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ABHANDLUNG

Feasible fraud and auditing probabilities for insurance companies and policyholders

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Abstract Insurance claims fraud is counted among the major concerns in the insurance industry, the reason being that excess payments due to fraudulent claims account for a large percentage of the total payments each year. We formulate optimization problems from the insurance company as well as the policyholder perspective based on a costly state verification approach. In this setting—while the policyholder observes his losses privately—the insurance company can decide to verify the truthfulness of incoming claims at some cost. We show simulation results illustrating the agreement range which is characterized by all valid fraud and auditing probability combinations both stakeholders are willing to accept. Furthermore, we present the impact of different valid probability combinations on the insurance company's and the policyholder's objective quantities and analyze the sensitivity of the agreement range with respect to a relevant input parameter. This contribution summarizes the major findings of a working paper written by Müller et al. (Working Papers on Risk Management and Insurance (IVW-HSG), No. 92, 2011).

Zusammenfassung Die Bekämpfung von Versicherungsbetrug gehört zu den zentralen Herausforderungen in der Versicherungswirtschaft. Für den Versicherungsnehmer besteht regelmässig die Möglichkeit einer Falschangabe bezüglich der tatsächlichen Schadenhöhe. Das Versicherungsunternehmen behält sich vor, den Wahrheitsgehalt eingehender Forderungen zu überprüfen. Im vorliegenden Beitrag werden zulässige Betrugs- sowie Prüfwahrscheinlichkeiten aus der Sicht beider Vertragspartner hergeleitet und in Form eines Einigungsbereichs (in einem solchen sind Versicherungsnehmer und -unternehmen bereit, einen Vertrag abzuschliessen) illustriert.

This contribution summarizes the major findings of a working paper written by Müller et al. (2011) which was presented by Katja Müller at the annual meeting of "Deutscher Verein für Versicherungswissenschaft e.V." in Hannover, March 2012.

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Auf dieser Grundlage wird im Anschluss das jeweilige optimale Verhalten ermittelt. Zusätzlich wird der Einfluss relevanter Parameter auf Form und Ausmass des Einigungsbereiches analysiert. Der vorliegende Beitrag stellt eine Zusammenfassung der zentralen Erkenntnisse aus einem Arbeitspapier von Müller et al. (Working Papers on Risk Management and Insurance (IVW-HSG), No. 92, 2011) dar.

1 Introduction

Insurance claims fraud arose to be one of the major concerns among the insurance industry. It occurs in all classes of insurance and accounts for a significant portion of the indemnity payments each year. The Insurance Research Council (IRC) published the study "Fraud and Buildup in Auto Injury Insurance Claims: 2008 Edition" in 2008[1] according to which the excess payments due to fraudulent claims added up to an estimated total of \$4.8 to \$6.8 billion in the auto injury insurance sector in the U.S. during the year 2007. Conferred to the five main private passenger auto injury coverages this corresponds to 13 to 18 percent of the total payments.

The phenomenon of insurance claims fraud is based on information being asymmetrically distributed between policyholder and the corresponding insurance company (see, e.g., Derrig 2002). Since insureds may hold private information about the actual amount of the loss suffered, there exists the possibility of misrepresentation. Consequently, the insurance companies can choose to audit incoming claims in order to determine their truthfulness. In case of detected fraudulent behavior, a penalty payment can be imposed on the policyholder. However, this verification process comes at some cost. Therefore, the costs for auditing have to be traded off against the savings resulting from detected fraud.

The goal of this contribution is to derive and analyze optimal auditing and fraud probabilities from both the insurance company and policyholder perspective respectively aiming to maintain a common agreement range. For this purpose, we introduce a model framework based on a costly state verification environment which considers the insurance company's net present value of future cash flows as well as the policyholder's expected utility of his terminal wealth position. In order to make sure that both stakeholders are willing to sign and remain in the insurance contract, we include participation constraints. At first, some analytical solutions to the formulated optimization problems are derived and interpreted. However, due to the complexity of the model it is not always possible to derive closed-form solutions. Therefore, we present a numerical approach using Monte Carlo methods. The simulation results and their implications for both stakeholders' optimal strategies are discussed and illustrated graphically.

The agreement range which we derive determines the optimal behavior of the insurance company and the policyholder to that effect that the respective other is still willing to participate in the insurance contract. Consequently, it consists of all valid fraud and auditing probability combinations both stakeholders are willing to accept. For which part of the agreement range the participants settle depends on their market

[1] See Insurance Research Council (2008).

power. Assuming a highly competitive market, it is likely for those fraud and auditing probability combinations to be performed which maximize the policyholder's objective while the insurance company is still willing to participate in the insurance contract. In case of an monopoly, other probability combinations are of interest since the insurance company will try to maximize its own objective quantity while making sure that potential policyholders accept an insurance contract.

The remainder of this paper is organized as follows: We start by giving an overview of the related literature in Sect. 2. Thereafter, we present the model framework and first analytical results in Sect. 3. In Sect. 4, the numerical approach and the corresponding program are introduced. We discuss the simulation results in Sect. 5 before concluding in Sect. 6.

2 Literature overview

A comprehensive discussion of the related literature can be found in Müller et al. (2011). In the following, we summarize the main references.

One of the first to present the costly state verification approach was Townsend (1979). In his paper, he discusses optimal insurance contracts between two agents holding different information on actual events which can be transfered at some cost. His analysis is based on deterministic auditing policies which are characterized by verifying incoming claims with a probability of either one or zero. Picard (2000) extends this approach and examines the impact of principal-agent relationships on the design of optimal insurance contracts when having exogenous agents perform the auditing. Furthermore, the possibility of policyholders being able to manipulate auditing costs is included. Carlier and Dana (2003) study the existence and characteristics of efficient contracts when introducing a fixed component in the costs per audit and hence observing non-convex auditing costs. Mookherjee and Png (1989) prove that under certain conditions random auditing is always the optimal strategy. Picard and Fagart (1999) pick up this idea and analyze questions regarding the form of the auditing probability function as well as the optimal coverage schedule when assuming different kinds of risk aversion. Furthermore, Picard (1996) adds the aspect of insurance companies not being able to commit to their audit strategy. He shows that this issue can be eliminated by introducing a third party to perform the auditing. Boyer (2000) however found that there are conditions under which centralizing the investigation and verification process with the help of insurance fraud bureaus would lead to an even higher amount of fraud.

Another way to approach the topic of insurance claims fraud is by applying costly state falsification models. In this case, the policyholder can manipulate at some cost the amount of the claimed loss in order to obtain higher indemnity payments. Crocker and Tennyson (2002) show that under these conditions claims auditing does not serve as a deterrent. This leads to the introduction of an optimal indemnification profile which consists of systematically underpaying claims. However, Hau (2008) argues that separating costly state verification and costly state falsification models does not map the reality. He proves that based on the assumption of imperfect auditing processes, i.e., fraudulent behavior is detected with a probability less than one in case of auditing, an optimal insurance contract consists of both falsification and verification. Similarly, Bond and Crocker (1997) have already found that the combination of overpaying claims which can be easily verified and underpaying claims which require a more costly auditing process can be used to effectively combat insurance claims fraud.

3 Model framework

In this section, we present the model framework which is applied in Müller et al. (2011).

An individual with initial wealth W_0 is offered the possibility to sign an insurance contract with a fixed premium P due by the time of inception of insurance cover in $t = t_0$. At the same time, he faces some uncertain loss θ of stochastic amount which, by the time of occurrence, is observed privately. In case he signed the insurance contract earlier, the policyholder can then chose to file a claim of some size $\hat{\theta}$. In case of honest behavior the amount of that claim will equal the actual loss, i.e., $\hat{\theta} = \theta$. If the policyholder decides to commit fraud, he reports some finite $\hat{\theta} > \theta$. The probability of the policyholder choosing to report a fraudulent claim is denoted by p.

The insurer on the other hand has no information about the actual occurred loss. He therefore audits incoming claims with some probability q and at the constant cost of k per audited claim. Depending on whether auditing took place or not and the outcome in case of an audit, a payment R is made from the insurance company to the policyholder.

Considering the different possible combinations of fraud and auditing probabilities p and q, the payment R can be defined as follows:

$$R(\theta, \hat{\theta}) = (1 - p)\,\theta + p\left[(1 - q)\,\hat{\theta} + q\,(\theta - B)\right], \tag{1}$$

with B being the penalty payment deducted from the claim amount θ.

Equation (1) can be interpreted as an indemnity payment if R is positive, whereas a negative R represents the penalty payment made from the insured to the insurance company in case of detected fraud when $B > \theta$. There are several possible cases to be considered: If the reported loss is not checked, the insured receives the payment of $\hat{\theta}$. In case of auditing, the payment depends on whether the policyholder committed fraud or not. Proven honesty leads to a payment of $\theta = \hat{\theta}$. If a misrepresentation of loss is determined, the policyholder faces some penalty B. In practice, B is mostly chosen such that $\theta - B = 0$.

In this framework setting, we take audits to be perfect, i.e., if a fraudulent claim is made it will surely be detected in case of auditing.

3.1 Insurance company: cash flow, net present value and participation constraint

In the framework introduced above, we observe the future cash flows from the insurance company perspective at the time of insurance inception in $t = t_0$ and the time of loss realization and settling in $t = t_1$ and analyze their resulting net present value.

In case of an insurance contract coming into existence, the insurance company receives the premium payment P in $t = t_0$. An incoming claim in $t = t_1$ which is audited with probability q and at some given cost $k(> 0)$ per analyzed claim, results in $-R(\theta, \hat{\theta}) - qk$.

The insurance company's net present value of its future incoming and outgoing cash flows is denoted by[2]

$$NPV = P - \mathbb{E}\big(R(\theta, \hat{\theta})\big) - qk, \tag{2}$$

where $R(\theta, \hat{\theta})$ denotes the indemnity payment as defined in (1).

Condition 1 *The insurance company is willing to participate in an insurance contract if its net present value is positive. Hence, one obtains the following participation constraint*:

$$NPV \geq 0. \tag{3}$$

Applying (2), participation constraint (3) can be formulated as

$$P \geq \mathbb{E}\big(R(\theta, \hat{\theta})\big) + qk. \tag{4}$$

Apparently, the expression on the right-hand side represents a lower bound for the premium payments the insurance company is willing to accept. Its value depends on the expected value of the indemnity payments which will be made and a loading which reflects the auditing effort.

3.2 Policyholder: wealth position, utility function and participation constraint

From the policyholder perspective, we analyze his wealth position and the corresponding expected utility at the time of inception of insurance cover $t = t_0$ and the time of loss realization and claiming $t = t_1$ for the framework introduced above.

An individual initially owns some wealth W_0. Its consecutive development depends on whether he signs an insurance contract prior to the occurrence of loss or not. In a situation without insurance contract, the individual holds the unchanged wealth position

$$W_0^B = W_0 \tag{5}$$

at time $t = t_0$. At the time of loss occurrence in $t = t_1$, this amount decreases to

$$W_1^B = W_0^B - \theta = W_0 - \theta. \tag{6}$$

The decision to sign an insurance contract is accompanied by the payment of an insurance premium P. Consequently, when signing the contract at time $t = t_0$ the individual owns the wealth position

$$W_0^A = W_0 - P. \tag{7}$$

[2]We consider the expected value of future cash flows discounted with the risk-free interest rate $r_f = 0$.

Assuming a loss θ of some stochastic level occurs and therefore a claim is filed at time $t = t_1$, the policyholder's wealth at that point in time is denoted by

$$W_1^A = W_0^A - \theta + R(\theta, \hat{\theta}) = W_0 - P - \theta + R(\theta, \hat{\theta}) \tag{8}$$

with $R(\theta, \hat{\theta})$ as defined in (1).

We assume the policyholder's utility being described by a standard mean-variance utility function of his individual wealth. For a given wealth position W and the risk aversion parameter $a(> 0)$ of the individual, this utility function is given by

$$U(W) = \mathbb{E}(W) - \frac{a}{2} \operatorname{Var}(W), \tag{9}$$

where $\mathbb{E}(W)$ denotes the expected value of the stochastic variable W.

In case no insurance contract was signed prior to the occurrence of loss, using (6) and definition (9) the final utility is written as:

$$\begin{aligned} U\big(W_1^B\big) &= \mathbb{E}(W_0 - \theta) - \frac{a}{2} \operatorname{Var}(W_0 - \theta) \\ &= W_0 - \mathbb{E}(\theta) - \frac{a}{2} \operatorname{Var}(\theta). \end{aligned} \tag{10}$$

For the setting in which an insurance contract was signed by applying the definition in (9) to (8), we obtain:

$$\begin{aligned} U\big(W_1^A\big) &= \mathbb{E}\big(W_0 - P - \theta + R(\theta, \hat{\theta})\big) - \frac{a}{2} \operatorname{Var}\big(W_0 - P - \theta + R(\theta, \hat{\theta})\big) \\ &= W_0 - P - \mathbb{E}\big(\theta - R(\theta, \hat{\theta})\big) - \frac{a}{2} \operatorname{Var}\big(\theta - R(\theta, \hat{\theta})\big). \end{aligned} \tag{11}$$

Comparing (10) and (11), one sees the difference in influencing factors for the final expected utility for each situation: In a setting without the existence of an insurance contract the final expected utility $U(W_1^B)$ solely depends on the extend of the actual loss θ and the policyholder's risk aversion parameter a. However, in a situation in which insurance coverage exists, the value of the corresponding expected utility $U(W_1^A)$ is not only influenced by θ, P and a. In fact, the policyholder's fraud strategy p and the insurer's auditing strategy q have an impact on that value due to the payment of R (see (1)). Moreover, if the insured decides to commit fraud, the size of $\hat{\theta}$ which he choses to claim is relevant as well as the enforced penalty payment B (see (1)) in case the fraudulent claim gets detected.

Condition 2 *The individual's decision to get insurance coverage in the first place depends on whether his utility by the time of loss occurrence is greater with having insurance than without it*, *i.e.*,

$$U\big(W_1^A\big) \geq U\big(W_1^B\big). \tag{12}$$

Using (11) and (10) this participation constraint (12) can be written as

$$-P+\mathbb{E}\big(R(\theta,\hat{\theta})\big)-\frac{a}{2}\operatorname{Var}\big(\theta-R(\theta,\hat{\theta})\big)\geq-\frac{a}{2}\operatorname{Var}(\theta)$$
$$\iff\quad P-\mathbb{E}\big(R(\theta,\hat{\theta})\big)\leq-\frac{a}{2}\operatorname{Var}\big(R(\theta,\hat{\theta})\big)+a\operatorname{Cov}\big(\theta,R(\theta,\hat{\theta})\big). \tag{13}$$

Based on the representation $P\leq\mathbb{E}(R(\theta,\hat{\theta}))-\frac{a}{2}\operatorname{Var}(R(\theta,\hat{\theta}))+a\operatorname{Cov}(\theta,R(\theta,\hat{\theta}))$, the inequality in (13) can be interpreted as an upper bound for the insurance premium the potential policyholder is willing to pay for his insurance coverage. It depends on the utility of the payment R and the covariance between actual loss θ and R. Furthermore, the individual's risk aversion parameter a has an influence on his willingness to pay.

3.3 Optimization of positions

So far, the model framework as well as the participation constraints for both the policyholder and the insurance company have been presented. Based on this information, we state the corresponding optimization problems.

The insurance company is aiming to maximize the net present value of the incoming and outgoing future payments with respect to its audit strategy such that both stakeholders are still willing to participate, i.e., (12) and (3) hold. Again, it is assumed that the other parameters are given. This objective function can be written as

Insurance company's optimization problem:

Find the optimal audit strategy q s.t. NPV is maximized (14)

and (12), (3) hold.

At the same time, the policyholder's aim is to maximize his final expected utility with respect to his fraud strategy such that both participation constraints (12) and (3) hold, i.e., an insurance contract exists. It is assumed that all the other parameters are given. We will denote this optimization problem by the following

Policyholder's optimization problem:

Find the optimal fraud behavior p s.t. $U\left(W_1^A\right)$ is maximized (15)

and (12), (3) hold.

Both stakeholders try to optimize their own position respectively. Our aim is to analyze these conflicting objectives and participation constraints from both insured's and insurer's perspective and find a common agreement range for the resulting fraud and auditing strategies.

3.4 Assumptions about the distribution of information

Before presenting the results of our analytical analyses, we summarize the assumptions regarding the distribution of information among the stakeholders.

3.4.1 Insurance company perspective

The insurance company is assumed to have full information about the distribution of the reported losses $\hat{\theta}$ due to having observed incoming claims in the past. Furthermore, we expect that it has an adequate estimate for the distribution of the actual losses θ based on the outcomes of previous auditing processes. Consequently, the insurer can derive the relative fraud amount $\alpha := \hat{\theta}/\theta$ in case fraudulent behavior occurs. Since the optimal auditing strategy also depends on the prevalent fraud probability p, the insurance company has to estimate this value. It then chooses the corresponding optimal audit probability which maximizes its *NPV* in response.

3.4.2 Policyholder perspective

The policyholder has no reliable information about the behavior of the insurance company regarding the verification process. He can only make assumptions about its auditing probability q. Based on this estimate, the policyholder decides upon his optimal fraud strategy p which maximizes $U(W_1^A)$.

3.5 Analytical results

In the course of this subsection, we present some analytical results for the presented optimization problems. The proofs can be found in the Appendix.

The following proposition characterizes optimal fraud and auditing strategies for the policyholder and the insurance company respectively in a general setting. For this purpose, we make two assumptions. First of all, the policyholder's risk aversion parameter a is supposed to be non-zero, i.e., $a > 0$. This means, his objective function is actually given as an expected utility function, i.e., the variance of the difference between indemnity payment $R(\theta, \hat{\theta})$ and actual loss θ, denoted by $\mathrm{Var}(\theta - R(\theta, \hat{\theta}))$, has an impact on the final result. Furthermore, whenever the policyholder decides to make a fraudulent claim, he reports $\hat{\theta} = \alpha\theta$ for some given finite $\alpha \geq 1$ to the insurance company. This setting implies that the amount of fraud is constant.

We will derive optimal fraud and auditing strategies, namely p^{opt} and q^{opt}, for the setting introduced above.

Proposition 1 *Assume p, q to be in the agreement range, i.e., an insurance contract exists. For $a > 0$, $B = \theta$, $\hat{\theta} = \alpha\theta$ with some given $\alpha \geq 1$, the respective optimal strategies p^{opt}, q^{opt} are given by:*

(i) *Insurance company perspective*

Let some p be given. In order for the net present value NPV to be maximized, choose

$$q^{\mathrm{opt}} = \begin{cases} \text{as large as possible} & \text{if } p > p^* \\ \text{as small as possible} & \text{if } p \leq p^* \end{cases}, \tag{16}$$

where $p^* := \dfrac{k}{\alpha \mathbb{E}(\theta)}$.

(ii) *Policyholder perspective*

Let some q be given. In order for the final expected utility $U(W_1^A)$ to be maximized, choose

$$p^{\text{opt}} = \begin{cases} \textit{as large as possible} & \textit{such that } \frac{-\mathbb{E}(\theta)}{ap(1-\alpha(1-q))\operatorname{Var}(\theta)} \geq 1 \\ \textit{as small as possible} & \textit{such that } \frac{-\mathbb{E}(\theta)}{ap(1-\alpha(1-q))\operatorname{Var}(\theta)} \leq 0 \end{cases}. \tag{17}$$

For the case $0 < \frac{-\mathbb{E}(\theta)}{ap(1-\alpha(1-q))\operatorname{Var}(\theta)} < 1$ *no general statement can be made.*

Proposition 1(i) looks at the optimization problem from the insurance company perspective. It states the optimal auditing strategy with respect to a given fraud probability. The insurance company has two general strategies to chose from. It can either decide to audit the incoming claims with the maximal probability possible, i.e., such that the participation constraints of both policyholder and insurance company hold true, or the auditing probability can be chosen as minimal as possible. This decision depends on an estimate of the policyholder's behavior p. Based on whether it exceeds or deceeds the threshold $\frac{k}{\alpha\mathbb{E}(\theta)}$, the insurance company opts for a high or low auditing probability respectively. According to Proposition 1(i), the exceed of the threshold is influenced by the costs per audit k. The lower these are, given some fixed α and θ, the more likely it is for the fraud probability to exceed the resulting threshold. In this case, it becomes optimal for the insurance company to verify the incoming claims with a high probability. The opposite relationship holds true for the expected loss amount θ and the degree of fraud which is represented by α. The higher their values are, the lower the threshold becomes and the more likely it is for the estimated fraud probability to exceed the latter. For the insurance company this implies auditing the incoming claims with the highest probability possible as well. For an illustration of the results obtained in Proposition 1(i) see Fig. 1a and the discussions in Sect. 5.1.

Proposition 1(ii) considers the policyholder point of view in this optimization problem. In this case, the decision whether to chose the fraud probability as large or small as possible given a certain auditing strategy, is not as clear as in the previous situation described in Proposition 1(i) especially since there are situations for which no forecast can made. Furthermore, difficulties arise when trying to interpret the impact of single model parameters on the value of the threshold which determines the optimal auditing behavior in the known cases. However, see Fig. 1b and the discussions in Sect. 5.1 for an illustration of the optimal fraud probability from the policyholder perspective.

The challenges which occur with finding a closed-form analytical solution to the introduced maximization problem emphasize the need for a numerical approach. In Sect. 4, we therefore present a method for deriving the agreement range for both policyholder and insurance company. Furthermore, the impact of valid $p-q$ combinations on the objective quantities $U(W_1^A)$ and *NPV* is analyzed and illustrated.

4 Computational aspects

As seen in the previous section, simple analytical solutions to the optimization problem can not be derived for all general settings. Moreover, the results may be hard to interpret both graphically and economically. In this section, we will approach these challenges by using numerical methods and Monte Carlo simulation. The aim is to compute the agreement range with respect to the fraud and auditing strategies for various parameterizations of the model. After having introduced the procedure, the results of the simulations will be analyzed and presented graphically.

4.1 Monte Carlo simulation and numerical methods

We use the Monte Carlo technique to find the optimal agreement range regarding the fraud and auditing strategies of the policyholder and the insurance company respectively. The main idea behind this approach is to generate a sufficiently large number of realizations N of the random variable θ. Furthermore, we consider all fraud and auditing probabilities p and q which are represented by $l \cdot \frac{1}{M}$ for $l = 0, 1, \ldots, M$ where M denotes the number of discretization points on the interval $[0, 1]$. Based on these assumptions, the resulting indemnity payments R, the policyholder's wealth positions with and without having signed the insurance contract W_1^A and W_1^B as well as the insurance company's value V are calculated for each outcome of the simulation and each fraud and auditing probability combination. Using (1), (6) and (8) for R, W_1^B and W_1^A respectively, this can written as follows

$$R[n,i,j] = \big(1 - p[i]\big)\theta[n] + p[i]\Big(\big(1 - q[j]\big)\alpha\theta + q[j]\big(\theta[n] - B[n]\big)\Big) \tag{18}$$

$$W_1^B[n,i,j] = W_0 - \theta[n] \tag{19}$$

$$W_1^A[n,i,j] = W_0 - P\theta[n] + R[n,i,j], \tag{20}$$

where $\theta[n]$ denotes the n^{th} realization of the random variable θ and $p[i]$ and $q[j]$ are the considered fraud and auditing probability represented by $i\frac{1}{M}$ and $j\frac{1}{M}$ for $i, j = 0, 1, \ldots, M$ respectively. Consequently, the term $[n, i, j]$ indicates for which combination of loss realization and fraud and auditing probabilities the quantities R, W_1^B and W_1^A are evaluated.

The next step to determining the agreement range is to derive the objective quantities, i.e., the policyholder's final utility depending on whether he signed the insurance contract prior to the loss or not and the insurance company's present value based on the corresponding wealth and value positions calculated before. For this purpose, we use arithmetic averaging with respect to the realizations of the random variable θ for each possible combination of p and q. Regarding the individual's final utility when having decided against insurance coverage, we use the following formula, derived from (10)

$$U\big(W_1^B\big)[i,j] = \hat{\mu}_n\big(W_1^B[n,i,j]\big) - \frac{a}{2}\hat{\sigma}_n^2\big(W_1^B[n,i,j]\big), \tag{21}$$

where $\hat{\mu}_n$ denotes the estimator for the expected value with respect to all realizations $n = 1, \ldots, N$ and $\hat{\sigma}_n^2$ the estimator for the variance with respect to all realizations $n = 1, \ldots, N$. The same procedure applies for the case when an insurance contract was signed, this time using (11):

$$U\big(W_1^A\big)[i,j] = \hat{\mu}_n\big(W_1^A[n,i,j]\big) - \frac{a}{2}\hat{\sigma}_n^2\big(W_1^A[n,i,j]\big). \tag{22}$$

From the insurance company point of view, the net present value of its future incoming and outgoing cash flows depending on the fraud and auditing probability can be derived as follows, based on (2)

$$NPV[i, j] = P - \hat{\mu}_n\big(R[n, i, j]\big) - q[j]k. \tag{23}$$

We are now in the position to check for the participation constraints of both policyholder and insurance company. Only if these hold true, an insurance contract comes into existence and merely in this case, the optimization problems are well defined. The idea here is to systematically analyze the participation constraints given in (12) and (3) for each combination of fraud and auditing probabilities. In case these are verified, we consider the corresponding p–q combination as valid. At the end of this procedure, we obtain the agreement range.

The actual aim is to find the optimal strategies p and q such that the objective quantities, i.e., the policyholder's final wealth position $U(W_1^A)$ and the present value of the insurance company's future incoming and outgoing cash flows NPV, are maximized from each of the participants perspectives. In order for these to be determined, we calculate the results for $U(W_1^A)$ and NPV evaluated with respect to the valid p–q combinations respectively. Once the maximal values have been found, we can retrace the corresponding fraud and auditing probabilities under which the maximum was attained. This procedure is performed separately for the two participants.

4.2 Choice of parameters

We analyze the implementation of the model for different parameterizations. The aim here is to study the influence of certain model parameters on the agreement range regarding the valid fraud and auditing probabilities.

Table 1 sums up the choices for the input parameters for a reference setting. In the course of this section, we base our simulations and studies on these values. A detailed description of the assumptions leading to these choices can be found in Müller et al. (2011).

Unless otherwise noted, the simulation results are based on $N = 100{,}000$ realizations of the loss variable θ and $M = 50$ discretization points in the interval [0, 1].

Table 1 Input parameters for the reference setting

Input parameter		Reference level
Initial wealth position	W_0	0
Insurance premium	P	1.45
Occurred loss	θ	$\ln\mathcal{N}(1, 0.4)$
Fraud amount	α	1.2
Risk aversion parameter	a	6
Auditing cost	k	0.1
Penalty payment	B	Realization of θ

5 Simulation results

This section summarizes the main findings of the working paper written by Müller et al. (2011). The discussion of the results from the perspective of both stakeholders, as well as further sensitivity analyses can be found in the working paper.

In this contribution, we present the agreement range resulting from the parametrization chosen in the reference setting. Furthermore, a sensitivity analysis with regard to the influence of the relative fraud amount is performed.

5.1 The reference setting

Figure 1 shows the agreement range from both the policyholder as well as the insurance company perspective based on the values for the input parameters which were presented above. Each point in the graphic represents a valid fraud and auditing probability combination.

In order to illustrate the dimension of the objective quantities $U(W_1^A)$ and *NPV* which result from the current parameter choice and a certain *p*–*q* combination, the points are displayed in different colors according to the value. For this purpose, given that the input parameters are fixed, the *p*–*q* combinations which lead to the lowest third of outcomes are presented in the lightest color whereas those combinations which result in the highest third of outcomes are shown in the darkest color. The remaining points are displayed in a medium color. This implies that the darker the color of a point, the higher is the relative value of the corresponding $U(W_1^A)$ or *NPV*.

5.2 Sensitivity analysis of relevant parameters

In the remainder of this section, we present and discuss the resulting agreement range, i.e., all valid $p - q$ combinations, based on different choices regarding the input parameter fraud amount α.

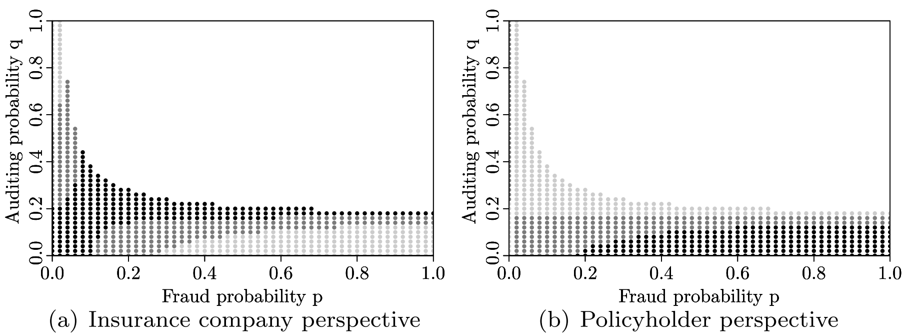

Fig. 1 Agreement range from both stakeholders' perspectives respectively. All parameters are chosen as presented in Table 1. *p*–*q* combinations which result in the highest third of *NPV* and $U(W_1^A)$ respectively are displayed in the darkest color, the ones which result in the lowest third of *NPV* and $U(W_1^A)$ respectively in the lightest color and the remaining ones in a medium color

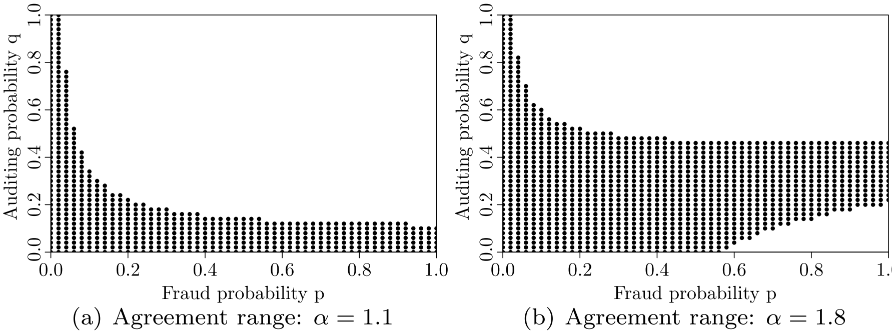

Fig. 2 Agreement range for different fraud amounts α. The remaining parameters are chosen as presented in Table 1

5.2.1 *Influence of fraud amount*

In Fig. 2 the agreement range is displayed for the fraud amounts $\alpha = 1.1$ and $\alpha = 1.8$ respectively, i.e., in case of fraudulent behavior the claimed loss is given by $\hat{\theta} = 1.1 \cdot \theta$ or $\hat{\theta} = 1.8 \cdot \theta$. Again, the remaining input parameters are chosen as displayed in Table 1.

Comparing the graphics for the different choices of α, we see that this time the upper bound of the agreement range as well as part of the lower bound shift in an upward direction when increasing the fraud amount. To be more precise: While the number of valid p–q combinations with high auditing probabilities q increases for all fraud strategies p, the change in the lower bound occurs only in the area of high fraud probabilities p. Summing up these effects, we can say that the higher the amount of fraud the broader the agreement range becomes. However, a change from $\alpha = 1.2$ in the reference setting to $\alpha = 1.1$ results in marginal modifications within the agreement range.

This outcome can be interpreted the following way: The higher the amount of fraud α per claim, the more likely it is for the policyholder to accept higher auditing probabilities q given that the own fraud probability p is fixed. In these cases, even though the auditing activity increased, the gain in final utility $U(W_1^A)$ due to excessive claiming is still positive despite the higher chance of being convicted and imposed with a penalty payment. On the other hand, it becomes unattractive from the insurance company perspective to audit the incoming claims with a low probability q when the amount of fraud is increased assuming a high fixed fraud behavior p. Such a strategy would imply that the majority of fraudulent claims remained undetected which consequently leads to an increase in outgoing cash flows due to excessive fraud amounts which is not covered by incoming positions like the insurance premiums or penalty payments. Therefore, if the fraud amount α goes up, p–q combinations with higher values for q become acceptable to both stakeholders whereas no insurance contract will come into existence with individuals who are expected to commit excessive fraud frequently.

6 Conclusive remarks

In this paper we build and analyze a model framework which depicts the handling of insurance claims fraud based on a costly state verification approach. We present analytical solutions as well as numerical methods for solving the resulting optimization problems which take both the insurance company and the policyholder perspective into account. Our focus is set on deriving an agreement range consisting of all valid fraud and auditing probability combinations. In addition, we discuss the impact of a relevant input parameter on the size of the agreement range.

One of our main findings is the derivation of optimal auditing and fraud strategies from stakeholders' perspectives respectively. Especially from the insurance company point of view, they seem intuitive: Summarizing, it can be said that the optimal answer to low fraud probabilities is to perform auditing with a small probability as well whereas medium and high fraud probabilities require the largest valid audit probability in order to maximize the net present value. However when audits are performed with a small constant probability, it is optimal to chose those policyholders whose claims contain fraud with a low probability. If the insurance company commits itself to a high audit probability, it is optimal to chose policyholders who report fraudulent claims with a high probability.

Based on our numerical approach, we are able to present and analyze the agreement range for different parameterizations as well as the optimality of different auditing and fraud probability combinations regarding the stakeholders' respective objective quantities. We find that a high relative amount of fraud results in broadening the agreement range, the latter becomes smaller whenever the value of this particular input parameter is chosen the opposite way. Furthermore, the simulation results support and illustrate our analytical findings regarding optimal fraud and auditing strategies.

Appendix: Proof of Proposition 1

(i) Using (1), (2) and the assumptions $B=\theta$, $\hat{\theta}=\alpha\theta$ with $\alpha \geq 1$, we get

$$\begin{aligned} NPV &= P-(1-p)\mathbb{E}(\theta)-p\big[(1-q)\mathbb{E}(\hat{\theta})+q\mathbb{E}(\theta-B)\big]-qk \\ &= P-(1-p)\mathbb{E}(\theta)-p(1-q)\alpha\mathbb{E}(\theta)-qk \\ &= P-\mathbb{E}(\theta)+p(1-\alpha)\mathbb{E}(\theta)+q\big[\alpha p\mathbb{E}(\theta)-k\big]. \end{aligned} \tag{24}$$

Deriving (24) with respect to q leads to

$$\frac{\partial}{\partial q}NPV=\alpha p\mathbb{E}(\theta)-k, \tag{25}$$

which can be distinguished into two cases with respect to its sign.

(a) If for the given fraud strategy $p>\frac{k}{\alpha\mathbb{E}(\theta)}$ holds, the NPV as defined in (2) has a positive slope with respect to the parameter q. Consequently, the optimal auditing strategy q^{opt} has to be chosen as large as possible in order to maximize the value of NPV.

(b) If the given fraud strategy p is given such that $p \leq \frac{k}{\alpha \mathbb{E}(\theta)}$ holds, the *NPV* has a negative slope with respect to the parameter q. Hence, the optimal auditing strategy q^{opt} has to be chosen as small as possible for the *NPV* to be maximized.

(ii) Applying the assumptions $a \neq 0$, $B = \theta$, $\hat{\theta} = \alpha\theta$ with $\alpha \geq 1$ to (1) and (11), we obtain

$$\begin{aligned} U\big(W_1^A\big) &= W_0 - P - \mathbb{E}(\theta) + (1-p)\mathbb{E}(\theta) + p\big[(1-q)\mathbb{E}(\hat{\theta}) + q\mathbb{E}(\theta - B)\big] \\ &\quad - \frac{a}{2}\operatorname{Var}\big[-\theta + (1-p)\theta + p(1-q)\hat{\theta} + pq(\theta - B)\big] \\ &= W_0 - P - \mathbb{E}(\theta) + (1-p)\mathbb{E}(\theta) + p(1-q)\mathbb{E}(\hat{\theta}) \\ &\quad - \frac{a}{2}\operatorname{Var}\big[-\theta + (1-p)\theta + p(1-q)\hat{\theta}\big] \\ &= W_0 - P - p\mathbb{E}(\theta) + p\alpha\mathbb{E}(\theta) - pq\alpha\mathbb{E}(\theta) \\ &\quad - \frac{a}{2}\operatorname{Var}(-p\theta + p\alpha\theta - pq\alpha\theta) \\ &= W_0 - P - p(1-\alpha+q\alpha)\mathbb{E}(\theta) - \frac{a}{2}p^2(1-\alpha+q\alpha)^2\operatorname{Var}(\theta). \end{aligned} \tag{26}$$

Deriving (26) with respect to p results in

$$\frac{\partial}{\partial p}U\big(W_1^A\big) = -(1-\alpha+q\alpha)\mathbb{E}(\theta) - ap(1-\alpha+q\alpha)^2\operatorname{Var}(\theta). \tag{27}$$

Based on (27), three cases can be identified:

(a) For $\frac{-\mathbb{E}(\theta)}{ap(1-\alpha(1-q))\operatorname{Var}(\theta)} \geq 1$, the policyholder can choose any fraud strategy $p \in [0, 1]$, especially any p in the agreement range, such that $p \leq \frac{-\mathbb{E}(\theta)}{a(1-\alpha(1-q))\operatorname{Var}(\theta)}$. Applying this inequality to (27), we obtain $\frac{\partial}{\partial p}U(W_1^A) \geq 0$. From this can be concluded that $U(W_1^A)$ has a positive slope. Consequently, the optimal fraud strategy p^{opt} has to be chosen as large as possible in order to maximize the value of $U(W_1^A)$.

(b) Similarly, for $\frac{-\mathbb{E}(\theta)}{ap(1-\alpha(1-q))\operatorname{Var}(\theta)} \leq 0$, the policyholder can choose any fraud strategy $p \in [0, 1]$, especially any p in the agreement range, such that $p \geq \frac{-\mathbb{E}(\theta)}{a(1-\alpha(1-q))\operatorname{Var}(\theta)}$. For (27) this implies that $\frac{\partial}{\partial p}U(W_1^A) \leq 0$. This means that in this case $U(W_1^A)$ has a negative slope and hence, the optimal fraud strategy p^{opt} needs to be chosen as small as possible for $U(W_1^A)$ to be maximized.

(c) For $0 < \frac{-\mathbb{E}(\theta)}{a(1-\alpha(1-q))\operatorname{Var}(\theta)} < 1$, no general statement about the corresponding optimal fraud strategy p^{opt} can be made. □

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