms: "Most importantly, he [the quantum theorist] has appropriated the word 'measurement' and used it to characterize a special type of interaction by means of which information can be obtained on the state of a definite object. [...] On the other hand, since he is unable to follow the path of the information until it enters his, or the observer's, mind, he considers the measurement completed as soon as a statistical relation has

conclude that quantum mechanics leads us to apply either Q or Q_C, with equal predictions. Following the later notion of a measurement, we might conclude that quantum mechanics leads us to apply Q_{OC} leaving us with different contradictions. If the Wigner's-friend setup can be realized, then it allows to empirically test this question and decide between the formalisms Q_{OC} from Q and Q_C.

Note, however, that the predictions of Q and Q_{OC} match as soon as Wigner gets to know the result of the friend's measurement, by either measuring the joint system in a basis $\{|0\rangle_S|0\rangle_F, |1\rangle_S|1\rangle_F, \ldots\}$, or S in a basis $\{|0\rangle_S, |1\rangle_S\}$, or F in a basis $\{|0\rangle_F, |1\rangle_F\}$. The problem merely occurs if we insinuate that the friend obtained a definite result without actually knowing it. So, can we establish that a measurement has happened without revealing the result? One might assume that the friend's memory was in an initial state $|\Delta\rangle$ orthogonal to both $|0\rangle$ and $|1\rangle$ and there was a unitary operator U with

$$
\begin{aligned}
U : \ & \mathcal{H}_M \otimes \mathcal{H}_S \to \mathcal{H}_M \otimes \mathcal{H}_S \\
& |\Delta\rangle \otimes |k\rangle \ \mapsto \ |k\rangle \otimes |k\rangle \quad \forall k = 1, 0.
\end{aligned}
$$

for an orthonormal basis $\{|k\rangle\}_k$. Then, a POVM could determine whether the memory was still in the initial state or not without revealing the actual result. If we were to build a consistent and universal theory, then any other unitary of the same form as U should qualify as a "measurement." And any such unitary should induce a collapse if we were to apply Q_{OC}. So far, however, *an isolated system showing a collapse* has not been observed[31], while we assume that unitaries of the form U have been experimentally investigated. Note that if there had been an observation of an isolated system showing a collapse, we would have regarded quantum mechanics generally in trouble, rather than as evidence for Q_{OC}. So, if

---

been established between the quantity to be measured and the state of some idealized apparatus. He would do well to emphasize his rather specialized use of the word 'measurement'." [127] Later, Bell suggested to remove the term measurement altogether: "[T]he word ['measurement'] comes loaded with meaning from everyday life, meaning which is entirely inappropriate in the quantum context. When it is said that something is 'measured' it is difficult not to think of the result as referring to some pre-existing property of the object in question. [...] When one forgets the role of the apparatus, as the word 'measurement' makes all too likely, one despairs of ordinary logic — hence 'quantum logic'. When one remembers the role of the apparatus, ordinary logic is just fine. [...] [T]he word ['measurement'] has had such a damaging effect on the discussion, that I think it should now be banned altogether in quantum mechanics." [9, p. 216] While Bell, in some sense, points already into the direction we are headed in Section 7, we suggest to remove neither the term "measurement" nor the burden to "say something about the results of measurements."

[31] More precisely, when we have observed a non-unitary evolution we commonly assume that the system was not properly isolated rather than "a measurement occurred inside."

one is to assume a general flag for the occurrence of a measurement, then the question whether Q or Q_{OC} applies seems to have been decided in favor of Q (or, equivalently, Q_C). This illustrates that characterizing a measurement by the form of an associated U is not sufficient. However, the flag in combination with a measurement insuring the initial state $|\Delta\rangle$ merely exhibits this form of U.

The flag is yet another formal way to say that "a measurement occurred"[32] that *per se* also applies to situations that have been experimentally tested. Either we take the results from these tests as evidence for deciding the measurement problem, or we impose further requirements on our notion of "a measurement occurred." If we build on the intuition that low-dimensional quantum systems do not qualify as observers, we are lead back to the dualistic picture discussed in Section 6.1. We return to the idea that there is a realm of sufficiently qualified observers protected against the perils of contextuality who may establish "objective facts" and a realm of proper quantum systems. Note that we are lead to these considerations merely by starting off with the idea of an "isolated observer." Usually we interact and communicate measurement results. Isolated observers in the sense of Wigner's friend are *not* part of our experience. Thus, the assumption of the existence of such a realm might rather reflect the hope for objective facts—facts without the need of being discursively established.

Let us pursue the idea for a criterion that qualifies observers: If we did observe an isolated proper quantum system showing a collapse, then how could we be sure that we did not actually look at a qualified observer? Would quantum mechanics be falsified if it did not follow *our* criterion for qualifying an observer? Or is nature just telling us that our criterion is off? Conversely, if a flag qualified an interaction within an isolated system as a measurement, should we not observe a collapse in order to be convinced of the flag working properly? So, either we insinuate the flag simply works according to our notion of which systems qualify as observers. Or we must assume the existence of a realm where Q_{OC} applies—where we can actually test our flag—, and the measurement problem

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[32]Note here Footnote 82.

is decided—by the existence of a realm where Q_{OC} applies.[33]
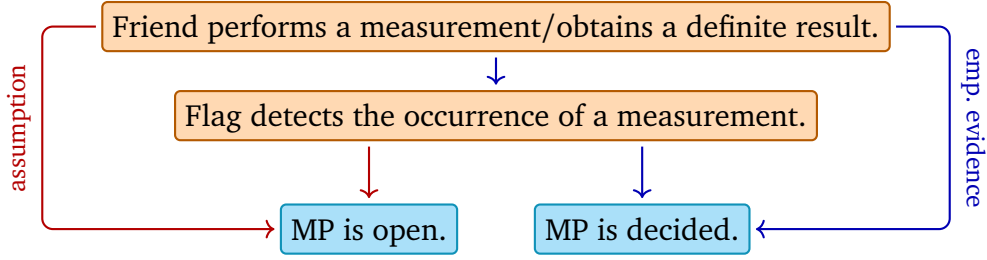
The measurement problem *is* already the question whether there can be experimental evidence for a criterion that qualifies observers.

It seems that the measurement problem is either already decided or it cannot shake off substantial assumptions. Whether the assumption that a flag detects the occurrence of a measurement is less of a commitment than the assumption that the friend obtained a definite result is surely debatable. And so is the need for the flag in the first place. It is tempting to regard the Wigner's-friend setup merely as interesting as an isolated system showing a collapse—irrespective of what is supposed to happen inside that system. The measurement problem attracts, however, attention as the commotion about [36, 37] illustrates. In this regard, these doubts are only first steps of a critical examination.[34] What remains from these considerations is that we might have to move from the question what the measurement tells us about quantum mechanics to the question what the measurement problems tells us about our ways of doing physics.

6.2.2   An alternative account

In this section, we present an alternative account of the Wigner's-friend experiment that is more aligned with the above established perspective on quantum mechanics. Wigner ensures that the friend is ready to perform a measurement by means of a measurement M_W^F.[35] He then prepares a system S with an initial measurement M_W^S and sends it to his friend. The friend performs a measurement M_F^S

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[33]The following figure does not distinguish between *available empirical evidence* and the *possibility of empirical evidence*.

[34]The motivation for this work is an observation about a discourse, rather than a somewhat "real" problem—even though such a distinction stems from a philosophic position we do not support.

[35]Subsequently, we adopt the following notation: The symbol M_O^S denotes a measurement performed by the observer O on the system S. Note that we refer to an observer O by a capital letter: In this regard, O is just another system and, thus, can be measured, too. If we refer to measurements on joint systems $S \otimes P$, we abbreviate $M_O^{S \otimes P}$ by M_O^{SP}.

on S. The joint system $S \otimes F$ remains however isolated, and, therefore, evolves unitarily. Finally, Wigner performs the measurement M_W^{SF} on the joint system.

unitary evolution/isolated system
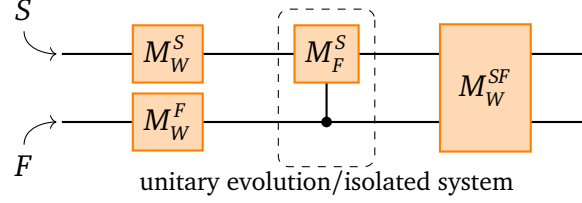
Commonly, the measurement M_W^S is replaced by a phrase of the sort "Wigner prepares a system in a state $\phi \in \mathcal{H}$." We grant Wigner the ability to choose whether to send the system S to the friend or not conditioned on the result of his measurement. Further, we allow for measurements such that eigenvalues to projectors orthogonal to $|\phi\rangle\langle\phi|$ are different from the eigenvalue λ associated with ϕ. Thus, when "Wigner prepares S in a state ϕ," he merely sends S to his friend if the measurement M_W^S yields the result λ. Similarly, the statement "The friend is ready to perform his measurement." means that Wigner knows that "the friend is ready." If Wigner aims to describe his friend by means of quantum mechanics, then he gets to know that "the friend is ready" by performing a measurement M_W^F.

The measurement problem in this setup amounts to the following: Quantum mechanics ostensibly allows for two different descriptions of the friend's measurement. For suitably chosen measurements, these result in different predictions for the probability distribution of Wigner's final measurement M_W^{SF}. If we ascribe one of these descriptions to the friend, and the other to Wigner, then we conclude that they disagree on the predictions for Wigner's final measurement. Straight solutions[36] to the measurement problem consist of restricting quantum mechanics as to not apply to measuring observers, to preclude the system F to be isolated, to choose one of the available descriptions as the correct one, or to embrace the predictions of quantum mechanics as subjective.

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[36]A *straight* solutions consists of "pointing out to the silly sceptic a hidden fact he overlooked" [71, p. 69], i.e., that (at least) one of the assumptions is unwarranted.

Ladders

In Section 7, we widen the perspective, and examine what characterizes theories that are troubled by a measurement problem. This leads us to an epistemological reflection on the measurement problem. In Section 8, we look more closely at the use of words in the usual framing of the quantum measurement problem. Reflections on the nature of language expose and challenge the assumption of an exhaustive language necessary to the contradiction in the measurement problem.

7   Measuring Measuring

Epistemological Concerns

The measurement problem is not so much a peculiarity—or defect—of quantum mechanics. Instead, we argue that it appears in theories that (a) account for interactions so that they are empirically significant, (b) require that an observation necessarily goes with *such* an interaction, (c) are falsifiable, and (d) in which experimental results have a minimal stability. The first two requirements render an observation itself empirically traceable. They are combined in the *interaction assumption*:

(IntA) Interactions are empirically traceable. An observation necessitates such an interaction.

The last requirement (d) is a generalization of Popper's characterization of physics being concerned with reproducible effects to the demand that asking the same question twice will yield the same answer:

(ISys) There exist conditions under which two equivalent, subsequent measurements performed on the same system yield the same answer. These conditions are independent of the questions asked.

A system satisfying these conditions will be called *isolated*.

7.1   Describing systems

In order to discuss the consequences of the interaction assumption for theories beyond quantum mechanics, we need to conceptualize physical theories more generally. Popper states in Ref. [92] that the "scientifically meaningful physical effect" is characterized by being *reproducible—regularly* and by *anybody who builds the experiment according to the instructions.*[37] To capture this notion formally, we assume a *set of questions* $\mathfrak{Q}$. In an experiment, we inquire about a question. If the answer obtained in the experiment differs from the one associated by the theory, then the theory can be considered falsified.

We assume the elements in $\mathfrak{Q}$ to carry a "time-stamp," i.e., there is an order relation on $\mathfrak{Q}$. If a measurement corresponding to α is performed *before* another one β, then $\alpha <_t \beta$. Therefore, any two inquiries about questions correspond to *different* elements in $\mathfrak{Q}$—even if the experimenter inquires "about the same question twice." In other words: With the set $\mathfrak{Q}$ we are able to resolve and order different inquiries.

Let us consider, for example, a Stern-Gerlach experiment [42]. In order to distinguish the inquiries, we number both the runs and the measurements in each run. The set of questions is then $\mathfrak{Q}^{\mathrm{st}} = \{(q,i,j) \mid q \in \mathscr{Q}, i,j \in \mathbb{N}^+\}$ where elements in $\mathscr{Q}$ represent different measurement bases.
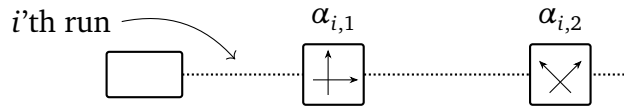
Elements in $\mathfrak{Q}^{\mathrm{st}}$ with coinciding first indices are questions referring to the identical system. The second index, in turn, distinguishes and orders the measurement within a given run.

A theory associates answers—or measurement results—from a set $\mathfrak{A}$ to the questions in $\mathfrak{Q}$. Thus, a theory refers to ordered pairs $(\alpha, \tau) \in \mathfrak{Q} \times \mathfrak{A}$. For the set of questions to allow for reproducible effects, we assume an equivalence relation $\sim$ on $\mathfrak{Q}$. We say that an effect (α_1, τ_1) is reproduced by a pair (α_2, τ_2) if and only if $\alpha_1 \sim \alpha_2$ and $\tau_1 = \tau_2$.[38] In the example of the Stern-Gerlach experiment above, this allows to reproduce effects within the same run, and across different runs. In order to distinguish these two cases, we introduce a second equivalence relation $\sim^s$ on $\mathfrak{Q}$ that associates questions referring to the same run or the identical system. For convenience, we denote the equivalence relation formed by the

---

[37]"Der wissenschaftlich belangvolle physikalische Effekt kann ja geradezu dadurch definiert werden, daß er sich regelmäßig und von jedem reproduzieren läßt, der die Versuchsanordnung nach Vorschrift aufbaut." [92, §I.8]

[38]The quotient set $\mathfrak{Q}/\sim$ is denoted by $\mathscr{Q}$ and its elements by Latin letters.

intersection of the two equivalence relations introduced before by $\equiv$. Then, two questions $\alpha_1 \equiv \alpha_2$ refer to the identical system *and* are associated to the same effect.
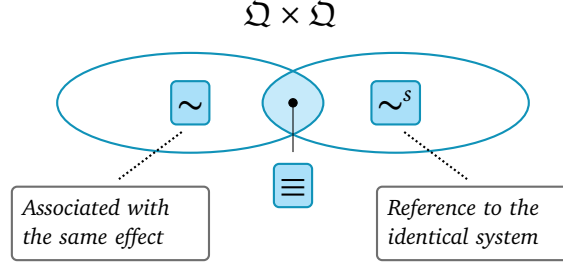
The equivalence on the left-hand side, $\sim$, is categorically different from the on the right hand side, $\sim^s$—as we discuss in detail in Section 11.4: The former compares *statements* that are assigned truth values, i.e., a relation or a proposition from, e.g., propositional or predicate logic. The latter compares *references* from, e.g., some object theory (see Section 11.4.5). Even though, we do not elaborate on entailing problems in this section, we add a word of warning: The derived equivalence relation $\equiv$ is deep in the quagmire.

If we return to the example of a Stern-Gerlach experiment with a set of questions

$$\mathfrak{Q} = \{(A, i, j) \,|\, A \in \mathrm{End}(\mathbb{C}^2), A^\dagger = A, i \in \mathbb{N}^+, j \in \mathbb{N}^+\},$$

the situation looks as follows where questions are denoted by their measurement basis:
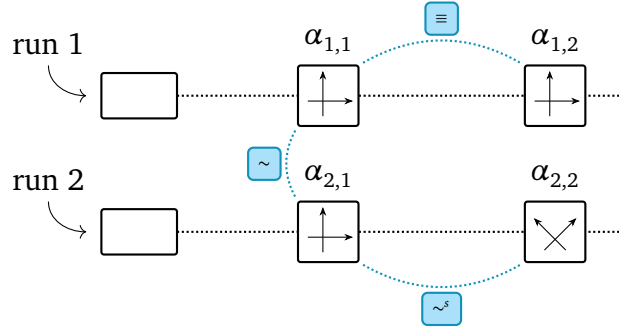
7.2   An interacting system

Following the Interaction Assumption, we assume that there is a correspondence between interactions and observations. Any observation goes with a necessary interaction, but also every interaction corresponds to a question inquired about. Imagine now there is an equivalence class $q_{\mathrm{int}} \in \mathcal{Q}$ that represents the question

"Did the system interact with its environment?" In light of the Interaction Assumption, inquiring about a question $\xi_i \in q_{\text{int}}$ necessitates an interaction as well. Let us consider a system under the condition specified in (ISys).

$$(6)$$

Independent of whether the inquiry about ξ_2 occurred or not, the answers are equal, $\tau_1 = \tau_3 \, (= \tau_2)$. Thus, the interaction associated with τ_2 does not make a difference: If the answers to ξ_1 let us conclude that the system interacted before, then it does so also after when inquiring about ξ_3; and similarly if the answer let us conclude that it did not interact. Whether or not the interaction corresponding to ξ_2 occurred: The answers to questions in q_{int} remain the same. Thus, the interaction assumption cannot be realized by a single inquiry about questions in a special equivalence class in $\mathcal{Q}$. Yet, we can establish within $\mathfrak{Q}$ whether an interaction occurred by how questions *relate* to one another. In particular, if we aim to position the characteristic "having interacted" dichotomously to "being isolated" as described in (ISys), then the former characteristic is expected to be relational as is the latter. To proceed in characterizing this relation, we need to conceptualize how a theory refers to effects.

7.3   Deterministic theories

In a first step, we assume that a theory associates deterministically answers to questions—or better sets of answers to sets of questions given that the theory has relationally realize the Interaction Assumption (IntA). This takes a theory as a map

$$\mathscr{P}(\mathfrak{Q}) \to \mathscr{P}(\mathfrak{Q} \times \mathfrak{A})$$

that associates to any set of time-ordered questions $Q \subset \mathfrak{Q}$ a set of effects $\{(\xi, \tau) \mid \xi \in Q, \tau \in \mathfrak{A}\}$. By the assumption (ISys), there are conditions for any system, so that any inquiry about subsequent equivalent questions $\xi_i \in q$ yield the same answer:
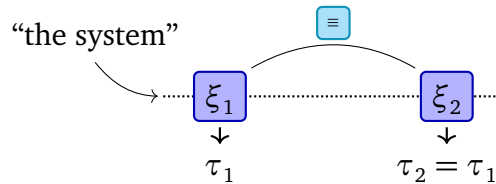
By transitivity, this does not change we there is an intermediate inquiry about another equivalent question $\xi' \in q$:
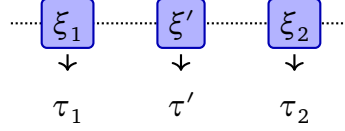
If, however, the intermediate question η is *not* in q, then, in order to satisfy (IntA), the conditions of (ISys) cannot be satisfied anymore: For a given η there must exist an equivalence class q such that the answer to questions $\xi_1, \xi_2 \in q$ with $\xi_1 <_t \eta <_t \xi_2$ trace the interaction associated with the inquiry about η:
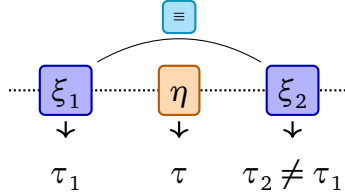
Let us consider successive inquiries about three questions $\xi_1, \xi_2, \xi_3 \in q$ interrupted by intermediary questions $\eta_1, \eta_1 \in p \neq q$ such that the questions in q detect interactions associated with questions in p:
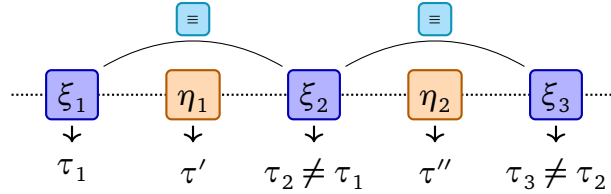
Say we inquired about the first question ξ_1 and obtained an answer τ_1. Any inquiry about a question η_1 with $\eta_1 \not<_t \xi_1$ alters the answer to questions $\xi_2 \equiv \xi_1$. If there is a binary set of answers, $\mathfrak{A} = \{\mathtt{t}, \mathtt{f}\}$, then answer τ_2 is $\neg\tau_1$ ("not τ_1"). With the same reasoning, we obtain that $\tau_3 = \neg\tau_2$, and, thus

$$\tau_3 = \neg\tau_2 = \neg(\neg\tau_1) = \tau_1.$$

The answers $\tau_1 = \tau_3$ to the questions $\xi_1 \equiv \xi_3$ fail to detect the interactions represented by η_1 and η_2. By the pigeon-hole principle, this extends to any finite set of answers. We arrive at an *inconsistent triad*: A theory cannot

(a) satisfy the interaction assumption,

(b) have sets of answers $\mathfrak{A}$ and questions $\mathfrak{Q}$, such that $|\mathfrak{A}| < \infty, |\mathfrak{A}| < |\mathfrak{Q}|$, and

(c) be deterministic.

In order to go ahead, we leave behind the last assumption and turn to probabilistic theories. At the same time, we tighten the second assumption and, henceforth, reduce $\mathfrak{A}$ to the binary set $\{\mathtt{t}, \mathtt{f}\}$.[39]

7.4   Probabilistic theories

Following the example of Gleason's theorem [46], we examine probabilistic theories. Note that the equivalence relation $\sim^s$ introduced in Section 7.1 allows to resolve different runs of an experiment, i.e., repetitions of the same experiment. To accommodate a notion of probability, we drop the reference to different runs within $\mathfrak{Q}$. Elements in $\mathfrak{Q}$ thus refer to the identical system rendering $\sim^s$ and $\equiv$ obsolete.[40] Different runs yield the statistical evidence for the assigned probabilities. This step is surely loaded with an interpretational baggage [55] that awaits further exploration.

To the end of examining probabilistic theories, we take theories to be a map

$$\mathscr{P}(\mathfrak{Q} \otimes \mathfrak{A}) \to [0, 1]$$

that assigns a set of time-ordered events a probability

$$\mathrm{P}\big((\alpha_1, \tau_1), (\alpha_2, \tau_2), \dots\big)$$
$$\text{with} \quad \alpha_i \in \mathfrak{Q}, \alpha_l <_t \alpha_{l+1}, \tau_i \in \{\mathtt{t}, \mathtt{f}\}.$$

Then, the requirement (ISys) that consecutive equivalent questions

$$\alpha \sim \beta, \ \alpha <_t \beta, \ \alpha, \beta \in \mathfrak{Q}$$

yield equal answers $A, B \in \{\mathtt{t}, \mathtt{f}\}$ translates to

$$\mathrm{P}(A = B) = \mathrm{P}(A = 1, B = 1) + \mathrm{P}(A = 0, B = 0) = 1 \,.$$

Let us consider the possibility that the probability derives from a unary function

$$\mu : \mathfrak{Q} \to [0, 1]$$

---

[39]Constraining ourselves to binary answers allows to regard the elements in $\mathfrak{Q}$ as propositions with truth values. Conversely, one might consider the cases of $\mathfrak{A}$ with cardinality three with three-valued logic [69, 101], and, beyond that, with many-valued logic.

[40]Reference to the identical system, however, remains a challenge for the measurement problem, as we discuss in Section 7.9.

such that $\mu(\alpha)$ is the probability for the answer to α being $\mathtt{t}$. From the above formulation of (ISys), it follows that

$$P(A = B) = \mu(\alpha)\mu(\beta) + (1 - \mu(\alpha))(1 - \mu(\beta)) = 1$$

which is the case if and only if $\mu(\alpha) = \mu(\beta) = 0$ or $\mu(\alpha) = \mu(\beta) = 1$. Thus, the function $\mu : \mathfrak{Q} \to \{0,1\}$ takes merely two values, and is constant within an equivalence class $a \in \mathscr{Q}$. Therefore, there is an induced function $\mu' : \mathscr{Q} \to \{0,1\}$. An immediate consequence is that if the question q_{int} from above is an element in $\mathscr{Q}$, then inquiring about the question "whether the system interacted" yields either always $\mathtt{t}$ or always $\mathtt{f}$, independently of inquiries about any other question. This is a contradiction with (IntA). A theory satisfying both (IntA) and (ISys) cannot allow for an assignment of probabilities to elements in $\mathfrak{Q}$ independent of inquiries about other, non-equivalent questions: The theory is contextual [70, 61].

To ensure a minimal detectability of inquiries and their corresponding interactions, we are lead to assume the following, similar to Heisenberg uncertainty: For any $\alpha \in \mathfrak{Q}$ there exist equivalent $\beta_1 \sim \beta_2, \beta_1 <_t \alpha <_t \beta_2$ such that

$$P(B_1 \neq B_2) = \sum_{A,B} P(B, A, \neg B) \geq \epsilon \qquad (7)$$

for some $\epsilon > 0$.

7.5   Lattice structures

So far, the set of questions $\mathfrak{Q}$ has not been particularly structured. Let us now consider the situation described above in terms of lattices, as introduced in Appendix 1. In order to endow $\mathfrak{Q}$ with more structure, let us consider the conditional, i.e., $\alpha \to \beta$, defined as follows:

> If the inquiry about α yields $\mathtt{t}$, then a subsequent inquiry about $\beta >_t$ α yields $\mathtt{t}$.

The conditional on $\mathfrak{Q}$ induces an order relation on the set of equivalence classes, $a < b$ for $a, b \in \mathscr{Q}$, if verifying the order relation does not break the equivalence: If we inquire about consecutive questions

$$\alpha <_t \beta <_t \alpha' <_t \beta', \text{ with } \alpha \to \beta, \ \alpha \sim \alpha', \ \beta \sim \beta',$$

then we assume to still obtain equal answers for α and α', as well as for β and β'. Thus, we demand that the sublattice generated by $\{a, \neg a, b, \neg b\}$ is distributive

if $a < b$. If there exist elements 0 and 1 such that $a \wedge \neg a = 0$ and $a \vee \neg a = 1$ for all $a \in \mathcal{Q}$, then the above sublattice requirement renders the complement defined above order-reversing and leaves us with an orthocomplemented lattice of classes of equivalent questions $(Q, <)$.[41] To ensure the distributivity of the sublattice defined above, we require the lattice to be orthomodular, i.e., to satisfy the following:

$$\text{If } a < b, \text{then } a \wedge (\neg a \vee b) = b \,.$$

We now turn to the question how to ensure the interaction assumption (IntA) given that the $\mathcal{Q}$ forms an orthomodular lattice. As done above, we can examine a potential equivalence class $q_{\text{int}} \in \mathcal{Q}$ corresponding to asking "whether the system interacted." If we inquire consecutively about $\alpha <_t \sigma$ with $\sigma \in q_{\text{int}}$, then we expect σ to yield $\mathfrak{t}$ independent of the result of the inquiry about α. This should also hold for $\neg \alpha$. It follows that $q_{\text{int}} > \alpha \vee \neg \alpha$, and, therefore, $q_{\text{int}} = 1$. Thus, the interaction assumption cannot be realized by a single inquiry about questions in a special equivalence class in $\mathcal{Q}$.

As done in Section 7.2, we can establish within $\mathfrak{Q}$ whether an interaction occurred by how questions *relate* to one another. In particular, if we aim to position the characteristic "having interacted" dichotomously to "being isolated" as described in (ISys), then the former characteristic is expected to be relational as is the latter. Following this path, we demand that equivalent questions $\sigma, \sigma' \in q_{\text{int}}$ inquired about before and after an interaction corresponding to an inquiry about a question $\alpha \in a$ with $\sigma <_t \alpha <_t \sigma'$ do not necessarily yield the same answer independent of what the result of the inquiry about α is. For the equivalence classes this entails[42]

$$(q_{\text{int}} \wedge a) \vee (q_{\text{int}} \wedge \neg a) \neq q_{\text{int}} \,. \tag{8}$$

That is, q_{int} is *incompatible* with a. Equivalently, the sublattice generated by q_{int} and a is not distributive (see Appendix). As compatibility in an orthomodular lattice is symmetric, the interaction corresponding to inquiries about questions in q_{int} can be traced inversely with inquiries about questions in a.

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[41]If $a < b$, then

$$b \wedge \neg a = (a \vee b) \wedge \neg a = (a \wedge \neg a) \vee (b \wedge \neg a)$$

by distributivity. With $a \wedge \neg a = b \wedge \neg b$,

$$b \wedge \neg a = (b \wedge \neg b) \vee (b \wedge \neg a) = b \wedge (\neg a \vee \neg b),$$

thus, $\neg a > \neg b$.

[42]Imagine inquiring about $\sigma <_t \alpha <_t \alpha' <_t \sigma'$ where $\sigma, \sigma' \in q_{\text{int}}, \alpha \in a, \alpha' \in \neg a$. Then, $\sigma \not\mapsto \sigma'$. The same is the case for $\sigma, \sigma' \in \neg q_{\text{int}}$.

To ensure that *all* elements in $\mathfrak{Q}$ correspond to *traceable* interactions, we require that in the orthomodular lattice $\mathscr{Q}$, the sublattice $\mathscr{Z}$ of elements *compatible with all other elements* in the lattice, called the *center,* contains merely 1 and 0. The requirement for $\mathscr{Q}$ to form an orthomodular lattice with trivial center is sufficient to satisfy the interaction assumption.

This links to the considerations in Section 7.4: What probability distributions can be assigned to an orthomodular lattice? Let us assume that $\mathscr{Q}$ forms such a lattice and that $\mu' : \mathfrak{Q} \to [0, 1]$ is a function that satisfies

$$\mu'(0) = 0, \qquad \mu'(1) = 1;$$
$$\text{if } a, b \text{ are compatible, then}$$
$$\mu'(a) + \mu'(b) = \mu'(a \wedge b) + \mu'(a \vee b);$$
$$\text{if } \mu'(a_i) = 1 \text{ then } \mu'\big(\wedge_i a_i \big) = 1.$$

If we impose (ISys), then, with the same reasoning as above, μ' is a *dispersion-free state* [66]. From Theorem I in Ref. [66] and Theorem 1 in Ref. [50], it follows that if there exists a dispersion-free state on $\mathscr{Q}$ then the center $\mathscr{Z}$ is not trivial. Again, we conclude: *If we require (IntA) and (ISys), then any assignment of probabilities to elements in $\mathfrak{Q}$ must be contextual.*

7.6   Isolated systems

After the discussion in Section 7.2, we are now able to explicate the notion of an *isolated system* consistent with the two assumptions (ISys) and (IntA): *A system is isolated if and only if inquiries about any two equivalent questions $\alpha \sim \beta$, $\alpha, \beta \in \mathfrak{Q}$ yield equal answers with certainty.*

To empirically verify whether a system is isolated—at least for the time between two inquiries—, one inquires about any two equivalent questions and compares the thus obtained answers. If the answers differ, then the inquiries detect an intermediate inquiry about a non-compatible question, and the system is *not isolated*. While the equality of the answers is necessary, it is, however, *not sufficient* for the system to be isolated.

This empirical test is an essential ingredient in a key-distribution protocol like [13]. Inversely, any theory satisfying the assumptions (IntA) and (ISys) allows for a similar protocol.

Note the interdependence of the equivalence relation and the notion of a system being isolated: If a system is isolated, then we can empirically verify the equivalence relation. For a system with an equivalence relation, we can empirically verify whether it is isolated. Conversely, we cannot say whether a system

is isolated without a pre-established equivalence relation, and, vice versa, we cannot verify the equivalence relation without the system being isolated.

7.7 Interactions within a joint system

We now turn to interactions *within* an isolated system, i.e., between different parts of a joint system. Two systems, S_1 and S_2, together can again be regarded as *one* system assuming that the ability to refer to S_1 and S_2 suffices to refer to the corresponding combined system. The joint system consisting of S_1 and S_2 is denoted $S_1 \times S_2$.

What does it mean for the two systems to interact? Along the above correspondence between interactions and measurements, we can put this into another perspective: One can think of S_2 as a friendly experimenter measuring S_1, inspired by the Wigner's-friend experiment [127, 24, 126].
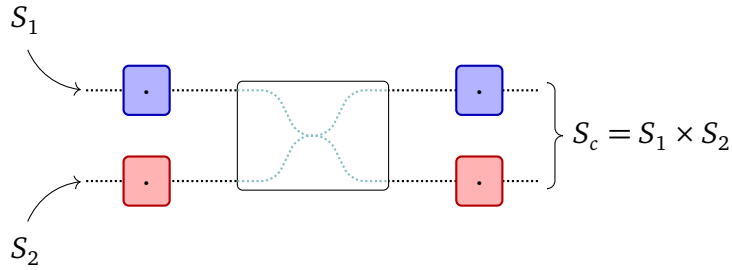
Let us, for now, merely consider S_1: Before and after our friend inquires about a non-trivial $\alpha \in \mathfrak{Q}_1$ we inquire about two equivalent $\beta_1 \sim \beta_2$ that belong to an equivalence class incompatible to the one represented by α, i.e., $\beta_1, \beta_2 \in b \neq a$.[43]
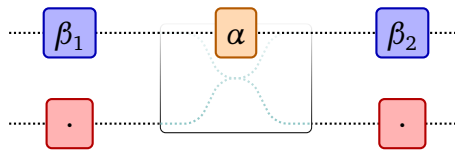
The joint system $S_1 \times S_2$ is however isolated. Thus, the equivalence classes of the joint system are not induced by the subsystems if they interact: Despite, $\beta_1 \sim \beta_2$ in $\mathfrak{Q}_1$, $(\beta_1, 1) \not\sim (\beta_2, 1)$ in $\mathfrak{Q}_c$. *The interaction between the subsystems shows in the equivalence classes of the joint system.*

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[43]Here, it is irrelevant whether a and b are incompatible in the sense that they satisfy the Heisenberg uncertainty 7 or, if $\mathscr{Q}$ forms an orthomodular lattice, the elements are incompatible in the sense established for lattices.

We are confronted by the measurement problem: As established in Section 7.6, to empirically verify an equivalence relation for questions, we must *take* a corresponding system to be isolated and *inquire* about any questions in *any* of the equivalence classes. This includes questions *incompatible with the equivalence class that allows to retrieve the measurement result from the friend.* By the Interaction Assumption (IntA), this incompatible equivalence class must exist. In other words: *We cannot at the same time empirically test whether the system interacted, and know about the result of the measurement.*[44]

7.7.1   Lattice considerations

We now turn to interactions *within* an isolated system, i.e., between different parts of a joint system, in terms of lattices as introduced in Appendix 1. Two systems, S_1 and S_2, together can again be regarded as *one* system assuming that the ability to refer to S_1 and S_2 suffices to refer to the corresponding combined system. The joint system consisting of S_1 and S_2 is denoted $S_1 \times S_2$.

In Ref. [91], Piron shows that an orthomodular lattice has a trivial center if and only if it is irreducible, i.e., the lattice cannot be written as a direct union, defined as follows: The direct product of orthocomplemented lattices L_i with $i \in I$, forms another orthocomplemented lattice L^p with the order relation

$$x > y, x, y \in L^p \iff x_i > y_i \; \forall i \in I$$

and the orthocomplementation

$$\neg x = (\neg x_1, \ldots, \neg x_i, \ldots).$$

It follows from (IntA) that the lattice $\mathcal{Q}_c$ cannot be the direct product of lattices $\mathcal{Q}_1$ and $\mathcal{Q}_2$.

We imagine S_2 to be a friendly experimenter measuring S_1, inspired by the Wigner's-friend experiment [127, 24, 126]. Let us, for now, merely consider S_1: Before and after our friend inquires about a non-trivial $\alpha \in \mathfrak{Q}_1$ we inquire about two equivalent $\alpha' \sim \alpha''$ that belong to an equivalence class incompatible to the one represented by α. The joint system $S_1 \times S_2$ is however isolated. Thus, the equivalence classes of the joint system are not induced by the subsystems if they interact: Despite, $\alpha' \sim \alpha''$ in $\mathfrak{Q}_1$, $(\alpha', 1) \not\sim (\alpha'', 1)$ in $\mathfrak{Q}_c$. The interaction between the subsystems shows in the equivalence classes of the joint system.

Let us characterize the friend's inquiry about a non-trivial $\alpha \in a \in \mathcal{Q}_1$, more specifically, as follows:

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[44]For an explication of the problem in terms of lattices, see [57].

If $(\alpha_1, \sigma) <_t (\alpha_2, \beta) \in \mathfrak{Q}_c$ with $\alpha_i, \alpha \in a \in \mathscr{Q}_1$, then $\alpha_1 \leftrightarrow \alpha_2$ and $\alpha_1 \leftrightarrow \beta$, for some σ.

A measurement effects an implication that reaches across systems.[45] It is a case not accounted for in a product lattice. In particular, the measurement establishes the equivalence between (σ, α_1) and (β, α_2) in $\mathfrak{Q}_c$ while σ and β might not be equivalent in $\mathfrak{Q}_2$. Let us denote $m \in \mathscr{Q}_c$ the equivalence class of (σ, α_1) and (β, α_2). The characterizations also implies: $(a, 1)$ and $(a, 0)$ are equivalence classes in $\mathscr{Q}_c$ with $(a, 0) < m < (a, 1)$. In particular, the equivalence class $n \in \mathscr{Q}_c$ represented by (σ, α'_1) is incompatible with $(a, 0)$ and $(a, 1)$ if $a' \in \mathscr{Q}_1$ is incompatible with $a \in \mathscr{Q}_1$.[46] Therefore, also n and m are incompatible.

To empirically test whether the two subsystems S_1 and S_2 interact with one another, one empirically tests the equivalence relation on Q_c by inquiring about questions in the same equivalence class and verifying that their answers match. That is, we test whether the joint system *is isolated under this equivalence relation* (see Section 7.6) and, therefore, whether equivalent questions yield same answers, *independent of the choice of the equivalence class*. Imagine, we initially inquired about a question in n. To verify that S_c is isolated, and, thus, the two subsystems interacted, we inquire about a later element in n. *By the incompatibility of n and $(a, 1)$, we cannot at the same time empirically test whether the system interacted, and know about the result of the measurement.*

We encounter the measurement problem: We cannot meaningfully—i.e., with the suitable empirical support—speak of the measurement as an interaction between two systems, while maintaining the idea of the measurement yielding definite results.

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[45]To illustrate this characterization, we resort to the relative-state formalism [30] of quantum mechanics (see Section 5.5.2). The unitary describing a measurement is characterized by the following effect: If an observer performs a measurement $\Pi_\phi \in \mathfrak{P}(\mathscr{H}_S)$ on S, the observer is initially attested to be in a "ready-state," and S is prepared in a state $\phi \in \mathscr{H}_S$, then the observer ends up in a state $\psi \in \mathscr{H}_O$ of "having obtained the correct result." If an observer performs a measurement $\Pi_\phi \in \mathfrak{P}(\mathscr{H}_S)$ on S, the observer is initially attested to be in a "ready-state," and S is prepared in a state $\phi^\perp \in \mathscr{H}_S$ *orthogonal* to ϕ, then the observer ends up in a state $\psi^\perp \in \mathscr{H}_O$ orthogonal to ψ.

[46]See Lemma 1 in Appendix 1. In quantum mechanics, this corresponds to preparing a superposition state with respect to the friend's measurement basis.

7.8　Problems

In the above account of the measurement problem, the notions of *equivalent questions*, *isolated systems*, and *interacting systems* are strongly interdependent.
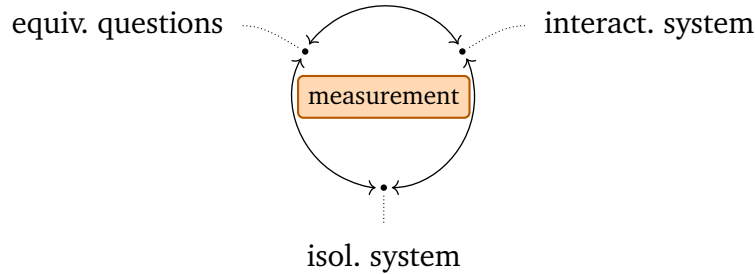
To establish what the measurement problem consists of, we have to posit that a system is isolated. This assumption, in turn, can merely be empirically grounded if we posit an equivalence relation for the questions the system can be inquired about. If we now place the notion of a measurement in this context, together with the assumption that a measurement yields a definite result, then we can rephrase the measurement problem as: An isolated system is supposed to behave as if it had been interacted with.

As hinted at in Section 7.1 when discussing the equivalence relation $\equiv$, there are already substantial problem if we merely relate equivalent questions with isolated systems, as done in (ISys) and discussed in Section 7.6. We return to these issues in Section 11.3.

7.9　The Epistemological Import: Spectator theories

The measurement problem unfolds if we compromise the interaction assumption (IntA) in order to save the measurement and its result from contextual dependences. Thus, the measurement problem exposes the idea that we can *read off* measurement results without effects for the measured system—the idea of a *spectator theory*.

> The theory of knowing is modelled after what was supposed to take place in the act of vision. The object refracts light to the eye and is seen; it makes a difference to the eye and to the person having an optical apparatus, but none to the thing seen. The real object is the object so fixed in its regal aloofness that it is a king to any beholding mind that may gaze upon it. A spectator theory of knowledge is the inevitable outcome. [25, §1, p. 26]

The idea of a spectator theory is the starting point of the article by Einstein, Podolski, and Rosen [28]:[47]

> The elements of the physical reality cannot be determined by *a priori* philosophical considerations, but must be found by an appeal to results of experiments and measurements. A comprehensive definition of reality is, however, unnecessary for our purposes. We shall be satisfied with the following criterion, which we regard as reasonable. *If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity*. [28]

The quest for certain measurement results leads to an epistemic problem:[48] If knowledge is scientific knowledge and science is natural science, then the anchor of our knowledge is observation and measurement. And if knowledge must be constituted of certainties, then these observations cannot carry contextual dependences.[49]

For Dewey in his critique of absolute knowledge, quantum mechanics promises to get the natural sciences back on the path instituted by Galileo: The path of a dissolution of the distinction between immutable absolute knowledge and practical activity guided by belief.

> The work of Galileo was not a development, but a revolution. It marked a change from a qualitative to the quantitative or metric;

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[47]The stance does not entirely comply with Einstein's take of a *system* (see Section 8.3). In fact, Einstein's involvement in the so-called EPR article remains debated.

[48]It is an old problem: "Greek thinkers saw clearly—and logically—that experience cannot furnish us, as respects cognition of existence, with anything more than contingent probability. Experience cannot deliver to us necessary truths; truths completely demonstrated by reason. Its conclusions are particular, not universal. Not being 'exact' they come short of 'science'. [...] [E]mpirical or observational sciences were placed in invidious contrast to rational sciences which dealt with eternal and universal objects and which therefore were possessed of necessary truth." [25, §2, p. 28]

[49]The spectator theory relates to correspondence theories of truth, as Habermas points out: "*The meaning of knowledge itself becomes irrational*—in the name of rigorous knowledge. In this way the naive idea that knowledge *describes* reality becomes prevalent. This is accompanied by the copy theory of truth, according to which the reversibly univocal correlation of statements and matters of fact must be understood as isomorphism. Until the present day this objectivism has remained the trademark of a philosophy of science that appeared on the scene with Comte's positivism." [53, §II, p. 68f, emphasis in original] The criticism of spectator theories relates to critiques of correspondence theories of truth as, e.g., in Ref. [87, 130, 111, 103].

> from the heterogeneous to the homogeneous; from intrinsic forms to
> relations; from aesthetic harmonies to mathematical formulae; from
> contemplative enjoyment to active manipulation and control; from
> rest to change; from eternal objects to temporal sequence. The idea
> of a two-realm scheme persisted for moral and religious purposes; it
> vanished for purposes of natural science. [25, §4, p.92]

Ironically, the core idea of spectator theories initially served to challenge estab-
lished certainties:[50]

> When he [Galileo] pointed his telescope towards the moon, he deter-
> mined that its surface was not as smooth and spherical, but uneven,
> rough, with dips and heights, not different from the surface of the
> earth. Had Galilei been a traditional philosopher, then he immedi-
> ately would have had to ponder why the observed is a deception.
> For according to the established cosmology, a celestial body was per-
> fect and spherical and consisted of a very different substance as the
> earth; to regard the moon to be of same quality as the earth was not
> a possibility. Instead, Galilei did something that in the eyes of the
> then philosophers was utterly absurd: relying on his senses, he took
> the observed as prima facie truth, and went about to determine the
> height of mountains on the moon. [85, §3, p. 211, own translation]

Not only did Galileo trust his observations in order to challenge existing cer-
tainties: He took his observations as the bearer of truth. Without necessity for
challenging established belief, he held onto the idea of certain knowledge.

For Dewey, Heisenberg uncertainty was less of a problem for maintaining
physics as a spectator theory, but rather a door opener towards overcoming the
idea itself.

> The element of indeterminateness is not connected with defect in the
> method of observation, but is instrinsic. The particle observed does

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[50]"Als er [Galilei] sein Fernrohr auf den Mond richtete, stellte er sehr rasch fest, dass dessen
Oberfläche nicht glatt und sphärisch war, sondern uneben, rauh, mit Senkungen und Erhebungen,
nicht anders als die Oberfläche der Erde. Wäre nun Galilei ein traditioneller Philosoph gewesen,
so hätte er sich sofort überlegen müssen, warum das Gesehene eine Täuschung sei. Denn nach
gängiger Kosmologie war ein Himmelskörper vollkommen und sphärisch und bestand aus einer
anderen Substanz als die Erde; im Mond einen Körper von gleicher Beschaffenheit wie die Erde
zu sehen, ging nicht an. Galilei tat stattdessen etwas in den Augen damaliger Philosophen völlig
Absurdes: er nahm im Vertrauen auf seine Sinne das Beobachtete als prima facie Wahrheit, ging
hin und bestimmte die Höhe eines Mondberges." [85, §3, p. 211]

not *have* fixed position or velocity, for it is changing all the time because of interaction: specifically, in this case, interaction with the act of observing, or more strictly, with the conditions under which an observation is possible; for it is not the 'mental' phase of observation which makes the difference. Since either position or velocity may be fixed at choice, leaving the element of indeterminacy on the other side, both of them are shown to be conceptual in nature. That is, they belong to our intellectual apparatus for *dealing with* antecedent existence, not to fixed properties of that existence. An isolation of a particle for measurement is essentially a device for regulation of subsequent perceptual experience. [25, §8, p. 194, emphasis in original]

The "intrinsic element of indeterminateness" is taken as an indicator for the participatory and practical elements of knowing.

The change [that Heisenberg uncertainty calls for] for the underlying philosophy and logic of science is, however, very great. In relation to the metaphysics of the Newtonian system it is hardly less than revolutionary. What is known is seen to be a product in which the act of observation plays a necessary rôle. Knowing is seen to be a participant in what is finally known. Moreover, the metaphysics of existence as something fixed and therefore capable of literally exact mathematical description and prediction is undermined. Knowing is, for philosophical theory, a case of specially directed activity instead of something isolated from practice. The quest for certainty by means of exact possession in mind of immutable reality is exchanged for search for security by means of active control of the changing course of events. Intelligence in operation, another name for method, becomes the thing most worth winning. The principle of indeterminacy thus presents itself as the final step in the dislodgement of the old spectator theory of knowledge. It marks the acknowledgment, within scientific procedure itself, of the fact that knowing is one kind of interaction which goes on within the world. [25, §8, p. 195f]

Dewey was not alone in assigning epistemological import to quantum mechanics. Bohr writes in an article published in the same year as Dewey's "Quest for Certainty":

The discovery of the quantum of action shows us, in fact, not only the natural limitation of classical physics, but, by throwing a new

> light upon the old philosophical problem of the objective existence
> of phenomena independently of our observations, confronts us with
> a situation hitherto unknown in natural science. As we have seen,
> any observation necessitates an interference with the course of the
> phenomena, which is of such a nature that it deprives us of the foun-
> dation underlying the causal mode of description. The limit, which
> nature herself has thus imposed upon us, of the possibility of speak-
> ing about phenomena as existing objectively finds its expression, as
> far as we can judge, just in the formulation of quantum mechan-
> ics. However, this should not be regarded as a hindrance to further
> advance; we must only be prepared for the necessity of an ever ex-
> tending abstraction from our customary demands for a directly visu-
> alizable description of nature. [15, p. 115, as reprinted in [16]]

Dewey, expecting the eye-opening effects of quantum mechanics, disregards
his own, earlier concern about the Galilean revolution:

> But—and this 'but' is of fundamental importance—in spite of the rev-
> olution, the old conceptions of knowledge as related to an antecedent
> reality and of moral regulation as derived from properties of this re-
> ality, persisted. [25, §4, p.92]

The idea of immutable knowledge did not only survive the Galilean but also the
quantum revolution. It merely changed *what* could be absolutely known. With
a theory that satisfies (IntA) and (ISys), we cannot expect to know answers to
all questions that a system can be inquired about. But we can still hope for
either some properties to be more fundamental than others—the Bohmian way
out of the problem—or for the theory to expose the real *structure* of the world—
the Everettian, or Parallel Lives, way out of the problem. In both cases, there
remains a real and absolute element antecedent to any act of knowing to which
we have merely limited access. One might think of this as an exploration of the
middle ground between the incommensurable concepts of a spectator theory and
the interaction assumption.
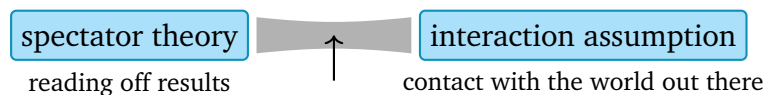
This saves an essential aspect of spectator theories, namely, the positivist idea
that we can gain access to something that is independent of our act of knowing.

If, however, the measurement problem reminds us that "Knowing is seen to be a participant in what is finally known." [25, §8, p.195][51]—just as antecedent "meaning" is a myth [111]—, then we are led to a very different reflection [58]. Following Habermas, we are in fact led to *unwind the positivist removal of reflection*.[52] The idea that cognition must be *scientific* cognition can be understood as a belief that we have put into the scientific act of knowing, and not something that the world imposes on us.[53] The measurement "problem" opens the door for reflection, for imagining new ways of looking at science and of doing physics [104].

8   The "Measurement" Problem
The Linguistic Approach

The usual account of the Wigner's-friend experiment goes as follows: *Wigner* measures his friend who, in turn, measures another system. With appropriately chosen measurements, the friend ends up being in a superposition of states after his measurement. This is at odds with the idea that measurements yield definite results. The definiteness is commonly translated to the case of Wigner's friend by demanding that the joint system $F \otimes S$ collapses to one of a number of orthogonal states representing different measurement results. In order to obtain a formal contradiction, it is crucial to describe the friend's measurement by means of quantum mechanics. This amounts to the following statement:

(A) For an observer O observing a result x when measuring a system S, it is sufficient that $O \times S$ is in a state ϕ for some $\phi \in \mathcal{H}_O \otimes \mathcal{H}_S$.

The measurement problem in its usual reading results from the discrepancy between $\phi_{\text{uni}} = U\phi_{\text{init}}$, i.e., the result of a unitary evolution U, and ϕ_{cps}, i.e., the "collapsed" post-measurement state associated with "definite measurement results" if one chooses a suitable ϕ_{init} and U.

The association of "having measured x" with "being in a state ϕ" is justified if physical theories are taken to describe an ontological reality. We adopt the

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[51]The idea has been established before by Fleck (see Footnote 2).

[52]"That we disavow reflection *is* positivism." [53, p.vii, emphasis in original]

[53]"From then on, the theory of knowledge had to be replaced by a methodology emptied of philosophical thought. For the philosophy of science that has emerged since the mid-nineteenth century as the heir of the theory of knowledge is methodology pursued with a scientistic self-understanding of the sciences. 'Scientism' means science's belief in itself: that is, the conviction that we can no longer understand science as *one* form of possible knowledge, but rather must identify knowledge with science." [53, §I, p. 4, emphasis in original]

weaker position that physics provides us with descriptions gauged by our (experimentally acquired) experience. Thus, we start from the following two characteristics which we regard as necessary (albeit not sufficient) for *doing physics*:

(C1) Physics strives for a (formal) *description* of the world.

(C2) *Experience* provides the basis for normative judgements about the correctness of the theory.

Then, the association expressed in (A) faces obstacles that we examine in this note: How can a general, contextual theory establish reference to a particular system? How can we expect to exhaustively describe the measurement while maintaining its role in deciding about the correctness of a theory? In light of these questions, the statement (A) constitutes an *implicit assumption* necessary to seeing a problem in the measurement. Conversely, maintaining that quantum mechanics has a problem concerning the measurement as displayed in the Wigner's-friend experiment is a commitment to this assumption.

8.1 Theories, sentences, experience

In light of Characteristic (C1), we take physical theories to aim for a formal language. A *formal language* is a set of sentences $P \subset S^*$, with S^* being the Kleene closure (the set of all finite strings or concatenations) of an alphabet S, that are *syntactically correct* with respect to a set of rules R.[54]

The relation between observations and a formal language $T = (S, R, P)$ is as follows [92]:

> *If* a formal language T forbids the observation of x, *and yet* I have observed x, then I deem a formal language T untenable as a suitable description of the world.

If, for instance, in two subsequent identical quantum measurements, one observed *different* values, quantum mechanics, $T^{\mathrm{qm}} = (S^{\mathrm{qm}}, R^{\mathrm{qm}}, P^{\mathrm{qm}})$, would be falsified: There are no quantum states and measurement operators that could, within the postulates of quantum mechanics, account for the corresponding result (x_1, x_2) with $x_1 \neq x_2$. The elements in S^{qm} together with rules in R^{qm} do not allow to conclude that the result (x_1, x_2) may be observed for either there is no

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[54]Tarski similarly characterizes "formalized language" in Ref. [118] and remarks the additional structure for "formalized deductive sciences," where the rules are specified by axioms and deductive rules.

sentence in P^{qm} corresponding to this result or an associated probability weight is zero.

Let $T = (S, R, P)$ be the formal language of some theory. In order to formally characterize the sentences that falsify T, we assume a formal language $T^{\mathrm{con}} = (S^{\mathrm{con}}, R^{\mathrm{con}}, P^{\mathrm{con}})$ whose syntactically correct sentences $p \in P^{\mathrm{con}}$ refer to *possible results* observable for a given context, i.e., an experimental setup. The language T^{con} can be thought of as to represent all possible value combinations that can be displayed on the screens showing the results of a given experiment. Furthermore, we assume that the formal language T can be restricted to some T' such that the corresponding sentences P' are the subset of P^{con} of sentences that T accounts for. For T to *be falsifiable* in a context T^{con}, the set of sentences P' has to be a proper subset of P^{con}, i.e., if the set of falsifying sentences $P^{\mathrm{fal}} := P^{\mathrm{con}} \setminus P'$ is not empty.
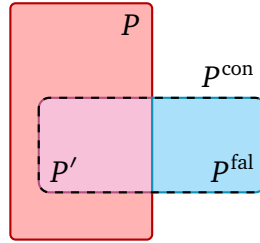
If a sentence in P^{fal} represents an actual observation, then T is falsified for the given context. In the previous example of two identical subsequent quantum measurements, the result (x_1, x_2) with $x_1 \neq x_2$ corresponds to a sentence in P^{fal}.

Falsification refers to theories, including semantic concepts beyond a formal language, rather than to the formal language directly.[55] The semantic concepts can be hidden in the condition "I have observed x" that connects to Characteristic (C2). When I say "I have observed x" then I *refer* to the experimental setup whose possible configuration are captured in T^{con}, and I *mean* that the experimental setup is in the configuration corresponding to x. This is the tension we have mentioned before: Between the individual and singular experience required in Characteristic (C2), and the formal language following universal rules in Characteristic (C1). To see a problem in the measurement requires to represent the individually experienced "definite measurement results" in the formal language of the theory. Subsequently, we examine this step of subsuming the individual and particular experience appearing in Characteristic (C2) under the paradigm of Characteristic (C1).

Let $v_O : P^{\mathrm{con}} \to \{\mathtt{true}, \mathtt{false}\}$ be a function—the so-called *verification function*—, for which $v_O(p) = \mathtt{true}$ if and only if the observer O has observed p. More

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[55] Popper in Ref. [92] refers to "empirical-scientific systems."

specifically, if $v_O(p) = \texttt{true}$ then O deems some apparatus fit to produce values in the context $\mathcal{M}$, and O is certain to have obtained the value p. The verification function establishes whether the condition "I have observed p" is satisfied or not. Thus, for the observer O, T is falsified for the context P^{con} if there exists $p \in P^{\mathrm{fal}}$ with $v_O(p) = \texttt{true}$.

There remains, however, the undesired dependence on the observer O. So the question above can be reframed as follows: Can we detach the verification function from the observer? Can we ensure the objectivity of the experienced results by demanding (at least in principle) the possibility to replace one observer for another, and expect the possibility to repeat the experiment thereby producing the "same experience"? Can we, if we know enough about the observer O, describe his experience, and, thus, what O may ever say? Is there a "single language sufficient to state all the truths there are to state" [104]?

These question anticipate the age-old tension between general, observer-independent, certain aspects and subjective, observer-dependent counterparts. Physical theories aspire for the former but draw legitimacy from the latter. How can we derive certain knowledge—about e.g., some thing-in-itself—from subjective experience that might—even worse—involve practical involvement? Later, we turn to the linguistic version of this question: How can different speakers mean or refer to the same despite subjective uses or subjective intentions?[56]

Before we turn to more general linguistic concerns we briefly consider the case of *formal* languages. *Self-reference* imposes limitations on formal languages. Problems arising from self-reference crystallize, e.g., in the *Liar's antinomy*:

<div align="center">This sentence is false.</div>

The sentence leads to a contradiction if one requires that a statement is true if and only if the claim *that the statement be true* is true. This particular *unquotation notion of truth*—illustrated by: "Snow is white" is true if and only if snow is white—is called *Schema T* [119]. For a truth predicate T and with $\langle \cdot \rangle$ denoting the name of a sentence, it can be written as

$$\phi \leftrightarrow T\langle \phi \rangle, \quad \forall \text{ sentences } \phi \,.$$

With the theorem on the *undefinability of truth*, Tarski showed that any formal language extending first-order arithmetic[57] with a truth predicate containing Schema T allows for such a contradiction. Self-reference, together with a

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[56]The appeal in evolutionary "explanations" of innate ideas, linguistic capabilities, etc. seems to stem from the desire to find some common, unquestionable element.

[57]A first-order arithmetic is an axiomatic system for the arithmetic of natural numbers relying merely on statements of first-order logic.

negation, thwarts the unification of different linguistic contexts as it precludes a consistent truth predicate.

The verification function above serves as a predicate. If it is formalized in a theory extending first-order arithmetic, we require it to contain the Schema T, and no sentences of the formal language are excluded from arising from observation, then there is a liar's antinomy. There are three escape routes: (1) Either nature miraculously removes all problematic sentences, and leaves us with an *incomplete* formal language in which certain syntactically correct sentences are excluded by assumption, or (2) we can run into glitches in our experience that we cannot account for without contradiction, undermining our confidence in experience as the appropriate normative authority (see also the reliability constraint in Ref. [51, 2B]), or (3) we avoid the reference to sentences within that language. Tarski follows the latter path: He confines the definition of truth to languages that are not semantically closed—languages that do not refer to their own sentences. The definition of a truth predicate then becomes part of an over-arching *meta-language*. If we follow Tarski's path, then we have to accept a dualism *before* addressing the problematic dualism between unitarily evolving states and definite measurement results in quantum mechanics.[58]

We are lead to question the entire approach we have taken thus far—i.e., the idea that experience provides us with a predicate over a set of available atomic sentences on which valid theories build a logical construct of meaningful statements.[59]

8.2 Reducible, or not?

In a first step towards challenging the program of reductionism, we retrace consideration from Wittgenstein. In Ref. [129], he examines how we acquire the ability to speak and finds a circular interdependence between the meaning of words and experience: For a child, that has no medium to establish meaning from explanation[60], the acquisition of language reduces to trimming[61]. This process, in turn, relies on observation. There is an issue of self-reference at the very root of meaning—or at the very root of any account of experience for that mat-

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[58]It feels like being in the quagmire, as discussed in Section 11.4.

[59]Quine describes the program as follows: "The other dogma is *reductionism:* the belief that each meaningful statement is equivalent to some logical construct upon terms which refer to immediate experience." [98]

[60]"One has already to know (or be able to do) something in order to be capable of asking a thing's name. But what does one have to know?" [129, §30]

[61]"Here the teaching of language is not explanation, but training." [129, §30]

ter. What we can meaningfully say, depends on our experience. And inversely, how experience can find its way into our language depends on the linguistic means available to us. Feyerabend observes similarly that we cannot draw a clear boundary between facts and theories.[62] Meaning and facts do not have their self-sufficient existence out there.

To break this circular dependence, one might place one before the other by assumption—either by assuming innate semantic capabilities[63] or by assuming self-evident sense data. This assumption, however, is tainted. Firstly, the dualism between innate capabilities and acquired or learnt ones raises epistemic concerns as discussed by Sellars [111] (see Footnote 88). Secondly, if we want to incorporate this into the approach above, we would require one consistent framework. On the one hand, the Liar's antinomy stands in the way. On the other, limitations on the translatability of languages suggest that ways of speaking can be radically different and can, therefore, not simply be pieced together into one entity. The Object/Relation Impedance Mismatch serves as an example here (see Section 11.4). And if we were not concerned with creating a basis for discussing the measurement problem, then the problem itself might be taken as a similar case. In fact, dualities, such as quantum/classical, practical activity/certain knowledge and meta-/object language, are the recognition of like incommensurabilities.

This repelling effect of seemingly insurmountable incompatibilities is contrasted[64] by an attraction through semantic holism (see Section 8.5.1): No statement has meaning in isolation.

By now, the reductionist approach above faces two obstacles: There is hardly a consistent way to incorporate different meaningful ways of speaking into one logical construct other than by privileging or prioritizing a subset by assumption. And the association of atomic sense data with atomic statements, thus, establishing their meaning, is averted by the apparent impossibility to atomize meaning.

So far we have been concerned mostly with how the world acts through experience on the meaning of the words we use. The following statement by Nietzsche summarizes the above developed scepticism:[65]

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[62]"Tatsachen und Theorien sind viel enger verknüpft, als es das Autonomieprinzip wahrhaben will." [31, §3] — "Facts and theories are much tighter interconnected than the principle of autonomie wants to admit." [31, §3, own translation]

[63]Fodor is to mention here. For a critical review of his position, we refer to Ref. [97].

[64]The two effects do hardly counteract one another.

[65]"Ein Maler, dem die Hände fehlen und der durch Gesang das ihm vorschwebende Bild ausdrücken wollte, wird immer noch mehr bei dieser Vertauschung der Sphären verrathen, als die empirische Welt vom Wesen der Dinge verräth. Selbst das Verhältnis eines Nervenreizes zu dem hervorgebrachten Bilde ist an sich noch kein nothwendiges; [...] [D]as Hart- und Starr-Werden einer Metapher verbürgt durchaus nichts für die Notwendigkeit und ausschliessliche Berechti-

> A painter without hands who wished to express in song the picture before his mind would, by means of this substitution of spheres, still reveal more about the essence of things than does the empirical world. Even the relationship of a nerve stimulus to the generated image is not a necessary one. [...] [T]he hardening and congealing of a metaphor guarantees absolutely nothing concerning its necessity and exclusive justification. [88]

The existence of any one true and exhaustive language is in doubt. The assumption of innate, non-contextual structures, an underlying *lingua mentis* that allows to deduce semantics and related experience if deciphered correctly can be contested. Well, it is an assumption (see Section 8.5). Consequently, it is questionable whether the friends acquisition of a measurement result can be described exhaustively.[66]

Furthermore, physical theories rely on singling out parts in our environment as orthogonality shows (see Sections 6.1 and 11.3.1). Thus, we must address the complementary question how the words we use refer to entities in the world around us. Orthogonality in this sense illustrates that doing physics is not based on a unidirectional imprint of nature on what we can say.

Putnam argues against the existence of exhaustive criteria that determine *reference* or *representation*: Does an ant's incidental "picture of Churchill" in the sand refer to Churchill? *Similarity* to the features of Churchill is *neither necessary nor sufficient* to refer to Churchill (see [96, §1]). Magritte's "Treachery of Images" exposes similar ambiguities: Foucault distinguishes in his discussion of Magritte's references to a pipe [35] *similitude* and *resemblance*, and regards the painter to bring "the former into play against the latter" [35, §5].[67]

We discuss the role of reference to systems in the context of the Wigner's-

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gung dieser Metapher." [87, p. 18]

[66] Schneider [108] considers the measurement as "the production of a *signifier*." This draws from Cassirer's work who observed: "What primarily distinghuishes linguistic concept from strictly logical concept formation is that it never rests solely on the static representation and comparison of contents but that in it the sheer form of reflection is always infused with *dynamic* factors; that its essential impulses are not taken solely from the world of being but are always drawn at the same time from the world of action. All linguistic concepts remain in the zone between action and reflection." [20, §4.1] There emerges a connection to Fleck's epistemology. Schneider subsequently employs the "auto-productive nature of signifier" to justify the collapse. The argument creates a problematic connection between semantic processes as essentially inter-subjective and an isolated subject, i.e., the friend.

[67] "Resemblance presupposes a primary reference that prescribes and classes. [...] Resemblance serves representation, which rules over it; similitude serves repetition, which ranges across it." [35, §5]

friend experiment in Section 8.3, and in the context of indistinguishable particles in Section 11.3.

8.3   The system

The system S in the Wigner's-friend experiment exposes issues with the *physical systems in general*. Wigner and his friend must agree to refer to the *same* system S. In the above scenario, this amounts to Wigner reading off by means of his measurement M_W^F that the friend refers to the same S as he does. If the friend is ready to "measure S," then the friend "has S in mind." We have to add to (A) the following aspect:

(ASys) For an observer O referring to a system S, it is sufficient that O is in a state ϕ for some $\phi \in \mathcal{H}_O$.

Before examining whether a measurement can establish reference to a system, let us take a closer look at the notion of a "system" itself. What is it that we commit to if we use the term "system"? *The notion builds on the idea that we can talk about a clearly confined part of the world around us, and distinguish it from other such parts.* We assume a *separability*, i.e., the possibility to make statements about one such part independent of other parts. Einstein places this separability at the core of his understanding of *physical reality*:[68]

> I just want to explain what I mean when I say that we should try to hold on to physical reality. We are, to be sure, all of us aware of the situation regarding what will turn out to be the basic foundational concepts in physics: the point-mass or the particle is surely not among them; the field, in the Faraday-Maxwell sense, might be, but not with certainty. But that which we conceive as existing ('actual') should somehow be localized in time and space. That is, the real in one part of space, *A*, should (in theory) somehow 'exist' independently of that which is thought of as real in another part of space, *B*. If a physical system stretches over the parts of space *A* and *B*, then

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[68]We omit the following part of the quote, which, in light of non-locality [10, 3, 78] is questionable. "If one adheres to this program, then one can hardly view the quantum-theoretical description as a complete representation of the physically real. If one attempts, nevertheless, so to view it, then one must assume that the physically real in *B* undergoes a sudden change because of a measurement in *A*. My physical instincts bristle at that suggestion." [64, §5 (translated quote from [27])] We do not regard separability as *sufficient* in the sense that measurements on parts, together with previously shared information, reveal the results of any measurement that can possibly be performed on the combined system.

what is present in *B* should somehow have an existence independent of what is present in *A*. What is actually present in *B* should thus not depend upon the type of measurement carried out in the part of space, *A*; it should also be independent of whether or not, after all, a measurement is made in *A*. [...] However, if one renounces the assumption that what is present in different parts of space has an independent, real existence, then I do not at all see what physics is supposed to describe. For what is thought to be a 'system' is, after all, just conventional, and I do not see how one is supposed to divide up the world objectively so that one can make statements about the parts. [64, §5 (translated quote from [27])]

Importantly, the ability to speak of separate systems does not imply an "objective division" of the world. Einstein's reservations towards such a division go beyond a temporal inability to conceive such an objective division:

Terms that have proven useful for the ordering of things attain easily such an authority over us so that we forget their worldly origin and we accept them as unalterable facts. They are, then, put down as 'thinking-necessities,' 'a priori given,' etc. The path of scientific progress is often made impassable for a long time by such misconceptions. [26, p. 102, own translation]

This joins Feyerabend's [31, 32] and Kuhn's [73] investigation into the history of science, refuting the idea of an overarching convergent trend, with the considerations by Wittgenstein, Sellars, and Rorty [130, 111, 103] on the contingency of language. If we assume that there is neither a final privileged language, i.e., a "truth out there" [103], nor that we are able to "step outside the various vocabularies we have employed" [103], then we must allow for a Kuhnian paradigm shift, i.e., a radical re-description. Adopting (A) and (ASys) is, in turn, a step towards assuming a privileged language, at the risk of petrifying scientific discourse. But what supports this suspicion towards the existence of, or convergence towards, an ultimate language that reflects the truth out there other than assuming uniformity in the history of science?[69] What suggests to repudiate (ASys)?

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[69]Arguments like the following-rule paradox against the existence of a privileged language (see Section 8.4.2) are based on the observation that such uniformity is not warranted. Thus, historical arguments are tainted.

8.3.1   Isolated systems, decoherence, and superselection rules

A problem with reference to systems is related to decoherence in the following sense: If Wigner observers a non-unitary time evolution of $S \otimes F$, then it is possible that the friend measures a system S' bigger than S, i.e., bigger than what Wigner thinks the system is. The "escaping photon" leading to decoherence can be seen as a problem of non-aligned reference: The photon is contained in S' but not in S. Conversely, we can see decoherence—provided that we take it as *inevitable* interaction with the environment—as the inability to sharply draw the boundary between one system and another, or between one system and its environment. This perspective also challenges how we understand *decoherence* itself: We cannot define decoherence as "a system interacting with its environment" because there is no clear-cut distinction between "system" and "environment." *Thus, decoherence is maybe better seen as the abandonment of the notion of a well-confined system.*

This also affects *environment-induced superselection rules*. The program of "einselection"[70] addresses the problem of many-worlds interpretations how to fix the basis corresponding to measurement results, and, thus, how "to split the worlds." Einselection is not primarily concerned with explaining how a single world splits into systems. Explaining the system-split yet poses a problem. Superselection rules might provide a basis on which one might attempt to divide the world into systems, against Einstein's above-mentioned concerns. These superselection rules cannot be induced by the environment,[71] because it assumes the notion of a system already, leaving us with a circularity. We are similarly faced by circularity, if we attempted to "measure" what qualifies as a system, because "measuring" here means again "measuring a system." A way out is to supplement quantum mechanics from the beginning by superselection rules.

The idea of introducing superselection rules has another interesting consequence: If we regard contextuality as an essential aspect of quantum mechanics [91, 50, 70], then the superselection rules effectively undermine quantum mechanics itself: With the superselection rules, we introduce observables in the center of the orthomodular lattice of allowed projectors $\mathfrak{P}'(\mathscr{H})$, i.e., elements that commute with all other elements in the lattice. This is *the cost of introducing a non-contextual notion of a system into contextual theory* [91]: There must be a *Heisenberg cut*, i.e., a line at which things become at least in part classical. This

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[70]This is the abbreviation of environment-induced superselection rules.

[71]On a side-note, let us remark that the idea of "being induced by the environment" resembles conceptually the epistemic idea of sense-data: There is something *given* in our environment that induces us to know (see later footnotes.)

might also affect the measurement problem directly: If in the Wigner's-friend experiment the notion of a system was established by suitable superselection rules, not all of the measurements leading to the contradiction are necessarily permitted. If, for instance, the measurement M_W^{SF} with ambiguous probability predictions was precluded by the superselection rules, then there would be no measurement problem.[72]

There are objections: Quantum mechanics with postulated superselection rules is *not* quantum mechanics. Or, put otherwise, introducing superselection rules undermines the assumption that quantum mechanics is *universal*.[73] We are left with a problem: How can quantum mechanics be universal and exhaustive without threatening the notion of a system? How can quantum mechanics *be* without the notion of system? How can the notion of a system avoid the above circularities without supplementing the theory by such a notion and threatening its universality? This necessarily levels all particular features of a particular system.[74]

8.4   The Friend

8.4.1   Quantum inquiries about intentions

Let us assume that, despite the above scepticism, there are theoretical means to *define* a system, e.g., by a suitable set of superselection rules added to quantum mechanics. The question remains whether there can exist an element $\Pi_W^F \in \mathfrak{P}(\mathscr{H}_F)$ that shows that "the friend means to measure S"—that the friend "has S in mind." This question carries two intricacies: Can we find something *inside* the friend that reveals

(Itnt1) the friend's reference to something outside of him, and

(Itnt2) the friend's intention to perform a measurement?

If we require that the friend's intention to measure S is before his actual contact with S, then we must expect to read off this intention by merely measuring F, and *not $S \otimes F$*.

We examine the possibility of asking the more specific question whether

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[72]This reminds of the Bohmian restriction to position measurements.

[73]Consider the following extreme case: Let us assume the world is entirely classical. Then, we might still describe it by a Boolean sub-lattice of $\mathfrak{P}(\mathscr{H})$. Is then quantum mechanics universal, given that we have to restrict the quantum mechanical description to a Boolean sub-lattice?

[74]See also the discussion in Section 8.4.2.

(Itnt3) the friend intends to perform the measurement $\Pi_F^S \in \mathfrak{P}(\mathscr{H}_S)$.

If Wigner can inquire about such a question, and answer it positively, then he can conclude that the friend has the intention "to measure S." It might be that one measurement Π_W^F with a suitable result reveals that the friend intends to "perform the measurement Π on S for some $\Pi \in P \subset \mathfrak{P}(\mathscr{H}_S)$." Let us, therefore, assume the map

$$\pi : \mathfrak{P}(\mathscr{H}_S) \to \mathfrak{P}(\mathscr{H}_F)$$

to formally capture the following: If Wigner can positively answer to $\Pi_W^F \in \mathfrak{P}(\mathscr{H}_W)$, then he can conclude that the friend intends to measure an element in $\pi^{-1}(\Pi_W^F)$.

The friend does not intend to measure *because* we asked him about it. Or, even worse, oscillate between different intentions, as we change our inquiries.[75] We,

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[75] In a sense, this is what we insinuated when we allowed for concluding from (Itnt3) to (Itnt1) and (Itnt2). To illustrate this, consider the following situation: You offer your friend one scoop of ice-cream, and you tell him to choose one flavor. To find out which flavor he would like to have, you ask him the following two questions (arbitrarily many times and in arbitrary order): (Q1) "Do you want chocolate ice-cream?" and (Q2) "Do you want vanilla ice-cream?" If he now says consistently "yes" to one, and "no" to the other, then you might conclude that he has a clear "intention." This conclusion is not justified if your friend gives changing answers as you repeat these questions. If he says consistently "yes" to both questions, then you doubt that he understood what you meant with "choosing one flavor." If he does not understand what it means to "choose a flavour," how can we conclude his intentions from his answers to the questions (Q1) and (Q2)?

Even though we are getting a little ahead of ourselves here, let us take this one step further and imagine that you give him a cone with a scoop of ice-cream. Your friend starts eating or not. In the case that the friend consistently gives a preference, and you offer him the respective ice-cream, you expect him to starting eating (and his face showing how he is enjoying it). Conversely, if you offer him the other flavor, then he should not eat and rather insist "But I meant chocolate ice-cream." Now, if you offer him the type that you concluded from his consistent answers to be his preference, and he does not start eating, we can imagine the friend to say "Oh sorry, I meant the vanilla." or, angrily "But I meant chocolate." In the latter case you might learn that he (consistently) permutes the words vanilla and chocolate. If your friend in the past always asked for vanilla, and always ate it with joy, and you now skip the question to directly offer him vanilla ice-cream, then he might still not eat it, e.g., because today he "felt like chocolate," and, thus, intended to have chocolate ice-cream. In the case of the friend giving changing answers, we can imagine that he (emotionlessly) eats the ice-cream, unless he said "no" and you give him the respective flavor nonetheless. Does this last case, despite being logically correct, still qualify for "having chosen a flavor"? Now, one might oppose that I deprived the friend of his emotions, to testify of his choice. But then I can easily imagine him showing disgust, even if he eats the logically correct flavor, e.g., because *this* chocolate ice-cream is horrible. Did I now fiddle with the *circumstances* too much? Should there not be a clear pattern, if we consider "normalized conditions"? But how are these conditions characterized? By the friend sticking to

therefore, demand that the image $\pi(\mathfrak{P}(\mathcal{H}_s))$ forms a distributive lattice avoiding that the friend "changes his mind" if we ask him questions in a different order. Thus, π effectively collapses the non-distributive lattice $\mathfrak{P}(\mathcal{H}_S)$ into a distributive sub-lattice of $\mathfrak{P}(\mathcal{H}_F)$. Thus, we re-encounter the preferred basis problem of the many-worlds interpretation and the "system" problem from Section 8.3.1: We are required to postulate a distributive sub-lattice that corresponds to "measuring intentions." Suspecting that the friend has "S in (his) mind" posits intentions as non-contextual. As in Section 8.3.1, we encounter issues anchoring non-contextual notions in a contextual theory that is supposedly universally valid. Thus, we do not have to go as far as the measurement problem for contextuality[76] to get into the way of an exhaustive quantum description.

8.4.2   Normativity and the nexus between past and future

The above-developed scepticism relates more specifically to quantum mechanics. The requirements (C1) and (C2) give rise to another, more general doubt. Recall that (C1) specifies the *descriptive task* of physics, while (C2) specifies that this description has to *succumb to a normative judgement*. Kripke, when discussing Wittgenstein's paradox of rule-following[77], observes:

> The relation of meaning and intention to future action is *normative*, not *descriptive*. [71, p.37, emphasis in original]

On the one hand, this statement reflects on the above-discussed reference to systems and the problem of Wigner aligning his reference to his friend's. On the other hand, it foreshadows a similar problem with *describing* the friends measurement—i.e., referring to the same measurement that supposedly provides the normative experience. If we "interchangeably use the words 'experience', 'observation,' and 'state of the observer'" [83], then we remove the room for

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the logical rule? This leaves us with a circularity. These are first steps towards the discussion in Section 8.4.2.

[76]This takes contextuality as essential to the measurement problem (see [57]).

[77]Wittgenstein summarizes: "This was our paradox: no course of action could be determined by a rule, because every course of action can be made out to accord with the rule. The answer was: if everything can be made out to accord with the rule, then it can also be made out to conflict with it. And so there would be neither accord nor conflict here." [129, §201] To put it in the words of the example in Footnote 75, imagine that your friend in the past always asked for vanilla ice-cream. We cannot tell whether he is following a rule that dictates him to choose vanilla again, or a rule that lets him change his choice for chocolate ice-cream this time, *before* he has uttered his choice.

normative judgement. We effectively postulate the theory to be true.[78] In the following, we elaborate on these concerns.

There is a link between Hume's problem of induction and Wittgenstein's paradox of rule-following:

> Both [Hume and Wittgenstein] develop a sceptical paradox, based on questioning a certain *nexus* from past to future. Wittgenstein questions the nexus between past 'intention' or 'meanings' and present practice: for example, between my past 'intentions' with regard to 'plus' and my present computation '68+57=125'. Hume questions two other nexuses, related to each other: the causal nexus whereby a past event necessitates a future one, and the inductive inferential nexus from the past to the future. [71, p. 62, emphasis in original]

The link between the two can be explicated as: *Nothing in the friend's past logically determines whether and how the friend means to account for his experience in a measurement.* That is: Nothing in the friend's past determines whether he intends to measure S or T, or whether he thinks to have successfully performed a measurement, or what result he obtained. Unless the friend's particular state a can be subsumed under a general category A. The extension into the future of this *general* category is not logically warranted [62]. Thus, all theories have a tentative character and we must resort to falsification [92].

The idea that quantum mechanics provides means to exhaustively describe the friend's measurement is to assume that quantum mechanics provides the general category that covers all future measurements. Then, however, the friend is no different from a brain-in-a-vat [96]: Employing the separability assumption from above, we can conclude that anything that we can say about the friend, including his intentions and possible accounts of experience, can be derived from the friend's state. With the help of some auxiliary environment T, we can simulate the friend's measurement without ever putting him in contact with S despite

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[78]This brings us back to reflect Einstein's concerns about the passibility on the path of scientific progress cited in Section 8.3.

his alleged initial intentions of measuring S.[79]
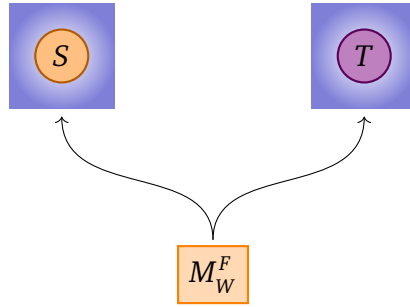
This renders the reference to the S that we struggled to ensure previously—as a necessary requirement for qualifying the friends acts as a measurement of S—invalid. The friend's reference does not conform to our notion of "referring to S," as in referring to *a particular* system S. There does not seem to be a general criterion for particular reference [97]. The problem translates to any other observer, also to us:[80] If there is no principle difference between the friend and us, we are thrown back to the question how we can ever refer to anything outside ourselves, if we must suspect some Wigner-like super-observer simulating us.[81] Similarly, a measurement is no more something we actually, or better *actingly*, have a part in, but something that *is said about us*.[82] Is there then any measurement that could provide us with the grounds to reject a theory as demanded in (C2)?
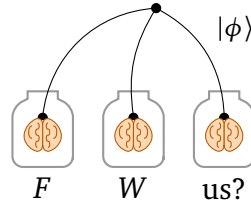
Kripke's argument against functionalism can be seen as a variant of the following-rule paradox. Translating it to Wigner's friend allows to explicate the issue. Let

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[79]We merely have to ensure the partial trace on $\mathcal{H}_F$ to remain the same.

[80]If quantum mechanics fully describes the friend's measurement, what then is "reading this text" other than a quantum-mechanically described observation?

[81]In the Truman Show, Christof tries to keep Truman from leaving with the following statement: "There is no more truth out there than there is in the world that I created for you." [125, ca. min. 133] Truman finds himself at the door because observing glitches lead him to question the exhaustiveness of Seahaven's normality. Similar to pre-Galilean philosophers, Truman might have taken the falling spotlight as a figment of his imagination and fall in line with Christof's exhaustiveness argument. If Truman had assumed the exhaustiveness of Seahaven's normality—including a travel agency that never takes you anywhere outside the city—, the question whether he is part of a staged play would not have been meaningful.

[82]In this regard, "measuring" becomes "being said to measure," analogue to [128, §202].

us first summarize the argument laid out in Ref. [18]: If we assume that physical computers can *break down*, then we can also imagine the following scenario: If a physical computer computes F and breaks down, it actually computes *another* function G. If, vice versa, a computer computes G and breaks down, it actually computes the function F.
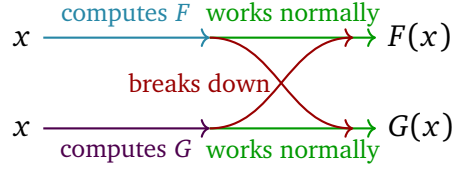
Thus, we are left with the following problem:

We cannot decide whether a physical computer physically computes $\left\{ \begin{matrix} F \\ G \end{matrix} \right\}$,

and $\left\{ \begin{matrix} \text{works normally} \\ \text{breaks down} \end{matrix} \right\}$.

We can stipulate that the computer works fine, and conclude that it computes F or G. Or we stipulate that it computes F, and conclude whether it works correctly or breaks down. But we lack the means to fix both.

The problem translates to the friend and his reference to systems S and T:
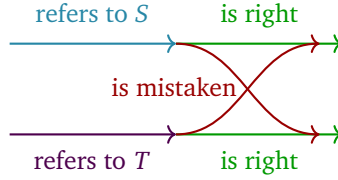
If the friend is mistaken, then he refers to the respective other system.

We cannot decide whether the friend refers to $\left\{ \begin{matrix} S \\ T \end{matrix} \right\}$, and is $\left\{ \begin{matrix} \text{right} \\ \text{mistaken} \end{matrix} \right\}$.

So as long as we allow the friend to be mistaken at times—or dreaming, or hallucinating—, and this results in swapping references, or measurements,
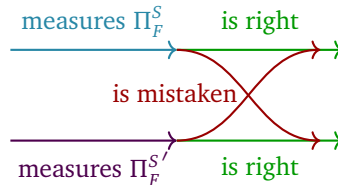
then we are faced with the problem of deciding what measurement or what system the friend meant, *and* whether he is right or mistaken. There is nothing we can expect to find in the state of the friend that will definitely decide both question at once. Again, we can stipulate that he is dreaming and establish whether he is referring to S or to T. Or we stipulate that he is referring to S and establish whether he is right or mistaken. The authority to establish either of these stipulations is usually delegated to a wider circle of persons that *intersubjectively* forms an agreement.[83]

In light of the challenges laid out above, we conclude: The common reading of the measurement problem relies on *fully describing* the friend's measurement, thus removing the friend from the ground of intersubjectivity.[84]

The "problem" then consists in the incommensurability of two different descriptions that quantum mechanics allows for. *The insistence that the measurement is a problem that needs a solution in form of another theory or an appropriate reading of quantum mechanics implicitly depends on removing the need to (inter-*

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[83] We can draw a connection to Habermas' criticism of positivism [52]: If quantum mechanics is taken to yield an exhaustive description, then the need for intersubjective agreement on norms is superfluous. Language and science becomes private in a way that Habermas regards intrinsic to Pierce's pragmatism. "Peirce would have had to come upon the fact that the ground of *intersubjectivity* in which investigators are always already situated when they attempt to bring about consensus about metatheoretical problems is not the ground of purpose-rational action, which is in principle solitary. [...] It is possible to think in syllogisms, but not to conduct a dialogue in them. [...] But the communication of investigators requires the use of language that is not confined to the limits of technical control over objectified natural processes. It arises from symbolic interaction between societal subjects who reciprocally know and recognize each other as unmistakable individuals. This communicative action is a system of reference that cannot be reduced to the framework of instrumental action." [53, §6, p. 137, emphasis in original] With the abandonment of intersubjectivity, Pierce cannot resort to a community of scientist vouching for methodologic means to yield certain knowledge as he effectively does: "Ontological propositions about the structure of reality unintentionally elucidate the process of mediation through which we come to know reality. Yet in fact this concept of reality was first introduced only as the correlate of a process of inquiry that guarantees the cumulative acquisition of definitively valid statements. As soon as we remember this point of departure, Scholastic realism of Peirce's stamp can be seen through as the ontologizing of an originally *methodological* problem. Indeed for Peirce the problem of the relation of the universal and the particular presented itself outside of the tradition. That is, it appeared not as a logical-ontological problem, but rather in connection with the methodological concept of truth as a problem of the logic of inquiry." [53, p. 109f, emphasis in in original] Irrespective of whether one adheres to Habermas' critical assessment of positivism or not, the connection illustrates the epistemological import of assuming an exhaustive language (see also Footnote 88).

[84]This shows in an unquestioned use of phrases like the following (see Section 6.2.2): "If Wigner aims to describe his friend by means of quantum mechanics, then he gets to know that 'the friend is ready' by performing a measurement M_W^F."

subjectively) settle normative questions.

What does this mean for the Wigner's-friend experiment? No matter the reading of quantum mechanics,[85] an actual Wigner's-friend experiment surprises if an *isolated system shows a collapse*—irrespective of what or who constitutes it: The surprise would be that we can meaningfully call a system isolated *despite* it showing a non-unitary evolution. Whatever evidence justifies us to qualify the system as isolated is then at odds with quantum mechanics. This takes $S \otimes F$ as just another quantum system. Quantum mechanics is universal insofar as it can be applied to $S \otimes F$. The "isolated friend" does, however, not provide any contribution to intersubjectively agreeing on the correctness of the theory. Whatever happens inside $S \otimes F$ is *not* a measurement that provides the experience required in (C2).

Hume's problem of induction shows that one can hardly rely on uniformity to support general claims. As such, the historical analysis of Feyerabend and Kuhn cannot serve as reason to reject the assumption of an exhaustive language as this requires a uniformity of history (see also Section 8.3, in particular Footnote 69). The idea that science aims for its own disintegration strikes us, however, as odd: For, can we not really learn when we have to listen carefully, or watch closely? When mere description gives way to metaphorical disruption—"suddenly breaking off the conversation long enough to make a face, or pulling a photograph out of your pocket and displaying it, or pointing at a feature of the surroundings, or slapping your interlocutor's face, or kissing him?" [103, §1] When experimental behavior is *not* covered by the established description? When emitted electrons do *not* get any faster if the intensity of light shun onto a metallic plate is increased?

8.5   Assumptions and contingencies

The discussion in Section 8.4 demonstrates that normativity affects the Wigner's-friend experiment in several, interdependent ways: On the one hand, there is Hume's nexus between existing experimental findings and the *general* validity of a theory. On the other hand, there is Wittgenstein's nexus between past "references" or "intentions" and a *general* rule for "reference" or "intention." These two aspects join in the idea of *reproducibility*:

> Indeed the scientifically significant *physical effect* may be defined as that which can be regularly reproduced by anyone who carries out

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[85]We consider GRW [43] to be a different theory [8].

the appropriate experiment in the way prescribed. [93, §I.8, emphasis in original]

Repeated successful applications of a theory to experimental setups lend legitimacy to physical theories. To allow for reproducibility we assume that different observers are able to agree on having performed *equivalent* measurements on the *same* system and thus obtained the *same* result.[86] Following Wittgenstein, different observers do *not* refer to the same system because they follow the *true* "rule of reference," but because they *establish* a contingent agreement on referring to the same system. The contingency of our description of the world out there is not a contemporary defect reflecting the inadequacy of our current description. In this perspective, Characteristic (C2) remains an aspect of science and is neither rendered obsolete nor less importance by following some "scientific method"— i.e., a *true* "rule for doing science."[87] Science retains its room for creativity, and remains itself a creative activity [33]. These observations carry a circularity as they take us back to our starting point, i.e., Characteristic (C2) that we ascribed to physics in the first place. Our arguments are subject to the contingency of language as well. Thus, the problems we see in seeing the measurement as a problem *are created* as much as, in our regards, is *any* language, at least in part. It seems that we must, at this point, retract from declaring our assumptions as "weaker," as we initially did. Such a comparison seems hardly justified.

If, on the contrary, one sees the quest of physics to excavate a *truth out there*, independent of any normative judgements or creative acts—a truth that imposes itself—, then the assumptions (A) and (ASys) can hardly be accepted as such, i.e., *as assumptions*. Identifying an assumption is a reflection on the creative steps that facilitate one's way of speaking: It constitutes the admission that it could have been done otherwise.[88] In this light, it is only consequent that pos-

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[86]This requirement is weaker than Popper's requirement of a rule that "prescribes" how to carry out an experiment.

[87]With upholding Characteristic (C2), we repudiate the empiricist reading: "*Experience* provides the basis for judgements about the correctness of the theory." We do not believe that, once we figure out the "language of sense-data," the room for normative judgements closes (see, e.g., [111]).

[88]There is the possibility to regard assumptions as a temporary evil until the evident foundations have been properly sorted out. Until the assumption is turned into an inevitable conclusion of self-imposing facts. This approach is, however, tainted by the tension within the idea of "learning the self-imposing," or, to put otherwise, "to acquire an unaquired ability." This is the core of Sellars inconsistent triad: "[classical sense-datum theories] are confronted by an inconsistent triad made up of the following three propositions: A. x senses red sense content s entails x non-inferentially knows that s is red. B. The ability to sense sense contents is unacquired. C. The ability to know facts of the form x is ø is acquired." [111, §6]

itivism removes such reflections (see [52]). We return to Wittgenstein's observation that the reader might not be easily convinced if he had not had similar thoughts before. The ladder can merely be climbed if one has already made the first steps onto it (see [128, preface and §6.53]). Embracing the contingencies of any language-game bars us from offering an ultimate argument for the contingency of language.[89]

8.5.1   Specker's assumption

Ernst Specker is credited for his contribution regarding the contextuality of quantum mechanics. In 1960, Specker published the article "Die Logik nicht gleichzeitig entscheidbarer Aussagen"—the logic of not simultaneously decidable propositions [113]. The article does not (yet) prove the contextuality of quantum mechanics, but discusses a non-distributive poset and, thus, prepares the ground for the latter findings on *contextuality* [66, 70, 50]. Specker begins with the following statement:[90]

> The motto put before the paper ['La logique est d'abord une science naturelle.' F. Gonseth, (Logic is first of all a natural science.)] is the subtitle of the chapter *La physique de l'objet quelconque* [The physics of any object...] from the work *Les mathématiques et la réalité* [The mathematics and the reality]; this physics turns out to be essentially a form of classical propositional logic which, in this way, on the one hand obtains its typical realization and, on the other, is stripped of the claim for absoluteness that has at times been appended to it. The following explanations follow this view and should be understood in the same empirical sense. [113, p. 1, own translation]

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[89]Putnam hints at this concern when he states: "Reichenbach, Carnap, Hempel, and Sellars gave principled reasons why a finite translation of material-thing language into sense-datum language was impossible. Even if these reasons fall short of a strict mathematical impossibility proof, they are enormously convincing [...]. In the same spirit, I am going to give principled reasons why a finite empirical definition of intentional relations and properties in terms of physical/computational relations and properties is impossible—reasons which fall short of a strict proof, but which are, I believe, nevertheless convincing." [97, §5]

[90]"Das der Arbeit vorangestellte Motto ['La logique est d'abord une science naturelle.' F. Gonseth] ist der Untertitel des Kapitels *La physique de l'objet quelconque* aus dem Werk *Les mathématiques et la réalité*; diese Physik erweist sich im wesentlichen als eine Form der klassischen Aussagenlogik, welche so einerseits eine typische Realisation erhält und sich andererseits auf fast selbstverständliche Art des Absolutheitsanspruches entkleidet findet, mit dem sie zeitweise behängt wurde. Die folgenden Ausführungen schliessen sich an diese Betrachtungsweise an und möchten in demselben empirischen Sinn verstanden sein." [113, p. 1]

In this first paragraph, Specker explicitly *assumes* a philosophic stance and urges the reader—almost as a hermeneutic prescription—to consider the following work in this perspective. The assumption is interesting because of the specific stance Specker endorses, and its placement *before* the explanations leading towards contextuality. Following Gonseth, Specker ascribes to logic, through physics, the character of an empirical natural science, thus, subject to revision and removed of any Platonist aspirations for *absoluteness*. At the very beginning of considerations that bore contextuality and quantum logic, two concepts that fundamentally question absoluteness claims appended to the natural sciences, stands the explicit abandonment of such claims. This reminds of Wittgenstein's cautioning remarks in the preface of the Tractatus:

> This book will perhaps only be understood by those who have themselves already thought the thoughts which are expressed in it—or similar thoughts. [128, preface]

Both, Wittgenstein and Specker, expose a circular dependence: Cognition builds on cognitive elements, meaning on other meaningful entities. Problems of this circular dependence become particularly evident when we turn cognition or meaning against the respective self. Gonseth observes the double role of the speaker with regards to such *reflexive* activities:[91]

> There is a per se not closed set of *reflexive* activities, that can be performed on themselves, or, as a condition for carrying out themselves, have to be presumed. In first place, one should mention here the thinking about thinking, the knowing about knowing, the researching about researching [and the speaking about speaking]. In all these cases, the person appears as a bearer of a double role. On the one side, this role is passive, insofar as the person is the object of the performed activity, on the other side, the role is active, insofar as the activity has to be carried out by the person. It is clear that this circumstance must have far-reaching consequences. [48, p. 180, emphasis in original, own translation]

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[91]"Es gibt eine an sich nicht abgeschlossene Menge von *reflexiven* Aktivitäten, die auf sich selbst ausgeübt werden können, oder als Vorbedingung für ihre eigene Ausübung vorausgesetzt werden müssen. An erster Stelle sind wohl das Denken über das Denken, das Wissen über das Wissen, das Forschen über das Forschen zu erwähnen. In all diesen Fällen erscheint die Person als Trägerin einer doppelten Rolle. Auf der einen Seite ist diese Rolle passiv, indem die Person das Objekt der auszuübenden Aktivität darstellt, auf der andern Seite ist sie aber aktiv, indem die Aktivität von der Person selber auszugehen hat. Es sollte klar sein, dass dieser Umstand sehr weittragende Konsequenzen haben muss." [48, p. 180, emphasis in original]

These reflexive activities amplify the problem of the reflexive dependence, i.e.,
the necessity for its own presumption. While the reflexive activities make the
problem clearer, the reflexive dependence has more serious consequences as it
affects our ways of speaking, knowing and researching more broadly. Whatever
meaning or cognitive content we assign, we inevitably do so in light of previously
established meaning or knowledge. Meaning and knowledge, thus, are charac-
teristically holistic. In consequence of such observations, Gonseth builds his *open
philosophy* on a principle of revisability that exposes all elements in a system of
knowledge, including logic, to revision.[92]

Rorty's notion of *contingency of language* characterized as

> I call the 'contingency of language'—the fact that there is no way to
> step outside the various vocabularies we have employed and find a
> metavocabulary which somehow takes account of all possible vocab-
> ularies, all possible ways of judging and feeling. [103, Introduction]

is closely related to the reflexive dependence. The inevitable presumption of
knowledge and meaning not only results in the inability to step outside language
and cognitive processes but also hinders the to step into a new language-game:

> A *picture* held us captive. And we could not get outside it, for it lay in
> our language and language seemed to repeat it to us inexorably. [129,
> §115]

The following observation by Fleck offers a picture of this captivity—clearly we
cannot have left language behind:[93]

> Unfortunately, we have the idiosyncrasy that we regard old, familiar
> trains of thought as particularly evident, so that they do require no
> proof and do not even admit such proof. They form the iron funda-
> tion on which one calmly builds on. [33, p. 46, own translation]

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[92]The idea of principles for philosophy, appears to be at odds with the above observations
on reflexive activities. Gonseth cannot aim for any abstract or general principles. Instead these
principles rather emerge from a tentatively assumed separation of a meta-level. Any considera-
tions from the meta- on the object-level, proceeds in a *dialectic* sense insofar as changes on one
side are not without effects on the other. (See the discussion on meta- and object-language in
Ref. [48], and further elaborations in Ref. [76, 29].)

[93]"Leider haben wir die Eigenheit, alte, gewohnte Gedankengänge als besonders evident zu
betrachten, so dass dieselben keines Beweises bedürfen und ihn nicht einmal zulassen. Sie bilden
das eiserne Fundament, auf dem ruhig weitergebaut wird." [33, p. 46]

It is in doubt that building on such a foundation is to be regarded as an ancient habit that can be eradicated rather than being an inevitable part of *saying something*. To establish *agreement* on the use of terms—or should I rather say on the *rules* how to use terms?—is a process troubled again by the reflexive dependence. In this sense, the "calmly building on" might rather be understood as "throwing ourselves" into any of these reflexive activities.

Specker's and Wittgenstein's introductory comments reflect that developing ideas questioning this iron foundation faces obstacles.[94] For one has to question what one necessarily has to build on. These obstacles become more insurmountable if the basis one is to question contains explicit notions that are held as "a priori" or "necessities."[95] Reductionism or foundationalism—and more generally the assumption of a privileged language, epistemology, or method—exacerbates the problems to overcome the captivity imposed by contingency—be it linguistic [98], epistemic [102] or methodologic [31]. In this light, Specker embracing Gonseth's open philosophy does not only caution the reader of subsequent thoughts that might question the reader's current iron foundation, but also testifies a scepticism towards clinging too tightly to *any* such foundation.[96]

Gonseth, recognizing the contingency and holism of any system of knowledge, was concerned that the consequent revisability of *any* part of such systems might undermine their the stability. He thus added to his open philosophy the

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[94]Whether Specker and Wittgenstein *meant* to express this concern can be questioned. Wittgenstein, however, uttered doubt that his Tractatus was and ever would be properly understood [110, §II.2].

[95]The vicinity of a "necessary foundation" and a "explicit necessary notions" as part of this "necessary foundation" seems confusing at first sight. To say that there exist some basis on which we build in cognitive processes and when establishing meaning is not to say that (parts of) this basis necessarily have to be given. The latter assumption, of necessary elements in the iron foundation, corresponds to the "dangerous fallacy" that Wittgenstein recognizes with hindsight in the Tractatus: To treat philosophical questions as if they could be given answers sometime in the future (compare [110, §II.2]).

[96]At the same time, this raises the question what it took Specker, Jauch, and Piron to consider quantum mechanically represented propositions outside the corset of propositional logic, ignoring for the moment the first steps by Birkhoff and von Neumann [14]. To this end, we suspect that the vicinity of Specker and Jauch to Gonseth and Bernays, as well as respective seminars at ETH Zurich, was helpful. Further, there seems to be a cultural difference: On the one hand, there were the "first generation" researchers that experienced the wake of quantum mechanics—i.e., a paradigm shift within which both the formal basis and the philosophic foundation were subject to *open* debates. On the other, the following generations of physicist employed the formalism and primarily *calculated* with it. For this latter generation that has been called by Andreas Buchleitner the "quantum field theorists," a resort to solutionism and a consequent idea of "solving the measurement problem" appears natural. In Section 10, we trace a similar genealogy with regards to influences of computers and computer sciences on doing physics.

principle of technicity:

> The second principle, the *principle of technicity*, is to counterbalance the principle of revisability. This principle is to impose a limit on what counts as a legitimate reason for starting a process of revising a position. The idea is that the motivation for a change has to come from within existing technical, experimental means including the technical language of a science. This principle brings into focus the way in which technical progress – in particular in developing experimental instruments – is relevant to progress in science. [29, p. 8, emphasis in original]

The principle of technicity acts, thus, as a *conservative* counterweight to revisions. Assuming a privileged layer to resort to removes concerns about the stability of a system of knowledge. This presents us with a spectrum of *certainty*: On the one side, we find the assumption of a privileged layer—whether we already know it or not—with its guaranteed stability at the cost of being held captive. On the other, there is the acknowledgement of the contingency arising with reflexive dependencies—Rorty terms this *irony*[97]—entailing questions about stability—culminating in a feeling of being left with sand running through the hands. In light of this spectrum, the quest for certain knowledge is contrasted by the con-

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[97]"I shall define an 'ironist' as someone who fulfills three conditions: (1) She has radical and continuing doubts about the final vocabulary she currently uses, because she has been impressed by other vocabularies, vocabularies taken as final by people or books she has encountered; (2) she realizes that argument phrased in her present vocabulary can neither underwrite nor dissolve these doubts; (3) insofar as she philosophizes about her situation, she does not think that her vocabulary is closer to reality than others, that it is in touch with a power not herself. Ironists who are inclined to philosophize see the choice between vocabularies as made neither within a neutral and universal metavocabulary nor by an attempt to fight one's way past appearances to the real, but simply by playing the new off against the old." [103, §4] Rorty contrasts irony by *common sense* in the following way: "The opposite of irony is common sense. For what is the watchword of those who unselfconsciously describe everything important in terms of the final vocabulary to which they and those around them are habituated. To be commonsensical is to take for granted that statements formulated in that final vocabulary suffice to describe and judge the beliefs, actions and lives of those who employ alternative final vocabularies." [103, §4, p. 74] This differs from the spectrum we suggest insofar that it does not place the assumption of the existence of a privileged vocabulary in contrast to irony, but rather the embracing of the current vocabulary as such. It might be true that the assumption of the existence causes one to suspect the current vocabulary to take the place of that privileged vocabulary.

cerns about the progress of science.

irony   quest for certainty

instability   captivity

It is tempting to also consider the measurement problem in the context of this spectrum. If we then replace quest for certainty with a quest for *the* solution, then the spectrum easily turns into a spectrum over the *quality* of solutions to the measurement problem. The quest for *the* solution is then contrasted by the quest for possible, contingent solutions. It seems, however, that attempts to locate approaches to the measurement problem on this spectrum still bear a taint of Wittgenstein's "dangerous fallacy" (see Footnote 95): We have not left behind the paradigm that *the measurement problem requires a solution*. A *reflection* on the measurement problem, thus, finds itself in a very different opposition to the quest for *the* solution.

possible, contingent solutions   quest for *the* solution
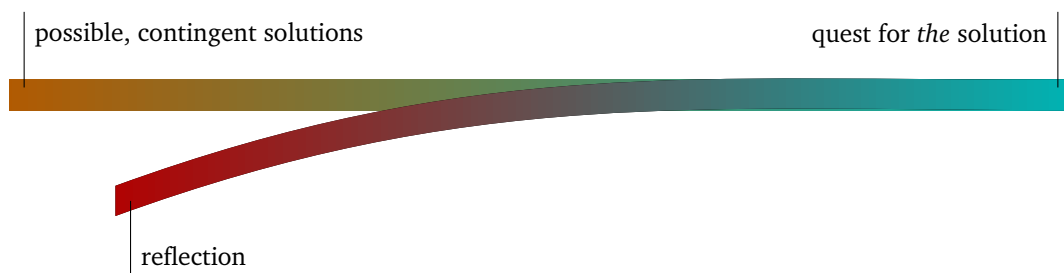
reflection

Ways of Doing Physics

In the perspective emerging in the previous Sections 7 and 8, the measurement problem appears as a symptom of a particular philosophic stance. It reflects the hope that we can exhaustively describe the world around us. The description turns into an unclouded mirror image. Concepts that aspire to be key to developing such an "understanding of nature"—rather than a "making sense of nature"—are the notion of *states*, of *information*, and of a *system*. In the following sections, we take a closer look at these terms and their role in doing physics.

9 The State

The perspective on quantum mechanics we have established in Section 5 puts emphasize on measurements rather than states. In a sense, we have degraded the quantum states, and more generally quantum density matrices, to a representation of probability distributions over measurement operators. From this perspective, the state hardly attains the role of a *state of being* or, as we discuss below, of a *state of knowing*. The state is rather a useful mathematical concept than an entity of ontological or epistemological import. The common framing of the measurement problem suggests the prevalence of a different understanding of quantum states. Taking a look at how the state is often introduced in text books, lets the measurement problem in its usual disguise appear as a reflection of a commonsensical understanding of quantum states among physicists.[98] In

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[98]"1. States: A state is a complete description of a physical system. In quantum mechanics, a state is a *ray* in a *Hilbert space*." [94, §2.1] "Postulate 1: Associated to any isolated physical system is a complex vector space with inner product (that is, a Hilbert space) known as the *state space* of the system. The system is completely described by its *state vector*, which is a unit vector in the system's stat space." [86, §2.2.1] "1. States: The set of states of an isolated physical system is in one-to-one correspondence to the projective space of a Hilbert space $\mathcal{H}$. In particular, any physical state can be represented by a *normalized vector* $\phi \in \mathcal{H}$ which is unique up to a phase factor. In the following, we will call $\mathcal{H}$ the *state space* of the system." [21, §4.2] "I. The state of a particle is represented by a vector $|\psi(t)\rangle$ in a Hilbert space." [112, §4.1] "*First Postulate:* At a

this regard, reflections about the measurement problem turn into an investigation of the effect of media (*Medienwirkungsforschung*).

On the one hand, there is the prominent placement of the state as subject of the *first* axiom or postulate of many axiomatic introductions. It might be a relict from classical mechanics to think of physical theories from the end of the state. The non-contextuality of classical mechanics allows for the absence of a concept of observables. Only in light of quantum mechanics, we see the urge to characterize subsets of phase-space forming a Boolean algebra to be regarded in analogy to projective operators.

On the other hand, the state is held—implicitly or explicitly—as (the basis of any) (complete) description of a physical system. Note that this is different from holding the state as the basis for a physical description of a system.[99] In the latter perspective, the state is a tool in a particular way of describing a given referent, termed the system. This description neither has to account for ways in which we establish reference to the system nor is there any restrictions on what counts as a system. Thus, also an observer can be subject to a physical description. The conclusion of Section 8 is that *describing* the observer removes him from the normative, intersubjective realm in which measurements can be made to serve as to provide legitimacy to physical descriptions.

The commonsensical notion of a state culminates in the following tacit assumption:

> Any system is in a state of being.

What defines and characterizes the existence of a system, is resolved in a state of some kind. This assumption already contains the idea that systems can be exhaustively described. The question merely is: What is the correct description? Does our description *correspond* with *the* description? Do we already *mirror* nature?

Physicists, being trained to assign states, understand the world through *systems with states*. The following statement provides an example of this perspectivity:

> A theory need not do anything more than enable us to make predictions about instrumental state **p** if we only want to make predictions

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fixed time t_0, the state of a physical system is defined by specifying a ket $|\psi(t_0)\rangle$ belonging to a state space $\mathscr{E}$." [22, §B.1]

[99]With a *physical description*, we do not mean to assume a neutral outside perspective onto a "language of physics." Instead, we emphasize a particular justificatory context that permits for particular formulations and approaches to problems.

about things we can measure. However, if we are interested in go-
ing beyond what we observe with our instruments, senses, etc., and
asking what reality is 'really' like then it seems we must consider the
ontic and associated epistemic states. [59, §1]

In this view, philosophic questions about the "real reality" might require different
classes of states, but states nonetheless. It is noteworthy that this rarely regarded
as a narrow perspective, unlike the perspective of someone who is trained to
use a hammer, and to whom everything looks like a nail. The statement is not
only interesting for its use of the term "state" but also for its reference to a "real
reality." The latter hints at the solutionism that Wittgenstein later regarded as the
"dangerous fallacy" he had committed in the Tractatus (see also Section 8.5.1).

This is not to say that there is clear line drawn between physical problem
that ask to be addressed under the assumption that a solution can be found and
philosophical problems that preclude such an approach. Instead, this serves to
illustrate that approaching problems under the assumption of solvability is *one
contingent* approach. At the same time, the implicit distinction between pre-
dictions about the mere apparent and the "real reality" puts the ontic-epistemic
divide into the light of the distinction between *practical activity* and *certain knowl-
edge* as criticized by Dewey. It is tempting to see the endorsement of an epistemic
view, as the rejection of the idea of certain knowledge. On a closer look, it seems
that epistemic views on quantum mechanics do not abandon the quest for cer-
tainty as we discuss subsequently.

The above assumption is challenged if the we are deprived of the transparency
of a spectator theory. One consequence of the grappling to commensurate con-
textuality with the idea of physics excavating the truth out there is the distinc-
tion between ψ-epistemic and ψ-ontic models. The reading of quantum states as
states of being is then complemented by a neo-Kantian conception of quantum
states as states of knowledge. In other words, the metaphysical interpretation
of states is complemented by an epistemological one, and the assumption turns
into:

$$\text{Any } \left\{ \begin{array}{c} \text{object} \\ \text{subject} \end{array} \right\} \text{ is in a state of } \left\{ \begin{array}{c} \text{being} \\ \text{knowing} \end{array} \right\}.$$

The debate about which of these models applies [59, 114, 60, 95, 105] has man-
ifested the distinction between the two branches in the above assumption as a
dichotomy. The assumption, however, remains and sets how to understand states
as "complete descriptions" or theories as "universal" [80]. With this, doubts con-
cerning the meaning of *assumption* in light of any privileged "state"-vocabulary

(see Section 8.5) that reflect the captivity of any *Absolutheitsanspruch*[100] are still pressing.

The Kantian turn to epistemology, manifested in the notion of epistemic states, is accompanied by a second development, i.e., the appearance of the notion of "information" in doing physics [114, 105]. In particular, epistemic models have employed or been argued for by reference to information. Quantum Bayesianism (Qbism) is located on the far end of the "informational" readings of quantum mechanics.

9.1   Quantum Bayesianism

Quantum Bayesianism is a subjectivist reading of quantum mechanics that understands quantum mechanics as a theory about information and aims for its representation within the framework of probability theory. In this reading, the quantum states "are not something out there, in the external world, but instead are expressions of information," [41, §II] or "compendia of beliefs." [41, §IV] In Fuchs' pragmatistic reading, quantum mechanics provides a users' manual:

> Qbism has a story to tell on both quantum *states* and quantum *measurements*, but what of quantum *theory* as a whole? The answer is found in taking it as a *universal* single-user theory in much the same way that Bayesian probability theory is. It is a users' manual that *any* agent can pick up and use to help make wiser decisions in this world of inherent uncertainty. To say it in a more poignant way: In my case, it is a world in which *I* am forced to be uncertain about the consequences of most of my actions; and in your case, it is a world in which *you* are forced to be uncertain about the consequences of most of *your* actions. [41, §III]

> The world is filled with all the same things it was before quantum theory came along, like each of our experiences, that rock and that tree, and all the other things under the sun; it is just that quantum theoy provides a calculus for gambling on each agent's own experiences—it doesn't give anything else than that. [41, §III]

This exposes Qbism to Habermas' critique of the monologistic traits present in Pierce's pragmatism (see Footnote 83). In Section 8.4, the need to intersubjectively establish agreement arose from the normative nexus between past to

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[100]We refer here to the claim for absoluteness, addressed by Specker, as discussed in Section 8.5.1.

future action.[101] Thus, the picture of a theory as a "universal manual" for single users is oblivious of the problematic nexus between particulars and universals that affects meaning as well as inductive inference.

These concerns turn into an issue that arises from within in the picture: Do agents have the freedom to perform unwise actions? In the case of the Wigner's-friend experiment this would amount to the friend changing the measurement he performed on the system S. So for the few certain cases, where we expect that we have a deterministic prediction—i.e., in the cases of performing what we think are equivalent measurements—,any deviation can be "explained" by an agent inside the system "acting unwisely." It is, however, not clear whether there would be the need to "explain" anything, because that would be a justificatory act for why a certain behavior or consequence is covered by a given rule—i.e., an *intersubjective* discursive act. Either universality or the single-user focus is in jeopardy. Finally, it seems that we must just regard it as "the consequences of wise actions in an inherently uncertain world," if there are no normative measures for this constituting an "unexpected outcome" in the universal single-user theory. We are left with a qbistic brain-in-a-vat.[102] The "unwise friend" resembles Maxwell's demon. In the common reading of the measurement problem, the friend has no such agency independent of our description of him and, thus, cannot obstruct matters like this.[103] One might suspect that it is the possibility for unwise behavior exceeding our description of the demon which turns him into an "intelligent being" [117]. At the same time, Bennett's solution for the puzzle caused by the demon [11] is to subject him to our description, and, thus, deprive him of such agency outside our words accounting for it.

For Fuchs, the above mentioned concerns are not as pressing as the doubt that Qbism might not be a realistic view. In other words, not the scepticism re-

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[101]Note that Fuchs uses "normative" in a very different sense when he remarks: "As probability theory is a *normative* theory, not saying what one *must* believe, but offering rules of consistency an agent should strive to satisfy within his overall mesh of beliefs, so it is the case with quantum theory." [41, §III] Normativity says here something about the quality of the statements of a theory, and nothing about how to understand these statements. This normativity occurs in a setting of mere description short of the need to establish meaning.

[102]It is possible to preclude this interference of another agent's actions "against the wise rules" by restricting the applicability of the manual, or by simply assuming all agents to act wisely. Again, if there is no intersubjective agreement on what constitutes rules of wise actions, then it might not even be justified to say that an agent broke the rules. This leads eventually to the questions how to understand any rules if we do not intersubjectively agree on them. And, thus, what justifies to speak about such rules.

[103]Recall that the initial measurement performed on the friend revealed the friend's intentions as to what measurement to perform on what system.

garding the above assumption is the problem but the scepticism regarding the question whether one is located in the correct branch of the assumption. If we regard this as a response to being discredited for being in the wrong branch of the assumption—as an indicator for ways of thinking and talking among the quantum theorists—, then the prospect of this work to be regarded as an interesting contribution is rather dim. To be clear, Fuchs does not abandon the quest for certainty as it becomes evident in the following statement:

> If quantum theory is a user's manual, one cannot forget that the world is its author. And from its writing style, one may still be able to tell something of the author herself. The question is how to tease out the psychology of the style, frame it, and identify the underlying motif. [41, §IV]

The statement is diametrically opposed by Rorty's interdependent rejections of the ideas of a "truth out there" and a "language of nature":

> We need to make a distinction between the claim that the world is out there and the claim that truth is out there. To say that the world is out there, that it is not our creation, is to say, with common sense, that most things in space and time are the effects of causes which do not include human mental states. To say that truth is not out there is simply to say that where there are no sentences there is no truth, that sentences are elements of human languages, and that human languages are human creations. Truth cannot be out there — cannot exist independently of the human mind — because sentences cannot so exist, or be out there. The world is out there, but descriptions of the world are not. Only descriptions of the world can be true or false. The world on its own — unaided by describing activities of human beings — cannot. [103, §1]

> The world does not speak. Only we do. The world can, once we have programmed ourselves with a language, cause us to hold beliefs. But it cannot propose a language for us to speak. Only other human beings can do that. The realization that the world does not tell us what language games to play should not, however, lead us to say that a decision about which to play is arbitrary, nor to say that it is the expression of something deep within us. The moral is not that objective criteria for choice of vocabulary are to be replaced with subjective criteria, reason with will or feeling. It is rather that the

> notions of criteria and choice (including that of 'arbitrary' choice)
> are no longer in point when it comes to changes from one language
> game to another. [103, §1]

The argument for why Qbism should not be denied the qualification of a "realist view,"[104] is the assumption of an a priori given structure of systems with its Hilbert-space dimension as intrinsic "universal capacity." Thus, the dimension of a system does not correspond to the number of different results a given experimental setup allows to distinguish. Instead, there is the assumption of a universal and ultimate limit on the "information" we can have about a system. This implies a particular notion of "information" and its relation to knowledge: Knowing corresponds to having information. Thus, we end up with a Cartesian immediate awareness of the "information we have" and a consequent removal of reflexive dependences of knowledge or information. Even if one refrains from assigning cognitive meaning to "information," the placement of an absolute limit on information maintains the idea of an objective distinguishability short of the need to be established.

10   Information

Imagine reprinting measurement results in different fonts. Is then `0` different from *0,* or `1` different from *1*? Is such a `description of an experiment` describing a different experiment from the one with this *description of an experiment*? It is an *established convention*[105] that `0` and *0,* `1` and *1* mean the respective

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[104]Maybe we should rather regard this as the fear of Qbists to be expelled as heretics from the state-religion of "realism" upheld by some physicists. For realism vouched for a priestly access to a higher authority. "[A]s long as we think that 'the world' names something we ought to respect as well as cope with, something personlike in that it has a preferred description of itself, we shall insist that any philosophical account of truth save the 'intuition' that truth is 'out there.' This intitution amounts to the vague sense that it would be *hybris* on our part to abandon the traditional language of 'respect for fact' or 'objectivity' — that it would be risky, and blasphemous, not to see the scientist (or the philosopher, or the poet, or *somebody*) as having priestly function, as putting us in touch with a realm which transcends the human." [103, §1] This quote might activate a view on religion oblivious of reflection—a view commonly held among (natural) scientists and left-wing thinkers assigning themselves the authority of rationality. We do not share such converse characterization in full generality. I owe to Pepe Elwert for insights into critical perspectives in Protestant theology.

[105]"Als der Grundzug alles menschlichen Daseins erscheint es, daß der Mensch in der Fülle der äußeren Eindrücke nicht einfach aufgeht, sondern daß er diese Fülle bändigt, indem er ihr eine bestimmte Form aufprägt, die letzten Endes aus ihm selbst, aus dem denkenden, fühlenden, wollenden Subjekt hervorgeht." [19, as quoted in [54]]

same symbol, while {0,*0*} and {1,*1*} mean different symbols. Landauer's "Information is physical"[106] suggests that there is one set of mutually different symbols that is exempt from being conventional and, thus, offers a resort free of reflexive dependences.[107] When Landauer's slogan is contrasted by the slogan "It from bit" attributed to Wheeler[108], then the existence of such an ultimate set of symbols is not questioned but merely where to find this set.[109] Wheeler acknowledgement of reflexive dependences and the mentioning of "observer-participancy," in the following statement allow to doubt whether this contrasting reflects Wheeler's own position:

> To endlessness no alternative is evident but loop, such a loop as this:
> Physics gives rise to observer-participancy; observer-participancy gives
> rise to information; and information gives rise to physics. [132]

At the same time, these observations did not hold Wheeler from proclaiming "supreme goals" with reference to an absolute understanding of existence:[110]

> Niels Bohr tells us from 'a radical revision of our attitude as regards
> physical reality' and a 'fundamental modification of all ideas regard-
> ing the absolute character of physical phenomena.' [...] Bohr's mod-

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[106]"Information is not a disembodied abstract entity; it is always tied to a physical representation. It is represented by engraving on a stone tablet, a spin, a charge, a hole in a punched card, a mark on paper, or some other equivalent. This ties the handling of information to all the possibilities and restrictions of our real physical wor[l]d, its laws and its storehouse of available parts." [75, §1]

[107]In light of the following statement, one might assume that Landauer himself suggests such a fundamentalist reading: "The view I have expounded here makes the laws of of physics dependent upon the apparatus and kinetics available in our universe, and that kinetics in turn depends o the laws of physics. Thus, this is a want ad for a self-consistent theory." [75]

[108]"every *it* — every particle, every field of force, even the spacetime continuum itself — derives its function, its meaning its very existence entirely — even if in some context indirectly — from the apparatus-elicited answers to yes or no questions, binary choices, *bits*." [132]

[109]Again, Landauer seems to have entertained such a dichotomy: "Wheeler in a number of discussion[s] has an adventurous view in which the laws of physics result form our observation of the universe. Wheeler's details are not my details, but we both depart from the notion that the laws were there at the beginning." [75]

[110]Wheeler attempts to resolve the tension between a reductionist reading of "It from bit" and the infinite regress involving possibly intersubjective elements of an "observer-participancy" seems to be addressed in the idea of a "law without law." It seems not so clear whether this attempt to retreat to a vanishing point joins both aspects or rather abandons both of them. Is a vanishing Archimedean point rather vanishing, and, thus, not an Archimedean point, or an Archimedean point and thus not vanishing? How can any point be vanishing enough to escape the reflexive dependence of "observer-participancy" and still be Archimedean?

est words direct us to the supreme goal: *Deduce the quantum* from
an understanding of *existence* [132]

Landauer and Wheeler represent merely two of the multiple ways of joining
computer sciences and information theory with physics. These joins have put
emphasis on the descriptive aspects of physics (see Characteristic (C1)). The
notion of computers being exhaustively described machines following the rules
they have been programmed to follow finds its correspondence in the "laws of
nature":

> The laws of physics are essentially algorithms for calculation. These
> algorithms are significant only to the extent that they are executable
> in our real physical world. [75]

The focus, thus, turns to *finding the true laws or programs* without considering
how we come to understand these laws, and whether and how we find empirical
evidence for the applicability of these laws.[111]

A different idea of a computer emerges if we inversely transfer normative
ideas of physical theories (see Characteristic (C2)). Then, computers appear as
physical experiments. If they show unexpected behavior (glitches), we are lead
to revise any of the parts of the system of knowledge involved in the description
of their behavior.

In the subsequent sections, we examine aspects of the connection between
information theory and physics in greater detail.

10.1   Erasure, and the 2nd Law

Landauer concluded from the observation that any computation is carried out
on a *physical* computer combined with the 2nd Law of thermodynamics that the
erasure of any single bit entails the dissipation of $kT \ln 2$ free energy.[112] Erasure

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[111]Interestingly, with the increase of computational resources and the alongside increase of
complexity the degree to which we understand the rules that we program computers to follow
decreases and becomes increasingly heuristic. Merely for well-controlled scenarios, we can fully
describe (components of) computers—similar to physical experiments. This has resulted in what
Felix von Leitner has termed the "bugwave" (Bugwelle), i.e., lists of reported bugs that grow
faster than bugs are removed [123, 122]. How do we suppose to understand the rules nature
is programmed to follow—assuming that the picture of a programmed nature is viable—, if we
cannot even understand the rules we came up with? Even if we could formulate them, we might
not be able to understand them.

[112]"Landauer's principle, often regarded as the basic principle of the thermodynamics of infor-
mation processing, holds that any logically reversible manipulation of data [. . .] must be accom-

here means to set the physical system under consideration into a particular low-entropy macrostate.[113] If we start off in a state with higher entropy, this means to *reduce the entropy* and, thus, by the 2nd Law we expect this to be merely possible by investing work and dissipating heat. The picture of erasure is here the following where we assume the gas to be coupled to a heat bath:

$$\tag{9}$$

The molecules on the left occupy a larger phase-space volume than on the right. More precisely, the setup on the left represents a configuration in which a larger phase-space volume is accessible than in the setup on the right. So far, erasure has merely a thermodynamic meaning, quite opposite to its common use as a concept for information. How can we understand the above as *erasure of information*?

10.1.1   What does "erasure" mean?

How can we understand the depicted process as *erasure of information*? The answer depends on how we understand *information*. Let us consider the data-based conception of information established in Ref. [34], building on the following notion of a datum.

> **The Diaphoric Definition of Data (DDD):** A datum is a putative fact regarding some difference or lack of uniformity within some context. [34, §1.3]

In this context, information consists of one or more data.[114] Let us assume that the differences characterizing data can be labelled.[115] Such a labelling is an

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panied by a corresponding entropy increase in non-information bearing degrees of freedom of the information processing apparatus or its environment." [12, §]

[113]Norton locates here one of two fallacies commonly committed when it comes to discussions of Maxwell's demon: "I also noted that arguments for Landauer's Principle repeatedly used the same incorrect assumptions: that erasure must compress phase volume or that additional thermodynamic entropy derives from the probabilities of so-called 'random' data." [89, §1] This foreshadows problems that we discuss subsequently.

[114]The complete definition of semantic information includes further necessary characteristics: "**The General Definition of Information (GDI):** σ is an instance of information, understood as semantic content, if and only if: (GDI.1) σ consists of one or more *data*; (GDI.2) the data in σ are *well-formed*; (GDI.3) the well-formed data in σ are *meaningful*." [34, §1.2]

[115]It is tempting to think of this labelling in terms of "telling things apart." Adding this linguistic aspect might, however, go too far in respect of the use of the word information in the context of thermodynamics.

(injective) mapping into the set of bit strings of a given length.[116] The question thus is: What are possible (injective) mappings of the above scenario into a set of bit strings? What are possible discretizations of the above scenario? How do we locate data in the above scenario?

A first map consist of assigning a single bit to each of the sides in the above scenario: Let us introduce a piston and assume that the bit 0 corresponds to the left and bit 1 to the right of the following picture.

In this arrangement, we map *the position of a piston*[117] to a single bit. Instead of employing the extremal positions of the pistons, we might as well use two disjoint intervals, possibly with each including one of the extremal positions. Let us call this mapping a *binary macroscopic mapping*. To turn this into a function from the entire range of possible piston positions to the set $\{0, 1\}$, we have to expand the intervals so that they form a partition of that range. More generally, we can partition the volume in the space of macrovariables that can be associated with the piston into $M = 2^m$ parts and associate with each such parts an m-bit string.

If the depicted thermodynamic process is irreversible, then moving the piston to the left comes at the cost of some irretrievable work. If the process is reversible (or made reversible by, e.g., coupling the piston to a reversible battery), then erasure amounts to reversibly setting the piston to the left if we find it in a position to the right and else do nothing. It seems that Landauer's principle is not referring to macrovariables of a piston as carriers of information, when devising a *necessary* thermodynamic cost of erasure.

Let us ignore the piston, and consider mappings that instead relate to the microstate of the N particles. If these particles are indistinguishable, and if we assume that we can merely tell whether particles are in the left or in the right half of the volume, then we obtain a so-called *Hamming weight mapping* into the n-bit strings with $N = 2^n$. This is the simplest case of partitioning the volume into K parts that allows for

$$C = \frac{(N + K - 1)!}{N!(K - 1)!}$$

[116]This assumes a finite memory. Whether or not this assumption is warranted is an own philosophical debate. More generally, we could turn to maps into the Kleene closure S^*.

[117]We neglect for the moment other thermodynamic variables, such as pressure.

different distributions of N particles among the K partitions. This allows to injectively map the distributions to bit strings of length $\lceil \log_2 C \rceil$. We call this a *K-partition mapping*. This mapping can further be generalized to partitions of a 6-dimensional phase-space. If the particles are distinguishable, then we are left with K^N different distributions. If, furthermore, $K = 2$, then we obtain a mapping into the N bit strings.

The microscopic association of states to information divides along the question whether particles are distinguishable or not. We elaborate on the *distinguishability* of particles in Section 11. For the simple case of a one-particle gas, however, both mappings join. In the following section, we discuss turn to this case.

10.1.2 Erasing a single bit, and Szilard's one-particle gas

Maroney discusses in Ref. [79] how erasure of a single bit can be transferred to a thermodynamic process, and shows that this process is not necessarily irreversible. The one-particle gas also considered by Szilard [117] is the simplest case with $N = 1$ and $K = 2$. We recapitulate the discussion here with in the perspective of the question where information is to be located. This means that we discuss step-by-step the process depicted in Figure 2. We consider of a one-particle gas in contact with a heat bath at temperature T. Instead of the piston, we imagine a barrier that can be moved horizontally, possibly while being coupled to a battery. Most importantly, the barrier can be removed entirely and be inserted somewhere else, unlike the piston above.

Initially, the barrier is in the middle of the volume, separating the volume into two equal halves. The particle is either in the left half of the volume, (1.z), or in the right one, (1.o). The case (1.z) is associated with the bit zero, and the case (1.o) with the bit one. There is no difference in associating the bit zero *because* the gas is to the left of the barrier, or *because* the one-particle is in the microstate "in the left half of the volume." The microstate perspective and the macrostate perspective yield the same bit assignment. This changes, however, in the second step.

In (2), we remove the barrier. The gas, thus, expands into the entire volume. As the phase-space volume accessible to the particle doubles, the entropy of the gas increases by $k \ln 2$ if we assume that the particle is uniformly distributed across $V/2$ before the removal of the barrier and uniformly distributed across V after the expansion. From the microstate perspective shown in (2.z.a) and (2.o.a), the bit assignment remains well-defined.[118] From the macrostate

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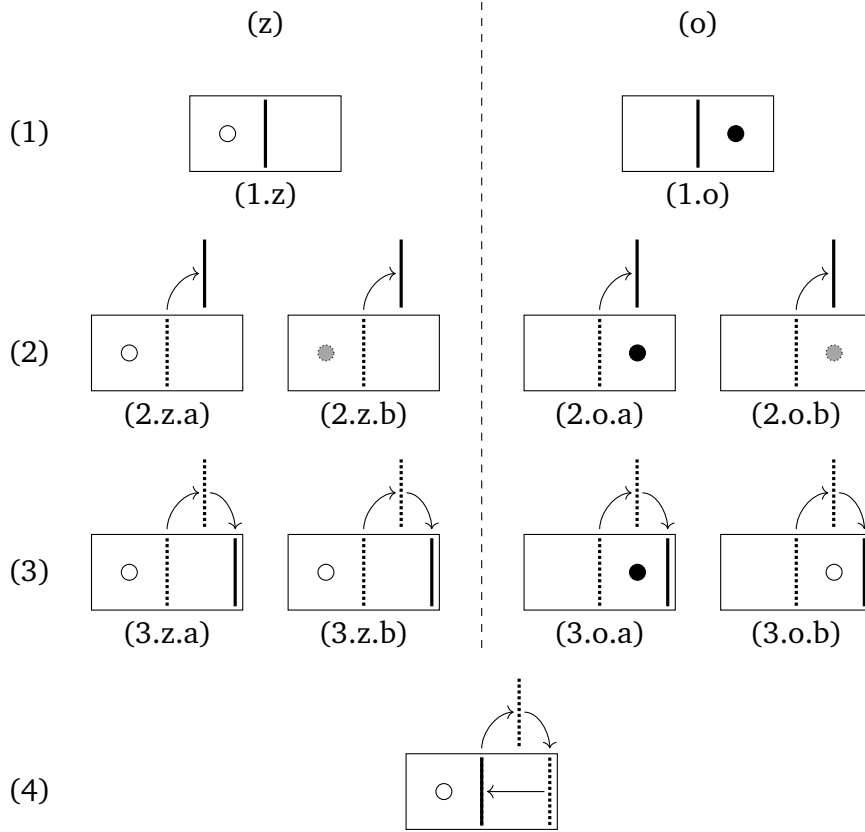[118]We take the center of mass as the reference point here.

Figure 2. A detailed resolution of an erasure process.

perspective shown in (2.z.b) and (2.o.b), however, the assignment of a bit by whether the particle is either to the left of the barrier or to the right of it is no longer well-defined.[119] In the latter case, one might say that we lost one bit of information. Maroney associates correspondingly the erasure to this step.

The barrier is inserted back into the volume in step (3). Inserting the barrier

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[119]There are other possible macroscopic bit assignments independent of the barrier. If we assume the bit 0 (1) to correspond to the gas being confined the left (right) half of the volume, then, as Xavier Coiteux-Roy points out, allowing the gas to expand into the entire volume leads to a third macrostate that one might associate with erasure. Like in ternary logic, we have a third state for "erased information," corresponding to the right-hand side in 12. Information-baring macrostates do, however, not have to be low-entropy states. We could also permute the assignment of the triple {0, 1, "erased info"} with drastic consequences for Landauer's principle. These considerations also show that the idea of locating erasure in step (2), i.e., associating it with the removal of the barrier, and, thus, with the loss of the ability to "talk about bits," is strongly dependent on the encoding. In a sense, there hardly seems to be a *natural* notion of erasure. In light of Fleck's statement in Footnote 2, this is not surprising.

at the far right of the volume, turns the bit to zero from the macrostate per-
spective. By compressing the volume in step (4), we join again the micro- and
that macrostate perspective. Maroney calls (4) correspondingly the "compres-
sion step." From the microstate perspective, however, the erasure happens only
now: If the temperature T of the heat bath is low enough, then a particle ini-
tially in a microstate corresponding to the bit one is only now reset to the bit
zero. Note that the isothermal compression is reversible.

Maroney describes the following inverse process: Let gas expand and load
the battery (or lift a weight) with the extracted $kT \ln 2$ amount of work. Then,
remove the barrier from the far right end of the volume and allow the gas to
expand. Finally, insert the barrier in the middle. This inverts the erasure process
if initially the microstates (1.z) and (1.o) were assigned the same probability
weight 1/2. We recovered the work used in the compression step (4).[120]

This exposes a tacit assumption we have made in step (1) and that is fatal
in two sense: We have assumed that the microstate and the macrostate perspec-
tive merge in the same bit assignment, (1.z) or (1.o). Firstly, from the macro-
perspective, this assignment is only warranted if we had previously gone through
step (3) and (4) for (1.z), or correspondingly (3') and (4') proceeding from the
far left end of the volume for (1.o). Secondly, if we know from the macro-
perspective that either (1.z) or (1.o) is the case, then we can reset to zero by
either doing nothing or by extracting the work when the barrier is push to the
left to then use it again to compress the gas from the right. In hindsight, the
introduction of a piston above was not as ungrounded as it might have initially
appeared.

We have, thus, discussed for two cases of initial thermodynamic states—i.e.,
probability distributions over microstates—how to reversibly proceed to the low-
entropy state in step (4). The argument generalizes to any distributions as laid
out in Ref. [79]. Thus, we are presented with a thermodynamically reversible
process that implements a logically irreversible erasure process. Note the differ-
ent meanings of erasure from the different perspectives: If one assumes a macro-
scopic perspective, then "erasure" rather means "loss of a distinction." This is
essentially a mixing process and hints at the path that we pursue in Section 10.2.
The idea of a "loss of distinction" is meaningless from a microstate perspective.
Instead, "erasure" turns into "setting to zero." Given that from a microstate per-
spective, we are already in a low-entropy state, we can isothermally extract the
work that is required to compress the gas again—if not already classical or quan-

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[120]Already in Ref. [77], it is noted that this is possible if the initial thermodynamical state is a
uniform distribution.

tum mechanics sufficiently describes the gas for a reversible manipulation. Thus, Landauer's principle merely applies if we somehow intertwine the microscopic and the macroscopic perspective.

10.1.3   Noumenal and phenomenal information

We started off by trying to understand the process in (9) as "erasing information." In the examination in Section 10.1.2, this process, however, acts as the last step in a larger picture. This last step alone can be understood as erasure— i.e., as putting all bits to zero—*only if* one considers the gas from a microscopic perspective. In fact, if one includes the macroscopic perspective, and if one understands the bit zero as "the one particle is to the left of a barrier" and the bit one as "the particle is to the right of a barrier," then the last step is rather understood as merging microscopic and macroscopic information. Note here the divergence implicitly present in the right-hand side of (9): Either the macroscopic bit assignment is not well-defined, because the barrier has been removed as in step (2). Or the macroscopic and the microscopic bit assignment differ with non-zero probability as in step (4). Only by compressing to a low entropy state, microscopic and macroscopic information are the same.

In thermodynamics, the "observables" are the macrovariables.[121] Microstates however are not, unless the gas is in a low-entropy macrostate for which the two coïncide. Thus, if we require that the data we are concerned with correspond to observable entities, then either we have to employ a macrostate mapping or restrict ourselves to low-entropy states. In the latter case, we might transition isothermal-reversibly to any other state with zero-entropy difference as discussed above. In the former case, erasure consists of removing the barrier as in step (2). Therefore, neither of the cases applies to "erasure" as depicted in (9). The right-hand side does not correspond to phenomenal information. The reading of (9) as

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[121]Jaynes remarks: "[T]he fundamental operational definitions of such terms as equilibrium, temperature, and entropy — and the statements of the first and second law — involve only the macrovariables observed in the laboratory. They make no reference to microstates, much less to any velocity distributions, probability distributions, or correlations. As Helmholtz and Planck stressed, this much of the field has a validity and usefulness quite independent of whether atoms or microstates exist." [68, §8] Thus, he concludes: "The entropy of mixing does indeed represent human information; *just the information needed to predict the work available from the mixing*." [68, §5] In a sense, the idea goes back to Wigner: "I have profited from discussions of these problems, over many years, with Professor E. P. Wigner, from whom I first heard the remark, 'Entropy is an anthropomorphic concept.'." [67, §VII] Therefore, we can regard Jaynes' stance as a reflection of Wigner's critical attitude towards the (anthropomorphic) use of the term measurement (see Footnote 30).

mapping any associated string to the all-zero string, thus, involves assigning the right-hand side *noumenal* information—"information-in-itself"—that is exempt from being observed.
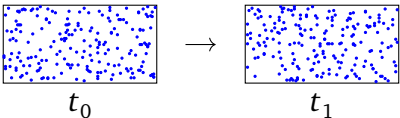
This exposes a crucial difference between thermodynamic entropy and Shannon entropy. Shannon considered bit strings emitted by a source. This setup allows to consider the probabilities characterizing the source in a frequentist reading, i.e., as the result of a statistical analysis of empirical data obtained from the source. Thermodynamic states in statistical mechanics as probability distributions over microstates cannot be understood as derived from an empirical sample of such states. This suggests to regard thermodynamic entropy as a function of phasespace volumes instead of probability distributions.

10.2   Gibbs' paradox, and tricking demons

Despite the critical perspective on Landauer's principle, information and thermodynamics might be brought together in an idea that could be termed "relative thermodynamics." The inspiration comes from Gibbs' thoughts on mixing processes and the ensuing paradox. The crucial observation for "thermodynamic information"—essentially information that is associated with macrovariables— is that it can be lost in the way we described in Section 10.1.2.
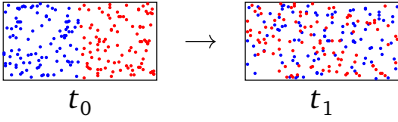
10.2.1   Mixing gases

Imagine a volume with N molecules. We assume that a molecule initially observed at a particular position x has an equal probability to be observed throughout the entire volume after a sufficiently long period of time has passed. This implies that molecules that are at an initial time t_0 on the left are at a later t_1 distributed across the entire volume; and so are the molecules that were on the right at time t_0:

$$\tag{10}$$

Let us compare this to another, slightly different scenario: We color the molecules in such a way that molecules initially on the left are blue, and the molecules initially on the right are red. The molecule positions and velocities are left unchanged. We merely make the distinction between "molecules on the left at

time t_0" and "molecules on the right at time t_0" visible.

$$\boxed{} \rightarrow \boxed{} \tag{11}$$
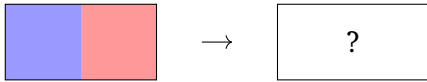$$t_0 t_1$$

The trajectories of the single molecules are the same in both scenarios, i.e., the kinematics coïncide: If we pick out a molecule, and follow it around, it behaves just the same, no matter its color.
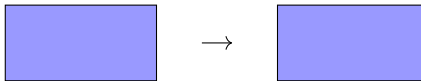
We now turn to the question which of the processes qualifies as *mixing* and why.[122] If we understand mixing as "molecules that are initially on the right are finally evenly distributed," then mixing occurs in both scenarios, (10) and (11): The gas mixes, no matter whether it is color-wise homogeneous or in-homogeneous. This rephrases Gibbs' observation.[123]

> Again, when such gases have been mixed, there is no more impos-
> sibility of separation of the two kinds of molecules in virtue of their
> ordinary motions in the gaseous mass without any especial external
> influence, than there is of the separation of a homogeneous gas into
> the same two parts into which it has once been divided, after these
> have once been mixed. [45]

Ostensibly, however, scenario (11) qualifies as mixing while (10) does not. Mixing here means the process of loosing the ability to tell the blue and the red molecules apart.

$$\boxed{} \rightarrow \boxed{\;?\;} \tag{12}$$

As scenario (10) does not express the ability in the first place, it cannot be lost.

$$\boxed{} \rightarrow \boxed{}$$

The initial *measurable* distinction between blue molecules on the left and red molecules on the right, is essential to this understanding of mixing. The reference to molecules, however, is not: It is sufficient to note that, at time t_0, the left half was blue while the right one was red. In other words, we merely require the ability to measure that "all molecules to the left are blue, and all to the right are red" which does *not* imply any statements about single particles. To summarize,

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[122]Jaynes' discussion of Gibbs' paradox in Ref. [68], has inspired our considerations strongly.

[123]The quote is explained in detail in Ref. [68]. It might be helpful to realize the following grouping: "Again, when such gases have been mixed, there is no more {...} than there is {...}."

we note: Mixing is, then, a process characterized by *loosing a distinction*—i.e., loosing a phenomenal datum as defined in Section 10.1.1.

An objection to the above scenario is the following: In thermodynamics the distinction is not between colors of molecules. So what does the above example say about the case where all molecules are initially in the left half of the volume?

t_0
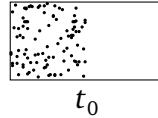
Then, mixing corresponds to the expansion of the molecules into the entire volume. This case compares to the colored scenario above, if we associate colors with the ability to measure *densities*: In the one-colored case, we can merely measure the molecule density of V, and not of smaller volumes. The two-colored case corresponds to a higher resolution of the density measurements. Conversely, when we assigned different colors to molecules we primarily introduced a *measurable difference* between the molecules on the left and on the right.

Again, there is a tacit assumption with important consequences: As in Section 10.1.2, we assume in the scenario (11) that the microstate and the macrostate join. If we can merely manipulate and observe macrostates, then we have to argue how this is warranted. In Section 10.1.2, the compression step (4) plays an important role: It allows to establish the macrostate datum while investing work. We can also imagine bringing two volumes in contact and, thereby, establishing a macrostate difference. Inversely, we can observe the datum by its ability to be turned into work. A piston, or solid barrier, implements a barrier for data that result in pressure differences. For molecules of different types, a piston would have to interact only with molecules of *one* type and not with molecules of the other. With this we re-establish a connection between information and thermodynamics: Entropy measures the amount of work that we can extract from a macroscopic difference by mixing. Therefore, entropy is *not the entropy of a system*, but rather of a set of observables for that system.[124] Consequently, the

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[124]Van Kampen remarks to this end:"Thus, whether such a process is reversible or not depends on how discriminating the observer is. The expression for the entropy (which he constructs by one or the other processes mentioned above [mixing of the same or of different gases]) depends on whether or not he is able and willing to distinguish between the molecules A and B. This is a paradox only for those who attach more physical reality to the entropy than is implied by its definition." [121, §4] Similarly, Jaynes states: "Consider, for example, a crystal of Rochelle salt. For one set of experiments on it, we work with temperature, pressure, and volume. The entropy can be expressed as some function $S_e(T, P)$. For another set of experiments on the same crystal, we work with temperature, the component e_{xy} of the strain tensor, and the component

2$^{\text{nd}}$ Law is a statement about how we describe the world and less about how the world is.

10.2.2   Gibbs' paradox

Gibbs' paradox is a variant of the above considerations. We recount the paradox following [68, §2]. Let us return to the previous setting, and interpret the colors as different types of ideal gases. On the left hand side, in a volume V_1, there are N_1 molecules of an ideal gas of type 1 at temperature T_1. On the right hand side, separated by a barrier, there are N_2 molecules of an ideal gas of type 2 in a volume T_2 at temperature V_2. If we assume that $T_1 = T_2 =: T$ and $V_1/V_2 = N_1/N_2$, then the pressure is equal, $P_1 = P_2 = NkT/V_i =: P$. When we remove the barrier, the gases mix, and we end up with $N = N_1 + N_2$ molecules in a volume $V = V_1 + V_2$.

$$V_1, N_1, T_1, P_1 \quad V_2, N_2, T_2, P_2 \qquad V, N, T, P$$
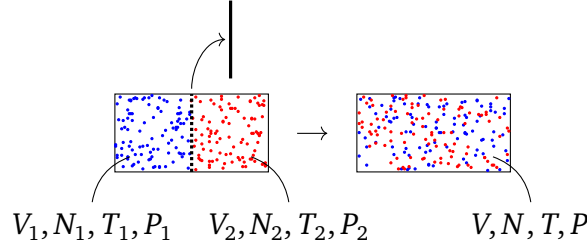
Temperature, pressure, and total energy remain the same. The difference in thermodynamic entropy is then

$$\Delta S = S_{\text{fin}} - S_{\text{init}} = Nk \log V - (N_1 R \log V_1 + N_2 R \log V_2)$$
$$= Nk \left[\frac{V_1}{V} \log\left(\frac{V_1}{V}\right) + \left(1 - \frac{V_1}{V}\right) \log\left(1 - \frac{V_1}{V}\right) \right].$$

If we the volume V_1 are V_2 are equal, and, thus, $V_1/V = 1/2$, then the entropy difference is

$$\Delta S = Nk \log 2.$$

P_z of electric polarization; the entropy as found in these experiments is a function $S_e(T, e_{xy}, P_z)$. It is clearly meaningless to ask, 'What is the entropy of the crystal?' unless we first specify the set of parameters which define its thermodynamic state. One might reply that in each of the experiments cited, we have used only part of the degrees of freedom of the system, and there is a 'true' entropy which is a function of all these parameters simultaneously. However, we can always introduce as many new degrees of freedom as we please. [...] There is no end to this search for the ultimate 'true' entropy until we have reached the point where we control the location of each atom independently. But just at that point the notion of entropy collapses, and we are no longer talking termodynamics!" [67, §VI]

The ability to tell apart particles on the left and particles on the right causes an entropy difference that is not present in the microscopically identical process depicted in (10). This is analog to the observation about erasure above: The removal of the barrier in step (2) only leads to an increase in entropy, if we know that the particle is either on the left or on the right of the barrier. This knowledge is merely thermodynamically described if it constitutes a macroscopic datum that can be used to extract the corresponding $kT \log 2$ of work.

The described scenario becomes paradoxical, if we assume that the difference between between red and blue molecules is continuous. The entropy difference, and, thus, the amount of extractable work does not depend on "how different" the particles on the left and on the right in (10.2.2) are. It merely matters that we can make out a difference. This reminds of the opposition of orthogonality and the continuity of the Hilbert space in quantum mechanics and Einstein's related concerns discussed in Section 6.1.

10.2.3   Tricking demons

While the macrovariables are the "observables" of thermodynamics and statistical mechanics—thus, an essential ingredient of these theories—, the microstates are of no necessity for a description of thermodynamic observables.[125] In the following, we aim to explore this observation by a thought experiment addressing the question what it takes to trick a demon that is fully aware of the coordinates of an N-particle system in $6N$-dimensional phase-space into seeing a violation of the second law.

As discussed in Ref. [68, §6], a difference in the ability to distinguish thermodynamic data can be exploited to trick an observer with lesser distinguishing abilities into seeing a violation of the second law. Imagine the gas from above, in (11), with a piston sensitive to the thermodynamic datum of color: The piston interacts with the blue molecules while it does not with the red ones. If the volume is coupled to a heat bath, then the extracted work W is compensated by a heatflow Q from the bath into the volume.
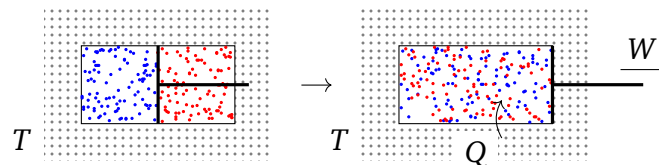
To an observer without the capability to distinguish the molecule types, the pro-

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[125]See Footnote 121.

cess looks as following.
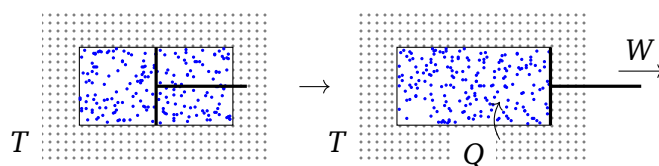
Thus, there from this second perspective no mixing occurs, and the decrease of entropy, $\Delta S = -Q/T$, through the heat flow from the bath, is *not* compensated by an entropy increase within the gas. The observer is left with a violation of the 2nd Law.

We can imagine a distinction between types of molecules that is independent of the kinematics described in classical mechanics. It seems that a reason why we cannot imagine tricking also a demon into seeing a violation of the 2nd Law lies in how we picture that the molecules interact with the piston: The demon can read off the distinction of the types by observing the particles' interaction with the piston.[126] As long as we assume that the way to extract work from a system is described by particles interacting with a piston combined with momentum conservation, any typical difference is resolved in terms of classical mechanics or quantum mechanics. While we expect that the extracted work is mechanical work, there is no necessity for particles to facilitate this extraction. In other words: Thermodynamics might apply also to differences that are not differences of (types of) particles. The picture of thermodynamics as an "information theory" about noumenal microscopic information ties the theory more tightly to the concept of particles than necessary. The abandonment of the particle picture might allow to develop thermodynamics in an independent description about work-extractable data.

11   Indistinguishability

In Section 10.1.2, we simplified the discussion about erasure by turning to a one-particle gas. This case removes the difference between *distinguishable* and *indistinguishable* particles. With multiple particles and the entailing need to resolve this difference, we turn to another aspect of Gibbs' paradox affecting the entropy in statistical mechanics—a problematic factor of $N!$—as we discuss in Section 11.2.

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[126]In light of Jaynes' comment quoted in Footnote 124, one might object that the demon does not "speak thermodynamics."

A question we hint at in Section 6.1 recurs: If we understand orthogonality to be closely related to the integrity of the system under consideration—and that system might be a single particle—, then the quantum mechanical description of measurements on a single particle require the ability to distinguish this particle from other such particles. Thus, if there are also systems that are merely accurately described if we assume the indistinguishability of their constituents, then the notion of a system comes with a contextual dependence. We discuss this in detail in Section 11.3.

We are turning from the Gibbs' paradox in thermodynamics with the connected discussion about the nature of information to the Gibbs' paradox in statistical mechanics and with it back to issues regarding the reference to systems.

11.1 Introduction to statistical mechanics

We consider a gas of N particles with a specified total energy E in a volume V, i.e., the particles form a so-called micro-canonical ensemble.[127] We postulate that a system in thermodynamic equilibrium is with equal probability in any of the possible states in phase space. Thus, the micro-canonical ensemble has a uniform density function on the $6N$-dimensional phase space Γ,

$$\rho(p,q) = \begin{cases} \text{const.} & \text{if } E < H(p,q) < E + \Delta, \\ 0 & \text{otherwise}. \end{cases} \tag{13}$$

The entropy of the gas is defined as the logarithm of the volume of the support of ρ, i.e.,

$$S(E,V) = k \log \Gamma(E) \qquad \text{with} \qquad \Gamma(E) = \int_{E<H(p,q)<E+\Delta} d^{3N}p \, d^{3N}q$$

where $k = 1.38 \times 10^{-23} J/K$ is Boltzmann's constant.

We now discretize the phase space Γ into cells of volume τ. Further, we partition the single particle energy spectrum into ranges $[\epsilon_s, \epsilon + \Delta\epsilon_s]$ and denote by C_s the number of cells in that energy range and by N_s the number of particles in that energy range.

C_s volume elements, containing N_s particles

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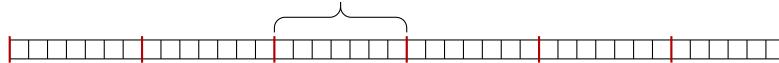
[127]For an in-depth discussion of statistical mechanics, we refer to [120, 65].

For a given set $\{N_s\}_s$ of *occupation numbers*, we denote the number of ways to distribute N particles among cells so that the distribution is consistent with these occupation numbers by $W\{N_s\}$. Replacing the cells by eigenstates of a quantum mechanical Hamiltonian, and regarding C_s as the degeneracy of the energy level ϵ_s, the scenario translate to quantum statistical mechanics.

If the particles are identical and non-interacting, then we can compute phase space volume $\Gamma(E)$ by summing over all sets of occupation numbers $\{N_s\}_s$ while imposing the constraints

$$E = \sum_s \epsilon_s N_s \qquad N = \sum_s N_s \,. \tag{14}$$

The volume $\Gamma(E)$ is well approximated by $W\{\overline{N}_s\}$ where the occupation numbers $\{\overline{N}_s\}$ maximise W subject to the constraints in (14), as laid out in Ref. [65].

We now turn to computing $W\{N_s\}$. If the particles are distinguishable, then each permutation of particles counts as a different distribution, and, thus, adds to W. For a given set of occupation numbers $\{N_s\}_s$, there are

$$\frac{N!}{N_1! \cdots N_s! \cdots} =: \begin{pmatrix} N \\ N_1 \ldots N_s \ldots \end{pmatrix}$$

ways of distributing the particles according to these occupation numbers corresponding to the *multinomial coefficient*. For each of the energy regions, there are $C_s^{N_s}$ ways of distributing the N_s distinguishable particles among the C_s available cells, and we obtain[128]

$$W^B\{N_s\} = \frac{N!}{N_1! \cdots N_s! \cdots} \prod_s C_s^{N_s} \,. \tag{15}$$

If we set the degeneracies $C_s = 1$, then we obtain up to a normalisation the Maxwell-Boltzmann statistics.

If, however, we regard the classical particles as indistinguishable, then we pass to the quotient space of phase space Γ/S_n. Then, there is merely one way of distributing the N particles according to a given set of occupation numbers $\{N_s\}_s$. And we correct the phase-space volume for each energy region, $C_s^{N_s}$, by a factor $1/N_s!$, yielding

$$W^{B.r} = \prod_s \frac{C_s^{N_s}}{N_s!} = \frac{W^B}{N!} \,. \tag{16}$$

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[128]In Ref. [106], Saunders makes use of this formula.

This corrected phase space volume is, however, *not* the number of distributions of N_s indistinguishable particles among C_s cells. The latter is

$$\frac{(N_s + C_s - 1)!}{N_s!(C_s - 1)!}$$

leading to the corresponding value of W of a Bose-Einstein-gas,

$$W^{BE}\{N_s\} = \prod_s \frac{(N_s + C_s - 1)!}{N_s!(C_s - 1)!} . \tag{17}$$

For the sake of complete, we add the Fermi-Dirac case. Each of the cells is then occupied by at most one particle, and, thus, we obtain

$$W^{FM}\{N_s\} = \prod_s \binom{C_s}{N_s} = \prod_s \frac{C_s!}{N_s!(C_s - N_s)!} . \tag{18}$$

The next step towards computing the entropy consist in solving the constraint optimization of the occupation numbers $\{N_s\}$. To this end, we refer to [65, §8.5]. Instead, we elaborate on the difference between $W^{B,r}$ in (16) corresponding to classical indistinguishable particles and W^{BE} in (17) corresponding to indistinguishable bosons. Following [106, §2.4], we consider the "occupation numbers" $\{n_l\}_{l=1}^{C_s}$ of the cells *within* a given region of energy ϵ_s. In (17), each factor is the number of ways to distribute N_s indistinguishable particles among C_s cells. This is the same as the count of possible occupation numbers $\{n_l\}_l$ that sum to N_s,

$$\sum_{\text{all } \{n_l\}_{l=1}^{C_s}, \text{ s.t. } \sum_l n_l = N_s} 1 \quad = \quad \frac{(N_s + C_s - 1)!}{N_s!(C_s - 1)!} . \tag{19}$$

While for the Bose-Einstein gas, each set of occupation numbers of the cells in a given energy region have the same weight, this is not the case for the classical indistinguishable particles. Consider the following identity[129]

$$\sum_{\text{all } \{n_l\}_{l=1}^{C_s}, \text{ s.t. } \sum_l n_l = N_s} \frac{1}{n_1! \cdots n_{C_s}!} \quad = \quad \frac{C_s^{N_s}}{N_s!} . \tag{20}$$

Thus, assigning multiple particles to a cell lowers the weight in the classical indistinguishable case.

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[129]This corresponds to equation (24) in Ref. [106, $2.4]. Saunders, in turn, refers to [100, 49–50].

To understand the difference in weights, we recall our initial intention to estimate a volume in $6N$-dimensional phase space Γ for the classical case. We silently omitted to multiply the count of distributions by a $6N$-dimensional volume element τ^N. When we move to the case of statistical quantum mechanics, we exchange the volume measure of phase space by a measure of dimension—i.e., we turn to counting orthogonal states.[130] There is however not only a difference between the quantum and the classical case, but also between the two classical cases that we distinguished above. For W^B in (15), we count volume elements of the phase space Γ, while for $W^{B.r}$ in (16), we count volume elements in the quotient space Γ/S_n. Gibbs explains the factor $1/N!$ which turns out to solve the Gibbs paradox as discussed in Section 11.2, by the difference in volume measures in Γ and Γ/S_n. The reduced weight in (20), results from the volume measure in the quotient space. The measure is lowered if multiple particles are assigned to the same cell. To understand how the volume measure in the quotient space is lowered, we consider a single cell of volume τ in the one-particle phase space μ containing two particles.[131]
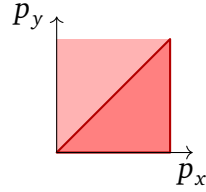
For every point in the volume there is another corresponding by the permutation of the particles.[132] Thus, merely the points below the diagonal, more precisely the diagonal plane in μ, count for the volume element in the quotient space. The volume is lower by a factor $1/2 = 1/2!$. In fact, if there are n_s particles in the volume element, we have to lower the volume by a factor $1/n_s!$. The volume in the quotient space is reduced by the number of ways we can exchange the particles in the element. In equation (20), we count volume elements of the quotient space Γ'/S_{N_s} with Γ' begin the phase-space of N_s particles. Thus, we are not referring to volume elements τ in μ, but to volume elements τ^{N_s}. The previous consideration translates to the volume of the diagonal elements in Γ' for which multiple particles occupy the same phase-space volume.

These differences in the measures appear in a different light if we assume the probabilistic perspective of Bach [5, 4, 6]. To this end, we assume that there

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[130]Again, we encounter orthogonality. We pursue this observation in Section 11.3.

[131]We only show the projection of the volume element onto the $p_x p_y$-plane.

[132]Except for some points in a zero-measure subset.

indices of particles in the respective cell

$\vec{X}$	1,2			1	2		2	1			1,2

MB	1/4	1/4	1/4	1/4
BE	1/3	1/6	1/6	1/3
FD	0	1/2	1/2	0

number of particles in the respective cell

$\vec{K}$	2	0		1	1		0	2

MB	1/4	1/2	1/4
BE	1/3	1/3	1/3
FD	0	1	0

Figure 3. We reproduce here the example given in Ref. [6, Fig. 3.2.].

are in total D bins, i.e., phase-space volume elements in classical mechanics, and energy eigenstates in quantum mechanics. Further, we assume a probability space (Ω, F, P), and define a first random variable

$$X_i : \Omega \to \{1, \ldots, D\} \qquad i \in \{1, \ldots, N\} \tag{21}$$

corresponding to the so-called *configuration* of the N particles. A given configuration $\vec{j} = (j_1, \ldots, j_N), j_i \in \{1, \ldots, D\}$ gives us for each particle the index of the bin we can find it in. We, thus, obtain events $[\vec{X} = \vec{j}]$ with associated probabilities. Given a configuration $\vec{j}$, we compute the occupation number $\vec{k} = (1, \ldots, D), k_i \in \{1, \ldots, N\}$ by the sum

$$k_i(\vec{j}) = \sum_{l=1}^{N} \delta_{i, j_l}.$$

The corresponding occupation number random

$$K_i : \Omega \to \{1, \ldots, N\} \qquad i \in \{1, \ldots, N\} \tag{22}$$

derives by means of the indicator random variable

$$1_{[X_m = i]} = \begin{cases} 1 & \text{if } X_m(\omega) = i, \\ 0 & \text{otherwise}, \end{cases}$$

and is defined as

$$K_i = \sum_{l=1}^{N} 1_{[X_l = i]} \qquad i \in \{1, \ldots, D\}.$$

Similarly, for a given partition of the D bins with C_s bins in the group c_s, the *group occupation number* is the sum of the occupation numbers of bins within that group,

$$n_s = \sum_{l \in c_s} k_l$$

with the corresponding random variable[133]

$$N_s : \Omega \to \{1, \ldots, N\} \qquad N_s := \sum_{l \in c_s} K_l. \tag{23}$$

The different ways of counting particle distributions now correspond to different probability distributions. The *Maxwell-Boltzmann distribution*, henceforth abbreviated by MB, corresponding to N distinguishable particles distributed over D bins—i.e., associated with W^B in (15)— is defined by a uniform distribution of the *configurations*, i.e.,

$$P_{MB}(\vec{X} = \vec{j}) := P_{MB}([\vec{X} = \vec{j}]) = D^{-N}. \tag{24}$$

This reflects the uniform density function above. The probability distributions of $\vec{K}$ and $\vec{N}$ are then

$$P_{MB}(\vec{K} = \vec{k}) = \binom{N}{k_1 \ldots k_D} D^{-N} \tag{25}$$

$$P_{MB}(\vec{N} = \vec{n}) = \binom{N}{n_1 \ldots n_F} \prod_s \left(\frac{C_s}{D}\right)^{n_s} \tag{26}$$

The Maxwell-Boltzmann distribution corresponds to counting *volume* elements. Correspondingly, we can regard the probability distribution of $\vec{N}$ to derive from the distribution over groups $\vec{n}$ where the probability to fall into a group s is proportional to the (relative) phase-space volume C_s/D occupied by that group (see [120, §1.7]).

The *Bose-Einstein distribution*, abbreviated by ME, in contrast, is defined by a uniform distribution over the occupation numbers, i.e.,

$$P_{BE}(\vec{K} = \vec{k}) := \binom{N+D-1}{N}^{-1}. \tag{27}$$

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[133]From this point on, we slightly abuse notation from above by using N_s for both, the number of particles per energy region, and its corresponding random variable.

The probability distributions of $\vec{X}$ and $\vec{N}$ are then

$$P_{BE}(\vec{X} = \vec{j}) = \binom{N}{k_1(\vec{j}) \dots k_d(\vec{j})}^{-1} \binom{N + D - 1}{N}^{-1} \tag{28}$$

$$P_{BE}(\vec{N} = \vec{n}) = \binom{N + D - 1}{N}^{-1} \prod_s \binom{n_s + C_s - 1}{n_s}. \tag{29}$$

For both, the MB case and the ME case, the uniformity assumption at the very beginning of this section holds with the crucial difference as to which random variable is concerned. Note that this uniformity is essential if we intend to interpret the entropy S as a form of Shannon entropy.

It still remains to integrate the case of classical indistinguishable particles corresponding to (16) into this probabilistic picture. The cases of W^B and $W^{B.r}$ correspond to the same probability distributions, for they only differ by a global factor $N!$. But instead of starting by means of postulating the uniform distribution of the configurations $\vec{X}$ as done in (24), it seems more appropriate for indistinguishable particles to start by postulating the distribution of the occupation number as done in the case of BE. As classically we are still referring to phasespace volumes, we must adjust this distribution accordingly as discussed above. Thus, we set the a priori distribution

$$P_{MB.r}(\vec{K} = \vec{k}) = (\text{norm.}) \times \frac{1}{k_1! \cdots k_D!}. \tag{30}$$

By means of (20), we can derive $P_{MB.r}(\vec{N} = \vec{n})$. Eventually, these distributions are no different from the ones given in (25) and (26). The quantum correspondent to these considerations is the definition of the configurations $\vec{X}$ and associated probability distributions $P_{BE}(\vec{X} = \vec{j})$.

In this probabilistic perspective, the distinction between distinguishable and indistinguishable particles has changed. In a sense with MB we might have talked about indistinguishable particles all along. And even if we have indistinguishable particles in mind—be they classical or quantum—, we can define configurations and assign probability distributions. Correspondingly Bach defines particles to be indistinguishable, if the probability distribution over particle-specific parameters, as the configurations, does not allow to make a difference, i.e., are uniform. This perspective fits to the perspective onto quantum mechanics we established in Section 5.6: The distinguishability of quantum particles is defined by the symmetry of the corresponding density matrix. With Gleason's Theorem this can be regarded as the symmetry of a probability distribution.[134]

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[134]"Indistinguishable quantum particles are characterized by the property that the statistical

On an interesting side note, we remark that the correlation of the configurations X_i and X_j show are bunching effect—i.e., condensation—for the BE case over the MB case. A system of classical particles that excludes the diagonal cells might show condensation, as well [49].

11.2 Gibbs' paradox in statistical mechanics

In classical thermodynamics, if we scale both the particle number N and the volume V of a gas by a factor λ, then we call a function $S(N, V, T)$ *extensive* if

$$S(\lambda N, \lambda V, T) = \lambda S(N, V, T).$$

The Gibbs paradox in statistical classical mechanics consists of the entropy $S = k \log W^B$ not being extensive: Employing (15) and Stirling's approximation, $\log x! \approx x \log x - x$ yields

$$S \approx kN \log N + k \sum_s N_s \log \frac{C_s}{N_s}. \tag{31}$$

If we double the particle number and the volume, then the second terms scales by a factor of two while the first does not. The latter picks up an additional term $kN \log 2$. Dividing W^B by $N!$ removes the first term and, thus, solves the issue.

What justifies this step? One response, following Gibbs[135], is to correct the volume measure: For if the classical particles are indistinguishable, then we have to compute the phase-space volume in the quotient space Γ/S_n and, thus, adjust the weight in counting distributions accordingly as discussed in Section 11.1. Another response is to recourse to statistical quantum mechanics and observe that for the dilute case—i.e., when $C_s \gg N_s$ and, therefore, with mostly no more than one particle per volume element—the expressions in (20) and (19) converge. In other words, statistical classical mechanics merely approximates how

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operator which determines the state of the system is invariant under any permutation of the particles. [...] Applying now the classical analog of the quantum definition of indistinguishability it turns out that the particles of the MB [Maxwell-Boltzmann] statistics are indistinguishable as any uniform distribution is invariant under permutations." [5, §1]

[135]"If two phases differ differ only in that certain entirely similar particles have changed places with one another, are they to be regarded as identical or different phases? If the particles are regarded as indistinguishable, it seems in accordance with the spirit of the statistical method to regard the phases as identical. In fact, it might be urged that in such an ensemble of systems as we are considering no identity is possible between the particles of different systems except that of qualities [...]." [44, §XV, p. 187]

things really are, i.e., how they are described in statistical quantum mechanics. The subtext of this argument is that

$$\text{classical particles are distinguishable, and,} \qquad (\text{D}_{\text{cl}})$$

$$\text{quantum particles are indistinguishable.} \qquad (\text{I}_{\text{qm}})$$

If one did not assume the former, D_{cl}, then the missing factor $1/N!$ would not be due to classical mechanics being wrong, but due to applying the wrong volume measure. The latter, I_{qm}, stems from the counting argument used when computing W^{BE} in Section 11.1, reflecting the symmetrization of the density matrices in statistical quantum mechanics.

Schrödinger remarks:

> It was a famous paradox pointed out for the first time by W. Gibbs, that the same increase of entropy must not be taken into account, when the two molecules are of the same gas, although (according to naïve gas-theoretical views) diffusion takes place then too, but unnoticeably to us, because all the particles are alike. The modern view solves this paradox by declaring that in the second case there is no real diffusion, because exchange between like particles is not a real event—if it were, we should have to take account of it statistically. [109, §VIII, p. 61]

With the statistical picture established in Section 11.1, one might wonder whether the in-/distinguishability differentiation—the distinction between the "classical" and the "modern" (quantum) view—deserves to figure so prominently in statistical mechanics: As discussed in Section 11.1, as long as particle-specific random variables are uniformly distributed, we can regard particles as indistinguishable—irrespective of whether we consider the classical or the quantum case (see also Footnote 134). What then is the basis for (D_{cl})? What warrants the exchange of particles in classical statistical mechanics to be a "real event"?

The picture in statistical classical mechanics is that a gas consists of N classical particles with their respective distinct N trajectories. We compare this picture to the following statement:[136] "The space-time continuity is our main criterion for the identity of material bodies." [110, §V.2, own translation] The center of mass of a particle i corresponds to a continuous trajectory in the one-particle phase space μ. The continuity of the trajectory warrants the identity of the particle.

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[136]"Die raum-zeitliche Kontinuität ist unser Hauptkriterium für die Identität materieller Gegenstände." [110, §V.2]

11.2.1   Trajectories in statistical mechanics

In statistical classical mechanics we do *not* describe a single particle, but an ensemble of particles. How much of the picture of particles with unique trajectories carries over from classical mechanics into statistical classical mechanics? The state of an ensemble in statistical classical mechanics is described by a probability density function on the N-particle phase-space Γ (see (13)). For $N = 1$, this turns into a probabilistic description of a single particle. One can regard classical mechanics as a probabilistic description with a concentrated distribution. With the uniformity assumption in (13), merely special cases of a single-particle ensemble qualify as a system in classical mechanics with the respective well-defined trajectory. Worse yet, if we discretize phase space—to introduce, e.g., the random variable for configurations, $\vec{X}$ (see (21)), or to meaningfully speak about information (see Section 10.1.1)—, then we loose the uniqueness of the trajectory as soon as the probability of two particles occupying the same one-particle phase space element[137] is non-zero. In short, while we may integrate classical mechanics into statistical classical mechanics, the notion of a unique trajectory associated with single particles seems to fade as we do so. Thus, as far as statistical classical mechanics is concerned, the idea of a unique trajectory appears to be rather the afterimage of a different description. Even if unique trajectories are not an inherent characteristic of statistical mechanics, it does not preclude to interpret the probability distribution function as uncertainty about the trajectories of N particles. Then, one might assume "classical particles"—i.e., particles that are not only described in classical mechanics, but rather particles that essentially are confined to a unique trajectory. Yet, these "classical particles" remain a contingent assumption with no necessity to statistical classical mechanics.

Besides the problems occurring when one attempts to transfer the picture of system identified by their trajectories in classical mechanics to statistical classical mechanics, there are doubts regarding the trajectory-picture itself. To this end, we turn to the following questions in the subsequent section: What role does the identity of a system play in physical descriptions? What is the prospect of deriving a criterion of identity from a physical theory? How does the identity of a system depend on the identity of its constituents?

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[137]Equivalently, we could speak about the probability of occupying diagonal elements in the N particle phase space.

11.3   Reference to systems

To test physical descriptions, we split the reference to the system under investigations from statements about that system. We separate *reference*—or "attributee"—and *attributes*. The theories then provide us with statements about attributes or properties of the system that can be experimentally tested. Thus, we usually assume that we can identify the system—at least within a particular experimental context. A theory is falsified if it yields an attribute about a given system different from the one actually observed—*not* because the theory gives statements about the wrong system. Moreover, the attributes crucially depend on what system they are attributed to, as becomes apparent by orthogonality.

11.3.1   Orthogonality revisited

In Section 6.1, we consider Einstein's concerns regarding the orthogonality of spatially separate effects. Recall, the following scenario of a particle deflected as it passes a slit
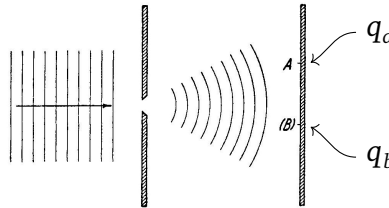

Fig. 1

with the effects at *A* and *B* corresponding to orthogonal propositions $q_a \perp q_b$. Thus, if q_a is true, then q_b is false, and vice versa. The orthogonality is warranted by the identity of the attributee, i.e., by q_a and q_b constituting statements about the *same* (identical) system. What seems strange in this scenario, is the spatial separation of the effect despite an observed orthogonality reassures us of the identity and integrity of the attributee. As we discuss below, spatial separation figures as an important criterion for separability, i.e., to distinguish different systems, even though quantum mechanics—if it were to provide us with a criterion of separability—does not inherently prefer a position measurement.

It might appear that the connection between orthogonality, i.e., how propositions relate to one another, and the identity of the system they are attributed to, is primarily a problem within quantum mechanics, for spatially separate effects associated to the same system do not seem to appear in, say, classical mechanics.[138]

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[138]Schneider in Ref. [108] separates the axioms of quantum mechanics into two groups, one about systems, the other about observables: "There is a kind of duality in these fundamental con-

To counter this view, we consider the following example by Frege:

> The discovery that the rising sun is not new every morning, but always the same, was one of the most fertile astronomical discoveries. Even today the identification of a small planet or a comet is not always a matter of course. [39, p. 36]

> If we now replace one word of the sentence by another having the same reference, but a different sense, this can have no bearing upon the reference of the sentence. Yet we can see that in such a case the thought changes; since, e.g., the thought in the sentence 'The morning star is a body illuminated by the Sun' differs from that in the sentence 'The evening star is a body illuminated by the Sun.' Anybody who did not know that the evening star is the morning star might hold the one thought to be true, the other false. The thought, accordingly, cannot be the reference of the sentence, but must rather be considered as the sense. [39, p. 41]

Putting aside that Frege employs the latter example to examine the *thought of a sentence*, as well as, the observation that the identification of a small planet or comet poses less of a challenge today, we note again the importance of identity for how propositions relate to one another. We can merely assign different truth values to equivalent attributes, if we assume that they apply to different systems.[139] Or, if we turn this around, how we identify systems affects which attributes we consider equivalent.

In the measurement problem, the importance of what we refer to shows as follows: If single photon escapes the friend's laboratory, i.e., if $S \times F$ (see Section 6.2) is not isolated, the evolution is no longer unitary. The scenario of the Wigner's-friend experiment is no longer problematic.

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cepts and rules, since rules R1–R3 [systems and Hilbert spaces, states and vectors, time evolution of isolated systems] deal with the description of the system, while rules R4–R6 [observables as operators, outcomes as eigenvalues, probabilities from inner products] deal with observables which appear heterogeneous with respect to the system. In this sense, the observables do not belong to the system." [108, §2.1] On the one hand, orthogonality binds both categories closely together. On the other, it is hardly an aspect of quantum mechanics: If we consider classical mechanics analogue to Section 5.6 starting from lattices, then classical observables are merely subsets of phasespace and can be separated similarly from systems. Finally, the distinction becomes further obfuscated, if we change to the Heisenberg picture. Thus, this separation is neither evident in quantum mechanics nor does it serve to distinguish quantum mechanics from classical mechanics.

[139]Depending on the attribute further assumptions are also needed, such as that the system is isolated.

These considerations illustrate the importance of the identity of systems for how we *do physics*.

11.3.2   Identity from properties

The trajectory picture carries the thought that the identity of a system might be guaranteed through a physical theory itself. The theory in question is taken to provide the means to identify the system. This attempt to incorporate the identification into the theory presents us with problems. Firstly, we are lead into a circularity: We either *suppose* the validity of the theory and derive a criterion to identify the system, *or* we *suppose* the reference to the system and test the validity of that theory. We are faced by the underdeterminism in semantic holism as discussed in Section 8.5.1: It seems we can hardly avoid one contingent assumption or the other. It might feel as if the basis we build on to say anything is rather shaky. In this regard, the appeal in assuming a fixed structure of systems which magically insures that we refer to the same system becomes palpable.[140] Down the line what offers an anchor here, is not that we circumvent *making an assumption*; but the quality of that assumption: If reference to the same system is warranted by how the world is split into systems—as opposed to, e.g., reference to the same system being discursively established—, allows to locate the cause for the coinciding reference in the external object and endows ones ways of speaking by objectivity.[141]

Secondly, there arise problems with identifying entities by their properties. Imagine we are given a trajectory in phase space $(\vec{x}(t), \vec{p}(t))$, and we observe a system S with its center of mass on this trajectory at time t_0. If the trajectory provides a criterion of identity, then whatever we find at some later time $t_1 > t_0$ at a phase space point $(\vec{x}(t_1), \vec{p}(t_1))$, *is the identical system*. From the phase space point alone we cannot infer *what* it is we find there.[142] What we mean, when we say that "we find something at the phasespace point," is that there exists a system with these phasespace coordinates. This presupposes that we know what system entities we have to examine with regards to their phasespace coordinates, i.e., with regards to their quantifiable attributes. This leads us to the question, whether we can examine the attributes of entities, that we *cannot* identify in the

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[140]Fuchs opts for such a system structure, as discussed in Section 9.1.

[141]Interestingly, this quest for objectivity leads to an assumption with a large scope: It is an assumption about the world rather than about "the systems we talk about."

[142]Saunders similarly observes the following: "[P]hysical theories are primarily about *quantities*, rather than things, so we cannot simply consult our best physical theories about what there is." [107, emphasis in original] This also relates back to the first paragraph in Section 11.3.

first place. For, Strawson remarks:

> Another possible interpretation of the slogan ['no entity without iden-
> tity'] [. . .] might run something like this: 'There is nothing you can
> sensibly talk about without knowing, at least in principle, how it
> might be identified.' I have nothing to say against *this* admirable
> maxim. [116, §1]

Then, we would have to solve the identifiability issue *before*, we can employ the
trajectory as a criterion for identifying a system. We find ourselves confronted
with yet another circularity.

11.4   The quagmire

There might emerge a deeper cleavage between ways of referring to (identical)
systems and statements about properties. To this end, let us recall from Sec-
tion 8.5.1 Gonseth's position that physics can be regarded as a form of propo-
sitional logic or, more generally, of predicate logic. We now dare a comparison
of this form of predicate logic with another one. At the beginning stands an ex-
tension to Gonseth's position: Another "typical realization" of formal languages,
including predicate logic, are programming languages.

Before trying to carry through a comparison between realizations, it is worth-
while pausing for a moment and reflecting on the step we are about to do. The *use*
of the "same" formal language in physics and in programming languages is prob-
ably quite different, and, thus, the *understanding* of the rules of the language.
This is a lesson from the consideration on following rules (see Section 8.4.2,
and [129, 71]). On the one hand, this is exactly what we hope for: Ideally, we
gain from this comparison a (new) understanding. Put in Feyerabend's words,
we hope to have found a contrast agent.[143] On the other, we must be wary of
overstretching this comparison: An understanding "transferred" along this com-
parison might turn out to not be tenable. Re-framing the latter concern is to
caution ourselves regarding *how* to compare different uses.[144] We mean to say

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[143]"Auch erkennt man einige formale Eigenschaften einer Theorie nicht durch Analyse, sondern
durch Kontrast." [31, §2] — "Also one recognizes formal properties of a theory not by analysis,
but through contrast." [31, §2, own translation]

[144]In a sense, we are drawing from Rorty's considerations on the contingency of languages
here. Any such comparison is merely one contingent bridge between uses of language—it is
itself only one use of language. This is a reminder that the step should not be seen as an outside
perspective onto uses of language.

here as well: In the following, we attempt a first step onto potentially unstable ground.[145]

We can say something about the difference in the use of language and anticipate what we might draw from this comparison: In programming languages, reference to (formal) objects is explicitly (and formally) expressed. Moreover, we find ample applications and can, thus, hope to draw from an empirical basis regarding the use of formal objects. The nexus of our comparison is exactly an empirical observation—namely the so-called *object/relation impedance mismatch*.[146]

11.4.1   The object/relation impedance mismatch

Attempts to interface between relational databases and object oriented programming lead into problems.[147]

> Although it may seem trite to say it, *Object/Relational Mapping is the Vietnam of Computer Science*. It represents a quagmire which starts well, gets more complicated as time passes, and before long entraps its users in a commitment that has no clear demarcation point, no clear win conditions, and no clear exit strategy. [84, emphasis in original]

The two sides of the problem are different ways of looking at a system—i.e., primarily a computer system, but we might learn something for ways of looking at physical systems. On the one hand, there is the objective perspective:

> Object systems are typically characterized by four components: *identity*, *behavior* and *encapsulation*. Identity is an implicit concept in most O-O languages, in that a given object has a unique identity that is distinct from its state (the value of its internal fields)—two objects with the same state are still separate and distinct objects, despite being bit-for-bit mirrors of one another. This is the 'identity vs. equivalence' discussion that occurs in languages like C++,C# or Java, where developers must distinguish between 'a==b' and 'a.equals(b)'. [84]

On the other hand, the relational perspective emerging from relational databases and representing predicate logic:

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[145]It can be questioned whether it has be any different in other parts of this text.

[146]I owe to Igor Moreno for pointing out the problem to me.

[147]We suspect that those who are concerned with the measurement problem can relate to the feeling of getting "entrapped in the quagmire."

> Relational systems describe a form of knowledge storage and re-
> trieval based on predicate logic and truth statements. In essence,
> each row within a table is a declaration about a fact in the world,
> and SQL allows for operator-efficient data retrieval of those facts us-
> ing predicate logic to create inferences from those facts. [84]

The impedance mismatch is the empirical observation that joining these perspectives creates problems. If we compare this with the following statement by Frege,

> Equality gives rise to challenging questions which are not altogether
> easy to answer. Is it a relation? A relation between objects, or be-
> tween names or signs of objects? [39]

then the question arises whether the woes of the impedance mismatch are—at least in part—a representation of philosophical concerns that also affect the identifiability of physical systems. If so, we can hope to leverage insights about programming languages to understand issues in philosophy and physics.[148]

One way of joining the impedance mismatch with Frege's statement is to observe that identity for objects is quite different from equivalence for propositions in predicate logic. The latter is a relation and, therefore, embeds naturally into predicate logic. Identity-as-a-relation for objects, as hinted at by Frege, requires to cross the problematic object/relation gap. And so does what we initially strived for, i.e., the identity-from-relations for objects.

So what is this object/relation gap? Let us consider the following example of a simple database scheme:
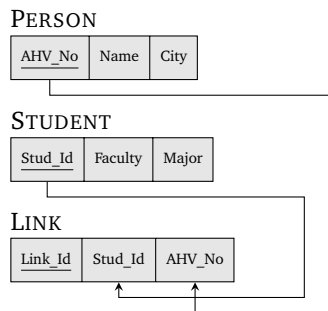

A row in the Person-table such as {756.0774.7623.28, Mustermann, Lugano}, is commonly read as "There exists a person called Mustermann, living in Lugano,

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[148]For readers rather sceptical of philosophical baggage, we add that this then exemplifies the "reality of philosophic concerns." Neward lists in Ref. [84] a number of "real" issues arising from the impedance mismatch.

and having an insurance number 756.0774.7623.28." In the same way, there "exists a link with student id 191178373 and insurance number 756.7299.9011.26" for a row {1,191178373,756.7299.9011.26} in the Link-table. We can already make out that this conflicts with our intuition that first there is student with a student id 191178373 and a person with insurance number 756.7299.9011.26 before we can link the two. In other words, we assume that we can only link entities that exist. Similarly, we expect that for every student there is person linked to it, i.e., every student *is a* person. We intuitively assume, there *are*[1] persons that are[2] maybe also students. The first "are[1]" is quite different from the second "are[2]." The former establishes what entities there are (ontologically speaking), and the second one establishes what may be said *about* some of these entities.

Object systems formalize aspects that we insinuated above, that are, however, not naturally present in relational systems. Let us consider the following example of a class person with a subclass student:
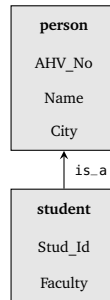
Every instantiation of the person-class has its unique identity, even if the fields have the same value. Calling twice

```
new person(756.0774.7623.28, Mustermann, Lugano)
```

yields two objects, each uniquely identified—commonly by a pointer to the associated memory address. Let us compare this to a scenario, in which we attempt to insert the same row {756.0774.7623.28, Mustermann, Lugano} twice into the same table: It seems this merely creates redundancy, for it repeats a statement with the same content—a statement with the same meaning.[149] In a sense, it is meaningless to speak of two statements (or two rows) if all their fields coincide. If one observes that we also invoke a notion of equality for fields, one can sense that we are already knee-deep in the quagmire. The sense in which there are no statements (or rows) with the same content, is the one of set equality: If we

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[149]Frege in Ref. [38] might call this the *thought* of a statement.

add an element to a set which already contains that element, the set remains the same.[150]

Database systems usually forbid replicated rows by enforcing a unique primary key for each row. This primary key does not name an entity for then also links are entities. Furthermore, there might be a person without a (unique) insurance number. The primary is neither necessary nor sufficient to identify entities.

If we aim to bridge between the above defined classes and tables—i.e., to establish an object/relation mapping (ORM)—, we have to impose constraints. The above observation of ORMs leading into the quagmire shows how these constraints are restraining. If ORMs are merely contingent connections between the relational and the object world—i.e., between the world of sentences with truth value and the world of referential sentences—, then there seems to be a fundamental difference. Whenever we traverse from the relation into the object world or vice versa, we must be suspicious.

11.4.2   Physical attributes and physical systems

If physical theories are a realization of predicate logic, i.e., comparable to (very large) relational data bases, whose relations depend on what objects we are referring to (see Section 11.3.1), we cannot avoid crossing the object/relation boundary. We consider two interrelated such cases that we mentioned before: On the one hand, when we think of a gas as N particles we aim to derive attributes for the gas from the features of its constituents, i.e., of the objects that constitute the gas. We call this the *constituent problem* (CP). On the other, we considered deriving the identity of a system—by which one is able to refer to it—from its attributes. We call this the *identity-from-attributes problem* (IfA). In a sense, the (IfA) gives rise to the (CP), for the idea that we can "identify particles by their trajectories" nurtures the hope to then describe a gas "through its constituting particles." Nonetheless, the two problems are quite distinct for they cross the boundary in different directions.

A first observation regarding (CP) is the following: As long as we speak *about* the gas—i.e., if the system we refer to is "the gas" (read: the object `new gas()`)—, any statements about its "constituents" are attributes (read: rows in a table) without the necessity to have any constituent-objects around.[151] In other words,

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[150]To speak of "adding an element" requires a reference.

[151]"Ich bin gegenüber allen sportlichen Geschwindigkeits- oder Geschicklichkeitsrekorden untentwegt auf dem Standpunkt des Schahs von Persien stehengeblieben, der, als man ihn animieren wollte, einem Derby beizuwohnen, orientalisch weise äusserte: 'Wozu? Ich weiss doch, dass ein Pferd schneller laufen kann als das andere. Welches, ist mir gleichgültig.'." [133] —

we can make an attribute statement about a gas involving "constituents," without the ability to identify its constituents. If, on the contrary, we are to make statements about the constituent C, then we must be able to refer to that constituent (read: that have an object `C = new constituent()`), and, thus, also be able to distinguish C from the other constituents.[152] Reference to a gas and reference to its constituents are substantially different. This difference becomes even more striking if we turn to quantum mechanics as discussed in Section 11.5.

11.4.3   Saunders ORM

We now consider Saunders' approach in Ref. [107]. The scenario is described in the following quotes:

> I take 'individual' to mean an object that (i) persists, somehow, in time, and (ii) can be uniquely identified throughout the time that it persists. I take 'object' (and interchangeably, 'thing') to mean anything that can stand in predicate position, typically the value of a bound variable; in this I follow Quine. [107]

> I see no safer way than to put questions of ontology into words, using simple declarative sentences and the standard apparatus of first-order quantifiers. [107]

> When I say an individual is *identifiable* (at a time or throughout a period of time), I mean it is absolutely discernible (at a time or throughout a period of time); thus individuals, in my sense, are always absolutely discernible. By 'indistinguishables' I mean things that are at most weakly discernible, if discernible at all. [107]

Thus, there are objects $x, y, z, \ldots, s, t, \ldots$ that are *discernible* to different degrees. Two objects s, t are

- *absolutely discernible* if there exists a predicate[153] P with Ps but not Pt,

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"Towards all sportive comparisons of velocity and skillfullness, I have always taken the stance of the Sha of Persia who as he was encouraged to attend a derby wisely uttered: 'What for? I know that one horse can run faster than another. Which one, does not concern me.'." [133, own translation]

[152]Here, we encounter a structural difference between object systems in programming languages and physical systems. Composite physical systems are again physical systems.

[153]Quantifying over predicates, we find ourselves here in 2nd order predicate logic, contrary to Saunders' statements in Ref. [107].

- *relatively discernible* if there exists a predicate F, such that Fst but not Fts, and

- *weakly discernible* if there exists a predicate F, such that Fst and Fts but neither Fss nor Ftt.

Consider two spheres of same color, radius, material, ... at a distance d from one another:

$$s \qquad t \tag{32}$$

If the only allowed predicate is the irreflexive, symmetric "being d apart" (F_d), then s and t are weakly discernible. But what is this a statement *about*? What are the relevant systems, here? And, thus, what notion of orthogonality applies down the line? So, far we have merely produced a single row $\{s, t\}$ in a table associated with F_d. What is the associated ORM? What are possible ORMs? Whatever "system-class" with instantiated objects we define: There is only one statement we can (or cannot) assign to the objects, namely "s and t are d apart," i.e., $F_d st$. In terms of a physicists' intuition, this is two say: We can merely assign attributes to the composite system.

There is no attribute that yields a statement justifying to speak of two system-objects. Merely the spatial separation present in the natural language meaning of F_d makes us believe that in assigning $F_d st$ to a system we are really referring to *two systems*. The idea that there "are" two spheres is rather the captivity of a particular picture. The objects s and t are primarily mathematical symbols used to generate statements. It seems that only as a predicate for a composite system we can speak of indistinguishable constituents. Differentiating between distinguishability and indistinguishability is merely possible by means of predicate applying to "ensembles of particles." In this regard, neither D_{cl} nor I_{qm} are tenable.

### 11.4.4   Orthogonality, yet again

To make our example slightly more interesting, we add a third object, so that we have the objects s, r, t and a binary predicate F_d, as well as its complement $\neg F_d$.

At this point, we get the following partial lattice structure:
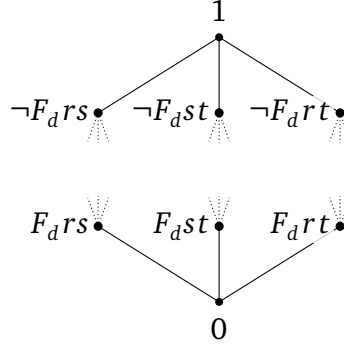
In the computer-linguistic picture above, the order relation corresponds to link tables. Thus, the order relation itself is merely a subset of the statements given in the relation picture.

Among the possible lattices satisfying these complementarity constraint, we consider the following two examples:
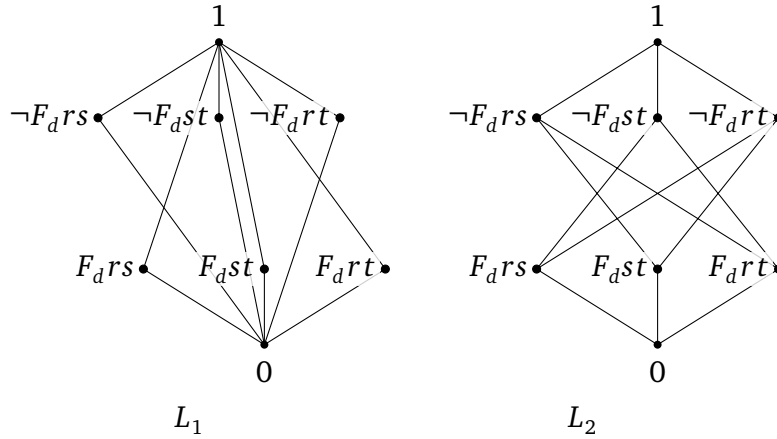
In the lattice L_1, we connect all the upper nodes with zero, and all the lower nodes with one. This corresponds to the picture of arranging three black spheres, labelled by s, r, and t, in a plane. Statements about any of the pairs are independent of one another. The other lattice, L_2, emerges if we assume two available positions for three spheres at a distance d: Placing a pair in these positions excludes the other two pairs from being at a distance d. Note that the complement in the lattice L_1 is not unique, were it not for us starting off with a notion of complementarity. So, in a sense, the notion of orthogonality does not arise from the relational (lattice) structure but is what we dictated from the beginning.
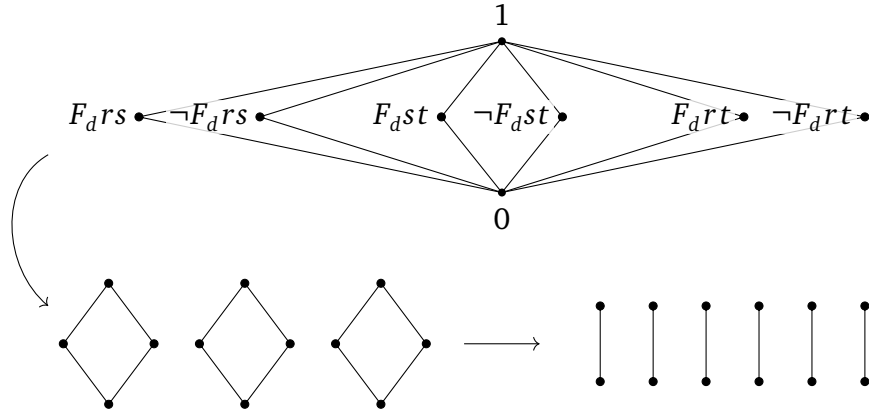
Figure 4. Reduction of L_1.
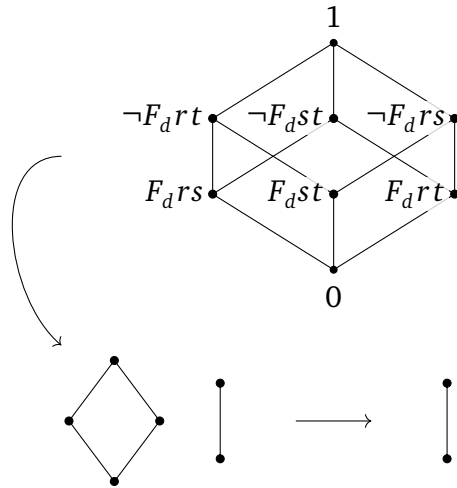
Figure 5. Reduction of L_2.

As visualized in Figure 4 and 5, both can step-by-step be reduced to binary lattices (see [91, Definition 2.41] and [23, §1.25]). This is no surprise as Boolean lattices are isomorphic to hypercubes lattices.[154]

The first step of the reduction comes with a good news and a bad one. First the bad one: The emerging structure has no connection to the variables r, s, t. There is nothing that supports that we should speak of them as objects in the sense of `new system()`. This reflects the considerations in Section 11.4.3. The good news is, however: There lattice seems to fall into "entities" with their own notion of orthogonality. This ray of hope—that some "object structure" crystallizes from the relational structure—is lost in the second step of the reduction: In both cases, we collapse to a number of trivial binary lattices. There is nothing that supports a meaningful notion of orthogonality that we might suspect in the first step. If classical mechanics is faithfully represented by a Boolean subset lattice, the hope that a meaningful system structure emerges is rather dim. Quantum mechanics presents us with a converse problem: As mentioned in Section 8.3.1, a lattice is merely reducible if the center is not trivial. The lattice emerging from quantum mechanics does not reduce at all.

If we cling to the idea that a system structure should be present in the relational structure of a theory—i.e., building an ORM from the relational side—, then the prospect in light of classical and quantum mechanics is as follows: On the one hand, classical mechanics requires us to impose some structure that *prevents* the reduction to trivial cases. On the other, quantum mechanics requires us to impose superselection rules that *allow* some reduction. If we assume the "universality" of either quantum mechanics or classical mechanics, then, it seems, we do not only have a measurement problem, but also a *system problem*.[155]

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[154]Boolean lattices are isomorphic to a subset lattices. And subset lattices are isomorphic to hypercubes. The lattice L_1 is a product of Boolean lattices, L_2 is the subset lattice of a three element set.

[155]To maintain that the notion of a system is necessarily vague, as Peres follows from considerations that resemble the case of brains-in-a-vat, seems to rather hide than solve the problem. "A *quantum system* is a useful abstraction, which frequently appears in the literature, but does not really exist in nature. In general, a quantum system is defined by an *equivalence class of preparations*. (Recall that "preparations" and "tests" are the primitive notions of quantum theory. Their meaning is the set of instructions to be followed by an experimenter.) For example, there are many equivalent macroscopic procedures for producing what we call a photon, or a free hydrogen atom, etc. [...] The ambiguity of these notions emerges as soon as we think of concrete examples. Is a hydrogen atom in a 2p state the same system as one in a 1s state? Or is it the same system as a hydrogen atom in a 1s state accompanied by a photon? [...] These examples show that we must be content with a vague "definition": A quantum system is whatever admits a closed dynamical description within quantum theory." [90, §2.1]

11.4.5   Object theory

A drawback of Saunders' approach is that it does not enforce singular predicate in defining objects. In the previous Section 11.4.3, this leads to the observation that an object in predicate logic is not necessarily something we can make statements about. There might not be statements about an object *s* in particular.

One way to respond to this, is to assume other predicates that are somehow "not part of a given theory" but are otherwise available and provide the means to make statements about particular objects. Effectively, this introduces a dualism with the usual side-effects. And deprives physical theories of any claim to universality. Another response is to explicitly include monadic predicates as in the approach discussed by Zalta [131]. The starting point is the Naive Object Theory characterised as follows:[156]

> A very simple statement of the theory upon which Meinong seemed to be relying in his early work is the following, which we will call Naive Object Theory. (NOT) For every describable set of properties, there is an object which exemplifies just the members of the set. [131, §0.2]

The idea is, thus, that the predicates single out what we are referring to.[157] Let us assume a second order predicate logic with objects $x, y, z, \ldots$ and n-place predicates $F^n, G^n, H^n, \ldots$ (together with common notions of predicate logic). We say that $x_1, \ldots, x_n$ *exemplify* F^n if the statement $F^n x_1 \ldots x_n$ is true. To understand what (NOT) means, let us approach it the other way around, i.e., from the object side. In light of the above considerations, this appears already fairly paradoxical: In order to build an ORM from the relational side we approach—through the quagmire—from the object side. As in Saunders' approach, we invoke the natural-language notion of a variable in predicate logic referring to an object in the sense of `new object()`. By now, we suspect that we never really left the quagmire. The real problem might not be to build an ORM but rather the untenability of the idea that we can cleanly separate the object from the relation side.[158]

Well then, let us assume that *s* "denotes an object." We think of *s* as the left sphere in the example considered in (32). So *s* exemplifies, e.g., roundness,

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[156]Subsequently, we follow [131, Introduction §2].

[157]In other words, the absorption of reference into a propositional framework can be regarded as a removal of "denotational" names in favour of "connotational" ones [72].

[158]"Philosophy is a battle against the bewitchment of our intelligence by means of language." [129, §109]

blackness, and being to the left in (32). These monadic predicates, also called properties, are then contained in a set

$$\{F^1 \mid F^1 s\}. \tag{33}$$

If t "denotes the right sphere in (32)," then we can form a set

$$\{F^1 \mid F^1 s \wedge F^1 t\}. \tag{34}$$

that contains properties like roundness and blackness. Following (NOT), this, yet again, singles out an object. There arises the question whether that object is the composite of two, namely s and t. If we are to establish reference from predicates, i.e., connotationally, then there is no referential difference between s and t, and we can hardly speak of two objects, unless we introduce a denotational difference between s and t—thereby undermining our initial goal—, or we assume some other formalism for the composition of systems. The prospect of a formalism for composition *without* the denotational use of names remains to be investigated.

   If the predicate logic comes with a notion of identity[159] for predicates, then we can also form sets such as

$$\{F^1 \mid F^1 = P \vee F^1 = Q\}. \tag{35}$$

which is (set-)identical to $\{P, Q\}$. The natural way to transfer orthogonality from proposition or statements to predicates, is to assume that

$$P \perp Q \; :\Longleftrightarrow \; \forall x (Px \Rightarrow \neg Qx).$$

Following (NOT), there exists an object for $\{P, Q\}$ with $P \perp Q$, threatening the falsifiability of our theory.

   To round off (NOT), we assume the Leibniz's Law

$$x = y \; \Longleftrightarrow \; \forall F^1 : (F^1 x \Longleftrightarrow F^1 y) \; \Longleftrightarrow \; \{F^1 \mid F^1 x\} = \{F^1 \mid F^1 y\}. \tag{36}$$

After the discussion in Section 11.4.1, we know that we are bound for trouble here: We started off with statements like "x denotes an object"—a referential statement. Thus, with (36), we establish the equivalence between a reference identity and a set/relation identity.[160]

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[159]Oh dear!

[160]Some of the differences to Saunders' approach become apparent, once more, if we compare (36) with Saunders' version of Leibniz's Law [107], i.e.,

$$x = y \; \Longleftrightarrow \; \forall F \in P : (F \dots x \dots \Longleftrightarrow F \dots y \dots) \tag{37}$$

where P is a set of predicates that may or may not contain monadic relations. At the core of Saunders attempt to establish indistinguishable objects stand a restricted notion of identity.

If we assume that every property G has a complement

$$\exists F : \forall x (Fx \Longleftrightarrow \neg Gx),$$

and that any two properties G and H have a conjunction

$$\exists F : \forall x (Fx \Longleftrightarrow Gx \wedge Hx),$$

then we also the composite object $\{F \mid Fx \wedge \neg Fx\}$ exists. Following (NOT), this object exemplifies the property $Fx \wedge \neg Fx$, and thus we are lead into a contradiction. We have arrived at the Liar sentence (see Section 8.1) of object theories:[161]

> The symbol s denotes the object that cannot be denoted.[162]

We observe here a mutual dependence between the Liar sentence of semantic theories—i.e., the one about sentences and their truth values—and the Liar sentence here: The former needs for its contradiction essentially a reference—i.e., "this sentence" or "the sentence on page..."—while the latter requires a statement with truth value—i.e., "x cannot be denoted." *Both contradictions illustrate the same quagmire.* The referential Liar sentence is less known for the lack of theories of objects/reference.[163]

11.4.6   Humean rescue?

By now, the situation appears fairly hopeless: We conclude that, on the one hand, reference "to the same system" is important, while, on the other hand, there does not seem to be a satisfying answer for the question how we can assure ourselves of such "same reference." Realizing the importance of aligning what we refer to leads to the urge to provide a general, extra-discursive explanation for such an alignment. In a sense, we are presented with another futile quest for certainty.

To some extent, this hopelessness is the result of a particular attitude according to which there are external criteria for correct reference—e.g., derived from some "real" object structure. An assumption that disguises as answer to

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[161] Similar to Tarski introducing the distinction between object- and meta-language, a dualism like the one adopted by Saunders can resolve the contradiction. Meinong and Mally suggest similar solutions.

[162] It is tempting start with "Let s denote...". This shows again the asymmetry in the use of relational theories and of referential theories.

[163] Saunders, in a similar vein, distinguishes the directions of implication in (37): "The implication from left to right follows from Leibniz's law (so the language is extensional). It is the implication from right to left that is controversial, enforcing, for physical objects, a version of the Principle of the Identity of Indiscernibles (PII)." [107]

the above dilemma might well be the root of it. For, if we take the Humean and Wittgensteinian doubt towards the nexus between past experience or available evidence and future events seriously (see Section 8.4.2), then whatever we can say about a real object structure or a truth out there is preliminary. So, on the one hand, any real object structure is not as reliable as it might appear on first sight. On the other hand, the particularity of any empirical test, the Humean predicament [99], reduces the requirements for the reference to systems. We do not need a general criterion of identity, merely the ability to agree on what we are talking about in a particular experimental setup.

11.5   Entanglement and criteria for separability

Let us consider the case of entanglement of two systems (see also Section 5.5). To this end, we assume a composite system with Hilbert space $\mathcal{H}_a \otimes \mathcal{H}_b$ that—put in the common phrasing—is in an entangled state

$$|\psi^+\rangle = \frac{1}{\sqrt{2}} \left(|0,1\rangle + |1,0\rangle \right).$$

This means that based on a measurement in an entangled basis, obtaining a result corresponding to the projector $|\psi^+\rangle\langle\psi^+| \in M_{ab}$ we assign the respective state (see Section 5.2). The story—about, e.g., non-locality (see Section 5.5)—continues with the separation of the subsystems and, eventually, concludes with measurements on the subsystems a and b:[164]
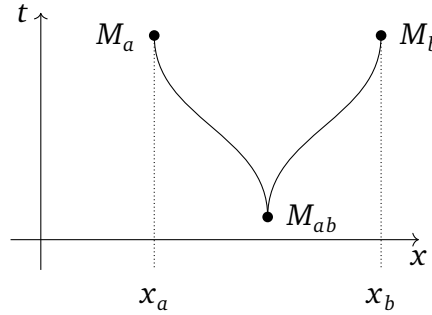
If we assume that the Hilbert spaces $\mathcal{H}_a$ and $\mathcal{H}_b$ are spanned by the two orthogonal vectors $|0\rangle_{a/b}$ and $|1\rangle_{a/b}$, then no PVMs M_a on $\mathcal{H}_a$ and M_b on $\mathcal{H}_b$ allow to distinguish the system a from the system b, for the reduced density matrix—obtained after performing the partial trace over the respective other system—is

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[164]In fact, in concludes with a predicate about the composite system—an insight that Stefan Wolf attributes to particular instances of late-night Matthias Fitzi.

in both cases maximally mixed. What allows to distinguish the systems, however, is the fact that the measurement M_a is performed at a location different from the one of measurement M_b. Implicitly, we also measure the position of the system. The projectors of M_a are rather of the form $\Pi_i^a \otimes \Pi_{\text{left}}$, and, respectively $\Pi_j^b \otimes \Pi_{\text{right}}$ for M_b, where Π_{left} and Π_{right} are orthogonal projectors corresponding to disjoint subsets of $\mathbb{R}^3$. Furthermore, x_a lies in the range of Π_{left}, and x_b in the range of Π_{right}.

This presents us with the complementary situation to the one discussed in Section 6.1: By orthogonality, we cannot observe the same particle in the same position at the same time. That is, while Π_{left} and Π_{right} are orthogonal, $\Pi_{\text{left}}^a :=$ $\Pi_{\text{left}} \otimes \mathbb{1}^b$ and $\Pi_{\text{right}}^b := \mathbb{1}^a \otimes \Pi_{\text{right}}$ are compatible. These observations reflect the separability assumption discussed in Section 8.3. Separability is—fairly intuitively— spatial separability. In Einstein's scenario considered in Section 6.1, however, two *spatially separate* effects are associated with the *same system*. This peculiarity carries over into the scenario here if we uniformly translate the measurements Π_{left}^a and Π_{right}^b back in time, thereby loosing the spatial separation while maintaining compatibility. We gain the possibility to imagine an incompatible measurement: Assume that $\Pi_{\text{left}}^a = |L\rangle\langle L|$ and $\Pi_{\text{right}}^b = |R\rangle\langle R|$ we can define a measurement $\Pi_x^+ := |\nu^+\rangle\langle\nu^+|$ with $|\nu^+\rangle = 1/\sqrt{2}(|L,R\rangle + |R,L\rangle)$.

Indeed, for any initial measurement Π_x with the range in the plane spanned by $|L,R\rangle$ and $|R,L\rangle$, we can distinguish two systems by their later position. Inversely, if we return with this picture to Einstein's scenario from Section 6.1, there is no difference to assuming that there are actually two particles that are initially measured by a Π_x' with range in the plane spanned by $|L,L\rangle$ and $|R,R\rangle$. We then speak of *one* system because *we cannot tell the two apart*.

The peculiarities arise because we equate *separability* with *spatial separability*—in the spirit of non-intersecting trajectories in classical mechanics. Similarly, we commit a fallacy when we think of the uniform translation as a translation "along the trajectories" as depicted above. Implicitly, this corresponds to singling out a preferred basis, the basis of position measurements. The position then becomes the warrant for "speaking about the same system." Theoretically, the $\{|0\rangle, |1\rangle\}$ basis and the $\{|L\rangle, |R\rangle\}$ basis are no different, and we may turn around the scenario by distinguishing particles by the former basis.[165] It seems that we can merely imagine performing arbitrary measurements on the joint system if we assume both parts to be spatially close.[166] To conclude, however, that,

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[165] We may go further and exchange any of the bases by a superposition, such as $\{1/\sqrt{2}(|L\rangle \pm |R\rangle\}$.

[166] Saunders consider the example of a spatially entangled state without starting from a spa-

therefore, we can merely distinguish systems if they are spatially separate or to exclude measurements with the diagonal basis $1/\sqrt{2}(|L\rangle \pm |R\rangle)$ is not necessary. In this sense, the notion of a system in quantum mechanics seems more flexible as commonly assumed. Whether we can refer to multiple constituents of a given system depends on the orthogonality and compatibility of the measurements performed on that system. Merely, if the orthogonality reflects a spatial separation, a lack of imagination makes us feels we *must* speak of multiple constituents.

As we have seen above, any identifying attribute as "being to the left" or "being to the right" can be dissolved in a contextual theory. And whenever we detect entanglement, by, e.g., violating a Bell inequality, then we must conclude that what we measure is not an isolated system and past identification by attributes in the span of the relevant basis are invalidated. Conversely, any assigned label "*L*" or "*R*" merely remains valid, as long as the system does not interact with other systems.

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tially closed system.

In Retrospective

12 Concluding Thoughts

I feel hesitant to write the customary conclusion, for I suspect it might tempt to skim off the results there [40]. What I consider the result is the above—surely incomplete—development of some thoughts.

What, I guess, can be said—if you want, as a distillate[167] of the above: (1) The measurement problem is a formidable starting point for (philosophic) reflections. Even if, ironically enough, it might be the denial of reflections that gives rise to the problem. (2) In the measurement problem, we can make out surfacing cleavages: On the one hand, there is the collision of conflicting epistemological stances. On the other, there are issues of language—e.g., the gap between propositional and referential ways of speaking, as well as between experience and meaning. (3) Sentences like "Consider a system in a state..." that are lightly said in discussions about physical theories hide a considerable commitment. It seems that rather than to acknowledge the contingency of the involved assumptions—thus, the contingency of any way of doing physics—we are tempted to overcompensate: An assumption that facilitates to say something about the world is replaced by an insinuation on how the world must be like in order for us to speak about it the way we do. On first sight, this unnecessarily broadens the assumption. But then it also allows to blur the assumption itself—obscuring the involved contingencies and allowing for a questionable certainty. The emergent idea of a master key [58] that grants access to the truth out there has unnecessarily normalising and narrowing effects.

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[167]Needless to say, eating grapes and drinking grappa are very different.

Bibliography

[1] Ulla Aeschbacher, Arne Hansen, and Stefan Wolf. *Invitation to Quantum Informatics*. Verein der Fachvereine, 2020.

[2] Ichiro Amemiya and Huzihiro Araki. A remark on Piron's paper. *Publications of the Research Institute for Mathematical Sciences, Kyoto University. Series A*, 2(3):423–427, 1966.

[3] Alain Aspect, Jean Dalibard, and Gérard Roger. Experimental test of bell's inequalities using time-varying analyzers. *Physical Review Letters*, 49(25):1804, 1982.

[4] Alexander Bach. Concept of indistinguishable particles in classical and quantum physics. *Foundations of Physics*, 18(6):639–649, 1988.

[5] Alexander Bach. *Indistinguishability, interchangeability, and indeterminism*, pages 149–166. Springer, 1990.

[6] Alexander Bach. *Indistinguishable Classical Particles*. Springer, 1997.

[7] Veronika Baumann, Arne Hansen, and Stefan Wolf. The measurement problem is the measurement problem is the measurement problem. November 2016.

[8] Veronika Baumann and Stefan Wolf. On formalisms and interpretations. *Quantum*, 2:99, October 2018.

[9] John Bell. Against 'measurement'. *Physics World*, 3(8):33–41, 1990.

[10] John S. Bell. On the Einstein-Podolsky-Rosen paradox. *Physics*, 1(3), 1964.

[11] Charles H. Bennett. The thermodynamics of computation—a review. *International Journal of Theoretical Physics*, 21(12):905–940, Dec 1982.

[12] Charles H. Bennett. Notes on Landauer's principle, reversible computation, and maxwell's demon. 2003.

[13] Charles H. Bennett and Gilles Brassard. Quantum cryptography: Public key distribution and coin tossing. In *Proceedings of IEEE International Conference on Computers, Systems, and Signal Processing*, page 175, 1984.

[14] Garrett Birkhoff and John von Neumann. The logic of quantum mechanics. *Annals of Mathematics*, 37(4):823–843, 1936.

[15] Niels Bohr. Atomteorien og Grundprincipperne for Naturbeskrivelsen. *Fysisk Tidsskrift*, 27:103–114, 1929.

[16] Niels Bohr. *Niels Bohr Collected Works*. Elsevier, 1985.

[17] Niels Bohr. VI - Discussion with Einstein on epistemological problems in atomic physics. In Jørgen Kalckar, editor, *Foundations of Quantum Physics II (1933–1958)*, volume 7 of *Niels Bohr Collected Works*, pages 339–381. Elsevier, 1996.

[18] Jeff Buechner. Does Kripke's argument against functionalism under the standard view of what computers are? *Minds & Machines*, 28:491–513, 2018.

[19] Ernst Cassirer. Naturalistische und humanistische Begründung der Kulturphilosophie. *Göteborg Kungl. Vetenskaps- och Vitterhets-Semhälles Handlingar*, 1939.

[20] Ernst Cassirer. *The Philosophy of Symbolic Forms: Language*. Yale University Press, 1955.

[21] Matthias Christandl and Renato Renner. *Lecture Notes — Quantum Information Theory*. 2012.

[22] Claude Cohen-Tannoudji, Bernard Diu, and Franck Laloë. *Quantum Mechanics*. Quantum Mechanics. Wiley, 1991.

[23] Brian A. Davey and Hilary A. Priestley. *Introduction to Lattices and Order*. Cambridge University Press, 2 edition, 2002.

[24] David Deutsch. Quantum theory as a universal physical theory. *International Journal of Theoretical Physics*, 24(1):1–41, 1985.

[25] John Dewey. *The Quest for Certainty: A Study of the Relation of Knowledge and Action*. Minton, Balch and Company, 1929.

[26] Albert Einstein. Ernst Mach. *Physikalische Zeitschrift*, 17, 1916.

[27] Albert Einstein, Hedwig Born, and Max Born. *Briefwechsel 1916-1955*. Rowohlt Verlag, Reinbek, 1972.

[28] Albert Einstein, Boris Podolsky, and Nathan Rosen. Can quantum-mechanical description of physical reality be considered complete? *Physical Review*, 47:777–780, May 1935.

[29] Michael Esfeld. Gonseth and Quine. *Dialectica*, 55(3):199–219, 2001.

[30] Hugh Everett III. "Relative state" formulation of quantum mechanics. *Reviews of Modern Physics*, 29(3):454, 1957.

[31] Paul Feyerabend. *Wider den Methodenzwang*. Suhrkamp Verlag, 1986.

[32] Paul Feyerabend. *Zeitverschwendung*. Suhrkamp Verlag, 1986.

[33] Ludwik Fleck. *Erfahrung und Tatsache*. Suhrkamp Verlag, 1983.

[34] Luciano Floridi. Semantic conceptions of information. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, spring 2017 edition, 2017.

[35] Michel Foucault. *This is Not a Pipe*. An art quantum. University of California Press, 1983.

[36] Daniela Frauchiger and Renato Renner. Single-world interpretations of quantum theory cannot be self-consistent. 2016.

[37] Daniela Frauchiger and Renato Renner. Quantum theory cannot consistently describe the use of itself. *Naure Communications*, 9(3711), 2018.

[38] Gottlob Frege. Über Sinn und Bedeutung. *Zeitschrift für Philosophie und philosophische Kritik*, 100:25–50, 1892.

[39] Gottlob Frege. Sense and reference. *The Philosophical Review*, 57(3):209–230, 1948.

[40] Erich Fromm. *Haben oder Sein*. Deutsche Verlags-Anstalt, 1976.

[41] Chris Fuchs. Quantum Bayesianism at the perimeter. 2010.

[42] Walther Gerlach and Otto Stern. Der experimentelle Nachweis der Richtungsquantelung im Magnetfeld. *Zeitschrift für Physik*, 9(1):349–352, Dec 1922.

[43] Giancarlo Ghirardi, Alberto Rimini, and Tullio Weber. A model for a unified quantum description of macroscopic and microscopic systems. In Luigi Accardi and Wilhelm von Waldenfels, editors, *Quantum Probability and Applications II*, pages 223–232. Springer, 1985.

[44] Willard Gibbs. *Elementary Principles in Statistical Mechanics*. Charles Scribner's Sons, 1902.

[45] Willard Gibbs. On the equilibrium of heterogeneous substances. *Transactions of the Connecticut Academy of Arts and Sciences*, 3, 1929.

[46] Andrew Gleason. Measures on the closed subspaces of a Hilbert space. *Indiana University Mathematics Journal*, 6:885–893, 1957.

[47] Hans-Johann Glock. *Quine and Davidson on Language, Thought and Reality*. Cambridge University Press, 2003.

[48] Ferdinand Gonseth. Über die Sprache sprechen. *Dialectica*, 27(3–4):179–217, 1973.

[49] Daniel Gottesman. Quantum statistics with classical particles. 2005.

[50] Stanley P. Gudder. Dispersion-free states and the exclusion of hidden variables. *Proceedings of the American Mathematical Society*, pages 319–324, 1968.

[51] Anil Gupta. *Empiricism and Experience*. Oxford University Press, 2006.

[52] Jürgen Habermas. *Erkenntnis und Interesse*. Suhrkamp Verlag, 1973.

[53] Jürgen Habermas. *Knowledge and Human Interests*. Heinemann Educational, 1978.

[54] Jürgen Habermas. *Vom sinnlichen Eindruck zum symbolischen Ausdruck*. Suhrkamp Verlag, 1996.

[55] Alan Hájek. Interpretations of probability. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, fall 2019 edition, 2019.

[56] Arne Hansen, Alberto Montina, and Stefan Wolf. Simple algorithm for computing the communication complexity of quantum communication processes. *Physical Review A*, 93:042315, Apr 2016.

[57] Arne Hansen and Stefan Wolf. Measuring Measuring. 2019.

[58] Arne Hansen and Stefan Wolf. No master (key) No (measurement) problem. 2019.

[59] Lucien Hardy. Quantum ontological excess baggage. *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics*, 35(2):267–276, 2004.

[60] Nicholas Harrigan and Robert W. Spekkens. Einstein, incompleteness, and the epistemic view of quantum states. *Foundations of Physics*, 40(2):125–157, 2010.

[61] Carsten Held. The Kochen-Specker theorem. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, fall 2016 edition, 2016.

[62] Leah Henderson. The problem of induction. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, winter 2019 edition, 2019.

[63] Grete Hermann. *Die naturphilosophischen Grundlagen der Quantenmechanik*. Abhandlungen der Fries'schen Schule. Verlag "Öffentliches Leben", 1935.

[64] Don A. Howard. Einstein's philosophy of science. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, fall 2017 edition, 2017.

[65] Kerson Huang. *Statistical mechanics*. Wiley, 1987.

[66] Josef-Maria Jauch and Constantin Piron. Can hidden variables be excluded in quantum mechanics? *Helvetica Physica Acta*, 36:827–837, 1963.

[67] Edwin T. Jaynes. Gibbs vs boltzmann entropies. *American Journal of Physics*, 33(5):391–398, 1965.

[68] Edwin T. Jaynes. *The Gibbs Paradox*, pages 1–21. Springer, 1992.

[69] Andreas Kamlah. The connexion between Reichenbach's three-valued and v. Neumann's lattice-theoretical quantum logic. *Erkenntnis (1975-)*, 16(3):315–325, 1981.

[70] Simon Kochen and Ernst Specker. The problem of hidden variables in quantum mechanics. *Journal of Mathematics and Mechanics*, 17(1):59–87, 1967.

[71] Saul A. Kripke. *Wittgenstein on Rules and Private Language*. Harvard University Press, 1982.

[72] Saul A. Kripke. *Reference and Existence: the John Locke Lectures*. Oxford University Press, 2013.

[73] Thomas S. Kuhn. *The Structure of Scientific Revolutions*. International encyclopedia of unified science. University of Chicago Press, 1962.

[74] Lev D. Landau and Evgeny M. Lifshitz. *Quantum Mechanics (Third Edition)*. Pergamon, third edition edition, 1977.

[75] Rolf Landauer et al. *Information is physical*. IBM Thomas J. Watson Research Division, 1992.

[76] Henri Lauener. Ferdinand Gonseth 1890-1975. *Dialectica*, 31(1–2):113–118, 1977.

[77] Harvey S. Leff and Andrew F. Rex, editors. *Overview*. Princeton University Press, 1990.

[78] Xiaosong Ma, Stefan Zotter, Johannes Kofler, Rupert Ursin, Thomas Jennewein, Časlav Brukner, and Anton Zeilinger. Experimental delayed-choice entanglement swapping. *Nature Physics*, 8(6):479–484, 2012.

[79] O. J. E. Maroney. The (absence of) relationship between thermodynamic and logical reversibility. 2008.

[80] Tim Maudlin. Three measurement problems. *Topoi*, 14(1):7–15, 1995.

[81] Alberto Montina and Stefan Wolf. Information-based measure of nonlocality. *New Journal of Physics*, 18(1):13035, 2016.

[82] Valter Moretti. *Fundamental Mathematical Structures of Quantum Theory: Spectral Theory, Foundational Issues, Symmetries, Algebraic Formulation*. Springer, 2019.

[83] Markus P. Müller. Law without law: from observer states to physics via algorithmic information theory. 2017.

[84] Ted Neward. `http://blogs.tedneward.com/post/the-vietnam-of-computer-science/`, 2006. [Online; accessed 24-Apr-2020].

[85] Ueli Niederer. Galileo Galilei und die Entwicklung der Physik. *Vierteljahrsschrift der Naturforschenden Gesellschaft in Zürich*, pages 205–229, 1982.

[86] Michael A. Nielsen and Isaac L. Chuang. *Quantum Computation and Quantum Information: 10th Anniversary Edition*. Cambridge University Press, 10th edition, 2011.

[87] Friedrich W. Nietzsche. *Über Wahrheit und Lüge im aussermoralischen Sinne*. 1873.

[88] Friedrich W. Nietzsche. *On truth and lies in a nonmoral sense*. Viking Press, 1976.

[89] John D. Norton. Waiting for Landauer. *Studies in History and Philosophy of Science Part B*, 42:184–198, 2011.

[90] Asher Peres. *Quantum Theory: Concepts and Methods*. Fundamental Theories of Physics. Springer, 1995.

[91] Constantin Piron. *Foundations of Quantum Physics*. Frontiers in Physics. W. A. Benjamin, Advanced Book Program, 1976.

[92] Karl Popper. *Logik der Forschung*. Mohr Siebeck, 11 edition, 1934.

[93] Karl Popper. *The Logic of Scientific Discovery*. Harper Torchbooks. HarperCollins Canada, Limited, 1958.

[94] John Preskill. *Lecture Notes for Ph219/CS219: Quantum Information*. 1998.

[95] Matthew F. Pusey, Jonathan Barrett, and Terry Rudolph. On the reality of the quantum state. *Nature Physics*, 8:476–479, June 2012.

[96] Hilary Putnam. *Reason, Truth and History*. Philosophical Papers. Cambridge University Press, 1981.

[97] Hilary Putnam. *Representation and Reality*. A Bradford book. A Bradford Book, 1991.

[98] Willard V. O. Quine. Two dogmas of empiricism. *Philosophical Review*, 60(1):20–43, 1951.

[99] Willard V. O. Quine. Epistemology naturalized. In *Ontological Relativity and Other Essays*. New York: Columbia University Press, 1969.

[100] Donald Rapp. *Statistical mechanics*. Holt, Rinehart and Winston, 1972.

[101] Hans Reichenbach. *Philosophic Foundations of Quantum Mechanics*. Dover Publications, 1944.

[102] Richard Rorty. *Philosophy and the Mirror of Nature*. Princeton University Press, 1979.

[103] Richard Rorty. *Contingency, Irony, and Solidarity*. Cambridge University Press, 1989.

[104] Richard Rorty. *Non-reductive Physicalism*, volume 1, pages 113–125. Cambridge University Press, 1990.

[105] Carlo Rovelli. Relational quantum mechanics. *International Journal of Theoretical Physics*, 35:1637–1678, 1996.

[106] Simon Saunders. *Indistinguishability*. Oxford University Press, 2013.

[107] Simon Saunders. *On the Emergence of Individuals in Physics,* chapter 9, pages 165–192. Oxford University Press, 2015.

[108] Jean Schneider. *Quantum measurement act as a speech act*, pages 345–354. 2005.

[109] Erwin Schrödinger. *Statistical Thermodynamics*. University Press, 1946.

[110] Joachim Schulte. *Wittgenstein, eine Einführung*. Reclams Universal Bibliothek, 1989.

[111] Wilfrid S. Sellars. Empiricism and the philosophy of mind. *Minnesota Studies in the Philosophy of Science*, 1:253–329, 1956.

[112] Ramamurti Shankar. *Principles of Quantum Mechanics*. Springer, 1994.

[113] Ernst Specker. Die Logik nicht gleichzeitig entscheidbarer Aussagen. *Dialectica*, 14:239–246, 1960.

[114] Robert W. Spekkens. In defense of the epistemic view of quantum states: a toy theory. *Physical Review A,* 75,, 2007.

[115] Otto Stern. Ein Weg zur experimentellen Prüfung der Richtungsquantelung im Magnetfeld. *Zeitschrift für Physik*, 7(1):249–253, Dec 1921.

[116] Peter Frederick Strawson. *Entity and Identity: And Other Essays*. Clarendon Press, 2000.

[117] Leo Szilard. Über die Entropieverminderung in einem thermodynamischen System bei Eingriffen intelligenter Wesen. *Zeitschrift für Physik*, 53:840–856, 1929.

[118] Alfred Tarski. Der Wahrheitsbegriff in den formalisierten Sprachen. *Studia Philosophica*, 1:261–405, 1936.

[119] Alfred Tarski. The semantic conception of truth: and the foundations of semantics. *Philosophy and Phenomenological Research*, 4(3):341–376, 1944.

[120] D. ter Haar. *Elements of Statistical Mechanics*. Butterworth-Heinemann, 1954.

[121] Nicolaas G. van Kampen. The Gibbs paradox. In W.E. Parry, editor, *Essays in Theoretical Physics*, pages 303 – 312. Pergamon, 1984.

[122] Felix von Leitner. Das Unbedenklich Unkompliziert Spektrum. `https://media.ccc.de/36c3/..../`, 2019. [accessed 08-Mar-2020].

[123] Felix von Leitner and Frank Rieger. Alternativlos 45. `https://alternativlos.org/45/`, 2020. [accessed 08-Mar-2020].

[124] Frank. W. Warner. *Foundations of Differentiable Manifolds and Lie Groups*. Graduate Texts in Mathematics. Springer, 1983.

[125] Peter Weir. The Truman show. Movie, 1998.

[126] Eugene P. Wigner. Remarks on the mind-body question. In I. J. Good, editor, *The Scientist Speculates*. Heineman, 1961.

[127] Eugene P. Wigner. The problem of measurement. *American Journal of Physics*, 31(1):6–15, 1963.

[128] Ludwig Wittgenstein. *Tractatus logico-philosophicus*. Routledge, 1922.

[129] Ludwig Wittgenstein. *Philosophical Investigations*. Basil Blackwell Ltd, 1953.

[130] Ludwig Wittgenstein. *Philosophische Untersuchungen*. Suhrkamp Verlag, 1953.

[131] Edward N. Zalta. *Abstract Objects: An Introduction to Axiomatic Metaphysics*. Synthese Library. Springer, 1983.

[132] Wojciech H. Zurek. *Complexity, Entropy, and the Physics of Information: The Proceedings of the 1988 Workshop on Complexity, Entropy, and the Physics of Information Held May-June, 1989, in Santa Fe, New Mexico*. Proceedings volume in the Santa Fe Institute studies in the sciences of complexity. Addison-Wesley, 1990.

[133] Stefan Zweig. *Die Welt von Gestern: Erinnerungen eines Europäers*. S. Fischer, 1955.

1   Introduction to Lattices

In this section, we give an introduction to lattices and summarize the results relevant for this thesis.

Definition 1 (Poset)**.** A set P is *partially ordered* if any elements $x, y, z \in P$ satisfy

(P1) $x < x$ (reflexivity),

(P2) if $x < y$ and $y < x$, then $x = y$ (antisymmetry),

(P3) if $x < y$ and $y < z$, then $x < z$ (transitivity).

A partially ordered set is often referred to as a *poset*.

Definition 2 (Supremum and infimum)**.** The *supremum* of a subset $S \subset P$ of poset P least upper bound, i.e.,

$$\sup S = \min\{s \in P \mid s > x \ \forall x \in S\}$$

and, vice versa, the *infimum* is the greatest lower bound, i.e.,

$$\inf S = \max\{s \in P \mid s < x \ \forall x \in S\}.$$

If the supremum or infimum exist, then they are unique.

Definition 3 (Join and meet)**.** If the supremum $\sup\{x, y\}$ exists, we call it *join*, and write $x \wedge y$. If the infimum $\inf\{x, y\}$ exists, we call it *meet*, and write $x \vee y$.

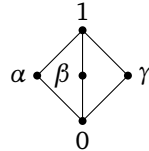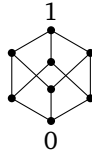**Definition 4** (Bounded Poset)**.** A poset is *bounded* if there exist a *null element* $0 \in P$ such that $0 < x \ \forall x \in P$ and a *universal element* $1 \in P$ such that $1 > x \ \forall x \in P$.

Definition 5 (Lattice)**.** A partially ordered set $(L, <)$ with unique greatest lower bound $a \wedge b$ and unique least upper bound $a \vee b$ is called a *lattice*.

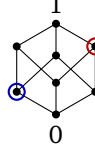The following is a bounded poset but not a lattice:
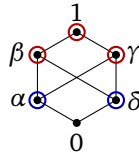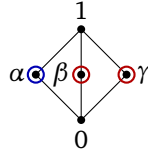

while these ones are:

Definition 6 (Complementation)**.** An *complementation* is a map $a \mapsto \neg a$ such that

$$a \wedge \neg a = 0, \ a \vee \neg a = 1 \,.$$

In the example above, the two marked elements are complements:

A lattice might allow for more than one complementation:


Then, the map is not necessarily bijective.

Definition 7 (Orthocomplementation)**.** If the complementation on a lattice is bijective and order reversing, i.e.,

$$\text{if } a < b : \neg a > \neg b \,,$$

then the we refer to it as an *orthocomplementation*.

Definition 8 (Orthogonality)**.** In an orthocomplemented lattice L, two elements a, b are *orthogonal*, denoted $a \perp b$, if $a < \neg b$, or, equivalently, $b < \neg a$—using the order reversing property of the ortho-complementation.

In the following example, a and b are orthogonal to one another:
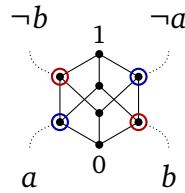
Definition 9 (Weak modularity)**.** An lattice L satisfies *weak modularity* if for all $a < b$:

$$a \vee (\neg a \wedge b) = a \,,$$
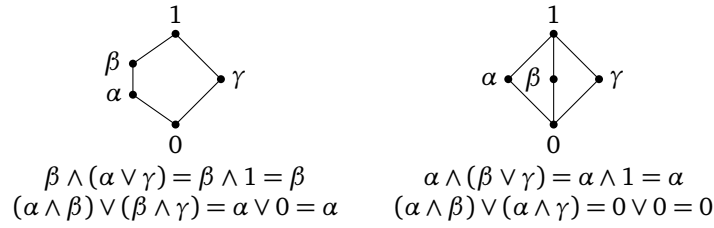
or, equivalently, if $a > b$:

$$a \wedge (\neg a \vee b) = a.$$

A orthocomplemented, weakly modular lattice is called *orthomodular*.

Definition 10 (Distributivity). A lattice L is *distributive* if for any $a, b, c \in L$

$$a \wedge (b \vee c) = (a \vee b) \wedge (a \vee c)$$

The following two lattices are the smallest non-distributive lattices:

$$\beta \wedge (\alpha \vee \gamma) = \beta \wedge 1 = \beta \qquad \alpha \wedge (\beta \vee \gamma) = \alpha \wedge 1 = \alpha$$
$$(\alpha \wedge \beta) \vee (\beta \wedge \gamma) = \alpha \vee 0 = \alpha \qquad (\alpha \wedge \beta) \vee (\alpha \wedge \gamma) = 0 \vee 0 = 0$$

Definition 11 (Compatibility). Two elements in a lattice, $a, b \in L$ are called *compatible*, if the lattice generated by $\{a, \neg a, b, \neg b\}$ is distributive.

If for $a, b \in L$

$$a \wedge (\neg a \vee b) = b \qquad \neg b \wedge (b \vee \neg a) = \neg a,$$

then the elements

$$0, a, \neg a, b, \neg b, a \wedge \neg b, \neg a \vee b, 1$$

form the distributive sublattice generated by $\{a, \neg a, b, \neg b\}$.

Theorem 2 (Compatibility I). *In an orthomodular lattice L, a and b are compatible if and only if*

$$(a \wedge b) \vee (\neg a \wedge b) = b. \tag{38}$$

The proof consists of combining Theorem (2.15) and (2.17) in Ref. [91].

Proof. If the sublattice is distributive, then

$$(a \wedge b) \vee (a \wedge \neg b) = a \quad \text{and} \quad (a \wedge b) \vee (\neg a \wedge b) = b.$$

It remains to show that (38) is sufficient. First we show that compatibility is symmetric: If $(a \wedge b) \vee (\neg a \wedge b) = b$, then, with $a \wedge (a \vee c) = a$ for any c,

$$a \wedge \neg b = a \wedge (\neg a \vee \neg b) \wedge (a \vee \neg b)$$
$$= a \wedge (\neg a \vee \neg b) = a \wedge \neg (a \wedge b).$$

As $a > (a \wedge b)$, we employ orthomodularity,

$$a = (a \wedge b) \vee (a \wedge \neg(a \wedge b)) = (a \wedge b) \vee (a \wedge \neg b),$$

which proves the symmetry of compatibility. Further, we have to show equalities of the form

$$(a \wedge b) \vee \neg b = a \wedge \neg b. \tag{39}$$

Note, first, that in any lattice $a \wedge b < b$ and, therefore,

$$\underbrace{(a \wedge b) \vee \neg b}_{=:c_1} < \underbrace{a \vee \neg b}_{=:c_2}.$$

Applying orthomodularity, i.e., $c_2 \wedge (\neg c_2 \vee c_1) = c_1$, yields

$$(a \vee \neg b) \wedge \left(\underbrace{(a \wedge b) \vee (\neg a \wedge b)}_{=b} \vee \neg b \right) = (a \wedge b) \vee \neg b$$

and, thus, we obtain (39).   □

Note the important role of weak modularity in the proof above.

Theorem 3 (Compatibility II). *In an orthomodular lattice L, a is compatible with b if and only if there exists mutually orthogonal elements $a', b', c \in L$ such that*

$$a = a' \vee c \qquad b = b' \vee c.$$

Proof. If a and b are compatible, then

$$(a \wedge b) \vee (\neg a \wedge b) = b$$
$$(a \wedge b) \vee (a \wedge \neg b) = a$$

and thus we can set

$$c := a \wedge b \quad a' := a \wedge \neg b \quad b' := b \wedge \neg a$$

which yields the orthogonal elements. If, inversely, a and b can be expressed in the above form, then

$$\neg a \wedge b = \neg a' \wedge \neg c \wedge (b' \vee c)$$
$$= b' \wedge \neg a' = b'$$

using weak modularity. Furthermore,

$$
\begin{aligned}
a \wedge b &= (a' \vee c) \wedge (b' \vee c) \\
&< (a' \vee c) \wedge (\neg a' \vee c) \\
&= (a' \vee c) \wedge \neg a' = c
\end{aligned}
$$

using weak modularity again, and, thus, with

$$
a \wedge b = (a' \vee c) \wedge (b' \vee c) > c
$$

we obtain $a \wedge b = c$. This yields 38. $\qquad\square$

Theorem 4 (Existence of dispersion-free states). *On an orthomodular lattice L, there exists a dispersion-free state if and only if there exists an atom in the center of the lattice.*

We follow the proof of Theorem I in Ref. [66] and the proof of Theorem 1 in Ref. [50].

Proof. We first show that the existence of a dispersion-free state μ implies that there is an element in the center of L different from 0 and 1. Let $L_\mu := \{a \in L \mid \mu(a) = 1\}$. Note that $L_\mu \neq \emptyset$. For any totally ordered subset T of L_μ, the element $a_0 = \wedge\{a \in T\} \in L_\mu$ by the properties of a dispersion-free state. Thus, with Zorn's Lemma, L_μ has a minimal element a_m.

Let us now show that $a_m \leq a \ \forall a \in L_\mu$: If $a_1 \in L_\mu$, then there must exist an $a_2 \neq 0$ with $a_2 < a_m$. Otherwise $a_m \wedge a_1 = 0$ which contradicts the requirement of a dispersion-free state. By weak modularity and Theorem 2, there exist an $b \in L$ orthogonal to a_2 such that $a_2 \vee b = a_m$.

If $\mu(a_2) = 0$, then $\mu(b) = 1$ and, thus, $b \in L_\mu$ and $b < a_m$, thus, $b = a_m$. But then, $a_2 = 0$ and we obtain a contradiction, and we conclude that $\mu(a_2) = 1$. This yields, again, $a_2 = a_m$, and then $a_m < a_1$.

We now show that a_m lies in the center of L: For any $b \in L_\mu$, $a_m < b$ and the two commute by weak modularity. If $b \notin L_\mu$, then with $\mu(b) + \mu(\neg b) = \mu(b \vee \neg b) = \mu(1) = 1$ it follows $\neg b \in L_\mu$. Thus, $a_m < \neg b$ and, therefore, a_m is compatible with $\neg b$ and thus with b.

Now, we show that a_m is an *atom* in L, i.e., $\forall b \in L, b < a_m$ either $b = a_m$ or $b = 0$. It suffices to show that if $b \notin L, b < a$ then $b = 0$: In this case, $\neg b \in L_\mu$, and $\neg b > a_m > b$. Therefore, $b \wedge b < b \wedge \neg b = 0$.

It remains to construct a dispersion-free state from an atom a in the center of L: Let $\mu(a) = 1$. Any $b \in L$ commutes with a, and thus $a \wedge b$ exists and $a \wedge b < a$. Since a is an atom, either $a \wedge b = 0$ and, therefore, $b = 0$ and $\mu(b) = 0$,

or $a \wedge b = a$. In the latter case, $a < b$ and $\mu(b) = 1$. Thus, μ is dispersion-free. □

Lemma 1 (Incompatible elements). *If $a < b$ and a are not compatible with c for some $a, b, c \in L$, then also b and c are incompatible.*

Proof. We have to show: If $a < b$ and b is compatible with c, then also a is compatible with c. Let us consider

$$
\begin{aligned}
a \wedge \neg c &= a \wedge \neg c' \wedge \neg d && \text{Thm } 3 : c = c' \vee d \\
&= a \wedge b' \wedge \neg d && c' \perp b' \\
&= a \wedge b' && d \perp b' \\
&= a \wedge b = a && a < b
\end{aligned}
$$

and thus

$$
(a \wedge c) \vee (a \wedge \neg c) = (a \wedge c) \vee a = a \,.
$$

□

2 Orthogonal projectors on a Hilbert space

Definition 12 (Inner product). Given a complex vector space V, the map $V \times V \to \mathbb{C}, (x, y) \mapsto \langle x|y \rangle$ is called an *inner product* if it satisfies

(IP1) $\langle x|x \rangle \geq 0 \quad \forall x \in V$,

(IP2) $\langle x|\alpha y + \beta z \rangle = \alpha \langle x|y \rangle + \beta \langle x|y \rangle \quad \forall x, y, z \in V, \alpha, \beta \in \mathbb{C}$,

(IP3) $\langle x|y \rangle = \overline{\langle y|x \rangle} \quad \forall x, y \in V$, and

(IP4) $\langle x|x \rangle = 0 \implies x = 0$.

Two vectors $x, y \in V$ are orthogonal if $\langle x|y \rangle = 0$. The *orthogonal complement* to a subspace $W \subset V$ is

$$
W^{\perp} := \{x \in V \mid \langle x|y \rangle = 0 \ \forall y \in W\} \,.
$$

Definition 13 (Hilbert space). An inner product space $(V, \langle \cdot|\cdot \rangle)$ is a *Hilbert space* if the V with the norm induced by the inner product is complete.
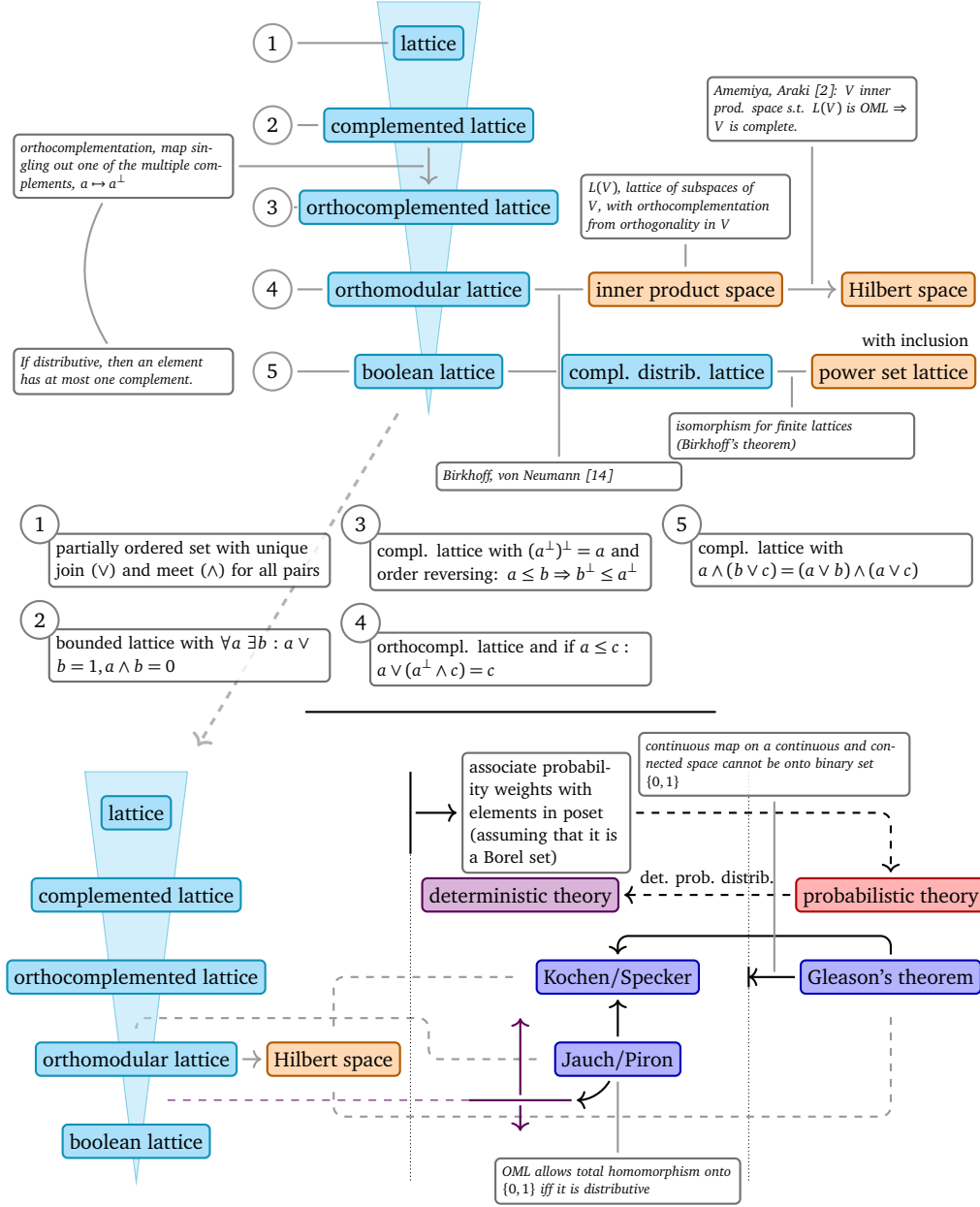
Figure 6. A brief summary of lattices and quantum logic.

Any Hilbert space $\mathcal{H}$ admits an orthonormal basis, i.e., a set of vectors N, such that for any $x, y \in N$ $\langle x|y \rangle = 0$ if $x \neq y$ and $\langle x|x \rangle = 1$ (see [82, Theorem 3.27]). For any linear operator $T \in \mathfrak{L}(\mathcal{H})$, there exists a unique operator $T^\dagger$ such that

$$\langle x|Ty \rangle = \langle T^\dagger x|y \rangle \ \forall x, y \in \mathcal{H} \tag{40}$$

(see [82, Proposition 3.36]).

Definition 14 (Hermitian adjoint)**.** The unique map $T^\dagger$ for which (40) holds is called the *Hermitian adjoint* of T.

With the Hermitian adjoint the set of continuous, linear operators $\mathfrak{L}(\mathcal{H})$ on a Hilberspace $\mathcal{H}$ forms a $C^\star$-algebra.

Definition 15 (Orthogonal projector)**.** An operator $P \in \mathfrak{L}(\mathcal{H})$ is an *orthogonal projector* if P is idempotent, i.e., $PP = P$, and self-adjoint, i.e., $P^\dagger = P$. We denote the set of orthogonal projectors $\mathfrak{P}(\mathcal{H})$.

The set of orthogonal projectors $\mathfrak{P}(\mathcal{H})$ forms an orthomodular, atomic lattice with the order relation that is equivalently characterized by any of the following conditions:

(OR1) $P \leq Q$, defined as $\langle x|Px \rangle \leq \langle x|Qx \rangle \ \forall x \in \mathcal{H}$.

(OR2) $P(\mathcal{H})$ is subspace of $Q(\mathcal{H})$.

(OR3) $PQ = QP = P$.

The following are equivalent characterizations of orthogonality (see [82, Proposition 7.16, Theorem 7.18]:

(OG1) $PQ = 0$.

(OG2) $QP = 0$.

(OG3) $P(\mathcal{H})$ and $Q(\mathcal{H})$ are orthogonal subspaces.

(OG4) $Q \leq \mathbb{1} - P$.

(OG5) $P \leq \mathbb{1} - Q$.

(OG6) $P \perp Q$, i.e., P and Q are orthogonal elements in the lattice $\mathfrak{P}(\mathcal{H})$.

We now characterize subsets of $\mathfrak{L}(\mathcal{H})$ by introducing Hilbert-Schmidt operators (see [82, §4.3]).

Definition 16 (Hilbert-Schmidt operators)**.** An operator $A \in \mathfrak{L}(\mathscr{H})$ is a *Hilbert-Schmidt operator* if there exists a basis U on $\mathscr{H}$ such that

$$\sum_{u \in U} \langle Au | Au \rangle < \infty \,.$$

The set of Hilbert-Schmidt operators $\mathfrak{L}_{\mathrm{HS}}(\mathscr{H}) \subset \mathfrak{L}(\mathscr{H})$ with the inner product

$$(A|B) := \sum_{u \in N} \langle Au | Bu \rangle$$

where N is a basis of $\mathscr{H}$, forms itself a Hilbert space. In particular, two projectors $P, Q \in \mathfrak{L}_{\mathrm{HS}}(\mathscr{H})$ are orthogonal in the sense above, if and only if, $(P|Q) = 0$.

Definition 17 (Trace class)**.** The set of *trace class operators* $\mathfrak{L}_{\mathrm{Tr}}(\mathscr{H}) \subset \mathfrak{L}(\mathscr{H})$ is the set of operators $T \in \mathfrak{L}(\mathscr{H})$ for which the trace $\mathrm{Tr}\, T$ is finite, i.e.,

$$\mathrm{Tr}\, T := \sum_{u \in N} \langle u | Tu \rangle < \infty \,,$$

where N is an orthonormal basis of $\mathscr{H}$. In particular, the trace is independent of the choice of basis.

The set of trace class operators is a subset of the Hilbert-Schmidt operators, i.e., $\mathfrak{L}_{\mathrm{Tr}}(\mathscr{H}) \subset \mathfrak{L}_{\mathrm{HS}}(\mathscr{H}) \subset \mathfrak{L}(\mathscr{H})$. The Hilbert-Schmidt inner product reduces for trace class operators to the *Frobenius inner product*, i.e.,

$$(A|B) = \mathrm{Tr}(A^{\dagger}B) \,.$$

Thus, trace-class projectors, in particular projectors on a finite dimensional Hilbert space are orthogonal, iff $\mathrm{Tr}(P^{\dagger}Q) = 0$.