WORKING PAPERS SES

A Machine Learning Approach for Flagging Incomplete Bid-rigging Cartels

Hannes Wallimann,
David Imhof
and
Martin Huber

A Machine Learning Approach for Flagging Incomplete Bid-rigging Cartels

Hannes Wallimann*, David Imhof** and Martin Huber***

* University of Applied Sciences and Arts Lucerne, Competence Center for Mobility, University of Fribourg, Dept. of Economics

** Corresponding Author, Swiss Competition Commission, University of Fribourg, Dept. of Economics and Unidistance (Switzerland)

*** University of Fribourg, Dept. of Economics

Abstract: *We propose a new method for flagging bid rigging, which is particularly useful for detecting incomplete bid-rigging cartels. Our approach combines screens, i.e. statistics derived from the distribution of bids in a tender, with machine learning to predict the probability of collusion. As a methodological innovation, we calculate such screens for all possible subgroups of three or four bids within a tender and use summary statistics like the mean, median, maximum, and minimum of each screen as predictors in the machine learning algorithm. This approach tackles the issue that competitive bids in incomplete cartels distort the statistical signals produced by bid rigging. We demonstrate that our algorithm outperforms previously suggested methods in applications to incomplete cartels based on empirical data from Switzerland.*

Keywords: Bid rigging detection, screening methods, descriptive statistics, machine learning, random forest, lasso, ensemble methods.

JEL classification: C21, C45, C52, D22, D40, K40, L40, L41.

1 Introduction

Bid rigging is a pervasive problem and has been considered in a range of studies. A few examples include bid rigging in a Swedish asphalt cartel (*Bergman et al.*, 2019), in public procurement auctions for construction works in Japan (*Ishii*, 2014), of stamp dealers in New York (*Asker*, 2010), among seafood processors in the US (*Abrantes-Metz et al.*, 2006), of construction firms in Korea (*Lee and Hahn*, 2002), for school milk contracts in Ohio (*Porter and Zona*, 1999), Florida and Texas (*Pesendorfer*, 2000) and related to US highway construction contracts (*Porter and Zona*, 1993; *Feinstein et al.*, 1985). When firms deviate from competitive bidding and form a bid-rigging cartel, they secretly conspire to raise prices or lower the quality of goods or services. As such, bid-rigging conspiracies directly harm taxpayers, buyers or sellers. In order to enhance the fight against bid rigging, the OECD recommends competition agencies to promote pro-active methods for uncovering bid-rigging cartels (*OECD*, 2014). Answering the need of competition agencies for pro-active statistical methods, *Imhof et al.* (2018), *Huber and Imhof* (2019) and *Imhof* (2020) have proposed different methods for uncovering bid-rigging cartels with screens. However, the three papers deal solely with one form of bid rigging: Complete bid-rigging cartels. The reality is more complex, and competitive bidders often participate in tenders in which a cartel is active. Competitive bids distort the statistical signals produced by cartels in the distribution of the bids so that conventional detection methods fail to recognize the track of bid rigging. With competitive bids, the correct classification rate of collusive and competitive tenders based on the methods suggested in the three previously mentioned papers shrinks. This study improves on this issue by constructing a more robust detection method obtaining a decent correct classification rate similar to *Huber and Imhof* (2019) even in the presence of competitive bids.

As methodological contribution, our approach considers a larger set of screens than *Huber and Imhof* (2019), likely more powerful for detecting incomplete cartels. Screens are statistics derived from the distribution of bids in a tender aiming to capture the distributional changes produced by bid rigging. However, since competitive bids distort the statistical signals produced by bid rigging, screens based on the total of bids in some tender are less effective in detecting incomplete bid-rigging cartels. Our approach overcomes this issue by calculating screens for subgroups of bids in a tender. For this purpose, we form all possible subgroups of three and four bids per tender and calculate screens for each subgroup. We then calculate the mean, median, minimum and maximum of each screen for the different subgroups within some tender. This permits restoring the original statistical signals produced by bid rigging in presence of competitive bids. We use data from Switzerland in which the incidence of collusive and competitive tenders is known in order to apply the screens as predictors for collusion and assess the correct classification rate of collusive and competitive tenders. As in *Huber and Imhof* (2019), we use machine learning for the optimal prediction (which does not

require the construction of explicit structural or causal models for collusion) and focus on the random forest, see *Breiman* (2001), a widely used algorithm for statistical learning (see *Athey and Imbens*, 2019).

We first apply our method to the Ticino bid-rigging cartel (hereafter: the Ticino cartel), which was a complete cartel involving all firms active in road construction in the canton of Ticino. The cartel members rigged all tenders from January 1999 until the end of March 2005 (see *Imhof*, 2020, for details). Since we also have the data from the post-cartel period, we use them to simulate competitive bids that we add to the collusive tenders. We create five new datasets with the collusive tenders: The first dataset contains one simulated competitive bid in each collusive tender. The second one includes two simulated competitive bids in each collusive tender, and so on. This stepwise approach permits investigating how competitive bids distort the statistical patterns of bid rigging in the distribution of the bids. We also note that the simulation process produces competitive bids exhibiting the same features as the bids in the post-cartel period. We compare the correct classification rate of three methods. The first method of *Imhof et al.* (2018) uses two screens applied with benchmarks. The second method of *Huber and Imhof* (2019) uses machine learning, namely lasso and an ensemble method (averaging over several machine learners, namely bagged trees, random forests and neural networks) to a range of screens in order to predict collusion. In this paper, we implement their suggested method using random forests as machine learner. The third method, which is an original contribution of this paper, includes summary statistics of the screens (*median*, *mean*, *maximum and minimum*) calculated in all possible subgroups of bids in a tender as predictors in the random forest.

We find that the method of *Imhof et al.* (2018) performs poorly when the number of competitive bids per tender increases in the data of the Ticino cartel. With three simulated competitive bids, its correct classification rate does not perform better than flipping a coin. It is particularly unsatisfactory in the case of collusive tenders, with a correct classification rate of only 15%. The method of *Huber and Imhof* (2019) entails a correct classification rate between 72% and 84%, again decreasing in the number of competitive bids added to the collusive tenders. Including summary statistics of screens calculated for all possible subgroups of bids in a tender, as suggested in this paper, exhibits a higher correct classification rate of 77% to 86%. We also observe that the correct classification rate does not decrease in the number of competitive bids added to the collusive tenders. Therefore, the relative performance of the method suggested in this paper when compared to *Huber and Imhof* (2019) increases in the number of competitive bids. With five simulated competitive bids, the difference between both methods accounts for over 10 percentage points. If we consider the error rate, defined as one minus the correct classification rate, it decreases by 43% with our approach. Cutting the error rate almost by half is substantial with regard to the heavy legal consequences of flagging firms as bid-rigging cartels. The detection method constructed in our paper can therefore flag both complete and incomplete bid-rigging cartels with a decent correct classification rate.

However, the simulation exercise based on the Ticino case is limited to the purpose of examining the correct classification rate of different methods. It does not account for the reaction of competitive and collusive bidders when they are aware of their reciprocal existence. In fact, competitive bidders might try to benefit from the umbrella effect of the cartel by bidding higher than they would have in a competitive situation (*Bos and Harrington*, 2010). Some may also try to join temporarily a cartel as observed in some cases. Conversely, collusive bidders react to the presence of competitive bidders. They might adopt a more competitive behavior by bidding more aggressively as attested in the Swiss case of See-Gaster in which cartel participants decided together on which tender they would form a cartel and on which they would compete. We therefore need to apply our methods to fully empirical data to verify the performance of our simulation exercise based on the Ticino cartel.

As an important empirical contribution, we therefore consider rather unique data from two investigations of the Swiss competition commission (hereafter: COMCO): See-Gaster and Strassenbau Graubünden. Both cases were characterized by well-organized bid-rigging cartels that were for a large share of tenders complete. However, from time to time, they faced competition from outsiders. We mainly focus on the collusive tenders with competitive bids, and again evaluate the performance of the three previously mentioned methods, namely the approach of *Imhof et al.* (2018), the method of *Huber and Imhof* (2019) using a random forest, and the method suggested in this paper that includes summary statistics calculated for subgroups of bids in a tender.

We find that in See-Gaster and Graubünden data, the approach of *Imhof et al.* (2018) again exhibits a low correct classification rate which roughly amounts to 50% for incomplete cartels (collusive tenders with competitive bids). Considering tenders with incomplete cartels only, the correct classification rate is even much lower and varies between 8% and 14%. Even for complete cartels, the correct classification rate of *Imhof et al.* (2018) barely improves. This result might suggest that the benchmarks applied in *Imhof et al.* (2018) for detecting collusive tenders in Swiss data are generally ill-posed, due to the substantial amount of false negative predictions (incorrectly proposing the absence of a cartel). However, benchmarks require few information to be implemented and perform relatively well in the data of the Ticino cartel (when complete), and they therefore remain a first step in developing screening methods for flagging cartels. Moreover, we also illustrate that one can use the decision tree to recalibrate the benchmarks used by *Imhof et al.* (2018) and increase the correct prediction rate.

Considering machine learning methods, we note that for complete cartels, there are very little differences across approaches in terms of the correct classification rate, which amounts to 81-83%. When competitive bidders are present, however, the correct classification rate is higher (amounting to 67% to 84% depending on the sample) when including summary screens for subgroups of bids in a tender than when using the screens of *Huber and Imhof* (2019) alone (61% to 77%). We note that the performance of all machine learning approaches decreases in the proportion of competitive bids.

From less to more competitive bids, the correct classification rate shrinks by 16 to 17 percentage points for the various methods. Contrary to the simulation based on the Ticino case, this result suggests that cartel participants partially endogenise the presence of competitive bidders by adopting in some cases a more competitive behavior.

Our paper is related to other studies using screens for uncovering bid-rigging cartels. *Huber and Imhof* (2019) and *Imhof* (2020) construct detection methods based on screens using data from Swiss bid-rigging cases, in which they can distinguish between collusive and competitive periods. Such analyses are *ex-post* and allow assessing the performance of the screens and detection methods. Our paper is also an *ex-post* analysis and contrasts with *Imhof et al.* (2018), who also develop a method based on screens, however without information on the existence of bid-rigging cartels in the data. By constructing different self-reinforcing tests, *Imhof et al.* (2018) uncovered a bid-rigging cartel in the region of See-Gaster in the canton of St Gallen. COMCO opened an investigation in 2013 essentially based on such a statistical analysis and sanctioned the firms involved in the cartel in 2016.[1] In a broader context, our paper is also related to contributions using screens for uncovering price-fixing conspiracies (see *Abrantes-Metz et al.*, 2006; *Esposito and Ferrero*, 2006; *Hueschelrath and Veith*, 2011; *Jimenez and Perdiguero*, 2012; *Abrantes-Metz et al.*, 2012; *Crede*, 2019).

Our study is also related to further papers on detecting bid-rigging cartels, however, not relying on screens. One seminal paper is *Bajari and Ye* (2003), who propose two econometric tests for classifying pairs of firms as collusive. The idea of the first test consists of verifying whether bids are independent conditional on the costs of each firm by estimating a bidding function and testing if residuals are correlated between firms. Correlation between firms would indicate that bids are not independent and thus point to potential collusion. The second test examines if firms react in the same way as a function of their own costs. The test verifies if estimated coefficients on cost variables are identical for each firm. Divergent coefficients between firms could indicate potential collusive issues. Subsequent papers apply and refine the econometric tests suggested by Bajari and Ye (2003) (see *Jakobsson*, 2007; *Aryal and Gabrielli*, 2013; *Chotibhongs and Arditi*, 2012a,b; *Imhof*, 2017; *Bergman et al.*, 2019). *Imhof* (2017) questions the performance of the econometric tests proposed by *Bajari and Ye* (2003) for detecting the Ticino cartel because the econometric tests produce too many false negative results by failing to classify pairs of firms as collusive. Moreover, econometric tests require data on the firm level, which are difficult to access especially if a competition agency wishes to apply such tests ex ante. In contrast, the test by *Bergman et al.* (2019) only uses data from the bid summaries[2] combined with a spatial model for uncovering bid-rigging cartels in Sweden. Our research is also related to papers analyzing the effect of bid rigging (*Pesendorfer*, 2000; *Ishii*, 2009) and to pa-

---

[1]See press release: https://www.weko.admin.ch/weko/fr/home/actualites/communiques-de-presse/nsb-news.msg-id-64011.html.

[2]Bid summaries are the official records of a bid opening. At a fixed date, the procurement agency open all received bids and writes a record of all the submitted bids for some specific contract. Then, the procurement agency examines all the submitted bids in detail.

pers investigating the change in bidding patterns when bid rigging occurs (*Porter and Zona*, 1993, 1999). As a further relevant approach, *Chassang et al.* (2020) investigate the occurrence of bid values with zero densities in the distribution of bids. Finding such gaps in the observed values of bids, especially between the first and the second lowest bid, might point to collusive behavior as bids with zero densities (between the two lowest bids) should not systematically occur in competitive markets.

The remainder of this study is organized as follows. Section 2 presents the bid-rigging cartels uncovered in Switzerland from which our data are drawn. Section 3 outlines the detection methods for flagging both complete and incomplete bid-rigging cartels. Section 4 applies our methods to a simulation of incomplete cartels based on data from the Ticino bid-rigging cartel and as well as to empirical data from the cases of See-Gaster and Strassenbau Graubünden. Section 5 concludes.

2 Bid-Rigging Cartels and Data

The Swiss Parliament revised the federal Cartel Act and introduced a sanction regime in April 2004, with an adaptation period of one year, alongside with a compliance program. This legislative change helped in initiating a change in the praxis towards economic harmful bid-rigging cartels. The same year, a strategic body of the federal administration for procurement of goods and services prepared a questionnaire for both procurement agencies and bidding firms. One result was that all surveyed persons declared to know the existence of bid rigging and 50% to have a concrete experience in bid rigging.[3] At the end of 2004, COMCO began to investigate the Ticino cartel and rendered its decision in 2007. The Ticino cartel dissolved without sanctions since it stopped its illegal conduct precisely before April 2005, after having consumed the whole adaptation period. It however stressed the damages and mischiefs of a bid-rigging cartel with a price increase over 30% (see *Imhof*, 2020). In 2008, COMCO decided to put priority to fight bid rigging with a strategy involving three pillars: prosecution, prevention, and detection.

In the pillar prosecution, COMCO treated many bid-rigging cases subsequent to the Ticino cartel. Table 1 lists the most important decisions of COMCO in bid-rigging cases and the sanctions for each case. Initially, COMCO rendered an important decision against bid rigging every other year. From 2015 on, however, COMCO rendered a decision each year and increased sanctioning, emphasizing the determination to prosecute bid-rigging conspiracies. This appeared necessary for disciplining firms and deterring the formation of bid-rigging cartels.

The sanctions presented in Table 1 depend on the proven facts of misconduct in the various cases. In Switzerland, the sanctions depend on the definition of the relevant market, which is derived from the proven facts. Whenever COMCO only dismantles the existence of single agreements between firms, it defines the relevant market as being restricted to the tenders in which firms have settled

---

[3]We suspect this rate to be underestimated since the bidding firms participating in a bid-rigging conspiracy might have an incentive for misreporting.

Table 1: Decisions of COMCO in bid-rigging cases

Decisions of COMCO (excerpt)	Year	Sanctions in CHF mio.	Number of firms
Road asphalting in Ticino	2007	–	17
Electric Installations Bern	2009	1,2	7
Road Construction and Civil Engineering Aargau	2011	7	18
Road Construction and Civil Engineering Zurich	2013	0,5	12
Tunnel Cleaning	2015	0,16	3
Road Construction and Civil Engineering See-Gaster	2016	5	8
Construction in Val Mustair	2017	–	5
Six short Decisions in Engadine	2017	1	12
Construction in Lower Engadine	2018	7,5	7
One short Decision in Engadine	2019	0,5	3
Road construction Graubünden	2019	11	12

the single agreements. However, whenever COMCO can prove a system agreement, which goes beyond the scope of a single tender implying contract allocation over time, then the relevant market comprises all market activities affected by bid rigging. For example, the Ticino cartel was a system agreement since all firms, active on the road construction and asphalting market, rigged all public and private contracts for more than five years. In such a case, COMCO considers all the revenues in the relevant market for sanctioning, not only in tenders for which COMCO has proved an agreement. The Ticino cartel went unsanctioned since it stopped its illegal conduct before April 2005, but if it had been sanctioned, the total fine would have amounted up to 30 million CHF. In the cases of road construction in Graubünden, building and civil engineering in Lower Engadine as well as road construction and civil engineering in See-Gaster, COMCO proved system agreements. Unsurprisingly, the sanctions are consistently higher than in the case of single agreements.

In parallel with the prosecution of bid rigging, COMCO repeatedly organized information sessions for procurement agencies in Switzerland to increase their awareness of bid rigging. Teaching procurement agencies how to recognize bid rigging is important since they are key players in organizing public procurement markets and may design procurement in a way that fosters competition and decreases the risk of bid rigging. If procurement agencies demonstrate that they care about competition and clearly stand against bid rigging, they send a clear deterrent signal toward bidders tempted to collude. Finally, procurement agencies are a source of information for COMCO in prosecuting bid-rigging cartels. The information and evidence gathered by procurement agencies can lead to the opening of an investigation. It is therefore advantageous for COMCO to make procurement agencies aware of relevant information of bid rigging so that they can transfer them to COMCO for assessment.

The last pillar concerns the detection of bid rigging. COMCO opens an investigation if there are reasonable grounds to assume the existence of a bid-rigging cartel. Compliance programs, whistle-

blowers and procurement agencies can provide insightful information leading to the opening of an investigation. However, COMCO decided to mitigate its dependency on those sources and started to develop statistical methods for detecting bid rigging based on screens (see *Imhof et al.*, 2018). Based on the latter method, COMCO opened an investigation of bid rigging in the region of See-Gaster in 2013.

Imhof et al. (2018) indicate that their screening method can detect partial cartels, as it succeeded in dismantling the quasi-complete bid-rigging cartel in the region of See-Gaster. However, as all firms in the road construction sector in that region were involved in the case, the term *"partial cartel"* in their study does not imply incomplete bid-rigging cartels as in our paper. The cartel only concerned one region out of eight regions in the canton of St. Gallen (and not the whole canton). In our simulation that combines competitive bids with the Ticino cartel, we show that the method of *Imhof et al.* (2018) is not suitable for flagging incomplete bid-rigging cartels. Considering the evolution of laws and practice of COMCO toward bid-rigging cartels, incomplete bid-rigging cartels occur more often than well-organized and complete cartels. Therefore, if COMCO desires to reinforce the detection pillar, it must continue to improve detection methods. The broader approach for flagging both incomplete and complete bid-rigging cartels proposed in this paper responds to that need and is likely of interest for competition agencies around the world.

In the empirical analyses, we use data from the three most important cases in Switzerland: the Ticino cartel, the cartel of See-Gaster and the asphalt cartel of Graubünden. After discussing procurement in Switzerland, we synthetize the key aspects of procurement data in Switzerland and each case below.

2.1 Procurement Data

Procurement agencies of cantons and cities in Switzerland follow the Agreement on Public Markets between cantons and their own cantonal laws for public procurement. A procurement agency can choose between four procedures: the open procedure, the procedure on invitation, the selection procedure and the discretionary procedure.[4] In the construction sector, a procurement agency generally uses either the open procedure or the procedure on invitation. The selection procedure is rare in the construction sector and when a procurement agency tenders a contract with a discretionary procedure, its application is restrictive since the laws authorize it only under specific conditions. The open procedure does not restrict the participation of submitting firms. In contrast, the procedure on invitation restricts participation as the procurement agency invites only a small number of firms to submit a bid, in general three to five firms. This changes the nature of competition, as the firms are

---

[4]The selection procedure allows procurement agency to select and qualify a set of bidders for participating in a tender. This procedure is useful when bidders are too numerous as for example in architecture designing where hundreds of architects are interested in submitting to the project. However, such a high number of bidders are rarely a problem in the construction sector.

aware of the restricted number of potential competitors.

A procurement agency announces future contracts along with the deadline for submitting bids (varying according to the procedure) in an official journal. If a firm is interested in submitting, the procurement agency provides the firm with all the relevant documents or information for the contract. Between the time of the announcement and the deadline, firms prepare their bids for submission. If they occur, then collusive agreements between firms are typically made during this period.

At a pre-announced date, the procurement agency gathers the incoming bids for the contract and opens them. It officially records all the bids received on time in a bid summary or so-called official record of the bid opening. The procurement agency registers the names of the firms, their addresses and their bids. After having written the official record of the bid opening, the procurement agency proceeds with the detailed examination of the bids. In the awarding of the contract, the agency does not only consider the price of the bids, but also other criteria as the quality, references, environmental or social aspects might play a role. However, as contracts are relatively homogeneous in the construction sector, especially in the road construction and the related civil engineering, the price remains the most important criterion for awarding the contract in practice. Furthermore, the differences in firms' criteria other than the price are typically small. We therefore consider the procurement process as an almost first-price sealed-bid auction.

2.2   The Ticino Cartel

The Ticino cartel started in January 1999 and dissolved itself at the end of March 2005, precisely when the adaptation year to the new cartel Act, entered in force in April 2004, terminated. The cartel was well-organized with a convention of two pages explaining the rules to follow (see *Imhof*, 2020). All firms active in the road construction sector participated in the cartel and rigged all public tenders and all private contracts above 20'000 CHF.[5] The convention allocated contracts among cartel participants according to different criteria. The first criterion was revenue, putting cartel participants with many contracts recently attributed at the bottom of a priority list for allocating new contracts that was updated each week, and those with few contracts at the top. The geographic distance between the firm and the location of the contract was the second most important criterion and played an important role in the decision of allocating small contracts. Ties with private clients was an important criterion in the attribution of private contracts. In particular, cartel participants that had already produced a quote for some private client were privileged. After allocating contracts, cartel participants decided on the price of the bid that the designated winner by the cartel should submit. COMCO stated in its decision that the Ticino cartel roused prices by 30% for contracts in

---

[5]Approximatively 23'500 USA dollars given an exchange rate of 0.85 (indirect quotation) in March 2005.

the road construction and asphalting market.[6]

We consider data from the cartel and the post-cartel periods. Table 2 summarizes key information about contracts with four or more bids in our sample. We observe 149 tenders in the collusive period, whose value amounts up to 160 million CHF. In total, we recorded 974 bids for the collusive period, henceforth referred to as collusive bids. For the post-cartel period, we observe only 33 tenders, accounting for a value of 23 million CHF, in which firms submitted 222 competitive bids.

Table 2: Overview sample Ticino cartel

Tenders in the cartel period	149
Volume of the collusive tenders in mio CHF	160.7
Collusive bids	974
Tenders in the post-cartel period	33
Volume of the competitive tenders in mio CHF	22.79
Competitive bids	222

Figure 1 visualizes the distribution of tenders for a predetermined number of bids. In either periods, most of the tenders consist of 4 to 8 bids (see also Table 14 in Appendix). Table 3 provides the empirical distribution of the bids for each period. Both periods contain contracts of different values varying from several tens of CHF to up 3 to 5 million CHF. The mean and the median of the cartel period are superior to those of the post-cartel period. In either periods, the contract values exhibit higher means than medians, indicating a right-skewed distribution with outliers of comparably high contract values.
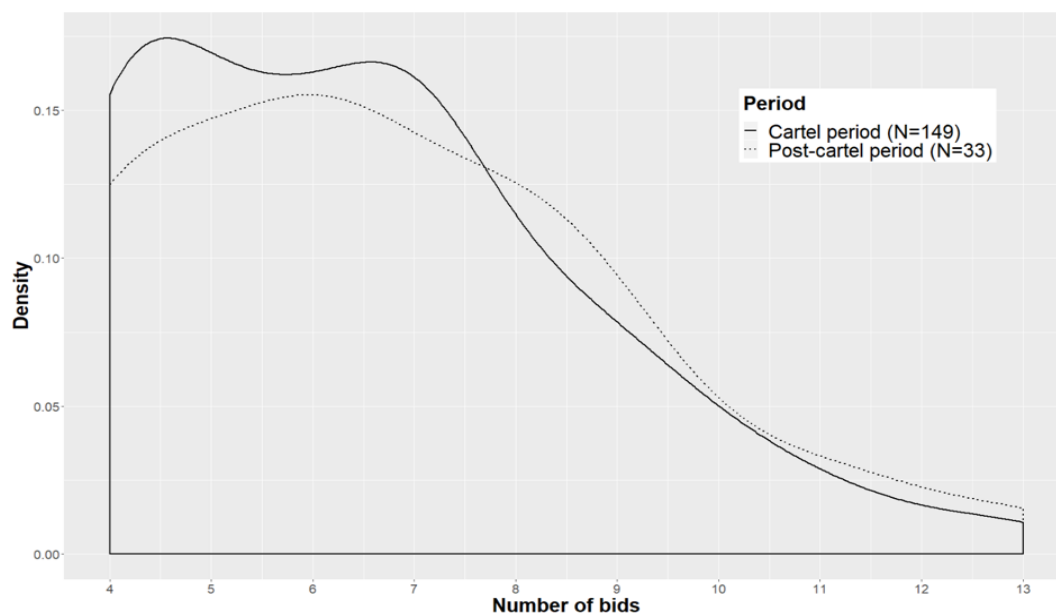

Figure 1: Distribution of tenders for a predetermined number of bids for the Ticino data

---

[6]For the decision of COMCO, see *Strassenbeläge Tessin* (LPC 2008-1, pp. 85-112).

Table 3: Empirical distributions of collusive and competitive bids in the Ticino data

	Cartel period	Post-cartel Period
Mean	1.08	0.69
Std	1.01	0.75
Min	0.02	0.04
Lower Q.	0.36	0.25
Median	0.78	0.44
Upper Q.	1.47	0.68
Max	4.85	2.95
N	149	33

Note: "Mean", "Std", "Min", "Lower Q.", "Median", "Upper Q.", "Max", and "N" denote the mean, standard deviation, minimum, lower quartile, median, upper quartile, maximum, and number of observations, respectively.

2.3 The Cartel in See-Gaster

COMCO opened its investigation against six firms in the region of See-Gaster mainly because of a statistical investigation based on the procurement data from 2004 to 2010, which were provided by the canton of St. Gallen (see *Imhof et al., 2018*).[7] Six months later, COMCO extended the investigation to two additional firms, undetected by *Imhof et al.* (2018). One firm did not bid in tenders of the canton of St. Gallen and had therefore not been detected previously. The second firm was small such that its participation in conspicuous contracts was insufficient for being flagged as potential cartel candidate.

In total, eight firms participated in bid-rigging conspiracies in the region of See-Gaster, including the district of See-Gaster in the canton of St. Gallen and the districts of March and Höfe in the canton of Schwyz.[8] Cartel participants regularly met once or twice per month. In their meetings, they mainly discussed about future contracts tendered, including road construction, asphalting and civil engineering, and exchanged their interest for them. Before each meeting, one cartel participant sent an actualized table to all others, listing all future contracts in the region of See-Gaster. Each cartel participant had a column and could put a star for each contract if interested in obtaining the contract or two stars in the case of very high interest.[9] When the tender procedure for a contract started, the cartel typically designated the cartel participant who should win the contract. The allocation mechanism was based on the interests announced and fairness in terms of allocation among participants for insuring cartel stability.[10] Besides, when two cartel participants had put two stars for a specific contract, they possibly formed a consortium for sharing the contract and the other

---

[7]Report release: See the decision *Bauleistung See-Gaster: Verfügung vom 8. Juli 2016*, available on the following internet page: https://www.weko.admin.ch/weko/fr/home/actualites/dernieres-decisions.html.

[8]See the decision *Bauleistung See-Gaster: Verfügung vom 8. Juli 2016*, available on the following internet page: https://www.weko.admin.ch/weko/fr/home/actualites/dernieres-decisions.html.

[9]See the decision *Bauleistung See-Gaster*, R. 809 ff.

[10]See the decision *Bauleistung See-Gaster*, R 587, R 608 and R 623.

participants covered the consortium.[11]

The cartel took the decision of contract allocation during the meetings in which they discussed the list, but organized separate meetings for discussing the price of the bids.[12] One reason for separate meetings is that not all cartel participants were interested in fixing the price since some did not necessarily participate in the tender. Second, discussions about price could have taken too much time such that the cartel preferred the designated winner to invite the other bidders to a separate meeting for discussing the price. COMCO found some evidence that the cartel from time to time used the mechanism of the mean for determining the bid of the designated winner,[13] which implies that the latter had to submit either his own bid or the mean of all exchanged bids in the separate meetings. With such a mechanism, the designated winner had some incentive to provide a relatively high bid to influence the calculated mean in the separate meeting. All other cartel participants whose announced bids were below the mean or below the bid of the winner increased their bids to cover the designated winner. They generally ensured a minimal price difference of 2-3% between the bid of the designated winner and their bids.[14]

Finally, the cartel also decided about contracts free for competitive bidding.[15] This decision was also determined by the presence of external bidders. The more external bidders, the smaller was the incentive to collude because the chances of success were lower. This was the case for some contracts of high value, for which more non-cartel firms were interested in bidding. Sometimes, the cartel also tried including such external firms in the agreement.

In June 2009, the cartel stopped its illegal conduct after COMCO launched house searches in the canton of Aargau, which to a certain extent explained the breakdown of the cartel. In its decision, COMCO attested that the cartel had discussed more than 400 contracts in the region of See-Gaster from 2004 to 2009 with a value of 198 million CHF. COMCO also proved that they attempted to rig at least 200 contracts for a value of 67.5 million CHF.[16] In the rendering of its decision, COMCO sanctioned the involved firms for bid-rigging conspiracies with more than 5 million CHF. Two firms applied to the leniency program and two other firms settled an agreement to close the case. Four firms appealed against the decision.

2.4 The Strassenbau Cartel in Graubünden

The cartel in the canton of Graubünden was organized by the members of the local trade association for road construction. The members were firms active in road construction and asphalting. COMCO proved in its decision that the cartel participants met regularly in the investigated period from 2004

---

[11]See the decision *Bauleistung See-Gaster*, R 620 ff. and R. 645.
[12]See the decision *Bauleistung See-Gaster*, R. 649 ff.
[13]See the decision *Bauleistung See-Gaster*, R. 714 ff.
[14]See the decision *Bauleistung See-Gaster*, R. 714 ff. and R 718.
[15]See the decision *Bauleistung See-Gaster*, R 681 ff. and R. 815 ff.
[16]See the decision *Bauleistung See-Gaster*, R. 797 ff. and table 15.

to the end of May 2010. The meetings called "allocation meeting" or "calculation meeting" were mainly held in the beginning of the year since the canton and the local municipalities tendered most of the contracts in spring.[17] They discussed contracts for road construction and asphalting tendered by the canton of Graubünden and the local municipalities. Since the geography of Graubünden is profoundly marked by mountains and valleys, the cartel was divided into firms operating in the North and South, respectively.

In the North of Graubünden, the cartel mostly organized the meetings in the office of the most important mixing plant in the canton, and to a lesser extent in the offices of cartel participants. The meetings included either all of the 12 to 13 cartel participants[18] or two different subgroups.[19] In the South, the all in all six cartel participants[20] also organized such meetings, however, with alternating locations.

COMCO stated in its press release that the cartel decided upon the allocation of contracts based on a contingent determined for all the cartel participants in the canton of Graubünden.[21] The cartel allocated contracts according to the interests of each firm and fixed the price of the designated winner following a specific calculation method.[22] The price of the designated winner was in general above the minimal announced bid in the respective meeting. The calculation method therefore contributed to raise the price.

During the investigated period from 2004 to the end of May 2010, the cartel allocated 70 to 80% of the total value of the cantonal and communal road construction contracts in North and South Graubünden among its participants. The cartel approximatively rigged 650 road construction contracts concerning in total 190 million of CHF of market volume.[23] The cartel ceased its illegal conduct in summer 2010 in both the North and South, since some firms decided to stop, mainly because arising concerns regarding the Cartel Act.[24]

2.5   Data from the Cases See-Gaster and Graubünden

We requested data on all bid summaries from the investigations of See-Gaster and Graubünden based on the Federal Act on Freedom of Information in the Administration (Freedom of information Act, FoIA).[25] COMCO approved the request and transmitted us the data, referred to as Swiss data hereafter. They contain the bids, a running number for each contract, a dummy variable for each of the anonymized cartel participants and a dummy variable indicating whether the tender took place in

---

[17]See the decision *Strassenbau Graubünden*, R. 139.

[18]See the decision *Strassenbau Graubünden*, R 247 ff.

[19]See the decision *Strassenbau Graubünden*, R 195 ff.

[20]See the decision *Strassenbau Graubünden*, R 248.

[21]See press release: https://www.newsd.admin.ch/newsd/message/attachments/58229.pdf.

[22]The online published decision *Strassenbau Graubünden*. and the press release give for now no precision about the calculation method.

[23]See press release: https://www.newsd.admin.ch/newsd/message/attachments/58229.pdf.

[24]See the decision *Strassenbau Graubünden*, R 197.

[25]https://www.admin.ch/opc/en/classified-compilation/20022540/index.html.

the cartel period (taking the value 1 for cartel and 0 otherwise), a categorical variable for the contract type (taking the value 1 for contracts in road construction and asphalting, 2 for mixed contracts including road construction and civil engineering and 3 for civil engineering contracts), as well as an anonymized date and year. The first year in our sample begins with the value of 1, the last year ends with the value of 14. The first anonymized date equals 42 and the last 4886. To ensure the anonymization of the bids, COMCO multiplied them with a factor between 1 and 1.2. Such a transformation does not affect the calculation of the screens.

Table 4: Overview for the Swiss data

Tenders with complete cartels	310
Volume of tenders with complete cartels in million CHF	111.74
Collusive bids in tenders with complete cartels	2031
Tenders with incomplete cartels	287
Volume of tenders with incomplete cartels in million CHF	114.73
Competitive bids in tenders with incomplete cartels	650
Collusive bids in tenders with incomplete cartels	1414
Competitive tenders	2398
Volume of competitive tenders	1735.91
Competitive bids in competitive tenders	13925
Tenders for road construction and asphalting	1389
Tenders for civil engineering	1286
Tenders for mixed contracts	273

Table 4 provides key information on the Swiss data. As for the data from Ticino, we only consider tenders with four bids or more. In total, there are 310 tenders with complete cartels with a total value of more than 110 million CHF and 2'031 bids submitted by the cartel participants. Furthermore, there are 287 tenders with incomplete cartels with a total value of more than 114 million CHF. In those tenders, cartel participants submitted 1'414 bids and external firms 650 bids. Finally, we observe 2'398 competitive tenders with a value of roughly 1'700 million CHF and 13'925 submitted bids.

Figure 2 visualizes the distribution of the number of bids per tender for complete cartels, incomplete cartels and competitive tenders, respectively. While tenders with four to seven bids dominate, there is also a sufficient number of tenders with eight or more bids (see also Table 18 in Appendix). Table 5 depicts the empirical distribution of the bids for each type of tenders. The empirical distributions for tenders with complete cartels and with incomplete cartels are similar. This is, however, not the case for competitive tenders. That have many more contracts, varying from one thousand CHF to 148 million CHF. As for the data from the Ticino cartel, all empirical distributions of the bids are right-skewed (such that the mean is higher than the median), but stronger so for competitive tenders than for complete and incomplete cartels.
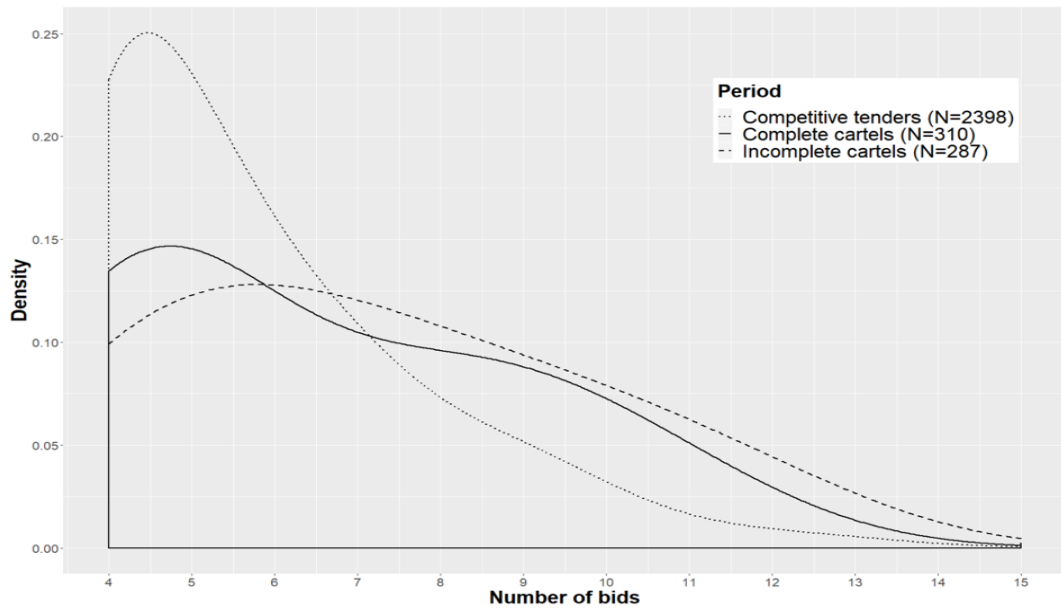
Figure 2: Distribution of tenders for a predetermined number of bids in the Swiss data

Table 5: Empirical distributions of bids in the Swiss data

	Complete cartels	Incomplete cartels	Competitive tenders
Mean	0.36	0.4	0.72
Std	0.36	0.47	3.81
Min	0.03	0.02	0.001
Lower Q.	0.16	0.12	0.13
Median	0.29	0.25	0.31
Upper Q.	0.44	0.50	0.66
Max	3.45	3.46	147.73
N	310	287	2398

Note: "Mean", "Std", "Min", "Lower Q.", "Median", "Upper Q.", "Max", and "N" denote the mean, standard deviation, minimum, lower quartile, median, upper quartile, maximum, and number of observations, respectively.

3   Detection Methods

This section outlines the suggested method to detect bid rigging. We first describe the concept of a random forest, the machine learning algorithm used for training and testing our predictive models for collusion (see *Ho*, 1995; *Breiman*, 2001). Second, we present in detail the screens that enter the algorithm as potential predictors. Third, we discuss five different predictive models applied to our data that differ in terms of included screens. Finally, we provide descriptive statistics for two important screens in each dataset.

3.1   Random Forest

We use the random forest as machine learning algorithm for predicting collusive and competitive tenders. In our data, the outcome takes the value 1 for collusive tenders, including both incomplete and complete bid-rigging cartels, and 0 for competitive tenders. Note that we intentionally do not distinguish between incomplete and complete cartels, as our aim is to construct a reliable detection method for any form of bid rigging. Tenders are therefore either collusive or competitive.

Machine learning requires randomly splitting the data into the so-called training data, used for developing the predictive model, and the test data, used for evaluating the performance of the model. We randomly split the data such that the training and test data consist of 75% and 25% of the observations, respectively. The random forest is a so-called ensemble method that averages over multiple decision trees for predicting the outcome. Tree-based methods split the predictor space (*i.e.* the values the screens might take) of the training data recursively into a number of non-overlapping regions. Each split aims at maximizing homogeneity of the dependent variable within the newly created regions according to a goodness of fit criterion like the Gini coefficient. The latter measures the average gain in purity (or homogeneity) of outcome values when splitting and is popular for binary variables like our collusion dummy. Splitting is continued until the decision tree reaches a specific stopping rule, *e.g.* a minimum amount of observations in a region or maximum number of splits. Tree-based prediction of bid rigging (1) or competition (0) is based on whether collusive or competitive tenders dominate in that region which contains the values of the screens for which the outcome is to be predicted.

Importantly, there exists a bias-variance trade-off in out of (training) sample prediction when using such tree-based (and other machine learning) methods w.r.t. model generality. More splits reduce the bias and increase the flexibility of the model specification, however, at the cost of a greater variance in unseen data (not used for training), as the test sample, due to smaller regions. The issue of a too large variance can be mitigated by repeatedly drawing many subsamples from the initial training data and estimating the predictive model, *i.e.* the tree (or splitting) structure, in each of the newly generated samples. In fact, a random forest consists of predicting the outcome by the majority

rule across the individual trees, *i.e.* based on whether the majority of the trees estimated in the various subsamples predict collusion or competition for specific values of the screens. A further feature of the random forest is that at each splitting step in a specific subsample, only a random subsample of possible predictors (*i.e.* screens) is considered, which reduces the correlation of tree structures across subsamples and thus further reduces the variance of prediction. In our application, we use the randomForest package by *Breiman and Cutler* (2018) for the statistical software R with growing 1'000 trees to estimate the predictive models in the training data and assess their performance in the test data based on the correct classification rate.

We note that we repeat the random sample splitting into 75% training and 25% test data as well as the assessment of the predictive performance in the latter 100 times. Our reported correct classification rate corresponds to the average of the correct classification rates across the 100 repetitions. This procedure likely entails a smaller variance in the estimation of the correct classification rate than when relying on a single random data split.

3.2 Predictors

Screens are statistics that permit analyzing economic data with the aim to flag anomalous outcomes that suggest potential anticompetitive issues. The literature usually differentiates structural from behavioral screens in cartel detection (see *Harrington*, 2006; *OECD*, 2014; *Froeb et al.*, 2014). Structural screens focus on the factors facilitating the emergence of collusive agreements and help to identify markets in which collusion is more likely. Among those factors, one distinguishes between market structure, demand-related and supply-related factors (*OECD*, 2014). In contrast, behavioral screens empirically measure the behavior of market participants and assess whether the observed behavior significantly departs from competition for flagging it as a potential issue worth scrutinizing deeper. Following *Huber and Imhof* (2019) we propose using various descriptive statistics as screens and combining them with machine learning, however, not only for uncovering complete, but also incomplete bid-rigging cartels.[26] We consider three classes of screens: Variance, asymmetry and uniformity.

As variance screens, we implement the coefficient of variation (CV) and the kurtosis statistic (KURTO), as suggested by *Huber and Imhof* (2019) and *Imhof* (2020). In addition, we also implement the spread (SPD) of the distribution of the bids as screen.

The coefficient of variation is widely discussed in the literature (see *Abrantes-Metz et al.*, 2006; *Esposito and Ferrero*, 2006; *Jimenez and Perdiguero*, 2012; *Abrantes-Metz et al.*, 2012; *Imhof*, 2020) and is defined as the standard deviation divided by the arithmetic mean of all bids submitted in a tender:

---

[26]In contrast to the context of causal inference, causality goes from the dependent variable (collusion or competition) to the predictors (screens) rather than the other way around. The incidence of collusion as explanatory variable affects the distribution of the bids and thus the screens in causal terms. As in *Huber and Imhof* (2019) our prediction problem consists of analyzing a reverse causality. By investigating the screens we infer the existence of their cause: Collusion.

$$CV_t = \frac{s_t}{\bar{b}_t}, \tag{1}$$

where s_t is the standard deviation and $\bar{b}_t$ is the mean of the bids in some tender t. As coordination and manipulation of bids by cartel participants might affect the convergence in the distribution of the bids, we also consider the following kurtosis statistic as screen:

$$KURTO_t = \frac{n_t(n_t+1)}{(n_t-1)(n_t-2)(n_t-3)} \sum_{i=1}^{n_t} (\frac{b_{it}-\bar{b}_t}{s_t})^4 - \frac{3(n_t-1)^3}{(n_t-2)(n_t-3)}, \tag{2}$$

where b_{it} denotes the bid i in tender t, n_t the number of bids in tender t, s_t the standard deviation of bids, and $\bar{b}_t$ the mean of bids in that tender. Note that we calculate the kurtosis statistic only for tenders with four bids or more. Furthermore, we estimate the spread using the following formula:

$$SPD_t = \frac{b_{max,t} - b_{min,t}}{b_{min,t}}, \tag{3}$$

where $b_{max,t}$ denotes the maximum bid and $b_{min,t}$ the minimum bid in some tender t.

Bid rigging may produce asymmetries in the distribution of the bids and we for this reason implement the following cover-bidding screens as in *Huber and Imhof* (2019): The percentage difference (DIFFP), the skewness (SKEW), the relative distance (RD) and the normalized distance (RDNOR). We also add an alternative measure for calculating the relative difference, the alternative relative distance (RDALT).

It seems plausible that cartel participants manipulate the difference between the lowest and second lowest bids to secure contract allocation with the designated winner by the cartel. To analyze the difference between the two lowest bids, we use the following formula to calculate the percentage difference:

$$DIFFP_t = \frac{b_{2t} - b_{1t}}{b_{1t}}, \tag{4}$$

where b_{1t} is the lowest bid and b_{2t} the second lowest bid in some tender t. We also consider the absolute difference between the first and second lowest bids $D_t = b_{2t} - b_{1t}$ in the empirical analysis.

The manipulation of the bids by cartel participants might simultaneously affect both the difference between the first and second lowest bids and the differences across the losing bids. Following Imhof et al. (2018), we calculate a relative distance (relative to a measure of dispersion) in a tender by dividing the difference between the first and second lowest bids by the standard deviation of the losing bids:

$$RD_t = \frac{b_{2t} - b_{1t}}{s_{losingbids,t}}, \tag{5}$$

where b_{1t} denotes the lowest bid, b_{2t} the second lowest bid, and $s_{t,losingbids}$ the standard deviation calculated among the losing bids in some tender t. In *Huber and Imhof* (2019) the RD was in terms of predictive power outperformed by the difference between the second and first lowest bids divided (or normalized) by the average of the differences between all adjacent bids. We also consider this normalized distance in our study:

$$RDNOR_t = \frac{b_{2t} - b_{1t}}{\frac{(\sum_{i=1, j=i+1}^{n_t-1} b_{jt} - b_{it})}{n_t - 1}}, \tag{6}$$

where b_{1t} is the lowest bid, b_{2t} the second lowest bid, n_t is the number of bids and b_{it}, b_{jt} are adjacent bids (in terms of price) in tender t, with bids being ordered increasing order.

We consider a further alternative measure for the relative distance, initially suggested by *Imhof et al.* (2018):

$$RDALT_t = \frac{b_{2t} - b_{1t}}{\frac{(\sum_{i=2, j=i+1}^{n_t-1} b_{jt} - b_{it})}{n_t - 2}}, \tag{7}$$

where b_{1t} is the lowest bid, b_{2t} the second lowest bid, n_t is the number of bids and b_{it}, b_{jt} are adjacent losing bids in a tender t, with bids being ordered increasing order. In contrast to the normalized distance, the mean of the differences in the denominator is calculated using only losing bids. Furthermore, bid manipulation might affect the symmetry of the distribution of bids. We therefore include the skewness as screen:

$$SKEW_t = \frac{n_t}{(n_t - 1)(n_t - 2)} \sum_{i=1}^{n_t} (\frac{b_{it} - \bar{b}_t}{s_t})^3, \tag{8}$$

where n_t denotes the number of the bids, b_{it} the ith bid, s_t the standard deviation of the bids, and $\bar{b}_t$ the mean of the bids in tender t.

Finally, we consider the nonparametric Kolmogorov-Smirnov statistic (KS) for verifying if the bids in a tender follow a uniform distribution:

$$D_t^+ = max_i(x_{it} - \frac{i_t}{n_t + 1}), D_t^- = max_i(\frac{i_t}{n_t + 1} - x_{it}), KS_t = max(D_t^+, D_t^-), \tag{9}$$

where n_t is the number of bids in a tender, i_t the rank of a bid and $x_i t$ the standardized bid for the ith rank in tender t. The standardized bids $x_i t$ are the bids $b_i t$ divided by the standard deviation of bids in tender t to facilitate the comparison of tenders with different contract values. We would suspect that competitive bids are closer to a uniform distribution than collusive bids such that the statistic should generally differ across cartels and competitive periods.

In incomplete cartels, competitive bidders distort the statistical signals produced by bid rigging in the distribution of bids in a tender. Therefore, the screens can fail to recognize bid rigging if calculated for all bids. We circumvent that distortion by calculating the screens not (only) for all

bids in a tender, but for all possible subgroups of three and four bids. Table 6 gives the number of possible subgroups of three or four bids, respectively, when the total number of bids in a tender varies between four to ten bids.

Table 6: Example of possible subgroups for three and four bids in a tender

Bids in a tender	Subgroups formed with three bids	Subgroups formed with four bids
4	4	1
5	10	5
6	20	15
7	35	35
8	56	70
9	84	126
10	120	210

For instance, in a tender with a total number of six bids, we calculate the same screen but for different 15 subgroups containing four bids and for 20 different subgroups containing three bids. In each tender, we then include summary statistics for each screen: the mean, the median, the minimum and the maximum of the respective screen across the various subgroups of three or four bids. These summary statistics are themselves used as predictive screens for flagging collusive and competitive tenders and permit comparing tenders with different numbers of bids. We subsequently exemplify the computation of such screens by means of the coefficient of variation for subgroups formed on four bids.

The mean of all coefficients of variation calculated for subgroups of four bids in each tender is:

$$MEAN4CV_t = \sum_{s=1}^{S_t} (\frac{s_{st}/\bar{b}_{st}}{S_t}),$$ (10)

where s and t denote the indices for some sub-group s and some tender t, respectively, S_t is the number of all the possible subgroups of four bids in tender t and s_{st} and $\bar{b}_t$ are the standard deviation and the mean of the bids, respectively. Likewise, the minimum and maximum of coefficients of variation across subgroups in a tender correspond to, respectively:

$$MIN4CV_t = min_s \frac{s_{st}}{\bar{b}_{st}},$$ (11)

$$MAX4CV_t = max_s \frac{s_{st}}{\bar{b}_{st}},$$ (12)

In order to calculate the median for subgroups of four bids in each tender, define the coefficient

of variation in subgroup s and tender t as $CV_{st} = \frac{s_{st}}{\bar{b}_{st}}$ and order the coefficients in so that

$$CV_{1t} \leq CV_{2t} \leq \dots \leq CV_{st} \leq \dots \leq CV_{S_t t}$$

. If the number of subgroups S_t in a tender is uneven, the median of the coefficient of variation in tender t is calculated as follows:

$$MEDIAN4CV_t = CV_{(S_t+1)/2,t}, \tag{13}$$

If the number of subgroups is even, the median corresponds to:

$$MEDIAN4CV_t = \frac{CV_{S_t/2,t} + CV_{S_t/2+1,t}}{2}, \tag{14}$$

These approaches are applied to all the screens discussed above across different tenders. Note also that we do not calculate screens for subgroups of two bidders because of the impossibility to calculate some screens as the RD, RDALT, RDNOR, KURTO, or SKEW. Moreover, cartel participants were in general more than two in tenders characterized by incomplete cartels. We also renounce calculating screens for sub-groups of five bidders or more. It makes sense only for tenders with six bids and more to use screens calculated for subgroups of five bidders. Using tenders with six bids or more would have restricted too much our sample and limited the application of our suggested methods in other cases. Calculating screens for subgroups of more than five bids would have only increased such limitation.

3.3 Model Specification

In the empirical analyses, we consider five different predictive models that vary in terms of screens considered, as well as the method of *Imhof et al.* (2018). Model 1 only applies screens calculated for all bids in a tender (rather than subgroups) as in *Huber and Imhof* (2019), but extends the set of predictors compared to the latter study by including the relative measure for the alternative distance (RDALT), the spread (SPD) and the Kolmogorov-Smirnov statistic (KS). In total, we use nine predictors and exclude any screens based on the absolute bid value to consider only scale-invariant screens in model 1. The results in *Huber and Imhof* (2019) suggest that scale-invariant screens work well for predicting collusive and competitive tenders and that the value added of statistics that are sensitive to the scale is limited. Furthermore, the transfer of a method to other cases (with possibly different contract values) is easier if screens are scale invariant. Model 1 therefore represents the baseline approach for detecting bid-rigging cartels based on focusing on scale-invariant screens at the tender level.

In contrast, model 2 exclusively includes the summary statistics (mean, minimum, maximum,

median) of the screens, which are calculated for all possible subgroups of three bids in a tender, as predictors. In total, we consider 32 of these summary statistics applied to eight screens, namely all of model 1 but the kurtosis (KURTO) which requires at least four bids. Model 3 uses summary statistics of the nine screens for all possible subgroups of four bids in a tender, thus in all 36 predictors (now including the kurtosis). Model 4 considers all predictors included in models 1, 2 and 3, this results in 77 statistics in total. Finally, model 5 additionally includes three screens that are based on absolute bid values (and thus, not scale-invariant) as well as the number of bids in a tender (NBRBIDS),[27] thus 81 predictors in total. The three value-based screens are the mean bid in a tender as proxy for the contract value (MEANBIDS), the standard deviation of the bids in a tender (STDBIDS) and the absolute difference between the first and the second lowest bids (D).

3.4   Descriptive Statistics for Predictors

In the appendix, we present the Tables of descriptive statistics for all the different samples used in the empirical analyses for both the Ticino simulation and the Swiss data. We review here the most important key information drawn from the descriptive statistics for the coefficient of variation (CV) and the normalized distance (RDNOR). Similar interpretation can be drawn with other screens.

For the Ticino cartel, the coefficient of variation exhibits a mean of 3.25 and a median of 2.97 with a low standard deviation of 1.18 (see Table 15). This contrasts with the post-cartel period in which the mean and the median of the coefficient of variation amount to 9.51 and 8.49, respectively, with a larger standard deviation of 5.38 compared to the collusive period (see Table 16). If a great majority of observations in the cartel period is below 3.83, we only find few CVs below 5.65 in the post-cartel period considering the upper and lower quartile, respectively. For the Swiss data, we find similar values for the cartel period with a mean of 3.66, a median of 3.29 and a standard deviation of 2.09 (see Table 19). All of them contrast with the values found for the post-cartel period (competitive tenders) in the Swiss data with a mean of 10.12, a median of 8.45 and a standard deviation of 7.89 (see Table 20). Note that we select in the following empirical analyses only competitive tenders with an anonymised year superior or equal to 8. Since collusive tenders are absent in the anonymised years superior or equal to 8, we conclude that both bid-rigging cartels has collapsed in this period called post-cartel period. It ensures that a competitive tender in the post-cartel period is really a "competitive" one.

If we look at the coefficient of variation for the incomplete bid-rigging cartel in sample 1 of the Swiss data, the CV is affected by the presence of competitive bids with a mean of 7.79, a median of 6.79 and a standard deviation of 3.89 (see Table 21). Looking more precisely at the minimum of all coefficients of variation calculated for subgroups of four bids in a tender (MIN4CV), we find a mean

---

[27] The motivation for including the number of bids is that it might be easier to settle an agreement in a tender with few bidders than with many.

of 3.16, a median of 2.26 and a standard deviation of 2.97 for the incomplete bid-rigging cartels in sample 1 (see Table 21). However, the MIN4CV for the competitive tenders exhibits higher values with a mean of 6.24, a median of 4.49 and a standard deviation of 6.77 (see Table 20). Noteworthy, the differences are weaker for the maxima of all coefficients of variation calculated for subgroups of four bids (MAX4CV), between incomplete cartels in sample 1 (mean of 10.63, median of 9.43 and a standard deviation of 5.46 in Table 21) and competitive tenders (mean of 12.14, median of 10.13 and a standard deviation of 9.73 in Table 20). This example is crucial to understand the benefit delivered by summary statistics of the screens. Even if the maxima of the coefficient of variation is high in both cases of incomplete bid-rigging cartels and competition, the minima notably diverge and could be used to differentiate between competition and collusion.

The normalized distance (RDNOR) take higher values in collusive periods than in competitive periods. For example, the RDNOR exhibits a mean of 2.93 and a median of 2.72 with a standard deviation of 1.35 for the Ticino cartel (see Table 15). In the post-cartel period, the values of the RDNOR are lower with a mean of 1.02, a median of 0.74, and a standard deviation of 0.80 (see Table 16). Although less notable, we find a divergence in the Swiss data between collusive tenders (with a mean of 1.38, a median of 1.24 and a standard deviation of 0.79 in Table 19) and competitive tenders (with a mean of 1.04, a median of 0.87 and a standard deviation of 0.82 in Table 20). We find similar values for the minima of the normalized distance (MIN4RDNOR) between incomplete bid-rigging cartels in sample 1 (mean of 0.37, median of 0.15, standard deviation of 0.54 in Table 21) and competitive tenders in the Swiss data (with a mean of 0.51, a median of 0.29 and a standard deviation of 0.56 in Table 20). The values are more divergent for the maxima (MAX4RDNOR) between both types of tenders. For the incomplete bid-rigging cartels in sample 1, we observe a mean of 2.18, a median of 2.37 and a standard deviation of 0.68 in Table 21 contrasting with competitive periods, which exhibit a mean of 1.62, a median of 1.74 and a standard deviation of 0.81 in Table 20. The result indicates that the maxima of the RDNOR could be used to discriminate between incomplete bid-rigging cartels and competition.

4 Flagging Incomplete Bid-rigging Cartels

4.1 The Ticino Simulation

We use the data from the Ticino cartel to investigate how the predictive models presented above perform in detecting bid-rigging cartels when competitive bids distort the statistical pattern produced by bid rigging. Since the Ticino cartel was complete, we use the data from the competitive periods to simulate competitive bids and progressively add them to the collusive tenders, creating five additional datasets for the cartel period. The first dataset includes only one simulated competitive bid in each collusive tender, the second two, and the fifth dataset includes five simulated competitive

bids. This stepwise approach permits investigating how different levels of partial collusion affect the performance of each model.

We generate simulated bids from competitive bids using the following formula:

$$b_{t,simulated} = \bar{b}_t(1 + \frac{b_{i,drawcomp} - \bar{b}_{drawcomp}}{\bar{b}_{drawcomp}}), \tag{15}$$

i and t are indices for bids and tenders, respectively, $\bar{b}_t$ is the mean bid of tender t (without the simulated bid), while $b_{i,drawcomp}$ and $\bar{b}_{drawcomp}$ are the bid the mean bid randomly drawn from competitive tenders, respectively. We simulate competitive bids to be added to collusive tenders in four steps. First, we generate the normalized mean differences for all bids in competitive tenders. Normalizing by the mean rather than the standard deviation avoids losing information about the dispersion of the bids. In a second step, we randomly draw normalized mean differences with replacement from their empirical distribution in the competitive period and randomly assign them a collusive tender. In a third step, we multiply the normalized mean difference with the mean bid in tender t. In a fourth step, we add the mean bid of the assigned collusive tender to the normalized mean difference of the competitive in order to simulate a competitive bid in a tender with bid rigging.

We end up with seven different datasets for the Ticino cartel: The dataset of the post-cartel period including only competitive bids, the dataset of the collusive period including only collusive bids and five different datasets including the collusive tenders with one to five competitive bids in each tender.

We verify if the simulated competitive bids are similar to the competitive bids of the post-cartel period. Since we generate five simulated competitive bids for each tender in the collusive period, we calculate the screens for those five simulated competitive bids only. We test if the screens based on the simulated competitive bids are statistically significantly different from the competitive bids of the post-cartel period. The results presented in the Appendix show that most statistical tests do not reject the null hypothesis of no distributional differences, such that our simulation process adequately generates competitive bids.

We first apply the screening methods of *Imhof et al.* (2018), using their benchmarks (rather than machine learning) for classifying conspicuous tenders,[28] and find a correct classification rate of 84.8% in the test data in the absence of competitive bids, see Table 7. However, when adding one simulated competitive bid to the collusive tenders, the correct classification rate falls to 66.7%. It continuously decreases in the number of competitive bids added such that with five competitive bids in the collusive tenders, the method suggested by *Imhof et al.* (2018) does not perform better than throwing a coin. As expected, the collusive tenders exclusively drive the decrease in the overall correct classification rate. With the addition of only one competitive bid, the correct classification rate among collusive tenders decreases to 48.5%. With five competitive bids, it decreases to 15.2%.

---

[28]Tenders with a CV below 6 and a RD above 1 are classified as conspicuous.

This suggests that while the screening method of *Imhof et al.* (2018) might be well suited for detecting complete bid-rigging cartels, it seems inappropriate for flagging incomplete bid-rigging cartels. This is explained by too many false negative predictions. Note that concerning the competitive tenders, the correct classification rate of 84.8% remains unaffected since the sample of the post-cartel period does not vary across the various simulations.

We turn now to the results for model 1, inspired by *Huber and Imhof* (2019), but using a larger set of screens and the random forest as machine learner. Applied to the Ticino simulation, we find that the correct classification rate of model 1 shrinks between 5.3 and 11.3 percentage points depending on the number of competitive bids added to the collusive tenders. Model 4 includes the screens calculated in all possible subgroups of three and of four bids in a tender, and with the change of the correct classification rate varying between -6.3 and +3.1 percentage points with the occurrence of competitive bids. Noticeably, the correct classification rate of model 4 outperforms that of model 1 in all simulated datasets. When competitive bids are absent, model 1 performs slightly better than model 4, while, model 4 outperforms model 1 by 10.1 and 10.3 percentage points when including four and five competitive bids, respectively. This result illustrates the gain of our approach based on calculating the screens for subgroups of bids in a tender in terms of statistical power. Considering the error rate, it amounts to 24.2% for model 1 such that almost one tender out of four is incorrectly classified. In model 4, the error rate is only 13.9%. Such a decrease of 42.6% in the error rate when compared to model 1 is substantial, especially when considering the legal consequences for firms being flagged as a potential cartel.

Models 2 and 3 include summary statistics applied to the screens calculated in subgroups of three and four bids, respectively. On average, model 2 performs slightly better than model 3, but the correct classification rates are very similar. The maximum difference in (overall) correct classification rates across model 4 and models 2 or 3 amounts to 2.2 percentage points in the sample with three competitive bids. Overall, the correct classification rates of models 2 and 3 thus hardly differ from model 4.

We examine variable importance in the random forest for predicting collusive and competitive tenders in each dataset according to the mean decrease in Gini index (hereafter: MDG) when omitting the respective predictor, which ranks variables according to their predictive power. It, however, does not allow for a direct comparison between models since the MDG depends on the number of predictors. As we use less variables in model 1 than in model 4, the MDG of the former model is therefore higher. For each dataset and models 1 to 4, we depict the five most important variables in Table 8.

We observe for model 1 almost the same important predictors in all six data sets (zero to five competitive bids), namely the Kolmogorov-Smirnov statistic (KS), the coefficient of variation (CV), the kurtosis statistic (KURTO) and the normalized distance (RDNOR). For zero or one competitive bid,

Table 7: Correct classification rate for the Ticino simulation

Comp.B	Tenders	Im.	M1	M2	M3	M4
	All	0.848	0.835	0.832	0.834	0.830
0	Comp.	0.848	0.816	0.811	0.806	0.811
	Coll.	0.848	0.863	0.861	0.87	0.859
	All	0.667	0.722	0.766	0.756	0.767
1	Comp.	0.848	0.701	0.781	0.739	0.777
	Coll.	0.485	0.758	0.765	0.788	0.769
	All	0.652	0.752	0.819	0.786	0.802
2	Comp.	0.848	0.751	0.856	0.786	0.832
	Coll.	0.455	0.758	0.79	0.803	0.784
	Overall	0.576	0.782	0.826	0.795	0.817
3	Comp	0.848	0.796	0.862	0.807	0.838
	Coll.	0.303	0.775	0.804	0.793	0.806
	All	0.561	0.727	0.808	0.832	0.828
4	Comp.	0.848	0.713	0.843	0.83	0.837
	Coll.	0.273	0.747	0.777	0.832	0.819
	All	0.500	0.758	0.871	0.871	0.861
5	Comp.	0.848	0.745	0.885	0.879	0.887
	Coll.	0.152	0.778	0.837	0.862	0.837

Note: "Comp.B", "Tenders", "Im.", "M1", "M2", "M3" and "M4" denote the number of competitive bids in the collusive tenders, the type of tenders, the results produced by the screening methods of *Imhof et al.* (2018), model 1, model 2, model 3 and model 4, respectively. For the type of tenders, "All", "Comp." and "Coll." denote the prediction for all types of tenders, the prediction for the competitive tenders and the prediction for the collusive tenders, respectively.

also the spread (SPD) is important, which is replaced by the alternative relative distance (RDALT) having more than one competitive bid. The order of importance changes with the number of competitive bids. With zero or one competitive bid, the Kolmogorov-Smirnov statistic (KS), the coefficient of variation (CV) and the kurtosis statistic (KURTO) are the most relevant variables with a MDG larger than three. In the presence of two or more competitive bids, the normalized distance (RDNOR) and the alternative distance (RDALT) exhibit a greater predictive power alongside with the Kolmogorov-Smirnov (KS) and the kurtosis statistic (KURTO).

In models 2 to 4, the Kolmogorov-statistic (KS), the spread (SPD), the difference in percentage (DIFFP) and the coefficient of variation (CV) are the best predictors. Contrary to model 1, screens based on the relative distance (RDNOR, RD and RDALT) or the skewness (SKEW) are less important, while the difference between the first and second lowest bids in percentage (DIFFP) are more important. As in model 1, the order of the importance changes with the number of competitive bids. In models 2 to 4 for zero and one competitive bid, the Kolmogorov-statistic (KS) is the most important variable. Different, in the data sets with four and five competitive bids the difference in percentage (DIFFP) comes first.

For models 2 to 4, the set of important variables changes with the number of competitive bids. The median or mean of certain screens calculated in subgroups of three and four bids in a tender is

most predictive in the presence of zero or one competitive bids, while the minima and the maxima of screens dominate under a larger number of competitive bids. Intuitively, the minimum or the maximum of some screen likely excludes competitive bids if the latter distort the distribution of collusive bids and should thus be relatively more predictive as the number of competitive bids increase. We also note that for three or more competitive bids, the random forest mainly selects screens calculated in subgroups of four bids in a tender (rather than three).

To sum up the results of the Ticino simulation, we find that our approach, based on screens for subgroups of three and four bids in a tender, is able to flag bid-rigging cartels even when we add competitive bids. When the number of competitive bids increases, the random forest puts more weight on minima or the maxima of screens across subgroups. For models 1 to 4, the algorithm selects a mix of screens from the three groups (variance, asymmetry and uniformity).

4.2 Application to the Swiss Data

A drawback of the simulation based on the Ticino case is that reactions of competitive and collusive bidders, aware of their reciprocal existence, are unconsidered. For instance, collusive bidders might adopt a more competitive behavior in the presence of competitive bidders, while the latter might try to benefit from the umbrella effect of a cartel by bidding higher than they would have in a fully competitive situation. We therefore apply our detection method to data of the cases See-Gaster and Strassenbau Graubünden, characterized by well-organized bid-rigging cartels that, however, faced competition from outsiders from time to time.

We investigate the performance of our various predictive models first considering complete cartels. As shown in Table 9, the correct classification rates do not differ importantly across machine learning-based models 1 to 5 and amount to 81.3% to 83.3%. For the screening method of *Imhof et al.* (2018), the correct classification rate of 61.7% is clearly below that of models 1 to 5 and in addition strongly differs between competitive and collusive tenders, amounting to only 33.4% in the latter case. Possible explanations for the poor performance are the reliance on only two screens, which are not necessarily the optimal predictors, as well as on benchmark values for these two screens from two previous investigations, that are not necessarily optimal in the data considered. In contrast, the machine learning approaches use a larger set of screens and weight their importance in a data-driven way.

However, if we adjust the benchmarks in *Imhof et al.* (2018), we can achieve a better prediction rate for complete cartels. In Appendix, we display a decision tree on figure 3 corresponding to the minimal cross-validation error. Our pruned tree, using as predictors only the RD and the CV as in *Imhof et al.* (2018), exhibits a correct classification rate of 81.6% for complete cartels. Since *Imhof et al.* (2018) have drawn their benchmarks from two previous investigations, one of them was the Ticino cartel, it is therefore coherent that the benchmarks produce better results for detecting complete

Table 8: Important predictors for the Ticino simulation

Comp.B	Model 1 IV	MDG	Model 2 IV	MDG	Model 3 IV	MDG	Model 4 IV	MDG
	KS	4.63	MEAN3KS	2.33	MEAN4KS	2.88	MEAN4KS	1.56
	CV	4.6	MEDIAN3SPD	1.88	MEDIAN4SPD	1.89	MEAN3KS	1.37
0	KURTO	3.37	MEAN3SPD	1.85	MEAN4SPD	1.86	MEDIAN3SPD	0.99
	SPD	2.42	MEAN3DIFFP	1.78	MEAN4CV	1.83	MEAN3SPD	0.97
	RDNOR	2.36	MEAN3CV	1.74	MEDIAN4KS	1.82	MEAN3DIFFP	0.97
	KS	3.88	MEAN3KS	1.92	MEAN4KS	2.08	MEAN3KS	1.16
	KURTO	3.65	MEDIAN3SPD	1.18	MEDIAN4SPD	1.15	MEAN4KS	1.08
1	CV	3.61	MEAN3SPD	1.17	MEAN4SPD	1.13	MEDIAN3SPD	0.65
	SPD	2.96	MEDIAN3KS	1.13	MEDIAN4CV	1.1	MEAN3SPD	0.65
	RDNOR	2.72	MEAN3CV	1.08	MIN4SPD	1.08	MEDIAN3KS	0.63
	RDNOR	4.2	MEAN3KS	1.45	MIN4DIFFP	1.45	MIN3DIFFP	0.82
	RDALT	3.37	MIN3DIFFP	1.4	MEAN4KS	1.4	MEAN3KS	0.81
2	KS	3.1	MIN3SPD	1.14	MIN4CV	1.14	MIN4DIFFP	0.76
	KURTO	2.99	MAX3KS	1.1	MIN4SPD	1.1	MEAN4KS	0.74
	CV	2.92	MIN3CV	1.1	MAX4KS	1.1	RDNOR	0.72
	RDNOR	3.67	MEAN3KS	1.64	MIN4CV	1.76	MAX4KS	0.99
	KURTO	3.47	MAX3KS	1.46	MAX4KS	1.75	MIN4CV	0.99
3	KS	3.45	MIN3CV	1.44	MIN4SPD	1.74	MIN4SPD	0.95
	RDALT	3.12	MIN3SPD	1.44	MIN4DIFFP	1.71	MEAN3KS	0.9
	CV	3.08	MIN3DIFFP	1.33	MEAN4KS	1.33	MIN4DIFFP	0.89
	KURTO	4.49	MIN3DIFFP	2.04	MIN4DIFFP	2.34	MIN4DIFFP	1.34
	RDNOR	3.72	MIN3SPD	1.69	MAX4KS	2.1	MIN4CV	1.27
4	KS	3.16	MIN3CV	1.59	MIN4CV	2.09	MAX4KS	1.27
	RDALT	2.68	MAX3KS	1.57	MIN4SPD	2.02	MIN4SPD	1.23
	CV	2.52	MAX3RDNOR	1.47	MAX4RDALT	1.28	MIN3DIFFP	1.07
	KURTO	5.08	MIN3DIFFP	2.15	MIN4DIFFP	2.44	MIN4DIFFP	1.53
	RDNOR	3.87	MIN3SPD	1.62	MIN4SPD	2.31	MIN4SPD	1.51
5	RDALT	2.77	MAX3KS	1.52	MAX4KS	2.05	MAX4KS	1.35
	KS	2.71	MIN3CV	1.5	MIN4CV	2.03	MIN4CV	1.34
	CV	2.15	MEAN3KS	1.5	MAX4RDALT	1.54	MIN3DIFFP	1.06

Note: "Comp.B", "IV" and "MDG" denote the number of competitive bids in the collusive tenders, the important variables selected by the random forest and the mean decrease in Gini index. "KS", "CV", "SPD", "RD", "RDNOR", "RDALT", "SKEW", "DIFFP" and "KURTO" denote the Kolmogorov-Smirnov statistic, the coefficient of variation, the spread, the relative distance, the normalized distance, the alternative relative distance, the skewness statistic, the percentage difference and the kurtosis statistic, respectively.

bid-rigging cartels in Ticino than in the Swiss data. Such discrepancy illustrates the fundamental difference between a benchmark method and machine learning. Benchmarks are given exogenous whereas machine learning outperforms benchmarks since it chooses the best predictors in each case. If a benchmark can still be adapted to different cases, machine learning algorithms are far more precise. Nonetheless, a benchmark method has the advantage of requiring few information to be implemented and therefore remains a first step for flagging cartels.

Considering incomplete cartels, the correct classification rates vary between 61.2% to 84.1% depending on the sample and the model. When the proportion of competitive bidders increases, the correct predictions generally decrease, as depicted in Table 9. This result suggests that cartel participants anticipated competitive bids and decided not to collude in some peculiar tenders, as attested for example in the case of See-Gaster. The models with screens, calculated for subgroups outperform model 1. Among them, models 3 and 4 (different to the Ticino simulation) slightly outperform model 2, indicating that in our case, screens calculated for subgroups of four bids exhibit a higher predictive power than those calculated for subgroups of three bids. The fact that we have four cartel participants per tender in most cases likely explains this result. In contrast, screens calculated for subgroups of three bids possibly work better if we mostly observe three cartel participants per tender.

Model 5 (which is the only to also include the number of bidders or the contract value as predictors) outperforms all other models and has a 5 to 10 percentage points higher correct classification rate than model 1. The advantage of models 3 or 4 over model 1 is 3 to 5.7 percentage points. The gain of calculating screens for subgroups is therefore not quite as high as for the Ticino simulation (4.5 to 10.3 percentage points). The result again suggests that cartel participants anticipated competitive bids and adapted their behavior, which was absent in the simulation of competitive bids for the Ticino case. However, models 2 to 4 still outperform model 1 and therefore competition agencies should consider screens for subgroups in the presence of incomplete bid-rigging cartels. Similar to the Ticino simulation, the method of *Imhof et al.* (2018) performs poorly for flagging incomplete bid-rigging cartels and does not better than throwing a coin. Specifically for truly collusive tenders, the correct classification rate is only between 8.7% and 14.7%.

When looking at the variable importance as reported in Table 10, we find for all models and samples that the Kolmogorov-Smirnov statistic (KS) is an important predictor. In many case, it is among the three most important variables. Even if both collusive and competitive tenders generally do not follow a uniform distribution, the collusive ones are generally less uniform such that the Kolmogorov-Smirnov statistic for deviations from the uniform distribution tends to exhibit higher values under bid rigging.

The random forest generally picks up a balanced set of screens for the variance and asymmetry along with the Kolmogorov-Smirnov statistic for all models in samples with incomplete cartels. How-

ever, for complete cartels and all models except model 1, the random forest selects screens for the variance, mainly the coefficient of variation (CV) and the spread (SPD), along with the Kolmogorov-Smirnov statistic (KS). In model 1, the random forest picks up screens for asymmetry as the percentage difference (DIFFP) and the normalized distance (RDNOR). Further, the results suggest that screens for asymmetry are less important in the case of incomplete cartels, especially if one uses the Kolmogorov-Smirnov statistic (KS).

For all the samples with incomplete cartels, the minima and the maxima of the screens calculated for subgroups are the most important variables, while under complete cartels it is the mean and the median. As for the Ticino case, the result suggests that a few competitive bids sufficiently disturb the statistical pattern produced by bid rigging so that it becomes difficult to detect collusion by screens based on the total of bids per tender. In contrast, the use of the minimum or the maximum of all possible subgroups mitigates the distortion by competitive bids.

Table 9: Correct classification rate in the Swiss data

Sample	Cart.F	Perc.Cart.F	Tenders	Im.	M1	M2	M3	M4	M5
0	> 1	71%	All	0.524	0.612	0.637	0.642	0.645	0.673
			Comp.	0.901	0.612	0.607	0.612	0.62	0.646
			Coll.	0.147	0.615	0.671	0.677	0.677	0.706
2	> 2	75%	All	0.525	0.648	0.665	0.675	0.678	0.708
			Comp.	0.901	0.652	0.643	0.645	0.65	0.683
			Coll.	0.148	0.647	0.691	0.71	0.709	0.737
3	> 3	79%	All	0.511	0.706	0.722	0.748	0.745	0.759
			Comp.	0.901	0.705	0.688	0.705	0.707	0.719
			Coll.	0.121	0.708	0.758	0.792	0.784	0.800
4	> 4	83%	All	0.506	0.743	0.770	0.8	0.798	0.814
			Comp.	0.901	0.755	0.751	0.764	0.771	0.783
			Coll.	0.111	0.735	0.791	0.835	0.826	0.846
5	> 5	88%	All	0.494	0.766	0.805	0.813	0.818	0.841
			Comp.	0.901	0.771	0.769	0.786	0.788	0.813
			Coll.	0.087	0.763	0.844	0.842	0.849	0.871
Compl. Cartel	All	100%	All	0.617	0.826	0.813	0.82	0.823	0.833
			Comp.	0.900	0.83	0.818	0.819	0.827	0.833
			Coll.	0.334	0.823	0.808	0.823	0.82	0.834

Note: "Sample", "Cartel.F", "Per.Cart.F", "Tenders", "Im.", "M1", "M2", "M3", "M4" and "M5" denote the sample, the number of cartel firms in the collusive tenders, the percentage of cartel firms in the collusive tenders, the type of tenders, the results produced by the screening methods of *Imhof et al.* (2018), model 1, model 2, model 3, model 4 and model 5, respectively. For the type of tenders, "All", "Comp." and "Coll." denote the prediction for all types of tenders, the prediction for the competitive tenders and the prediction for the collusive tenders, respectively.

Table 10: Important predictors for the Swiss data

Sample	Model 1 IV	MDG	Model 2 IV	MDG	Model 3 IV	MDG	Model 4 IV	MDG	Model 5 IV	MDG
1	SKEW	22.48	MIN3CV	9.98	MIN4CV	9.87	MIN4CV	5.8	STDBIDS	5.94
	RDNOR	21.73	MAX3KS	9.93	MAX4KS	9.55	MAX4KS	5.56	MEANBIDS	5.67
	SPD	21.67	MIN3SPD	9.66	MIN4SPD	9.34	MIN4SPD	5.39	MIN4CV	5.27
	KS	21.31	MAX3DIFFP	6.56	MIN4SKEW	8.14	MAX4RD	4.66	MAX4KS	5.11
	DIFFP	20.75	MIN3DIFFP	6.47	MAX4RD	8.08	MIN4SKEW	4.61	MIN4SPD	4.92
2	KS	19.64	MIN3CV	9.27	MIN4CV	9.26	MIN4CV	5.61	MIN4CV	5.23
	RDNOR	19.56	MAX3KS	9.23	MAX4KS	9.12	MAX4KS	5.54	STDBIDS	5.03
	SPD	19.51	MIN3SPD	8.98	MIN4SPD	8.87	MIN4SPD	5.32	MAX4KS	5.02
	SKEW	18.66	MIN3DIFFP	5.62	MAX4RD	7.71	MAX4RD	4.63	MEANBIDS	4.97
	CV	18.38	MAX3DIFFP	5.51	MIN4SKEW	7.67	MIN4SKEW	4.56	MIN4SPD	4.78
3	RDNOR	16.31	MAX3KS	8.42	MIN4CV	8.32	MIN4CV	5.46	NBRBIDS	5.88
	SPD	15.04	MIN3CV	8.32	MAX4KS	8.08	MAX4KS	5.43	MIN4CV	4.9
	KURTO	14.62	MIN3SPD	8.2	MIN4SPD	7.52	MAX4RD	5.01	MAX4KS	4.87
	KS	14.35	MAX3RD	5.09	MAX4RD	7.5	MIN4SPD	4.96	MAX4RD	4.53
	RDALT	14.02	MAX3RDALT	5.09	MIN4SKEW	7.4	MIN4SKEW	4.94	MIN4SKEW	4.49
4	RDNOR	13.28	MIN3SPD	8.15	MIN4CV	7.6	MIN4CV	5.15	NBRBIDS	7.49
	SPD	12.12	MAX3KS	8.13	MAX4KS	7.49	MAX4KS	5.09	MIN4CV	4.52
	KS	11.52	MIN3CV	8.09	MIN4SPD	7.46	MIN4SPD	4.99	MAX4KS	4.45
	RDALT	11.42	MAX3RD	4.51	MIN4SKEW	7.39	MIN4SKEW	4.96	MIN4SKEW	4.38
	CV	11.06	MIN3SKEW	4.49	MAX4RD	6.92	MAX4RD	4.66	MIN4SPD	4.33
5	KS	10.35	MIN3SPD	6.43	MAX4KS	6.05	MIN4CV	4.08	NBRBIDS	7.27
	SPD	10.14	MIN3CV	5.98	MIN4CV	6.05	MAX4KS	3.99	MIN4CV	3.51
	CV	9.83	MAX3KS	5.94	MIN4SPD	6.02	MIN4SPD	3.96	MIN4SPD	3.4
	RDNOR	9.53	MAX3RD	3.75	MIN4SKEW	5.09	MAX4RDALT	3.25	MAX4KS	3.39
	RDALT	8.03	MAX3RDALT	3.74	MAX4RDALT	4.96	MAX4RDNOR	3.25	MAX4RDALT	2.86
Compl. Cartel	KS	54.65	MEDIAN3CV	20.04	MEDIAN4KS	20.23	MEDIAN4KS	11.4	MEDIAN4KS	10.86
	CV	50.9	MEDIAN3SPD	18.68	MEDIAN4SPD	20.04	MEDIAN4SPD	11.27	MEDIAN4CV	10.82
	SPD	32.96	MEAN3CV	18.49	MEDIAN4CV	19.56	MEDIAN4CV	11.04	MEDIAN4SPD	10.75
	DIFFP	18.37	MEDIAN3KS	18.46	MEAN4KS	16.91	MEDIAN3CV	10.29	MEDIAN3CV	9.96
	RDNOR	17.1	MEAN3SPD	17.64	MEAN4CV	16.67	MEDIAN3KS	9.37	MEDIAN3SPD	9.22

Note: "Sample", "IV" and "MDG" denote the sample, the important variables selected by the random forest and the mean decrease in Gini index. "KS", "CV", "SPD", "RD", "RD-NOR", "RDALT", "SKEW", "DIFFP" and "KURTO" denote the Kolmogorov-Smirnov statistic, the coefficient of variation, the spread, the relative distance, the normalized distance, the alternative relative distance, the skewness statistic, the percentage difference and the kurtosis statistic, respectively.

4.3 Robustness Analysis

We investigate the robustness of our correct classification rates when discarding the most important predictors and applying the random forest to the remaining predictors. Since model 1 makes use of fewer predictors than the other ones, we leave out the three most important variables, while for models 2 to 5, we drop the five best predictors. Table 11 reports the difference in percentage points in the correct classification rates when keeping vs. dropping the respective predictors.

Table 11: Differences in percentage points in correct classification rates of original random forest minus the correct classification rates of the random forest with discarded variables

Sample	Cart.F	Perc.Cart.F	Tenders	M1	M2	M3	M4	M5
1	> 1	71%	All	3.4	1.1	-0.2	0	1.8
			Comp.	3.6	-0.9	-1.2	-0.3	1.8
			Coll.	3.2	3.1	0.9	0.3	1.8
2	> 2	75%	All	1.8	1.3	-0.4	0	1.4
			Comp.	2.9	-0.4	-1.1	-0.3	1.4
			Coll.	0.6	3	0.3	0.3	1.3
3	> 3	79%	All	4.8	0.1	0.1	0.1	0.5
			Comp.	6.3	-0.4	-0.2	-0.2	0.1
			Coll.	3.2	0.6	0.6	0.5	0.9
4	> 4	83%	All	3.4	1.2	0.2	0.9	1.8
			Comp.	4.1	0.9	-0.2	0.5	0.9
			Coll.	2.6	1.6	0.6	1.3	2.7
5	> 5	88%	All	0.6	2	0.2	0.2	1.7
			Comp.	1.3	0.9	0.5	0.2	1.5
			Coll.	-0.2	3.1	-0.4	0	1.7
Compl. Cartel	All	100%	All	1	-0.2	0.4	0	0
			Comp.	0.7	0.5	0.6	0.2	-0.2
			Coll.	1.3	-0.8	0.3	-0.1	0.2

Note: "Sample", "Cartel.F", "Per.Cart.F", "Tenders", "M1", "M2", "M3", "M4" and "M5" denote the sample, the number of cartel firms in the collusive tenders, the percentage of cartel firms in the collusive tenders, the type of tenders, model 1, model 2, model 3, model 4 and model 5, respectively. For the type of tenders, "All", "Comp." and "Coll." denote the prediction for all types of tenders, the prediction for the competitive tenders and the prediction for the collusive tenders, respectively.

The overall correct classification rate of model 1 in samples 1, 3 and 4 keeping all variables dominates that when dropping the best three predictors by 3.4 to 4.8 percentage points. Considering the other models and samples, we observe more or less the same predictive power when discarding the most important variables. This suggests that the remaining predictors seem to be good substitutes for the discarded ones. Other variables become more important when the most important predictors are omitted, and the correct classification rate is hardly affected, which is in line with previous findings of *Huber and Imhof* (2019).

Furthermore, we investigate the robustness with respect to the type of contracts. For both the cartel and post-cartel periods, we subsequently only consider contracts for road construction and asphalting. We exclude contracts for civil engineering and mixed contracts, combining civil engi-

neering and road construction or asphalting. The motivation is that some specific characteristics of contracts in civil engineering could possibly affect the screens and therefore the correct classification rate. Dropping mixed contracts and contracts for civil engineering permits verifying whether this importantly affects the correct classification rate among the remaining contracts for road construction and asphalting. Table 12 reports the difference in percentage points in the correct classification rates when using all contracts vs. using contracts for road construction and asphalting only.

Table 12: Differences in percentage points in correct classification rates of original random forest minus the correct classification rates of the random forest using only contracts for road construction and asphalting

Sample	Cart.F	Perc.Cart.F	Tenders	M1	M2	M3	M4	M5
			All	-6.2	-5.2	-6.7	-5.5	-3.6
1	> 1	81%	Comp.	-7.2	-4.6	-6.9	-5.7	-3.8
			Coll.	-5.2	-5.8	-6.2	-5.2	-3.3
			All	-2.8	-3	-4.3	-3.6	-1.2
2	> 2	82%	Comp.	-3.6	-3.1	-4.7	-3.9	-1.4
			Coll.	-1.9	-2.9	-3.8	-3.2	-1
			All	0.1	-0.6	-0.9	-0.6	0.2
3	> 3	84%	Comp.	-1.9	-1.5	-2.7	-2.1	-1.9
			Coll.	2	0	0.5	0.5	1.9
			All	1.5	2.2	2	2.4	3.4
4	> 4	86%	Comp.	2.2	2.7	1.9	2.5	3.1
			Coll.	1	1.9	1.9	2.2	3.6
			All	2.6	3.3	2	2.1	2.7
5	> 5	88%	Comp.	3.1	3.3	2.2	1.8	3
			Coll.	1.9	3.2	1.9	2.4	2.3
			All	0.7	0.3	0	0.3	0.3
Compl. Cartel	All	100%	Comp.	0.9	-0.3	-0.4	0	-0.2
			Coll.	0.4	0.9	0.4	0.5	0.8

Note: "Sample", "Cartel.F", "Per.Cart.F", "Tenders", "M1", "M2", "M3", "M4" and "M5" denote the sample, the number of cartel firms in the collusive tenders, the percentage of cartel firms in the collusive tenders, the type of tenders, model 1, model 2, model 3, model 4 and model 5, respectively. For the outcome classification, "All", "Comp." and "Coll." denote the prediction for all types of tenders, the prediction for the competitive tenders and the prediction for the collusive tenders, respectively.

In samples 1 and 2, we find the correct classification rates of the random forest for road construction and asphalting contracts to be superior to the classification rate of the random forest with all types of contracts. For example, the difference of the (overall) classification rate of model 1 in samples 1 and 2 accounts for 6.2 and 2.8 percentage points, respectively. A possible explanation could be that when we keep only the road construction and asphalting contracts, we implicitly suppress some competitors. For example in sample 1, the relative amount of collusive bidders of 80.9% is considerable higher as in the situation with all type of contracts (71.1%, see Table 9). The cartel percent is therefore higher for this restricted sample of exclusively road construction and asphalting contracts and explains the higher performance in samples 1 and 2. In sample 3, the situation begins

to change for both types and the correct classification rates are quasi identical. Noticeably for all models in samples 3 and 4, the differences rise again (not as strong as before), but in the opposite direction. That means for an almost identical relative amount of cartel participants, the correct classification rates of the random forest for all types of contracts are slightly superior to the ones for road construction and asphalting.

To investigate the robustness of the correct classification rate across different classes of machine learning, we also assess the performance of lasso regression and an ensemble method (including bagged trees, random forests and neural networks) as applied and outlined in *Huber and Imhof* (2019) for all models and samples, which are explained in more detail in the appendix. Table 13 reports the difference in the correct classification rates of the random forest minus the correct classification rates of the lasso and the ensemble method.

Table 13: Differences in percentage points in correct classification rates of original random forest minus the correct classification rates of the lasso and the ensemble method

	Tenders	Sample 1		Sample 2		Sample 3		Sample 4		Sample 5		Compl. Cart.	
		lasso	ens.	lasso	ens.	lasso	ens.	lasso	ens.	lasso	ens.	lasso	ens.
M1	All	-1.5	-2	-1.6	-2.3	-3.7	-2.7	-5.6	-6.7	-6	-4.3	0.9	0.4
	Comp.	-3.5	-1.4	-3.1	-0.7	-4.3	0.4	-5.9	0.9	-4	2.2	4.7	0.6
	Coll.	0.1	-2.9	-0.4	-3.9	-3.1	-5.9	-5.4	-14.1	-8.3	-11	-3	0.1
M2	All	-1.3	-1.1	-0.8	-0.7	-1.8	-1.3	-1.6	-1.8	-0.8	0.4	0.2	-0.8
	Comp.	-13	-9.9	4.8	1.5	2.2	1.4	2.7	1.3	2	2.4	4	-0.8
	Coll.	10.3	7.5	-6.5	-2.9	-5.9	-4	-5.7	-4.7	-3.7	-2	-3.6	-0.9
M3	All	-1.4	-0.9	-1.5	-1.2	-1.5	-1	-0.9	0.2	-0.3	0.6	0.1	0.1
	Comp.	0.9	-0.8	0.5	-0.6	1.9	0.7	0.3	1.9	2.6	-6.1	4.6	1.2
	Coll.	-3.8	-1.1	-3.5	-1.8	-4.9	-2.6	-2.2	-1.7	-3.4	7.5	-4.4	-1
M4	All	-0.7	-0.7	-0.7	-0.6	-1.4	-0.8	-0.9	-0.5	-0.3	1	0.2	-0.1
	Comp.	2.7	0.3	2.9	0.2	2.2	0.9	1.5	1.2	2.6	2.8	4.7	0.9
	Coll.	-4.3	-1.8	-4.4	-1.5	-5	-2.4	-3.2	-2	-3.4	-0.9	-4.3	-1.1
M5	All	-1.5	-2.9	-1.3	-1.4	-2.5	-1.1	-1.6	-1.8	1	-2.4	0.6	-0.1
	Comp.	-3.4	-3.4	-2.4	-2.2	-2.2	-1.4	-3	-0.1	2.4	5.9	4.4	0.6
	Coll.	0.3	-2.2	-0.1	-0.6	-2.7	-0.8	-0.3	-3.3	-3.5	-13.8	-3.2	-0.9

Note: "Tenders", "lasso", "ens.", "M1", "M2", "M3", "M4" and "M5" denote the type of tenders, the lasso, the ensemble of method, model 1, model 2, model 3, model 4 and model 5, respectively. For the type of tenders, "All", "Comp." and "Coll." denote the prediction for all types of tenders, the prediction for the competitive tenders and the prediction for the collusive tenders, respectively.

Considering samples 1 and 2 in Table 13, we find that the lasso and ensemble method slightly outperform the random forest. The maximum difference in (overall) correct classification rates across models and samples amounts to 2.9 percentage points. While the slightly lower rates speak against the random forest, it shows a more uniform performance and therefore less divergence across competitive and collusive periods, which may be an important aspect for practitioners. For samples 3, 4 and 5 the lasso and ensemble method in general slightly outperform the random forest, too, in two cases even more profoundly with a 4.3 to 6.7 percentage points higher correct classification rate for

model 1 in samples 4 and 5. This implies that in samples 4 and 5 (with a high amount of collusive bidders), considering screens for subgroups does not significantly improve the predictive power of lasso and ensemble method, in contrast to the random forest. On the other hand and as for samples 1 and 2, the random forest shows a more uniform performance (*i.e.* correct classification rates are not too different for competitive and collusive tenders). In complete cartels, we find a similar performance of (overall) correct classification rates between the random forest and the ensemble method. However, the random forest slightly dominates the lasso regression. Considering the deviation across prediction of competitive and collusive periods, the random forest and the ensemble method show a less divergent performance than the lasso. To conclude, the random forest shows in Table 13 a somewhat lower correct classification rate than the lasso and the ensemble of method, but exhibits a more homogeneous correct classification rate across competitive and collusive tenders.

5 Conclusion

In this paper, we suggested a method for flagging bid rigging in tenders that is likely more powerful for detecting incomplete cartels than previously suggested methods. Our approach combined screens, *i.e.* statistics derived from the distribution of bids in a tender, with machine learning to predict the probability of collusion. As a methodological innovation, we calculated the screens for all possible subgroups of three or four bids within a tender and considered summary statistics as the mean, median, maximum, and minimum for each screen as predictors in the machine learning algorithm. By this approach, we tackled the issue that competitive bids in incomplete cartels distort the statistical signals produced by bid rigging.

We first applied the methods to the Ticino bid-rigging cartel and found the method to attain a correct out-of-sample classification rate of of 77% to 86% even in the presence of simulated competitive bids. Our approach increasingly outperformed other methods based on conventional screens using all bids in a tender as the number of competitive bids per tender increased. In this simulation, there was by design no strategic reaction or interaction of competitive and collusive bidders. To allow for such reactions, we also applied our method to rather unique data from the investigations involving partial cartels in the regions See-Gaster and Graubünden in Switzerland. The out of sample performance of machine learning using summary statistics of screens (calculated for all possible subgroups of three and four bids) as predictors outperformed other screening methods. However, the performance of all machine learning-based methods decreased in the number of competitive bids, indicating that cartel participants partially adapt their bids in the presence of competitive bidders.

Compared to screens calculated with all bids in a tender, summary statistics of screens calculated for subgroups increased the correct classification rate by 5 to 10 and 3 to 7.5 percentage points for incomplete cartels in the Ticino simulation and the Swiss data from See-Gaster and Graubünden, re-

spectively. This implies a substantial decrease in error rate (which is one minus the correct classification rate) of 42.6% and 22.2% for the Ticino simulation and the Swiss data, respectively. As screening by competition agencies can trigger investigations with legal consequences for potential cartel candidates, such decreases in the error rate appears more than desirable. Our results demonstrate the usefulness of combining machine learning with an improved set of statistical screens reducing distortions of competitive bids in partial cartels. The method appears promising for detecting collusion in other industries or countries as an agenda for future research.

Appendix

Appendix 1: Descriptive Statistics for the Ticino Cartel

Table 14: Numbers of bids in a tender for the cartel and post-cartel periods in Ticino

Number of bids in a tender	4	5	6	7	8	9	10	10+
Cartel Period	32	24	23	28	15	12	7	8
Post-cartel Period	8	2	8	3	5	4	2	1

Table 15: Descriptive statistics for the collusive tenders in Ticino (without simulated bids)

Predictors	Mean	Std	Min	LowerQ.	Median	UpperQ.	Max	N
NBRBIDS	6.54	2.16	4	5	6	8	13	149
MEANBIDS	1134.58	1048.71	23.16	379.12	836.59	1547.76	4967.50	149
STDBIDS	34.08	30.13	0.82	12.77	25.84	44.68	136.90	149
CV	3.25	1.18	1.52	2.45	2.97	3.83	10.2	149
KURTO	2.71	2.12	−3.08	1.26	2.84	3.8	8.14	149
SKEW	−1.13	0.96	−2.76	−1.85	−1.34	−0.6	2.21	149
SPD	0.1	0.04	0.04	0.07	0.09	0.12	0.37	149
D	49.15	43.01	1.64	15.90	39.10	70.17	272.45	149
RD	4.09	3.37	0.31	1.77	3.14	5.13	23.02	149
RDNOR	2.93	1.35	0.53	1.95	2.72	3.6	6.95	149
RDALT	7.06	6.4	0.43	2.73	5.47	8.22	40.02	149
DIFFP	5.09	1.98	1.03	4.19	5.13	5.55	21.74	149
KS	34.27	9.93	9.77	26.5	34.26	41.17	66.05	149
MIN3CV	0.7	0.8	0	0.13	0.41	0.99	4.36	149
MAX3CV	5.03	1.87	1.99	3.71	4.72	5.87	16.64	149
MEAN3CV	2.89	1.12	1.24	2.11	2.63	3.39	8.61	149
MEDIAN3CV	2.87	1.29	0.4	1.93	2.94	3.49	6.58	149
MIN3SKEW	−1.67	0.17	−1.73	−1.73	−1.73	−1.71	−0.29	149
MAX3SKEW	1.33	0.72	−1.38	1.31	1.7	1.73	1.73	149
MEAN3SKEW	−0.44	0.48	−1.56	−0.76	−0.39	−0.12	0.94	149
MEDIAN3SKEW	−0.68	0.69	−1.69	−1.27	−0.74	−0.27	1.26	149
MIN3D	7.67	18.44	0.00	0.21	1.17	5.89	106.92	149
MAX3D	74.23	60.37	1.73	25.78	60.09	101.12	319.13	149
MEAN3D	37.58	36.04	1.03	11.60	28.73	45.99	186.08	149
MEDIAN3D	36.60	41.83	1.10	7.51	21.92	51.49	272.45	149
MIN3RD	0.43	0.75	0	0.02	0.1	0.46	4.86	149
MAX3RD	3177.54	31051.89	1.72	22.81	73.49	181.85	376725.28	149
MEAN3RD	48.87	279.28	0.75	5.09	8.67	13.66	3060.42	149
MEDIAN3RD	4.47	14.02	0.49	1.71	2.6	4.39	171.76	149
MIN3RDNOR	0.32	0.38	0	0.03	0.14	0.49	1.55	149
MAX3RDNOR	1.89	0.15	1.1	1.88	1.96	1.98	2	149
MEAN3RDNOR	1.2	0.22	0.61	1.05	1.17	1.35	1.74	149
MEDIAN3RDNOR	1.27	0.28	0.5	1.09	1.27	1.49	1.85	149
MIN3RDALT	0.3	0.53	0	0.02	0.07	0.33	3.44	149
MAX3RDALT	2246.86	21957	1.22	16.13	51.97	128.59	266385	149
MEAN3RDALT	34.56	197.48	0.53	3.6	6.13	9.66	2164.04	149
MEDIAN3RDALT	3.16	9.92	0.34	1.21	1.84	3.1	121.45	149
MIN3DIFFP	0.51	0.76	0	0.05	0.16	0.63	4.03	149
MAX3DIFFP	8	3.34	1.87	6.08	7.18	9.49	34.62	149
MEAN3DIFFP	3.68	1.58	1.19	2.56	3.35	4.55	15.43	149
MEDIAN3DIFFP	3.42	2.12	0.35	1.35	3.23	5.03	10.15	149

Predictors	Mean	Std	Min	LowerQ.	Median	UpperQ.	Max	N
MIN3SPD	0.01	0.02	0	0	0.01	0.02	0.09	149
MAX3SPD	0.1	0.04	0.04	0.07	0.09	0.12	0.37	149
MEAN3SPD	0.06	0.02	0.02	0.04	0.05	0.07	0.19	149
MEDIAN3SPD	0.06	0.03	0.01	0.04	0.06	0.07	0.14	149
MIN3KS	22.12	6.72	6.04	17.08	21.28	26.94	50.12	149
MAX3KS	851.31	3306.73	23.21	101.26	244.9	742.41	39320	149
MEAN3KS	88.19	88.53	17.01	40.86	64.31	100.72	863.76	149
MEDIAN3KS	49.42	38.86	15.83	29.28	34.73	52.31	247.55	149
MIN4CV	1.35	1.31	0.04	0.28	0.86	2.29	6.49	149
MAX4CV	4.21	1.6	1.69	3.07	3.92	4.94	13.92	149
MEAN4CV	3.06	1.16	1.35	2.27	2.81	3.55	9.2	149
MEDIAN4CV	3.29	1.46	0.54	2.55	2.99	4.06	12.88	149
MIN4SKEW	−1.67	0.58	−2	−1.99	−1.95	−1.62	1.47	149
MAX4SKEW	0.82	1.29	−1.92	−0.29	1.47	1.95	2	149
MEAN4SKEW	−0.65	0.66	−1.92	−1.12	−0.57	−0.2	1.47	149
MEDIAN4SKEW	−0.82	0.76	−1.96	−1.48	−0.9	−0.35	1.47	149
MIN4D	18.48	35.07	0.00	0.28	1.54	15.88	171.35	149
MAX4D	61.85	49.83	1.70	23.35	52.45	89.22	307.84	149
MEAN4D	40.84	38.74	1.34	12.48	29.47	52.38	229.44	149
MEDIAN4D	43.70	43.33	0.98	9.98	31.25	58.21	272.45	149
MIN4RD	1.15	2.28	0	0.04	0.19	1.14	13.56	149
MAX4RD	46.81	160.33	0.4	5.27	16.33	42.7	1865.04	149
MEAN4RD	5.2	4.81	0.4	2.47	3.79	6.54	34.88	149
MEDIAN4RD	3.27	2.83	0.32	1.44	2.34	4.24	19.6	149
MIN4RDNOR	0.65	0.75	0	0.06	0.28	1.11	2.62	149
MAX4RDNOR	2.47	0.53	0.53	2.22	2.69	2.87	3	149
MEAN4RDNOR	1.59	0.41	0.53	1.29	1.55	1.84	2.62	149
MEDIAN4RDNOR	1.66	0.5	0.44	1.29	1.65	2.05	2.73	149
MIN4RDALT	1.2	2.33	0	0.04	0.2	1.18	13.86	149
MAX4RDALT	49.69	170.21	0.43	5.68	17.11	43.97	1966.14	149
MEAN4RDALT	5.48	5.21	0.43	2.57	4.02	6.8	40.06	149
MEDIAN4RDALT	3.41	2.88	0.34	1.51	2.53	4.31	20.01	149
MIN4DIFFP	1.28	1.87	0	0.06	0.25	1.59	6.99	149
MAX4DIFFP	6.84	3.12	1.03	5.6	6.26	7.84	34.1	149
MEAN4DIFFP	4.05	1.79	1.03	2.88	3.58	5.02	18.1	149
MEDIAN4DIFFP	4.4	2.46	0.35	3.02	4.6	5.42	21.74	149
MIN4KURTO	−2.38	3.14	−6	−5.32	−3.19	0.35	3.76	149
MAX4KURTO	3.17	1.37	−3.08	2.93	3.83	3.98	4	149
MEAN4KURTO	1.47	1.18	−3.08	0.86	1.55	2.31	3.76	149
MEDIAN4KURTO	1.93	1.27	−3.08	1.45	2.09	2.77	3.87	149
MIN4SPD	0.03	0.03	0	0.01	0.02	0.06	0.17	149
MAX4SPD	0.1	0.04	0.04	0.07	0.09	0.12	0.37	149

See next page

Predictors	Mean	Std	Min	LowerQ.	Median	UpperQ.	Max	N
$MEAN4SPD$	0.07	0.03	0.03	0.05	0.07	0.08	0.24	149
$MEDIAN4SPD$	0.08	0.04	0.01	0.06	0.07	0.1	0.34	149
$MIN4KS$	26.58	8.13	7.22	20.48	25.64	32.53	59.33	149
$MAX4KS$	267.41	363.93	15.73	43.86	116.15	355.69	2643.96	149
$MEAN4KS$	55.64	36.47	15.73	31.62	43.12	64.75	195.25	149
$MEDIAN4KS$	38.72	25.53	7.82	24.73	33.39	39.56	184.51	149

Note: "Mean", "Std", "Min", "Lower Q.", "Median", "Upper Q.", "Max", and "N" denote the mean, standard deviation, minimum, lower quartile, median, upper quartile, maximum, and number of observations, respectively. The value for "MEANBIDS", "STDBIDS", "D", "MIN3D", "MAX3D", "MEAN3D", "MEDIAN3D", "MIN4D", "MAX4D", "MEAN4D" and "MEDIAN4D" are expressed in thousand CHF. "KS", "CV", "SPD", "RD", "RDNOR", "RDALT", "SKEW", "DIFFP", "KURTO", "D", "STDBIDS", "MEANBIDS" and "NBRBIDS" denote the Kolmogorov-Smirnov Statistic, the coefficient of variation, the spread, the relative distance, the normalized distance, the alternative relative distance, the skewness statistic, the percentage difference, the kurtosis statistic, the difference in absolute between the first and second lowest bids, the standard deviation of the bids in a tender, the mean of the bids in a tender and the number of the bids in a tender, respectively.

Table 16: Descriptive statistics for the Ticino cartel in the post-cartel period

Predictors	Mean	Std	Min	LowerQ.	Median	UpperQ.	Max	N
NBRBIDS	6.73	2.34	4	5	6	8	13	33
MEANBIDS	756.79	785.43	43.97	279.90	482.05	695.68	3191.78	33
STDBIDS	54.83	49.50	2.79	27.78	39.64	61.86	209.64	33
CV	9.51	5.38	1.71	5.65	8.49	12.7	21.12	33
KURTO	−0.08	1.78	−2.83	−1.31	−0.16	1.19	6.06	33
SKEW	0.24	0.85	−1.46	−0.31	0.31	0.93	2.36	33
SPD	0.31	0.2	0.04	0.16	0.26	0.47	0.84	33
D	29.67	36.08	0.68	7.87	17.42	31.26	149.28	33
RD	0.77	0.89	0.02	0.24	0.41	0.95	4.03	33
RDNOR	1.02	0.8	0.06	0.45	0.74	1.33	3.67	33
RDALT	1.22	1.23	0.05	0.42	0.72	1.46	4.84	33
DIFFP	5.16	5.02	0.23	2.21	3.73	5.07	20.6	33
KS	16.52	12.23	5.4	8.86	12.4	18.49	58.56	33
MIN3CV	2.58	2.8	0.07	1.03	1.73	3.11	12.23	33
MAX3CV	14.45	8.63	2.15	7.75	13.09	21.45	36.56	33
MEAN3CV	8.72	4.76	1.65	5.13	7.87	11.43	19.63	33
MEDIAN3CV	8.67	4.73	1.68	5.21	7.97	10.7	20.71	33
MIN3SKEW	−1.48	0.55	−1.73	−1.72	−1.7	−1.53	0.7	33
MAX3SKEW	1.48	0.47	−0.27	1.6	1.69	1.73	1.73	33
MEAN3SKEW	0.1	0.51	−1.05	−0.2	0.21	0.42	1.2	33
MEDIAN3SKEW	0.14	0.79	−1.26	−0.5	0.29	0.74	1.2	33
MIN3D	14.25	32.20	0.16	1.29	2.58	8.54	149.28	33
MAX3D	107.42	100.76	2.88	48.31	76.20	133.87	471.50	33
MEAN3D	47.30	47.20	1.75	23.50	31.57	48.04	221.08	33
MEDIAN3D	41.00	38.93	1.44	19.67	28.88	39.00	183.76	33
MIN3RD	0.26	0.38	0.01	0.03	0.12	0.23	1.7	33
MAX3RD	39.4	59.34	0.86	6.69	17.33	39.98	274.21	33
MEAN3RD	4.37	4.62	0.51	1.85	3.4	4.4	25.42	33
MEDIAN3RD	1.65	1.17	0.53	0.83	1.16	2.09	4.24	33
MIN3RDNOR	0.25	0.28	0.01	0.04	0.16	0.28	1.09	33
MAX3RDNOR	1.74	0.29	0.76	1.65	1.85	1.93	1.99	33
MEAN3RDNOR	0.96	0.22	0.49	0.83	0.91	1.09	1.43	33
MEDIAN3RDNOR	0.96	0.29	0.53	0.74	0.9	1.17	1.49	33
MIN3RDALT	0.19	0.27	0.01	0.02	0.08	0.16	1.2	33
MAX3RDALT	27.86	41.96	0.61	4.73	12.25	28.27	193.89	33
MEAN3RDALT	3.09	3.27	0.36	1.31	2.4	3.11	17.97	33
MEDIAN3RDALT	1.17	0.83	0.37	0.59	0.82	1.48	3	33
MIN3DIFFP	1.53	2.45	0.05	0.34	0.58	1.75	13.25	33
MAX3DIFFP	21.91	14.6	2.6	10.75	17.43	30.98	62.88	33
MEAN3DIFFP	8.32	4.73	1.43	4.44	7.63	10.3	20.24	33
MEDIAN3DIFFP	7.1	4.23	1.3	3.98	6.12	8.64	20.6	33

See next page

Predictors	Mean	Std	Min	LowerQ.	Median	UpperQ.	Max	N
MIN3SPD	0.05	0.06	0	0.02	0.03	0.06	0.28	33
MAX3SPD	0.31	0.2	0.04	0.16	0.26	0.47	0.84	33
MEAN3SPD	0.19	0.11	0.03	0.11	0.16	0.25	0.47	33
MEDIAN3SPD	0.19	0.11	0.03	0.11	0.17	0.23	0.49	33
MIN3KS	11.84	10.28	3.14	5.06	8.04	13.19	46.56	33
MAX3KS	113.2	230.7	8.41	32.52	58.13	97.09	1360.3	33
MEAN3KS	22.18	16.84	5.85	11.68	15.88	25.57	74.31	33
MEDIAN3KS	16.75	11.89	5.34	9.69	12.79	19.4	60.05	33
MIN4CV	4.4	4.32	0.5	1.85	2.93	6.33	20.66	33
MAX4CV	12.55	7.62	1.91	6.57	11.2	18.1	32.31	33
MEAN4CV	9.09	5	1.7	5.36	8.27	11.97	20.66	33
MEDIAN4CV	9.01	4.89	1.78	5.25	8.14	11.76	20.66	33
MIN4SKEW	−1.16	0.91	−2	−1.81	−1.46	−0.79	1.32	33
MAX4SKEW	1.14	0.99	−1.46	0.81	1.57	1.89	2	33
MEAN4SKEW	0.11	0.7	−1.46	−0.35	0.22	0.58	1.32	33
MEDIAN4SKEW	0.19	0.73	−1.46	−0.24	0.22	0.77	1.34	33
MIN4D	20.50	38.09	0.16	1.62	3.62	15.10	149.28	33
MAX4D	76.56	77.30	1.26	36.48	62.49	92.32	435.92	33
MEAN4D	37.81	35.59	1.26	17.18	25.70	42.78	149.28	33
MEDIAN4D	34.53	35.39	1.26	15.94	23.39	35.73	149.28	33
MIN4RD	0.49	0.92	0.01	0.04	0.17	0.48	4.03	33
MAX4RD	10.2	22.25	0.17	1.9	4.15	8.5	128.38	33
MEAN4RD	1.55	1.44	0.17	0.83	1.03	1.71	7.36	33
MEDIAN4RD	1.03	0.94	0.1	0.48	0.67	1.32	4.03	33
MIN4RDNOR	0.43	0.52	0.02	0.06	0.23	0.59	2.01	33
MAX4RDNOR	1.95	0.68	0.23	1.48	2.07	2.46	2.95	33
MEAN4RDNOR	0.97	0.41	0.23	0.71	0.87	1.18	2.01	33
MEDIAN4RDNOR	0.91	0.49	0.15	0.59	0.77	1.26	2.01	33
MIN4RDALT	0.51	0.92	0.01	0.04	0.17	0.49	4.03	33
MAX4RDALT	10.47	22.63	0.17	1.94	4.48	9.13	130.64	33
MEAN4RDALT	1.62	1.48	0.17	0.87	1.06	1.75	7.68	33
MEDIAN4RDALT	1.07	0.96	0.11	0.49	0.68	1.45	4.03	33
MIN4DIFFP	2.43	4.4	0.16	0.4	0.84	2.75	20.6	33
MAX4DIFFP	16.11	11.32	0.54	7.23	15.77	20.11	45.9	33
MEAN4DIFFP	6.78	4.45	0.54	3.91	6.06	9.05	20.6	33
MEDIAN4DIFFP	5.97	4.5	0.54	3.28	5.07	7.1	20.6	33
MIN4KURTO	−3.77	2.63	−6	−5.62	−4.78	−2.83	2.2	33
MAX4KURTO	2.65	1.73	−2.83	2.2	3.29	3.83	4	33
MEAN4KURTO	−0.08	1.17	−2.83	−0.8	−0.08	0.48	2.2	33
MEDIAN4KURTO	0.17	1.52	−3.68	−0.53	0.69	1.18	2.2	33
MIN4SPD	0.11	0.13	0.01	0.04	0.07	0.13	0.63	33
MAX4SPD	0.31	0.2	0.04	0.16	0.26	0.47	0.84	33

<div align="center">See next page</div>

Predictors	Mean	Std	Min	LowerQ.	Median	UpperQ.	Max	N
MEAN4SPD	0.24	0.15	0.04	0.13	0.2	0.32	0.63	33
MEDIAN4SPD	0.23	0.14	0.04	0.13	0.2	0.31	0.63	33
MIN4KS	13.88	12.22	3.79	6.05	9.27	15.47	52.64	33
MAX4KS	43.49	37.89	5.4	16.37	34.46	54.38	201.12	33
MEAN4KS	17.88	12.29	5.4	9.86	13.17	22.37	60.1	33
MEDIAN4KS	16.68	12.04	5.4	9.1	12.71	19.31	56.12	33

Note: "Mean", "Std", "Min", "Lower Q.", "Median", "Upper Q.", "Max", and "N" denote the mean, standard deviation, minimum, lower quartile, median, upper quartile, maximum, and number of observations, respectively. The value for "MEANBIDS", "STDBIDS", "D", "MIN3D", "MAX3D", "MEAN3D", "MEDIAN3D", "MIN4D", "MAX4D", "MEAN4D" and "MEDIAN4D" are expressed in thousand CHF. "KS", "CV", "SPD", "RD", "RDNOR", "RDALT", "SKEW", "DIFFP", "KURTO", "D", "STDBIDS", "MEANBIDS" and "NBRBIDS" denote the Kolmogorov-Smirnov statistic, the coefficient of variation, the spread, the relative distance, the normalized distance, the alternative relative distance, the skewness statistic, the percentage difference, the kurtosis statistic, the difference in absolute between the first and second lowest bids, the standard deviation of the bids in a tender, the mean of the bids in a tender and the number of the bids in a tender, respectively.

Table 17: Descriptive statistics for collusive tenders of the Ticino cartel with five competitive bids

Predictors	Mean	Std	Min	LowerQ.	Median	UpperQ.	Max	N
NBRBIDS	10.79	2.49	6	9	11	12	18	184
MEANBIDS	1417.60	1328.38	19.00	383.19	897.16	1962.42	6080.35	184
STDBIDS	112.94	140.47	1.83	21.68	60.74	143.96	859.50	184
CV	7.38	3.23	2.77	5.16	6.89	8.95	23.83	184
KURTO	1.78	2.37	−1.91	−0.11	1.24	3.27	9.87	184
SKEW	0.11	1.24	−2.6	−0.87	−0.08	1.01	3.07	184
SPD	0.29	0.14	0.08	0.18	0.26	0.36	0.89	184
D	633.31	93.76	0.11	6.86	25.49	69.55	585.65	184
RD	0.93	1.04	0	0.23	0.64	1.13	5.77	184
RDNOR	1.84	1.55	0	0.66	1.52	2.57	8.19	184
RDALT	2.51	2.94	0	0.64	1.62	3.17	18.31	184
DIFFP	5.27	4.81	0.01	1.79	4.23	7.66	27.56	184
KS	16.68	5.95	5.44	12.41	15.44	19.79	37.34	184
MIN3CV	0.49	0.52	0	0.12	0.29	0.7	2.9	184
MAX3CV	13.83	6.03	4.57	9.3	12.54	17.06	37.62	184
MEAN3CV	6.2	2.41	2.4	4.54	5.9	7.49	19.05	184
MEDIAN3CV	5.6	2.23	1.48	4.04	5.08	6.52	13.65	184
MIN3SKEW	−1.72	0.03	−1.73	−1.73	−1.73	−1.73	−1.51	184
MAX3SKEW	1.71	0.06	1.22	1.73	1.73	1.73	1.73	184
MEAN3SKEW	−0.06	0.4	−1.19	−0.33	−0.12	0.18	1.09	184
MEDIAN3SKEW	−0.1	0.77	−1.56	−0.67	−0.25	0.44	1.73	184
MIN3D	4.61	10.12	0.00	0.16	0.66	2.96	70.32	184
MAX3D	239.79	245.60	4.90	62.47	146.83	354.79	1327.34	184
MEAN3D	82.36	91.89	1.86	20.25	45.74	109.63	391.88	184
MEDIAN3D	71.06	82.74	1.55	14.69	41.10	92.43	394.87	184
MIN3RD	0.05	0.08	0	0	0.01	0.05	0.52	184
MAX3RD	5367.51	51859.16	6.4	53.19	129.19	604.61	693080.61	184
MEAN3RD	43.17	245.44	0.95	4.22	7.71	16.32	2710.21	184
MEDIAN3RD	1.82	1.01	0.25	1.11	1.69	2.28	7.49	184
MIN3RDNOR	0.06	0.09	0	0.01	0.02	0.06	0.54	184
MAX3RDNOR	1.96	0.06	1.64	1.95	1.98	2	2	184
MEAN3RDNOR	1.04	0.17	0.6	0.94	1.06	1.16	1.56	184
MEDIAN3RDNOR	1.05	0.26	0.3	0.88	1.08	1.23	1.68	184
MIN3RDALT	0.03	0.06	0	0	0.01	0.03	0.37	184
MAX3RDALT	3795.41	36669.96	4.53	37.61	91.35	427.52	490082	184
MEAN3RDALT	30.53	173.55	0.67	2.98	5.45	11.54	1916.41	184
MEDIAN3RDALT	1.29	0.72	0.18	0.79	1.19	1.61	5.29	184
MIN3DIFFP	0.23	0.34	0	0.02	0.11	0.29	2.01	184
MAX3DIFFP	20.05	8.11	7.05	13.94	18.21	24.27	53.73	184
MEAN3DIFFP	6.33	2.43	2.48	4.4	5.8	7.74	13.45	184
MEDIAN3DIFFP	5.21	2.33	0.87	3.55	5	6.2	14.71	184

See next page

Predictors	Mean	Std	Min	LowerQ.	Median	UpperQ.	Max	N
MIN3SPD	0.01	0.01	0	0	0.01	0.01	0.06	184
MAX3SPD	0.29	0.14	0.08	0.18	0.26	0.36	0.89	184
MEAN3SPD	0.13	0.05	0.05	0.09	0.12	0.15	0.45	184
MEDIAN3SPD	0.11	0.05	0.03	0.08	0.1	0.14	0.3	184
MIN3KS	8.83	3.51	3.06	6.22	8.1	11	22.3	184
MAX3KS	938.54	3130.28	34.68	144.12	339.85	830.93	39320	184
MEAN3KS	39.88	25.59	8.37	23.92	32.91	46.72	192.41	184
MEDIAN3KS	20.92	8.4	7.62	15.44	19.97	25.03	67.92	184
MIN4CV	0.94	0.95	0.04	0.28	0.73	1.27	7.67	184
MAX4CV	12.03	5.14	4.23	8.2	11.03	14.48	33.57	184
MEAN4CV	6.57	2.64	2.53	4.7	6.26	7.89	20.74	184
MEDIAN4CV	6.27	2.82	1.9	4.49	5.74	7.44	26	184
MIN4SKEW	−1.92	0.19	−2	−2	−1.99	−1.93	−0.67	184
MAX4SKEW	1.87	0.29	0.22	1.89	1.99	2	2	184
MEAN4SKEW	−0.08	0.53	−1.53	−0.48	−0.17	0.26	1.2	184
MEDIAN4SKEW	−0.06	0.7	−1.54	−0.59	−0.14	0.33	1.74	184
MIN4D	5.32	11.79	0.00	0.16	0.68	3.10	70.32	184
MAX4D	196.40	202.67	3.82	56.66	112.66	278.55	903.60	184
MEAN4D	75.31	87.29	1.35	18.73	41.48	96.57	449.88	184
MEDIAN4D	70.54	87.18	0.82	14.65	37.46	87.74	449.27	184
MIN4RD	0.06	0.11	0	0	0.02	0.06	0.74	184
MAX4RD	117.38	392.24	1.49	14.96	33.85	95.96	4710.41	184
MEAN4RD	3.59	4.07	0.37	1.59	2.58	4.27	44.88	184
MEDIAN4RD	1.31	0.78	0.12	0.72	1.21	1.76	4.63	184
MIN4RDNOR	0.09	0.14	0	0.01	0.03	0.1	0.81	184
MAX4RDNOR	2.74	0.29	1.32	2.65	2.84	2.94	3	184
MEAN4RDNOR	1.18	0.31	0.4	0.98	1.23	1.4	2.21	184
MEDIAN4RDNOR	1.13	0.41	0.17	0.83	1.18	1.43	2.12	184
MIN4RDALT	0.07	0.12	0	0.01	0.02	0.07	0.74	184
MAX4RDALT	124.1	415.85	1.56	15.04	34.79	99.1	4965.75	184
MEAN4RDALT	3.73	4.24	0.39	1.69	2.66	4.48	47.93	184
MEDIAN4RDALT	1.37	0.81	0.12	0.77	1.28	1.81	4.82	184
MIN4DIFFP	0.27	0.41	0	0.03	0.11	0.3	2.5	184
MAX4DIFFP	16.67	6.63	6	11.57	15.6	20.15	40.38	184
MEAN4DIFFP	5.99	2.63	1.71	3.99	5.49	7.6	14.87	184
MEDIAN4DIFFP	5.32	2.7	0.58	3.57	4.86	6.48	19.21	184
MIN4KURTO	−5.57	0.76	−6	−5.96	−5.84	−5.55	−0.04	184
MAX4KURTO	3.9	0.2	2.88	3.91	3.98	4	4	184
MEAN4KURTO	0.64	0.77	−1.74	0.15	0.72	1.11	2.39	184
MEDIAN4KURTO	1.32	0.98	−2.78	0.88	1.51	1.86	3.24	184
MIN4SPD	0.02	0.02	0	0.01	0.02	0.03	0.18	184
MAX4SPD	0.29	0.14	0.08	0.18	0.26	0.36	0.89	184

See next page

Predictors	Mean	Std	Min	Lower Q.	Median	Upper Q.	Max	N
$MEAN4SPD$	0.16	0.07	0.06	0.11	0.15	0.19	0.57	184
$MEDIAN4SPD$	0.15	0.08	0.05	0.11	0.13	0.18	0.75	184
$MIN4KS$	10.09	3.78	3.67	7.33	9.25	12.34	23.85	184
$MAX4KS$	288.31	349.99	13.07	79.29	137.96	363.99	2643.96	184
$MEAN4KS$	26.2	11.64	6.8	17.79	23.37	32.35	72.53	184
$MEDIAN4KS$	19.03	7.35	4.49	13.87	17.82	22.57	53.22	184

Note: "Mean", "Std", "Min", "Lower Q.", "Median", "Upper Q.", "Max", and "N" denote the mean, standard deviation, minimum, lower quartile, median, upper quartile, maximum, and number of observations, respectively. The value for "MEANBIDS", "STDBIDS", "D", "MIN3D", "MAX3D", "MEAN3D", "MEDIAN3D", "MIN4D", "MAX4D", "MEAN4D" and "MEDIAN4D" are expressed in thousand CHF. "KS", "CV", "SPD", "RD", "RDNOR", "RDALT", "SKEW", "DIFFP", "KURTO", "D", "STDBIDS", "MEANBIDS" and "NBRBIDS" denote the Kolmogorov-Smirnov statistic, the coefficient of variation, the spread, the relative distance, the normalized distance, the alternative relative distance, the skewness statistic, the percentage difference, the kurtosis statistic, the difference in absolute between the first and second lowest bids, the standard deviation of the bids in a tender, the mean of the bids in a tender and the number of the bids in a tender, respectively.

Appendix 2: Descriptive Statistics for the Swiss Data

Table 18: Numbers of bids in a tender in the Swiss data

Number of bids in a tender	4	5	6	7	8	9	10	10+
Tenders with complete cartels	94	50	29	24	33	33	23	24
Tenders with incomplete cartels	56	36	38	40	27	28	24	38
Competitive tenders	786	559	365	257	158	129	74	70

Table 19: Descriptive statistics for the collusive tenders including only cartel participants in the Swiss data

Predictors	Mean	Std	Min	LowerQ.	Median	UpperQ.	Max	N
NBRBIDS	6.57	2.44	4	4	6	9	13	308
MEANBIDS	379.88	376.09	34.42	172.13	305.81	460.98	3509.71	308
STDBIDS	13.15	13.67	0.49	5.59	9.75	15.88	109.94	308
CV	3.66	2.09	0.6	2.22	3.29	4.51	15.73	308
KURTO	0.16	1.65	−5.4	−0.99	0.21	1.33	4.37	308
SKEW	0.08	0.81	−1.94	−0.42	0.07	0.7	1.78	308
SPD	0.11	0.07	0.01	0.06	0.09	0.13	0.5	308
D	9.20	13.32	0.14	2.93	6.02	10.11	121.30	308
RD	1.16	1.36	0.01	0.46	0.75	1.31	13.66	308
RDNOR	1.38	0.79	0.02	0.78	1.24	1.72	5.03	308
RDALT	1.83	1.73	0.01	0.74	1.33	2.2	13.89	308
DIFFP	2.76	2.91	0.06	1.3	1.94	3.24	34.11	308
KS	36.54	20.15	6.59	22.69	31.2	45.55	167.93	308
MIN3CV	1.08	1.01	0.04	0.48	0.81	1.26	8.46	308
MAX3CV	5.49	3.41	0.73	3.14	4.72	6.88	21.17	308
MEAN3CV	3.41	1.94	0.57	2.1	3.04	4.18	15.32	308
MEDIAN3CV	3.46	2	0.63	2.1	3	4.25	16.87	308
MIN3SKEW	−1.48	0.44	−1.73	−1.73	−1.7	−1.42	0.69	308
MAX3SKEW	1.41	0.63	−1.49	1.43	1.68	1.73	1.73	308
MEAN3SKEW	0	0.49	−1.61	−0.26	0.05	0.33	1.16	308
MEDIAN3SKEW	0.03	0.69	−1.7	−0.41	0.07	0.55	1.53	308
MIN3D	2.67	6.13	0.00	0.38	1.03	2.99	73.33	308
MAX3D	26.81	27.14	0.59	10.58	20.08	32.20	200.26	308
MEAN3D	11.62	13.40	0.43	4.63	8.44	13.24	118.06	308
MEDIAN3D	10.88	13.28	0.42	3.89	7.87	12.01	121.30	308
MIN3RD	0.38	0.73	0	0.04	0.14	0.38	6.13	308
MAX3RD	117.7	665.15	0.87	5.26	16.67	54.56	9157.71	308
MEAN3RD	7.53	23.86	0.55	1.92	3.04	5.7	291.4	308
MEDIAN3RD	1.95	2.79	0.29	0.97	1.37	1.98	39.01	308
MIN3RDNOR	0.3	0.33	0	0.06	0.17	0.42	1.63	308
MAX3RDNOR	1.73	0.28	0.76	1.58	1.85	1.95	2	308
MEAN3RDNOR	1	0.22	0.51	0.86	0.98	1.11	1.8	308
MEDIAN3RDNOR	0.99	0.26	0.33	0.81	0.98	1.14	1.85	308
MIN3RDALT	0.27	0.52	0	0.03	0.1	0.27	4.34	308
MAX3RDALT	83.23	470.33	0.62	3.72	11.79	38.58	6475.48	308
MEAN3RDALT	5.32	16.87	0.39	1.36	2.15	4.03	206.05	308
MEDIAN3RDALT	1.38	1.97	0.2	0.68	0.97	1.4	27.58	308
MIN3DIFFP	0.69	0.85	0	0.15	0.44	0.91	8.62	308
MAX3DIFFP	8.1	6.07	1.13	4.26	6.76	10.05	47.45	308
MEAN3DIFFP	3.37	2.32	0.77	1.99	2.77	4.08	22.76	308
MEDIAN3DIFFP	3.13	2.31	0.69	1.79	2.51	3.95	23.91	308

See next page

Predictors	Mean	Std	Min	LowerQ.	Median	UpperQ.	Max	N
MIN3SPD	0.02	0.02	0	0.01	0.02	0.03	0.18	308
MAX3SPD	0.11	0.07	0.01	0.06	0.09	0.13	0.5	308
MEAN3SPD	0.07	0.04	0.01	0.04	0.06	0.08	0.37	308
MEDIAN3SPD	0.07	0.04	0.01	0.04	0.06	0.08	0.41	308
MIN3KS	26.02	17.06	4.76	14.83	21.48	31.93	137.18	308
MAX3KS	181.77	213.13	12.09	79.31	124.09	206.54	2751.23	308
MEAN3KS	48.26	25.64	7.44	30.53	42.03	60.32	199.62	308
MEDIAN3KS	38.19	20.28	6.19	24.04	33.58	47.94	158.95	308
MIN4CV	1.93	1.66	0.22	0.91	1.5	2.37	15.73	308
MAX4CV	4.74	2.95	0.6	2.75	4.07	5.83	19.16	308
MEAN4CV	3.54	2.01	0.6	2.18	3.13	4.35	15.73	308
MEDIAN4CV	3.6	2.07	0.6	2.22	3.21	4.39	15.73	308
MIN4SKEW	−1.07	0.98	−2	−1.89	−1.5	−0.41	1.78	308
MAX4SKEW	1.07	1.02	−1.93	0.46	1.48	1.87	2	308
MEAN4SKEW	0.04	0.68	−1.93	−0.35	0.07	0.5	1.78	308
MEDIAN4SKEW	0.05	0.71	−1.93	−0.3	0.03	0.54	1.78	308
MIN4D	4.47	11.64	0.00	0.50	1.36	3.86	121.30	308
MAX4D	20.38	20.98	0.36	7.07	14.66	25.33	121.30	308
MEAN4D	10.11	12.76	0.36	3.81	6.99	11.59	121.30	308
MEDIAN4D	9.73	13.27	0.17	3.48	6.40	10.73	121.30	308
MIN4RD	0.68	1.38	0	0.06	0.23	0.67	13.66	308
MAX4RD	9.14	19.83	0.15	1.51	4.31	10.73	298.7	308
MEAN4RD	1.66	1.53	0.15	0.87	1.26	1.94	13.66	308
MEDIAN4RD	1.26	1.37	0.07	0.61	0.87	1.35	13.66	308
MIN4RDNOR	0.53	0.58	0	0.1	0.31	0.77	2.62	308
MAX4RDNOR	1.9	0.72	0.22	1.31	2.09	2.55	2.98	308
MEAN4RDNOR	1.07	0.4	0.22	0.8	1.02	1.27	2.62	308
MEDIAN4RDNOR	1.02	0.44	0.1	0.72	0.93	1.23	2.62	308
MIN4RDALT	0.71	1.44	0	0.07	0.23	0.69	13.89	308
MAX4RDALT	9.67	22.08	0.15	1.55	4.57	11.2	340.47	308
MEAN4RDALT	1.74	1.62	0.15	0.91	1.33	2.02	13.89	308
MEDIAN4RDALT	1.31	1.42	0.07	0.64	0.9	1.38	13.89	308
MIN4DIFFP	1.19	1.93	0	0.19	0.67	1.55	23.91	308
MAX4DIFFP	6.2	5.28	0.64	2.59	4.98	7.89	39.3	308
MEAN4DIFFP	2.95	2.42	0.61	1.66	2.29	3.56	24.48	308
MEDIAN4DIFFP	2.82	2.74	0.2	1.49	2.08	3.28	34.11	308
MIN4KURTO	−2.96	2.98	−6	−5.63	−4.23	−0.28	3.75	308
MAX4KURTO	2.44	1.83	−5.4	1.66	3.14	3.77	4	308
MEAN4KURTO	0.14	1.36	−5.4	−0.58	0.09	0.95	3.75	308
MEDIAN4KURTO	0.34	1.54	−5.4	−0.28	0.61	1.31	3.75	308
MIN4SPD	0.05	0.04	0.01	0.02	0.03	0.05	0.47	308
MAX4SPD	0.11	0.07	0.01	0.06	0.09	0.13	0.5	308

Predictors	Mean	Std	Min	LowerQ.	Median	UpperQ.	Max	N
$MEAN4SPD$	0.08	0.05	0.01	0.05	0.07	0.1	0.47	308
$MEDIAN4SPD$	0.09	0.05	0.01	0.05	0.08	0.1	0.47	308
$MIN4KS$	30.68	20.89	5.42	17.65	24.8	36.76	167.93	308
$MAX4KS$	88.22	69.07	6.59	42.47	67.21	110.66	458.02	308
$MEAN4KS$	39.97	20.41	6.59	26.34	35.08	50.1	167.93	308
$MEDIAN4KS$	36.66	19.93	6.59	23.01	31.38	45.53	167.93	308

Note: "Mean", "Std", "Min", "Lower Q.", "Median", "Upper Q.", "Max", and "N" denote the mean, standard deviation, minimum, lower quartile, median, upper quartile, maximum, and number of observations, respectively. The value for "MEANBIDS", "STDBIDS", "D", "MIN3D", "MAX3D", "MEAN3D", "MEDIAN3D", "MIN4D", "MAX4D", "MEAN4D" and "MEDIAN4D" are expressed in thousand CHF. "KS", "CV", "SPD", "RD", "RDNOR", "RDALT", "SKEW", "DIFFP", "KURTO", "D", "STDBIDS", "MEANBIDS" and "NBRBIDS" denote the Kolmogorov-Smirnov statistic, the coefficient of variation, the spread, the relative distance, the normalized distance, the alternative relative distance, the skewness statistic, the percentage difference, the kurtosis statistic, the difference in absolute between the first and second lowest bids, the standard deviation of the bids in a tender, the mean of the bids in a tender and the number of the bids in a tender, respectively.

Table 20: Descriptive statistics for the competitive tenders in the Swiss data

Predictors	Mean	Std	Min	LowerQ.	Median	UpperQ.	Max	N
NBRBIDS	5.73	1.86	4	4	5	7	13	1082
MEANBIDS	828.06	1803.84	13.63	203.87	423.97	858.67	37786.87	1082
STDBIDS	87.35	216.52	0.41	13.96	32.74	81.27	3996.24	1082
CV	10.12	7.89	0.76	5.91	8.45	11.64	128	1082
KURTO	0.25	2.27	−6	−1.27	0.13	1.75	8.03	1082
SKEW	0.26	0.97	−2.68	−0.37	0.28	0.92	2.47	1082
SPD	2.5	29.79	0.02	0.16	0.24	0.35	730.71	1082
D	54.94	223.18	0.00	5.18	14.19	39.12	4656.85	1082
RD	1.16	2.45	0	0.23	0.57	1.13	41.26	1082
RDNOR	1.04	0.82	0	0.43	0.87	1.44	6.95	1082
RDALT	1.61	3.19	0	0.36	0.84	1.66	47.49	1082
DIFFP	176.79	2246.36	0	1.84	4.36	8.43	50228.95	1082
KS	15.07	10.98	1.48	9.12	12.24	17.44	132.33	1082
MIN3CV	3.37	3.13	0.02	1.16	2.33	4.68	24.05	1082
MAX3CV	14.22	11.46	0.93	8.19	11.77	16.52	122.06	1082
MEAN3CV	9.37	6.9	0.73	5.56	7.92	10.9	91.8	1082
MEDIAN3CV	9.7	7.83	0.6	5.61	8.19	11.41	121.69	1082
MIN3SKEW	−1.38	0.62	−1.73	−1.73	−1.67	−1.34	1.68	1082
MAX3SKEW	1.47	0.53	−1.61	1.48	1.69	1.73	1.73	1082
MEAN3SKEW	0.13	0.58	−1.66	−0.22	0.16	0.51	1.71	1082
MEDIAN3SKEW	0.22	0.83	−1.73	−0.35	0.26	0.83	1.73	1082
MIN3D	13.03	30.15	0.00	1.10	3.79	11.64	364.58	1082
MAX3D	154.93	433.88	0.72	22.68	52.94	132.40	7506.55	1082
MEAN3D	69.05	178.70	0.44	11.05	25.66	60.66	2536.05	1082
MEDIAN3D	64.40	171.21	0.36	10.29	23.09	56.37	2640.97	1082
MIN3RD	0.32	0.68	0	0.03	0.12	0.34	8.83	1082
MAX3RD	69.28	265.38	0.13	4.44	12.27	38.7	5315.77	1082
MEAN3RD	8.2	25.53	0.08	1.55	2.78	6.05	421.57	1082
MEDIAN3RD	2.22	4.95	0.01	0.78	1.21	1.98	72.77	1082
MIN3RDNOR	0.27	0.3	0	0.05	0.16	0.39	1.72	1082
MAX3RDNOR	1.68	0.34	0.17	1.52	1.8	1.93	2	1082
MEAN3RDNOR	0.94	0.27	0.11	0.77	0.93	1.11	1.85	1082
MEDIAN3RDNOR	0.92	0.34	0.01	0.7	0.91	1.12	1.94	1082
MIN3RDALT	0.22	0.48	0	0.02	0.09	0.24	6.24	1082
MAX3RDALT	48.99	187.65	0.1	3.14	8.67	27.36	3758.82	1082
MEAN3RDALT	5.8	18.05	0.06	1.1	1.96	4.27	298.09	1082
MEDIAN3RDALT	1.57	3.5	0	0.55	0.85	1.4	51.46	1082
MIN3DIFFP	2.07	2.77	0	0.36	0.98	2.56	22.67	1082
MAX3DIFFP	226.26	2740.18	0.89	9.94	15.42	23.4	63802.49	1082
MEAN3DIFFP	92.86	1118.61	0.64	4.6	7.07	10.73	23734.93	1082
MEDIAN3DIFFP	60.05	1011.71	0.54	4.08	6.52	10.25	25112.51	1082

See next page

Predictors	Mean	Std	Min	LowerQ.	Median	UpperQ.	Max	N
MIN3SPD	0.07	0.07	0	0.02	0.05	0.09	0.58	1082
MAX3SPD	2.5	29.79	0.02	0.16	0.24	0.35	730.71	1082
MEAN3SPD	1.14	12.52	0.01	0.11	0.16	0.23	279.23	1082
MEDIAN3SPD	0.76	10.69	0.01	0.11	0.17	0.25	264.32	1082
MIN3KS	10.96	8.72	1.1	6.33	8.72	12.45	107.96	1082
MAX3KS	82.65	179.47	4.21	21.54	43	86.79	4045.44	1082
MEAN3KS	21.45	21.35	2.62	11.62	16.51	24.77	463.06	1082
MEDIAN3KS	15.78	13.43	1.2	9.07	12.57	18.27	167.49	1082
MIN4CV	6.24	6.77	0.07	2.29	4.49	8.14	128	1082
MAX4CV	12.14	9.73	0.76	7.07	10.13	14.17	128	1082
MEAN4CVB	9.82	7.64	0.76	5.74	8.29	11.45	128	1082
MEDIAN4CV	10.24	8.21	0.76	6	8.55	11.74	128	1082
MIN4SKEW	−0.75	1.09	−2	−1.72	−1.02	0	2	1082
MAX4SKEW	1.02	1	−1.99	0.34	1.4	1.85	2	1082
MEAN4SKEW	0.19	0.8	−1.99	−0.31	0.22	0.74	2	1082
MEDIAN4SKEW	0.22	0.86	−1.99	−0.27	0.19	0.79	2	1082
MIN4D	22.95	76.73	0.00	1.76	5.64	18.51	1226.06	1082
MAX4D	111.58	328.01	0.02	13.13	34.32	86.30	5764.43	1082
MEAN4D	59.42	181.05	0.02	8.14	19.23	46.70	3035.79	1082
MEDIAN4D	59.94	224.26	0.00	6.98	18.35	45.41	4656.85	1082
MIN4RD	0.71	2.05	0	0.06	0.2	0.62	41.26	1082
MAX4RD	6.73	17.51	0	0.88	2.55	6.3	266.72	1082
MEAN4RD	1.69	3.23	0	0.56	1	1.72	67.61	1082
MEDIAN4RD	1.24	2.28	0	0.39	0.74	1.25	41.26	1082
MIN4RDNOR	0.51	0.56	0	0.09	0.29	0.76	2.87	1082
MAX4RDNOR	1.62	0.81	0	0.96	1.74	2.3	2.98	1082
MEAN4RDNOR	0.95	0.51	0	0.61	0.9	1.21	2.87	1082
MEDIAN4RDNOR	0.9	0.55	0	0.51	0.84	1.19	2.87	1082
MIN4RDALT	0.75	2.18	0	0.06	0.21	0.67	43.5	1082
MAX4RDALT	7.07	18.5	0	0.93	2.69	6.57	267.55	1082
MEAN4RDALT	1.77	3.36	0	0.59	1.05	1.8	68.45	1082
MEDIAN4RDALT	1.3	2.4	0	0.41	0.78	1.31	43.5	1082
MIN4DIFFP	20.35	549.37	0	0.51	1.54	4.44	18073.22	1082
MAX4DIFFP	206.69	2603.79	0.03	5.44	9.88	16.43	62175.31	1082
MEAN4DIFFP	116.56	1450.99	0.03	3.24	5.46	9.08	30433.18	1082
MEDIAN4DIFFP	140.96	2011.38	0	2.73	5.24	8.95	50228.95	1082
MIN4KURTO	−2.61	3.02	−6	−5.39	−3.37	0.07	4	1082
MAX4KURTO	2.07	2.28	−6	1.43	2.94	3.72	4	1082
MEAN4KURTO	0.12	1.83	−6	−0.83	0.1	1.31	4	1082
MEDIAN4KURTO	0.29	2.06	−6	−0.75	0.6	1.62	4	1082
MIN4SPD	0.34	6.03	0	0.05	0.1	0.2	198.43	1082
MAX4SPD	2.5	29.79	0.02	0.16	0.24	0.35	730.71	1082

See next page

Predictors	Mean	Std	Min	Lower Q.	Median	Upper Q.	Max	N
$MEAN4SPD$	1.54	17.19	0.02	0.14	0.2	0.29	388.02	1082
$MEDIAN4SPD$	1.78	23.58	0.02	0.14	0.21	0.3	621.75	1082
$MIN4KS$	12.89	10.47	1.38	7.42	10.24	14.5	132.33	1082
$MAX4KS$	36.64	66.05	1.48	12.61	22.53	44.04	1433.11	1082
$MEAN4KS$	16.65	11.9	1.48	9.73	13.55	19.78	132.33	1082
$MEDIAN4KS$	14.87	11.3	1.48	8.9	12.14	16.99	132.33	1082

Note: "Mean", "Std", "Min", "Lower Q.", "Median", "Upper Q.", "Max", and "N" denote the mean, standard deviation, minimum, lower quartile, median, upper quartile, maximum, and number of observations, respectively. The value for "MEANBIDS", "STDBIDS", "D", "MIN3D", "MAX3D", "MEAN3D", "MEDIAN3D", "MIN4D", "MAX4D", "MEAN4D" and "MEDIAN4D" are expressed in thousand CHF. "KS", "CV", "SPD", "RD", "RDNOR", "RDALT", "SKEW", "DIFFP", "KURTO", "D", "STDBIDS", "MEANBIDS" and "NBRBIDS" denote the Kolmogorov-Smirnov statistic, the coefficient of variation, the spread, the relative distance, the normalized distance, the alternative relative distance, the skewness statistic, the percentage difference, the kurtosis statistic, the difference in absolute between the first and second lowest bids, the standard deviation of the bids in a tender, the mean of the bids in a tender and the number of the bids in a tender, respectively.

Table 21: Descriptive statistics for incomplete bid-rigging cartels in sample 1

Predictors	Mean	Std	Min	LowerQ.	Median	UpperQ.	Max	N
NBRBIDS	7.49	2.54	4	5	7	9	13	252
MEANBIDS	435.90	476.67	18.01	146.59	296.62	538.73	3460.91	252
STDBIDS	35.12	50.17	1.54	9.60	19.62	33.72	362.86	252
CV	7.79	3.89	1.77	5.14	6.79	9.6	23.92	252
KURTO	0.41	2.12	−5.9	−0.94	0.04	1.54	6.97	252
SKEW	−0.07	0.99	−2.59	−0.67	−0.06	0.5	2.57	252
SPD	0.26	0.16	0.05	0.16	0.21	0.32	0.89	252
D	21.07	39.03	0.08	4.33	9.10	20.87	351.83	252
RD	1.39	2.67	0.01	0.26	0.58	1.42	28.37	252
RDNOR	1.41	1.11	0.01	0.59	1.14	2	5.48	252
RDALT	2.32	3.52	0.01	0.55	1.17	2.65	28.54	252
DIFFP	6.34	8.85	0.03	1.8	3.69	7.02	73.53	252
KS	16.62	8.04	4.15	10.99	15.24	19.97	57.54	252
MIN3CV	1.68	1.73	0	0.65	1.24	2.14	14.77	252
MAX3CV	12.26	6.39	2.9	7.87	10.66	15.21	38.48	252
MEAN3CV	7.12	3.48	1.55	4.7	6.32	8.98	22.96	252
MEDIAN3CV	7.06	3.81	1.01	4.59	6.28	8.51	29.82	252
MIN3SKEW	−1.6	0.36	−1.73	−1.73	−1.72	−1.65	0.97	252
MAX3SKEW	1.53	0.48	−1.1	1.64	1.72	1.73	1.73	252
MEAN3SKEW	−0.04	0.5	−1.43	−0.3	−0.03	0.25	1.47	252
MEDIAN3SKEW	−0.05	0.77	−1.71	−0.59	−0.05	0.51	1.73	252
MIN3D	3.64	8.58	0.00	0.43	1.31	3.23	80.15	252
MAX3D	76.68	107.56	1.58	18.87	41.15	80.60	842.70	252
MEAN3D	29.56	44.27	0.62	8.00	16.57	28.64	437.36	252
MEDIAN3D	26.54	40.57	0.41	7.70	14.60	27.21	424.30	252
MIN3RD	0.23	0.45	0	0.02	0.07	0.19	3.35	252
MAX3RD	1203.97	9993.9	0.68	10.98	31.05	96.88	122393.82	252
MEAN3RD	27.54	191.86	0.31	2.5	4.5	9.16	2742.11	252
MEDIAN3RD	2.09	2.42	0.21	1.02	1.49	2.21	27.08	252
MIN3RDNOR	0.21	0.28	0	0.03	0.09	0.24	1.41	252
MAX3RDNOR	1.83	0.23	0.65	1.77	1.92	1.97	2	252
MEAN3RDNOR	1.02	0.22	0.33	0.89	1.02	1.12	1.73	252
MEDIAN3RDNOR	1.02	0.3	0.26	0.83	1.02	1.2	1.89	252
MIN3RDALT	0.16	0.32	0	0.02	0.05	0.14	2.37	252
MAX3RDALT	851.33	7066.76	0.48	7.76	21.95	68.51	86545.5	252
MEAN3RDALT	19.47	135.66	0.22	1.77	3.18	6.48	1938.96	252
MEDIAN3RDALT	1.48	1.71	0.15	0.72	1.06	1.56	19.15	252
MIN3DIFFP	0.97	1.58	0	0.17	0.47	1.15	12.6	252
MAX3DIFFP	19.61	12.81	2.29	11.28	16.77	25.1	87.65	252
MEAN3DIFFP	7.76	5.62	1.39	4.79	6.4	8.94	49.6	252
MEDIAN3DIFFP	7.11	6.59	1.14	3.9	5.72	8.21	73.53	252

<div align="center">See next page</div>

Predictors	Mean	Std	Min	LowerQ.	Median	UpperQ.	Max	N
MIN3SPD	0.03	0.04	0	0.01	0.02	0.04	0.32	252
MAX3SPD	0.26	0.16	0.05	0.16	0.21	0.32	0.89	252
MEAN3SPD	0.15	0.08	0.03	0.09	0.13	0.19	0.62	252
MEDIAN3SPD	0.15	0.09	0.02	0.09	0.13	0.18	0.83	252
MIN3KS	10.96	6.18	3	6.88	9.62	12.9	34.52	252
MAX3KS	357.52	3429.85	6.79	47.14	80.58	155	54476.99	252
MEAN3KS	27.25	34.52	6.03	15.6	21.55	29.85	495.16	252
MEDIAN3KS	18.68	11.16	3.36	12.03	16.1	22.21	99.02	252
MIN4CV	3.16	2.97	0.12	1.31	2.26	4.2	23.92	252
MAX4CV	10.63	5.46	2.38	6.97	9.43	13.25	33.94	252
MEAN4CV	7.43	3.65	1.64	4.89	6.54	9.33	23.92	252
MEDIAN4CV	7.65	3.91	1.69	5.02	6.76	9.66	25.38	252
MIN4SKEW	−1.42	0.79	−2	−1.95	−1.78	−1.22	1.87	252
MAX4SKEW	1.28	0.97	−1.98	1.09	1.79	1.94	2	252
MEAN4SKEW	−0.07	0.69	−1.98	−0.43	−0.05	0.38	1.87	252
MEDIAN4SKEW	−0.07	0.79	−1.98	−0.47	−0.03	0.32	1.87	252
MIN4D	4.98	11.53	0.00	0.48	1.63	4.68	102.18	252
MAX4D	60.49	87.82	0.18	13.66	33.62	66.93	771.65	252
MEAN4D	25.08	38.97	0.18	6.81	14.32	25.80	410.17	252
MEDIAN4D	24.93	42.34	0.14	6.02	13.17	25.38	424.30	252
MIN4RD	0.59	2.26	0	0.03	0.09	0.31	28.37	252
MAX4RD	25.25	166.8	0.04	3.1	7.14	16.87	2627.72	252
MEAN4RD	2.25	2.96	0.04	0.91	1.52	2.37	28.37	252
MEDIAN4RD	1.53	2.43	0.04	0.53	1.01	1.66	28.37	252
MIN4RDNOR	0.37	0.54	0	0.05	0.15	0.42	2.8	252
MAX4RDNOR	2.18	0.68	0.07	1.83	2.37	2.7	3	252
MEAN4RDNOR	1.11	0.46	0.07	0.8	1.08	1.33	2.8	252
MEDIAN4RDNOR	1.06	0.54	0.07	0.66	1.05	1.38	2.8	252
MIN4RDALT	0.61	2.29	0	0.03	0.1	0.32	28.54	252
MAX4RDALT	27.16	186.06	0.04	3.13	7.47	18.11	2933.56	252
MEAN4RDALT	2.35	3.08	0.04	0.96	1.62	2.48	28.54	252
MEDIAN4RDALT	1.59	2.46	0.04	0.56	1.08	1.7	28.54	252
MIN4DIFFP	1.83	4.55	0	0.19	0.58	1.58	57.15	252
MAX4DIFFP	16.09	11.96	0.3	8.81	13.53	20.12	77.43	252
MEAN4DIFFP	7.05	6.61	0.3	3.75	5.57	7.76	60.05	252
MEDIAN4DIFFP	6.8	7.49	0.24	3.19	5.09	7.32	73.53	252
MIN4KURTO	−4	2.64	−6	−5.86	−5.28	−3.15	3.94	252
MAX4KURTO	2.99	1.68	−5.9	2.79	3.68	3.91	4	252
MEAN4KURTO	0.16	1.38	−5.9	−0.48	0.04	0.84	3.94	252
MEDIAN4KURTO	0.51	1.67	−5.9	−0.08	0.64	1.49	3.94	252
MIN4SPRD	0.08	0.08	0	0.03	0.05	0.09	0.81	252
MAX4SPD	0.26	0.16	0.05	0.16	0.21	0.32	0.89	252

See next page

Predictors	Mean	Std	Min	LowerQ.	Median	UpperQ.	Max	N
$MEAN4SPD$	0.19	0.11	0.04	0.12	0.16	0.23	0.81	252
$MEDIAN4SPD$	0.19	0.12	0.04	0.12	0.16	0.24	0.89	252
$MIN4KS$	12.65	7.28	3.64	7.85	10.91	14.56	42.15	252
$MAX4KS$	64.97	79.07	4.15	24.21	44.41	76.85	862.88	252
$MEAN4KS$	19.96	11.22	4.15	12.92	17.51	23.75	97.43	252
$MEDIAN4KS$	16.85	8.23	3.87	11.12	15.26	20.29	59.16	252

Note: "Mean", "Std", "Min", "Lower Q.", "Median", "Upper Q.", "Max", and "N" denote the mean, standard deviation, minimum, lower quartile, median, upper quartile, maximum, and number of observations, respectively. The value for "MEANBIDS", "STDBIDS", "D", "MIN3D", "MAX3D", "MEAN3D", "MEDIAN3D", "MIN4D", "MAX4D", "MEAN4D" and "MEDIAN4D" are expressed in thousand CHF. "KS", "CV", "SPD", "RD", "RDNOR", "RDALT", "SKEW", "DIFFP", "KURTO", "D", "STDBIDS", "MEANBIDS" and "NBRBIDS" denote the Kolmogorov-Smirnov statistic, the coefficient of variation, the spread, the relative distance, the normalized distance, the alternative relative distance, the skewness statistic, the percentage difference, the kurtosis statistic, the difference in absolute between the first and second lowest bids, the standard deviation of the bids in a tender, the mean of the bids in a tender and the number of the bids in a tender, respectively.

Table 22: Descriptive statistics for incomplete bid-rigging cartels in sample 2

Predictors	Mean	Std	Min	LowerQ.	Median	UpperQ.	Max	N
NBRBIDS	7.77	2.47	4	6	8	10	13	223
MEANBIDS	405.24	372.35	27.84	149.78	302.21	535.40	3002.37	223
STDBIDS	30.96	37.64	1.63	9.65	19.51	33.19	270.82	223
CV	7.6	3.77	1.77	5.14	6.66	9.07	23.92	223
KURTO	0.44	2.08	−5.75	−0.94	0.04	1.47	6.97	223
SKEW	−0.07	0.97	−2.59	−0.62	−0.08	0.46	2.57	223
SPD	0.26	0.16	0.05	0.16	0.21	0.31	0.89	223
D	20.13	36.04	0.08	4.13	8.90	21.08	351.83	223
RD	1.29	2.12	0.01	0.27	0.58	1.36	18.04	223
RDNOR	1.45	1.14	0.01	0.61	1.14	2.06	5.48	223
RDALT	2.3	3.24	0.01	0.58	1.17	2.83	20.74	223
DIFFP	6.3	9.13	0.03	1.88	3.69	6.78	73.53	223
KS	16.85	7.91	4.15	11.89	15.54	19.99	57.54	223
MIN3CV	1.47	1.22	0	0.62	1.15	1.93	7.05	223
MAX3CV	12.24	6.43	2.93	8	10.57	14.98	38.48	223
MEAN3CV	6.92	3.33	1.55	4.69	6.09	8.33	22.96	223
MEDIAN3CV	6.75	3.56	1.01	4.58	6	8.06	29.82	223
MIN3SKEW	−1.63	0.29	−1.73	−1.73	−1.72	−1.68	0.62	223
MAX3SKEW	1.58	0.38	−1.1	1.68	1.72	1.73	1.73	223
MEAN3SKEW	−0.04	0.45	−1.33	−0.28	−0.04	0.25	1.31	223
MEDIAN3SKEW	−0.05	0.74	−1.69	−0.58	−0.06	0.45	1.73	223
MIN3D	2.76	5.42	0.00	0.38	1.17	2.87	56.23	223
MAX3D	69.79	84.97	2.38	20.13	42.35	79.53	670.57	223
MEAN3D	26.13	30.47	1.42	8.28	16.72	28.29	235.67	223
MEDIAN3D	23.11	25.43	1.52	7.92	14.63	26.60	171.87	223
MIN3RD	0.18	0.37	0	0.02	0.06	0.15	3.35	223
MAX3RD	1355.45	10617.14	0.92	14.74	37.36	128.17	122393.82	223
MEAN3RD	30.39	203.82	0.43	2.65	4.66	9.35	2742.11	223
MEDIAN3RD	1.92	1.72	0.26	1.06	1.5	2.14	18.63	223
MIN3RDNOR	0.18	0.25	0	0.03	0.08	0.19	1.41	223
MAX3RDNOR	1.85	0.2	0.79	1.82	1.93	1.98	2	223
MEAN3RDNOR	1.02	0.2	0.41	0.9	1.02	1.12	1.66	223
MEDIAN3RDNOR	1.02	0.28	0.3	0.85	1.02	1.2	1.85	223
MIN3RDALT	0.13	0.26	0	0.01	0.04	0.11	2.37	223
MAX3RDALT	958.45	7507.45	0.65	10.42	26.42	90.63	86545.5	223
MEAN3RDALT	21.49	144.12	0.3	1.88	3.29	6.61	1938.96	223
MEDIAN3RDALT	1.36	1.21	0.18	0.75	1.06	1.51	13.18	223
MIN3DIFFP	0.81	1.25	0	0.14	0.43	0.99	10.47	223
MAX3DIFFP	19.73	12.93	2.29	11.81	16.81	24.42	87.65	223
MEAN3DIFFP	7.59	5.58	1.39	4.82	6.35	8.75	49.6	223
MEDIAN3DIFFP	6.92	6.67	1.14	3.91	5.71	8.1	73.53	223

See next page

Predictors	Mean	Std	Min	LowerQ.	Median	UpperQ.	Max	N
MIN3SPD	0.03	0.02	0	0.01	0.02	0.04	0.15	223
MAX3SPD	0.26	0.16	0.05	0.16	0.21	0.31	0.89	223
MEAN3SPD	0.15	0.08	0.03	0.09	0.13	0.18	0.62	223
MEDIAN3SPD	0.14	0.09	0.02	0.09	0.12	0.17	0.83	223
MIN3KS	10.9	6.01	3	6.98	9.7	12.52	34.52	223
MAX3KS	394.81	3645.21	14.27	52.14	87.43	161.06	54476.99	223
MEAN3KS	27.97	36.27	6.25	16	21.99	30.38	495.16	223
MEDIAN3KS	19.22	11.35	3.36	12.64	16.97	22.22	99.02	223
MIN4CV	2.8	2.48	0.12	1.2	2.17	3.87	23.92	223
MAX4CV	10.63	5.48	2.48	7.05	9.36	13.08	33.94	223
MEAN4CV	7.22	3.5	1.64	4.88	6.32	8.58	23.92	223
MEDIAN4CV	7.41	3.75	1.69	5.01	6.51	8.9	25.38	223
MIN4SKEW	−1.51	0.71	−2	−1.95	−1.82	−1.42	1.84	223
MAX4SKEW	1.38	0.85	−1.96	1.23	1.82	1.94	2	223
MEAN4SKEW	−0.06	0.64	−1.96	−0.4	−0.07	0.35	1.84	223
MEDIAN4SKEW	−0.07	0.74	−1.96	−0.46	−0.04	0.29	1.84	223
MIN4D	3.82	8.45	0.00	0.43	1.43	3.77	96.23	223
MAX4D	58.39	72.08	0.18	16.06	37.52	68.97	535.26	223
MEAN4D	23.35	29.62	0.18	7.26	14.45	25.83	274.73	223
MEDIAN4D	23.01	32.93	0.14	6.07	13.49	25.40	351.83	223
MIN4RD	0.41	1.4	0	0.03	0.09	0.25	18.04	223
MAX4RD	27.52	177.13	0.04	3.91	7.86	17.75	2627.72	223
MEAN4RD	2.19	2.53	0.04	0.96	1.62	2.37	24.7	223
MEDIAN4RD	1.42	1.77	0.04	0.57	1.02	1.52	18.04	223
MIN4RDNOR	0.32	0.46	0	0.04	0.14	0.36	2.7	223
MAX4RDNOR	2.25	0.61	0.07	2	2.42	2.71	3	223
MEAN4RDNOR	1.11	0.42	0.07	0.82	1.09	1.3	2.7	223
MEDIAN4RDNOR	1.06	0.51	0.07	0.69	1.06	1.31	2.7	223
MIN4RDALT	0.42	1.42	0	0.03	0.09	0.27	18.05	223
MAX4RDALT	29.63	197.6	0.04	4	8.35	18.57	2933.56	223
MEAN4RDALT	2.29	2.67	0.04	1.02	1.72	2.49	27.4	223
MEDIAN4RDALT	1.48	1.8	0.04	0.6	1.1	1.55	18.05	223
MIN4DIFFP	1.48	4.34	0	0.17	0.51	1.23	57.15	223
MAX4DIFFP	16.59	11.97	0.3	9.6	13.87	20.28	77.43	223
MEAN4DIFFP	7	6.68	0.3	3.97	5.51	7.55	60.05	223
MEDIAN4DIFFP	6.74	7.66	0.24	3.24	5.08	7.23	73.53	223
MIN4KURTO	−4.32	2.38	−6	−5.88	−5.48	−3.92	3.85	223
MAX4KURTO	3.16	1.43	−5.75	3.04	3.74	3.91	4	223
MEAN4KURTO	0.15	1.23	−5.75	−0.48	0.04	0.79	3.85	223
MEDIAN4KURTO	0.52	1.56	−5.75	−0.05	0.63	1.42	3.85	223
MIN4SPD	0.07	0.07	0	0.03	0.05	0.09	0.81	223
MAX4SPD	0.26	0.16	0.05	0.16	0.21	0.31	0.89	223

See next page

Predictors	Mean	Std	Min	LowerQ.	Median	UpperQ.	Max	N
$MEAN4SPD$	0.18	0.11	0.04	0.12	0.15	0.22	0.81	223
$MEDIAN4SPD$	0.19	0.11	0.04	0.12	0.16	0.22	0.89	223
$MIN4KS$	12.55	7.02	3.64	7.95	11.06	14.33	40.78	223
$MAX4KS$	69.75	82.58	4.15	26.17	46.65	84.1	862.88	223
$MEAN4KS$	20.49	11.41	4.15	13.41	18.45	23.97	97.43	223
$MEDIAN4KS$	17.14	8.1	3.87	11.75	15.89	20.31	59.16	223

Note: "Mean", "Std", "Min", "Lower Q.", "Median", "Upper Q.", "Max", and "N" denote the mean, standard deviation, minimum, lower quartile, median, upper quartile, maximum, and number of observations, respectively. The value for "MEANBIDS", "STDBIDS", "D", "MIN3D", "MAX3D", "MEAN3D", "MEDIAN3D", "MIN4D", "MAX4D", "MEAN4D" and "MEDIAN4D" are expressed in thousand CHF. "KS", "CV", "SPD", "RD", "RDNOR", "RDALT", "SKEW", "DIFFP", "KURTO", "D", "STDBIDS", "MEANBIDS" and "NBRBIDS" denote the Kolmogorov-Smirnov statistic, the coefficient of variation, the spread, the relative distance, the normalized distance, the alternative relative distance, the skewness statistic, the percentage difference, the kurtosis statistic, the difference in absolute between the first and second lowest bids, the standard deviation of the bids in a tender, the mean of the bids in a tender and the number of the bids in a tender, respectively.

Table 23: Descriptive statistics for incomplete bid-rigging cartels in sample 3

Predictors	Mean	Std	Min	LowerQ.	Median	UpperQ.	Max	N
NBRBIDS	8.5	2.15	5	7	8	10	13	173
MEANBIDS	439.36	388.89	30.24	182.64	329.61	572.01	3002.37	173
STDBIDS	33.50	40.38	2.01	11.89	20.48	33.44	270.82	173
CV	7.54	3.26	1.77	5.24	6.76	9.12	21.7	173
KURTO	0.46	1.97	−2.69	−0.88	−0.09	1.23	6.97	173
SKEW	−0.07	0.95	−2.59	−0.56	−0.08	0.43	2.57	173
SPD	0.26	0.14	0.06	0.17	0.22	0.31	0.88	173
D	21.95	39.01	0.08	4.53	9.46	22.94	351.83	173
RD	1.14	1.76	0.01	0.29	0.58	1.19	12.47	173
RDNOR	1.54	1.17	0.02	0.66	1.17	2.17	5.48	173
RDALT	2.33	3.2	0.02	0.63	1.19	2.81	20.74	173
DIFFP	5.91	7.48	0.1	2.13	3.7	6.74	45.42	173
KS	16.55	7.64	6.19	11.84	15.48	19.69	57.54	173
MIN3CV	1.2	0.9	0	0.52	1.03	1.56	3.95	173
MAX3CV	12.6	5.86	2.93	8.78	10.99	15.19	38.48	173
MEAN3CV	6.82	2.83	1.55	4.79	6.11	8.34	16.5	173
MEDIAN3CV	6.51	2.79	1.01	4.67	5.92	8.02	14.2	173
MIN3SKEW	−1.69	0.12	−1.73	−1.73	−1.73	−1.71	−0.88	173
MAX3SKEW	1.67	0.15	0.73	1.69	1.73	1.73	1.73	173
MEAN3SKEW	−0.03	0.39	−1.02	−0.22	−0.04	0.23	0.88	173
MEDIAN3SKEW	−0.03	0.69	−1.48	−0.47	−0.06	0.41	1.73	173
MIN3D	2.35	4.05	0.00	0.34	1.03	2.40	30.19	173
MAX3D	78.60	90.43	3.70	24.82	51.54	89.92	670.57	173
MEAN3D	28.43	32.44	1.58	9.40	17.96	32.97	235.67	173
MEDIAN3D	24.31	26.46	1.52	8.53	15.79	26.36	171.87	173
MIN3RD	0.11	0.19	0	0.02	0.04	0.12	1.76	173
MAX3RD	1737.96	12034.74	2.68	20.39	48.9	163.12	122393.82	173
MEAN3RD	37.63	231.03	0.92	2.91	5.15	9.8	2742.11	173
MEDIAN3RD	1.72	1.05	0.32	1.07	1.48	1.95	6.05	173
MIN3RDNOR	0.12	0.16	0	0.02	0.06	0.16	1	173
MAX3RDNOR	1.9	0.13	1.31	1.87	1.94	1.98	2	173
MEAN3RDNOR	1.02	0.18	0.61	0.91	1.03	1.11	1.46	173
MEDIAN3RDNOR	1.02	0.25	0.37	0.86	1.02	1.16	1.62	173
MIN3RDALT	0.08	0.14	0	0.01	0.03	0.09	1.25	173
MAX3RDALT	1228.92	8509.85	1.9	14.42	34.57	115.34	86545.5	173
MEAN3RDALT	26.61	163.36	0.65	2.06	3.64	6.93	1938.96	173
MEDIAN3RDALT	1.22	0.74	0.22	0.76	1.05	1.38	4.28	173
MIN3DIFFP	0.58	0.71	0	0.12	0.37	0.81	4.49	173
MAX3DIFFP	20.62	11.43	3.84	13.06	17.99	25.01	72.01	173
MEAN3DIFFP	7.33	3.89	1.55	5	6.42	8.68	22.8	173
MEDIAN3DIFFP	6.18	3.26	1.33	3.91	5.67	7.45	21.76	173

See next page

Predictors	Mean	Std	Min	LowerQ.	Median	UpperQ.	Max	N
MIN3SPD	0.02	0.02	0	0.01	0.02	0.03	0.08	173
MAX3SPD	0.26	0.14	0.06	0.17	0.22	0.31	0.88	173
MEAN3SPD	0.14	0.07	0.03	0.1	0.13	0.18	0.38	173
MEDIAN3SPD	0.13	0.06	0.02	0.09	0.12	0.17	0.3	173
MIN3KS	10.03	5.06	3	6.91	9.18	11.63	34.52	173
MAX3KS	489.07	4136.34	25.56	63.95	97.45	192.63	54476.99	173
MEAN3KS	29.3	40.7	9.3	16.34	21.99	30.15	495.16	173
MEDIAN3KS	19.58	12.05	7.28	12.7	17.13	21.76	99.02	173
MIN4CV	2.14	1.45	0.12	1.03	1.76	2.94	8.25	173
MAX4CV	11.02	5.04	2.48	7.92	9.6	13.24	33.94	173
MEAN4CV	7.12	2.98	1.64	4.99	6.32	8.58	18.01	173
MEDIAN4CV	7.27	3.09	1.69	5.15	6.57	8.91	17.17	173
MIN4SKEW	−1.69	0.44	−2	−1.96	−1.88	−1.64	−0.04	173
MAX4SKEW	1.62	0.52	−0.4	1.49	1.84	1.95	2	173
MEAN4SKEW	−0.05	0.55	−1.42	−0.34	−0.07	0.31	1.32	173
MEDIAN4SKEW	−0.06	0.67	−1.86	−0.42	−0.05	0.27	1.72	173
MIN4D	2.86	4.84	0.00	0.34	1.13	3.02	30.28	173
MAX4D	66.68	76.42	1.56	20.02	43.87	78.43	535.26	173
MEAN4D	25.49	31.63	1.22	8.73	15.85	27.51	274.73	173
MEDIAN4D	24.96	35.20	0.96	7.98	15.12	27.02	351.83	173
MIN4RD	0.16	0.26	0	0.02	0.06	0.17	1.93	173
MAX4RD	33.76	200.71	1.09	5.32	10.34	20.54	2627.72	173
MEAN4RD	2.22	2.44	0.35	1.04	1.68	2.38	24.7	173
MEDIAN4RD	1.31	1.24	0.16	0.69	1.05	1.39	9.81	173
MIN4RDNOR	0.2	0.25	0	0.03	0.1	0.25	1.51	173
MAX4RDNOR	2.42	0.44	1.07	2.2	2.53	2.75	3	173
MEAN4RDNOR	1.11	0.35	0.43	0.85	1.1	1.28	2.06	173
MEDIAN4RDNOR	1.07	0.44	0.22	0.79	1.07	1.25	2.5	173
MIN4RDALT	0.16	0.26	0	0.02	0.07	0.18	2.03	173
MAX4RDALT	36.41	223.95	1.1	5.54	10.76	21.56	2933.56	173
MEAN4RDALT	2.31	2.61	0.37	1.09	1.75	2.51	27.4	173
MEDIAN4RDALT	1.37	1.27	0.16	0.72	1.11	1.43	9.91	173
MIN4DIFFP	0.71	0.9	0	0.13	0.41	0.98	6.52	173
MAX4DIFFP	17.69	10.69	3.03	11.06	15.55	21.32	70.09	173
MEAN4DIFFP	6.67	4.5	1.18	4.22	5.62	7.44	26.42	173
MEDIAN4DIFFP	6.33	5.07	0.98	3.44	5.23	7.15	34.35	173
MIN4KURTO	−4.9	1.6	−6	−5.92	−5.55	−4.6	1.44	173
MAX4KURTO	3.56	0.61	1	3.49	3.82	3.93	4	173
MEAN4KURTO	0.17	0.89	−1.78	−0.46	−0.05	0.63	3.19	173
MEDIAN4KURTO	0.63	1.19	−4.42	0	0.63	1.34	3.57	173
MIN4SPD	0.05	0.03	0	0.02	0.04	0.07	0.2	173
MAX4SPD	0.26	0.14	0.06	0.17	0.22	0.31	0.88	173

See next page

Predictors	Mean	Std	Min	LowerQ.	Median	UpperQ.	Max	N
MEAN4SPD	0.18	0.08	0.04	0.12	0.15	0.22	0.48	173
MEDIAN4SPD	0.18	0.09	0.04	0.12	0.16	0.22	0.48	173
MIN4KS	11.44	5.86	3.64	7.8	10.66	12.95	40.78	173
MAX4KS	81.53	89.76	12.72	34.28	57.17	97.8	862.88	173
MEAN4KS	20.95	12.1	8.31	13.75	18.47	23.97	97.43	173
MEDIAN4KS	16.96	7.85	5.9	11.68	15.89	19.94	59.16	173

Note: "Mean", "Std", "Min", "Lower Q.", "Median", "Upper Q.", "Max", and "N" denote the mean, standard deviation, minimum, lower quartile, median, upper quartile, maximum, and number of observations, respectively. The value for "MEANBIDS", "STDBIDS", "D", "MIN3D", "MAX3D", "MEAN3D", "MEDIAN3D", "MIN4D", "MAX4D", "MEAN4D" and "MEDIAN4D" are expressed in thousand CHF. "KS", "CV", "SPD", "RD", "RDNOR", "RDALT", "SKEW", "DIFFP", "KURTO", "D", "STDBIDS", "MEANBIDS" and "NBRBIDS" denote the Kolmogorov-Smirnov statistic, the coefficient of variation, the spread, the relative distance, the normalized distance, the alternative relative distance, the skewness statistic, the percentage difference, the kurtosis statistic, the difference in absolute between the first and second lowest bids, the standard deviation of the bids in a tender, the mean of the bids in a tender and the number of the bids in a tender, respectively.

Table 24: Descriptive statistics for incomplete bid-rigging cartels in sample 4

Predictors	Mean	Std	Min	LowerQ.	Median	UpperQ.	Max	N
NBRBIDS	9.08	1.87	6	8	9	11	13	135
MEANBIDS	448.31	384.31	41.46	219.13	337.02	567.20	3002.37	135
STDBIDS	31.74	34.92	2.26	13.09	21.49	32.73	270.82	135
CV	7.19	2.93	1.77	5.22	6.3	8.62	18.9	135
KURTO	0.47	1.97	−2.26	−0.88	−0.18	1.18	6.97	135
SKEW	−0.09	0.94	−2.59	−0.54	−0.12	0.44	2.57	135
SPD	0.26	0.13	0.06	0.17	0.21	0.31	0.76	135
D	22.85	42.70	0.26	4.76	9.34	22.35	351.83	135
RD	1.05	1.5	0.01	0.3	0.57	1.12	10.58	135
RDNOR	1.6	1.18	0.04	0.7	1.22	2.28	5.48	135
RDALT	2.33	3.02	0.04	0.67	1.27	2.86	20.74	135
DIFFP	5.82	7.82	0.1	2.03	3.36	6.58	45.42	135
KS	16.91	7.21	6.62	12.48	16.29	19.83	57.54	135
MIN3CV	0.94	0.62	0	0.43	0.82	1.28	3.04	135
MAX3CV	12.33	5.32	3.36	8.87	10.64	15.02	32.54	135
MEAN3CV	6.47	2.56	1.55	4.69	5.85	7.72	15.38	135
MEDIAN3CV	6.07	2.49	1.01	4.52	5.67	7.29	13.93	135
MIN3SKEW	−1.71	0.07	−1.73	−1.73	−1.73	−1.72	−1.13	135
MAX3SKEW	1.69	0.1	0.93	1.71	1.73	1.73	1.73	135
MEAN3SKEW	−0.04	0.37	−1.02	−0.24	−0.04	0.21	0.87	135
MEDIAN3SKEW	−0.03	0.66	−1.45	−0.47	−0.07	0.4	1.73	135
MIN3D	1.96	3.70	0.00	0.30	0.87	1.99	30.19	135
MAX3D	81.04	93.45	5.67	30.62	55.61	88.77	670.57	135
MEAN3D	28.50	33.46	2.14	11.22	18.00	30.40	235.67	135
MEDIAN3D	23.43	25.63	2.24	9.00	16.39	25.58	171.87	135
MIN3RD	0.09	0.18	0	0.02	0.04	0.1	1.76	135
MAX3RD	2203.8	13597.91	3.46	27.6	62.53	192.17	122393.82	135
MEAN3RD	45.46	261.04	0.92	3.13	5.48	10.57	2742.11	135
MEDIAN3RD	1.69	0.96	0.45	1.08	1.5	1.95	5.61	135
MIN3RDNOR	0.1	0.14	0	0.02	0.06	0.13	1	135
MAX3RDNOR	1.92	0.1	1.42	1.91	1.96	1.99	2	135
MEAN3RDNOR	1.02	0.17	0.61	0.92	1.04	1.11	1.46	135
MEDIAN3RDNOR	1.02	0.24	0.49	0.87	1.02	1.16	1.6	135
MIN3RDALT	0.06	0.13	0	0.01	0.03	0.07	1.25	135
MAX3RDALT	1558.32	9615.17	2.44	19.52	44.22	135.89	86545.5	135
MEAN3RDALT	32.15	184.58	0.65	2.22	3.88	7.48	1938.96	135
MEDIAN3RDALT	1.2	0.68	0.32	0.76	1.06	1.38	3.97	135
MIN3DIFFP	0.44	0.47	0	0.11	0.27	0.56	2.32	135
MAX3DIFFP	20.71	11.13	3.84	13.33	17.81	25.2	63.38	135
MEAN3DIFFP	7.11	3.99	1.55	4.82	6.24	8.15	22.8	135
MEDIAN3DIFFP	5.68	2.93	1.33	3.81	5.29	6.99	21.76	135

See next page

Predictors	Mean	Std	Min	LowerQ.	Median	UpperQ.	Max	N
MIN3SPD	0.02	0.01	0	0.01	0.02	0.02	0.06	135
MAX3SPD	0.26	0.13	0.06	0.17	0.21	0.31	0.76	135
MEAN3SPD	0.14	0.06	0.03	0.1	0.12	0.16	0.35	135
MEDIAN3SPD	0.12	0.05	0.02	0.09	0.11	0.15	0.3	135
MIN3KS	9.9	4.37	3.47	7	9.69	11.51	30.19	135
MAX3KS	603.24	4679.33	32.93	77.85	122.26	231.34	54477	135
MEAN3KS	30.89	43.76	10.24	18.79	23.63	31.72	495.16	135
MEDIAN3KS	20.34	11.39	7.28	14.06	17.92	22.59	99.02	135
MIN4CV	1.7	1.03	0.12	0.91	1.5	2.18	5.53	135
MAX4CV	10.83	4.53	2.8	8	9.4	13.15	28.01	135
MEAN4CV	6.76	2.69	1.64	4.93	6.08	8	16.54	135
MEDIAN4CV	6.88	2.88	1.69	5.01	6.17	8.11	17.17	135
MIN4SKEW	−1.81	0.29	−2	−1.97	−1.92	−1.76	−0.28	135
MAX4SKEW	1.72	0.4	−0.26	1.7	1.86	1.96	2	135
MEAN4SKEW	−0.06	0.52	−1.37	−0.33	−0.1	0.28	1.18	135
MEDIAN4SKEW	−0.07	0.65	−1.69	−0.38	−0.07	0.26	1.72	135
MIN4D	2.38	4.46	0.00	0.33	1.04	2.48	30.28	135
MAX4D	69.99	79.50	5.47	25.08	47.56	78.76	535.26	135
MEAN4D	25.93	33.56	1.39	9.45	16.57	26.24	274.73	135
MEDIAN4D	25.13	37.53	0.96	7.99	16.21	25.40	351.83	135
MIN4RD	0.12	0.19	0	0.02	0.05	0.14	1.47	135
MAX4RD	41.1	226.79	1.09	6.95	13.21	23.81	2627.72	135
MEAN4RD	2.27	2.46	0.35	1.11	1.72	2.51	24.7	135
MEDIAN4RD	1.25	0.95	0.16	0.69	1.06	1.36	5.95	135
MIN4RDNOR	0.16	0.19	0	0.03	0.09	0.21	1.02	135
MAX4RDNOR	2.53	0.35	1.07	2.34	2.61	2.79	3	135
MEAN4RDNOR	1.11	0.33	0.43	0.87	1.11	1.28	2.05	135
MEDIAN4RDNOR	1.07	0.42	0.22	0.8	1.07	1.25	2.27	135
MIN4RDALT	0.12	0.17	0	0.02	0.06	0.15	1.04	135
MAX4RDALT	44.43	253.08	1.1	7.07	13.34	26.04	2933.56	135
MEAN4RDALT	2.37	2.65	0.37	1.16	1.79	2.62	27.4	135
MEDIAN4RDALT	1.31	1	0.16	0.73	1.11	1.43	6.3	135
MIN4DIFFP	0.53	0.59	0	0.12	0.37	0.81	3.21	135
MAX4DIFFP	18.02	10.4	3.03	11.48	15.55	22.07	56.74	135
MEAN4DIFFP	6.55	4.74	1.18	4.08	5.51	7.01	26.42	135
MEDIAN4DIFFP	6.13	5.32	0.98	3.26	4.82	6.87	34.35	135
MIN4KURTO	−5.26	1.06	−6	−5.93	−5.69	−5.08	−0.16	135
MAX4KURTO	3.71	0.39	2.18	3.62	3.86	3.95	4	135
MEAN4KURTO	0.16	0.8	−1.21	−0.4	−0.07	0.62	2.41	135
MEDIAN4KURTO	0.72	0.98	−3.63	0.06	0.68	1.38	3.11	135
MIN4SPD	0.04	0.02	0	0.02	0.03	0.05	0.13	135
MAX4SPD	0.26	0.13	0.06	0.17	0.21	0.31	0.76	135

See next page

Predictors	Mean	Std	Min	LowerQ.	Median	UpperQ.	Max	N
MEAN4SPD	0.17	0.08	0.04	0.12	0.15	0.2	0.45	135
MEDIAN4SPD	0.17	0.08	0.04	0.12	0.15	0.2	0.48	135
MIN4KS	11.21	5.03	4.25	7.95	10.99	12.79	36.12	135
MAX4KS	91.93	91.08	18.25	46.29	67.17	110.97	862.88	135
MEAN4KS	21.62	10.72	8.43	15.07	19.14	24.63	97.28	135
MEDIAN4KS	17.62	7.69	5.9	13.03	16.55	20.31	59.16	135

Note: "Mean", "Std", "Min", "Lower Q.", "Median", "Upper Q.", "Max", and "N" denote the mean, standard deviation, minimum, lower quartile, median, upper quartile, maximum, and number of observations, respectively. The value for "MEANBIDS", "STDBIDS", "D", "MIN3D", "MAX3D", "MEAN3D", "MEDIAN3D", "MIN4D", "MAX4D", "MEAN4D" and "MEDIAN4D" are expressed in thousand CHF. "KS", "CV", "SPD", "RD", "RDNOR", "RDALT", "SKEW", "DIFFP", "KURTO", "D", "STDBIDS", "MEANBIDS" and "NBRBIDS" denote the Kolmogorov-Smirnov statistic, the coefficient of variation, the spread, the relative distance, the normalized distance, the alternative relative distance, the skewness statistic, the percentage difference, the kurtosis statistic, the difference in absolute between the first and second lowest bids, the standard deviation of the bids in a tender, the mean of the bids in a tender and the number of the bids in a tender, respectively.

Table 25: Descriptive statistics for incomplete bid-rigging cartels in sample 5

Predictors	Mean	Std	Min	LowerQ.	Median	UpperQ.	Max	N
NBRBIDS	9.42	1.81	7	8	9	11	13	104
MEANBIDS	434.09	290.66	86.12	232.84	345.62	534.23	1559.96	104
STDBIDS	29.66	26.44	4.58	15.43	21.89	32.70	191.92	104
CV	6.87	2.53	1.77	5.19	6.22	8.05	14.4	104
KURTO	0.52	2.06	−2.26	−0.89	−0.15	1.17	6.97	104
SKEW	−0.1	0.94	−2.59	−0.62	−0.16	0.43	2.57	104
SPD	0.25	0.12	0.06	0.18	0.21	0.28	0.66	104
D	23.61	46.99	0.26	5.33	8.88	21.49	351.83	104
RD	1.03	1.56	0.01	0.3	0.57	1.09	10.58	104
RDNOR	1.65	1.24	0.04	0.72	1.25	2.39	5.48	104
RDALT	2.4	3.23	0.04	0.69	1.29	3.01	20.74	104
DIFFP	5.91	8.38	0.1	1.77	3.22	6.14	45.42	104
KS	17.25	7.04	6.93	13.12	16.66	19.81	57.54	104
MIN3CV	0.86	0.6	0	0.38	0.75	1.21	3.04	104
MAX3CV	11.97	4.77	3.36	8.96	10.54	13.64	26.24	104
MEAN3CV	6.19	2.19	1.55	4.7	5.74	7.34	12.47	104
MEDIAN3CV	5.71	2.04	1.55	4.39	5.55	6.88	11.38	104
MIN3SKEW	−1.71	0.08	−1.73	−1.73	−1.73	−1.72	−1.13	104
MAX3SKEW	1.71	0.05	1.48	1.71	1.73	1.73	1.73	104
MEAN3SKEW	−0.04	0.35	−1.02	−0.25	−0.05	0.16	0.87	104
MEDIAN3SKEW	−0.05	0.62	−1.45	−0.47	−0.07	0.36	1.32	104
MIN3D	1.47	2.09	0.00	0.31	0.81	1.65	13.14	104
MAX3D	76.95	75.24	9.36	35.44	56.91	86.41	530.84	104
MEAN3D	27.35	30.06	3.76	12.85	18.95	28.64	235.67	104
MEDIAN3D	21.88	21.69	2.24	10.65	16.45	24.90	144.14	104
MIN3RD	0.06	0.07	0	0.01	0.04	0.09	0.34	104
MAX3RD	2673.49	15390.37	3.46	35.67	75.98	246.78	122393.82	104
MEAN3RD	53.44	294.68	0.92	3.42	5.6	10.88	2742.11	104
MEDIAN3RD	1.66	0.88	0.45	1.11	1.48	1.95	5.61	104
MIN3RDNOR	0.08	0.08	0	0.02	0.06	0.11	0.39	104
MAX3RDNOR	1.93	0.09	1.42	1.92	1.96	1.99	2	104
MEAN3RDNOR	1.02	0.16	0.61	0.92	1.04	1.11	1.46	104
MEDIAN3RDNOR	1.02	0.23	0.49	0.88	1.02	1.16	1.6	104
MIN3RDALT	0.04	0.05	0	0.01	0.03	0.06	0.24	104
MAX3RDALT	1890.44	10882.63	2.44	25.22	53.73	174.5	86545.5	104
MEAN3RDALT	37.79	208.37	0.65	2.42	3.96	7.69	1938.96	104
MEDIAN3RDALT	1.17	0.62	0.32	0.78	1.05	1.38	3.97	104
MIN3DIFFP	0.37	0.4	0	0.09	0.24	0.5	2.32	104
MAX3DIFFP	20.4	11.4	3.84	13.56	17.28	24.31	63.38	104
MEAN3DIFFP	6.93	4.04	1.55	4.74	5.94	7.75	22.8	104
MEDIAN3DIFFP	5.4	2.68	1.33	3.54	5	6.68	14.62	104

See next page

Predictors	Mean	Std	Min	LowerQ.	Median	UpperQ.	Max	N
MIN3SPD	0.02	0.01	0	0.01	0.01	0.02	0.06	104
MAX3SPD	0.25	0.12	0.06	0.18	0.21	0.28	0.66	104
MEAN3SPD	0.13	0.05	0.03	0.1	0.12	0.15	0.3	104
MEDIAN3SPD	0.12	0.04	0.03	0.09	0.11	0.14	0.25	104
MIN3KS	9.91	4.06	3.91	7.59	9.79	11.48	30.19	104
MAX3KS	748.1	5328.42	32.93	82.68	133.82	260.68	54476.99	104
MEAN3KS	32.57	48.85	11.03	20.32	24.65	32.78	495.16	104
MEDIAN3KS	20.44	8.91	9.05	14.76	18.19	22.87	65.09	104
MIN4CV	1.54	0.97	0.12	0.85	1.34	2.04	5.53	104
MAX4CV	10.53	4.01	2.8	8.13	9.12	12.21	21.92	104
MEAN4CV	6.46	2.3	1.64	4.9	5.96	7.61	13.1	104
MEDIAN4CV	6.52	2.52	1.69	4.99	6.07	7.47	17.17	104
MIN4SKEW	−1.82	0.31	−2	−1.98	−1.94	−1.82	−0.28	104
MAX4SKEW	1.78	0.31	0.11	1.75	1.88	1.96	2	104
MEAN4SKEW	−0.06	0.5	−1.37	−0.34	−0.09	0.24	1.18	104
MEDIAN4SKEW	−0.07	0.62	−1.69	−0.38	−0.07	0.24	1.72	104
MIN4D	1.95	3.57	0.00	0.31	0.93	2.26	30.28	104
MAX4D	68.42	69.32	8.37	31.49	50.56	780.10	495.98	104
MEAN4D	25.58	33.77	2.31	10.61	17.47	25.20	274.73	104
MEDIAN4D	24.81	39.62	0.96	8.97	15.70	23.58	351.83	104
MIN4RD	0.09	0.1	0	0.02	0.05	0.11	0.45	104
MAX4RD	49.4	257.96	1.09	8.05	15.98	26.88	2627.72	104
MEAN4RD	2.33	2.67	0.35	1.15	1.76	2.51	24.7	104
MEDIAN4RD	1.24	0.92	0.16	0.74	1.07	1.35	5.95	104
MIN4RDNOR	0.13	0.14	0	0.03	0.09	0.18	0.58	104
MAX4RDNOR	2.56	0.37	1.07	2.44	2.67	2.81	3	104
MEAN4RDNOR	1.11	0.31	0.43	0.92	1.12	1.28	2.05	104
MEDIAN4RDNOR	1.07	0.4	0.22	0.84	1.08	1.25	2.27	104
MIN4RDALT	0.1	0.11	0	0.02	0.06	0.13	0.48	104
MAX4RDALT	53.65	287.9	1.1	8.68	16.19	29.49	2933.56	104
MEAN4RDALT	2.46	2.92	0.37	1.19	1.81	2.61	27.4	104
MEDIAN4RDALT	1.3	0.98	0.16	0.78	1.12	1.43	6.3	104
MIN4DIFFP	0.46	0.52	0	0.1	0.31	0.6	2.5	104
MAX4DIFFP	18.1	10.77	3.03	11.85	15.64	20.51	56.74	104
MEAN4DIFFP	6.47	4.92	1.18	3.87	5.34	6.87	26.42	104
MEDIAN4DIFFP	5.98	5.32	0.98	3.21	4.61	6.49	34.35	104
MIN4KURTO	−5.39	0.94	−6	−5.94	−5.77	−5.33	−1.12	104
MAX4KURTO	3.75	0.33	2.27	3.66	3.88	3.95	4	104
MEAN4KURTO	0.17	0.79	−1.07	−0.4	−0.06	0.56	2.41	104
MEDIAN4KURTO	0.71	0.98	−3.63	0.11	0.62	1.3	3.11	104
MIN4SPD	0.03	0.02	0	0.02	0.03	0.05	0.13	104
MAX4SPD	0.25	0.12	0.06	0.18	0.21	0.28	0.66	104

See next page

Predictors	Mean	Std	Min	LowerQ.	Median	UpperQ.	Max	N
$MEAN4SPD$	0.16	0.07	0.04	0.12	0.14	0.19	0.39	104
$MEDIAN4SPD$	0.16	0.07	0.04	0.12	0.14	0.17	0.48	104
$MIN4KS$	11.18	4.58	4.66	8.45	11.24	12.71	36.12	104
$MAX4KS$	101.52	99.91	18.25	49.53	75.09	118.27	862.88	104
$MEAN4KS$	22.21	10.63	9.43	16.22	19.85	25.26	97.28	104
$MEDIAN4KS$	18.1	7.29	5.9	13.93	17.06	20.66	59.16	104

Note: "Mean", "Std", "Min", "Lower Q.", "Median", "Upper Q.", "Max", and "N" denote the mean, standard deviation, minimum, lower quartile, median, upper quartile, maximum, and number of observations, respectively. The value for "MEANBIDS", "STDBIDS", "D", "MIN3D", "MAX3D", "MEAN3D", "MEDIAN3D", "MIN4D", "MAX4D", "MEAN4D" and "MEDIAN4D" are expressed in thousand CHF. "KS", "CV", "SPD", "RD", "RDNOR", "RDALT", "SKEW", "DIFFP", "KURTO", "D", "STDBIDS", "MEANBIDS" and "NBRBIDS" denote the Kolmogorov-Smirnov statistic, the coefficient of variation, the spread, the relative distance, the normalized distance, the alternative relative distance, the skewness statistic, the percentage difference, the kurtosis statistic, the difference in absolute between the first and second lowest bids, the standard deviation of the bids in a tender, the mean of the bids in a tender and the number of the bids in a tender, respectively.

Appendix 3: Classification tree adjusting the benchmarking rule of *Imhof et al.* (2018) in the Swiss data in sample only with complete cartels
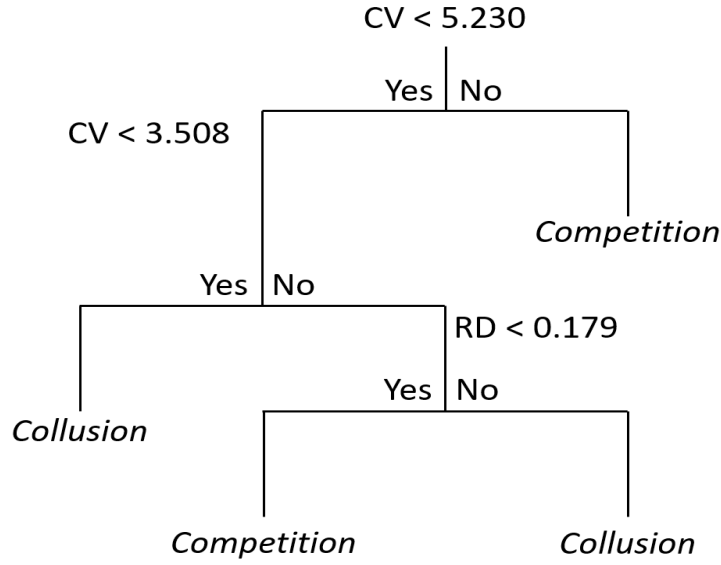
Figure 3: Adjusted classification tree

Appendix 4: Details about lasso regression and the ensemble method

We subsequently discuss in more detail the machine learning approaches of the lasso regression and ensemble method applied by *Huber and Imhof* (2019). Similar to the random forest, the lasso regression and ensemble method require randomly splitting the data into training (used for estimating the model parameters) and test data (used for out of sample prediction and performance evaluation). Again, our training and test samples contain 75% and 25% of the observations, respectively. Lasso regression corresponds to a penalized logit regression, where the penalty term restricts the sum of absolute coefficients on the regressors. Coefficients of less predictive variables shrink towards or even exactly to zero depending on the penalty term. Therefore, lasso regression may perform predictor selection. The estimation of the lasso logit coefficients is based on the following optimization problem:

$$\max_{\beta_0,\beta}\left\{\sum_{i=1}^{n}\left[y_i\left(\beta_0 + \sum_{j=1}^{p}\beta_j x_{ij}\right) - log\left(1 + e^{\beta_0 + \sum_{j=1}^{p}\beta_j x_{ij}}\right)\right] - \lambda\sum_{j=1}^{p}|\beta_j|\right\}. \tag{16}$$

where β_0 denotes intercept and slope coefficients on the predictors, β the slope coefficients on the predictors, x the vector of predictors, i indexes an observation in our data set (with n being the number of observations), j indexes a predictor (with p being the number of predictors), and λ a penalty term larger than zero. We use the same predictors as described in the main text for the different models. In our application, we use the hdm package by *Chernozhukov et al.* (2016) for

68

the statistical software R. We apply 15-fold cross-validation to select the penalty term λ based on minimizes the mean squared error of prediction.

For the ensemble method, we apply as in *Huber and Imhof* (2019) the "SuperLearner" package for "R" by *van der Laan et al.* (2008) with default values for bagged regression tree, random forest, and neural network algorithms in the "ipredbagg", "cforest", and "nnet" packages, respectively. Ensemble method also relies on training data for estimating the model parameters and test data for prediction and performance evaluation. However, any estimation step now consists of a weighted average of bagged classification trees, random forest and neural networks. Bagged trees consist of estimating single trees (rather than random forests) repeatedly using the outcome residuals of the respective previous tree as outcome. Rather than splitting the predictor space, neural networks aim at fitting a system of nonlinear functions that models the influence of the predictors on collusion in a flexible way. To do so, we model the association between the predictors and the outcomes via a network of non-linear intermediate functions, so-called hidden notes. Several layers of hidden nodes allow modelling associations as well as interactions between the predictors in a flexible way, with more nodes and layers increasing the variance but reducing the bias.

Appendix 5: Results for the statistical tests between the simulated bids and the competitive bids in the Ticino case

In the following, we test if the simulated competitive bids are similar to the competitive bids of the post-cartel period. We calculate the screens for the five simulated competitive bids for each collusive tender. We test if the screens differ from the screens calculated with the competitive bids of the post-cartel period. Since the screens are not normally distributed, we apply non-parametric tests to our data, which do not assume any particular distribution in the test procedures (see also *Imhof et al.*, 2018; *Imhof*, 2020). First, we apply the Mann-Whitney test (also called the Wilcoxon rank sum test).[29] To insure the robustness of the results, we second use the Kolmogorov-Smirnov test, a more general test examining for any kind of differences between samples.[30]

Table 26 indicates the test results. We find no rejection for all the tests (at the 5% significance level), except for the Kolmogorov-Smirnov test applied to the percentage difference (DIFFP). To sum up, the screens calculated with the simulated competitive bids do not significantly differ with the screens calculated with the "real" competitive bids of the post-cartel period. Therefore, the simulated competitive bids exhibit more or less the same statistical pattern as the "real" competitive bids. This result indicates that our simulation process adequately generates competitive bids for the purpose of our analyses.

---

[29]See *Rice* (2007) chapter 11, page 435 ff.; *Hollander et al.* (2014) chapter 4, page 115 ff.
[30]See *Hollander et al.* (2014) chapter 5, page 190 ff.

Table 26: Statistical tests for the screens calculated with the simulated competitive bids against the competitive bids of the post-cartel period

Screens	z-statistic	p-value MW	KSa	p-value KS
CV	-1.14	0.2525	1.24	0.0934
KURTO	-0.93	0.3545	1.04	0.2311
SPD	-0.45	0.6541	1.12	0.1623
DIFFP	-1.64	0.1014	1.90	0.0015
SKEW	-0.06	0.9524	0.87	0.4377
RD	-0.10	0.9215	0.78	0.5820
RDNOR	0.1290	0.9874	0.83	0.4913
RDALT	0.1179	0.9061	0.77	0.5901
KS	1.31	0.1890	1.21	0.1084

Note: "Screens", "z-statistic", "p-value MW" denote the screens tested, the z-statistic of the Mann-Whitney test and the p-value of the Mann-Whitney test, respectively. "KSa" and "p-value KS" denote the asymptotic Kolmogorov-Smirnov statistic and the p-value of the Kolmogorov-Smirnov test, respectively.

References

Abrantes-Metz, R. M., L. M. Froeb, J. F. Geweke, and C. T. Taylor, A variance screen for collusion, *International Journal of Industrial Organization*, *24*, 467–486, 2006.

Abrantes-Metz, R. M., M. Kraten, A. D. Metz, and G. Seow, Libor manipulation, *Journal of Banking and Finance*, *36*, 136–150, 2012.

Aryal, G., and M. F. Gabrielli, Testing for collusion in asymmetric first-price auctions, *International Journal of Industrial Organization*, *vol 31*, 26–35, 2013.

Asker, J., A study of the internal organization of a bidding cartel, *American Economic Review*, *100*, 724–762, 2010.

Athey, S., and G. W. Imbens, Machine learning methods that economists should know about, *Annual Review of Economics*, *11*, 685–725, 2019.

Bajari, P., and L. Ye, Deciding between competition and collusion, *The Review of Economics and Statistics*, *85*, 971–989, 2003.

Bergman, M. A., J. Lundberg, S. Lundberg, and J. Y. Stake, Interactions across firms and bid rigging, *Review of Industrial Organization*, 2019.

Bos, I., and J. E. Harrington, Endogenous cartel formation with heterogenous firms, *The RAND Journal of Economics*, *41*(1), 92–117, 2010.

Breiman, L., Random forests, *Machine Learning*, *45*(1), 5–32, 2001.

Breiman, L., and A. Cutler, Random forests for classification and regression, cran.r-project.org, 2018.

Chassang, S., K. Kawai, J. Nakabayashi, and J. Ortner, Data driven antitrust: Theory and application to missing bids, *Working Paper*, 2020.

Chernozhukov, V., C. Hansen, and M. Spindler, hdm: High-dimensional metrics, *The R Journal*, *8*((2)), 185–199, 2016.

Chotibhongs, R., and D. Arditi, Analysis of collusive bidding behavior, *Construction Management and Economics*, *30*, 221–231, 2012a.

Chotibhongs, R., and D. Arditi, Detection of collusive behavior, *Journal of Construction Engineering and Management*, *138*, 1251–1258, 2012b.

Crede, C. J., A structural break cartel screen for dating and detecting collusion, *Review of Industrial Organization*, *54*(3), 543–574, 2019.

Esposito, F., and M. Ferrero, Variance screens for detecting collusion: An application to two cartel cases in italy, *Working Paper*, 2006.

Feinstein, J. S., M. K. Block, and F. C. Nold, Asymmetric information and collusive behavior in auction markets, *The American Economic Review*, *75*, 441–460, 1985.

Froeb, L. M., D. S. Sibley, M. J. Doane, and B. P. Pinto, Screening for collusion as a problem of inference, *The Oxford Handbook of International Antitrust Economics*, *2*, 2014.

Harrington, J. E., *Handbook of Antitrust*, chap. 6 Detecting Cartel, 2006.

Ho, T. K., Random decision forests, *Proceedings of the 3rd International Conference on Document Analysis and Recognition Montreal*, pp. 278–282, 1995.

Hollander, M., D. A. Wolfe, and E. Chicken, *Nonparametric Statistical Methods*, Wiley Series in Probability and Statistics, third edition ed., Wiley, 2014.

Huber, M., and D. Imhof, Machine learning with screens for detecting bid-rigging cartels, *International Journal of Industrial Organization*, *65*, 277–301, 2019.

Hueschelrath, and Veith, Cartel detection in procurement markets, *Center for European Economic Research*, *Discussion Paper No. 11-067*, 2011.

Imhof, D., Econometric tests to detect bid-rigging cartels: Does it work?, *Working Papers SES 483, Faculty of Economics and Social Sciences, University of Fribourg (Switzerland)*, 2017.

Imhof, D., Detecting bid-rigging cartels with descriptive statistics, *Journal of Competition Law and Economics, forthcoming in printed version; avalaible online*, 2020.

Imhof, D., Y. Karagoek, and S. Rutz, Screening for bid rigging, does it work?, *Journal of Competition Law and Economics*, *14*(2), 235–261, 2018.

Ishii, R., Collusion in repeated procurement auction: a study of paving market in japan, *International Journal of Industrial Organization*, *27*, 137–144, 2009.

Ishii, R., Bid roundness under collusion in japanese procurement auctions, *Review of Industrial Organization*, *44*, 241–254, 2014.

Jakobsson, M., Bid rigging in swedish procurement auctions, *Working Paper*, 2007.

Jimenez, J. L., and J. Perdiguero, Does rigidity of pprice hide collusion?, *Review of Industrial Organization*, *41*, 223–248, 2012.

Lee, I. K., and K. Hahn, Bid-rigging in auctions for korean public-works contracts and potential damage, *Review of Industrial Organization*, *21*, 73–88, 2002.

OECD, Roundtable on ex officio cartel investigations and the use of screens to detect cartels, 2014.

Pesendorfer, M., A study of collusion in first-price auction, *The Review of Economic Studies*, *67*, 381–411, 2000.

Porter, R. H., and J. D. Zona, Detection of bid rigging in procurement auctions, *The Journal of Political Economy*, *101*, 518–538, 1993.

Porter, R. H., and J. D. Zona, Ohio school milk markets: an analysis of bidding, *RAND Journal of Economics*, *30*, 263–288, 1999.

Rice, J. A., *Mathematical Statistics and Data Analysis*, third edition ed., BROOKS/COLE CENGAGA Learning, 2007.

van der Laan, M., E. C. Polley, and A. E. Hubbard, Super learner, *Statistical Applications of Genetics and Molecular Biology*, 6, 2008.

Authors

Hannes WALLIMANN
University of Fribourg, Department of Economics, Ph.D. student
Lucerne University of Applied Sciences and Arts, Institute of Tourism, Research Associate
Phone: +41 41 228 99 30; Email: hannes.wallimann@hslu.ch; Website: https://www.hslu.ch/en/lucerne-university-of-applied-sciences-and-arts/about-us/people-finder/profile/?pid=4076

David IMHOF
Economist, Secretariat of the Swiss Competition Commission. Email: david.imhof@weko.admin.ch. Affiliated to the University of Fribourg, Faculty of Economics and Social Sciences, Chair of Applied Econometrics - Evaluation of Public Policies (Switzerland) and Unidistance (Formation universitaire à distance, Switzerland)

Martin HUBER
University of Fribourg, Faculty of Economics and Social Sciences, Chair of Applied Econometrics - Evaluation of Public Policies, Bd. de Pérolles 90, 1700 Fribourg, Switzerland.
Phone: +41 26 300 8255; Email: martin.huber@unifr.ch; Website: http://www.unifr.ch/appecon/en/

Abstract

We propose a new method for flagging bid rigging, which is particularly useful for detecting incomplete bid-rigging cartels. Our approach combines screens, i.e. statistics derived from the distribution of bids in a tender, with machine learning to predict the probability of collusion. As a methodological innovation, we calculate such screens for all possible subgroups of three or four bids within a tender and use summary statistics like the mean, median, maximum, and minimum of each screen as predictors in the machine learning algorithm. This approach tackles the issue that competitive bids in incomplete cartels distort the statistical signals produced by bid rigging. We demonstrate that our algorithm outperforms previously suggested methods in applications to incomplete cartels based on empirical data from Switzerland.

506 Fernandes A., Huber M., Plaza C.: The Effects of Gender and Parental Occupation in the Apprenticeship Market: An Experimental Evaluation; 2019
507 Wallimann H.: Substantial Discount on Ski Passes: A Synthetic Control Analysis; 2019
508 Huber M., Schelker M., Strittmatter A.: Direct and Indirect Effects based on Changes-in-Changes; 2019
509 Songane M.: Patenting Government Funded Innovations as a Strategy to Increase Budget for Academic Vaccine R&D; 2019
510 Songane M., Grossmann V.: The Patent Buyout Price for Human Papilloma Virus (HPV) Vaccine and the Ratio of R&D Costs to the Patent Value; 2019
511 Ducret R., Isakov D.: The Korea discount and chaebols; 2019
512 Wu J., Morschett D.: M&A by Chinese POEs in Developed Countries. Acquiring and Bundling Strategic Assets; 2020