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STATE-OF-THE-ART-ARTIKEL

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How to invest over the life cycle: Insights from theory

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Abstract We analyse the state of the art in the field of life cycle portfolio choice, a recent strand of the literature on intertemporal portfolio selection. Life cycle models are designed to identify optimal savings and portfolio policies over the lifetime of investors. They can help to improve pension schemes by showing how these could be specifically tailored to the individual employee's circumstances to overcome the 'one-size-fits-all' philosophy still prevailing in parts of the mandatory retirement savings system. To facilitate comparison, we first describe set-up, solution method and characteristic results for a basic model and then derive a general framework to classify existing contributions. We highlight the models' strengths and weaknesses and assess their ability to resolve existing portfolio puzzles. Lessons from the literature are summarized and promising areas for further research identified.

Keywords Personal finance · financial planning · life cycle model · portfolio choice

JEL classifications G11, D14, D91, H55

1 Introduction

Young people and families in European countries are nowadays aware of the fact that they have to privately accumulate savings if they want to preserve their standard of living after retirement. The investment horizon of these households is necessarily long-term, and the investment period extends across different stages in their life cycle. Under these circumstances, it is not intuitive to set up an optimal investment

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and consumption strategy. This article reviews the stream of literature which models investment and consumption with special regard to the life cycle of investors.

The standard portfolio theory introduced by Markowitz (1952) is static in nature, since it explores investment decisions for only one period. A more realistic setting must account for the multiperiod dimension of the portfolio choice problem. Only under very specific circumstances, the optimal portfolio structure is time invariant. In this special case, a one-period optimization suffices to characterize the optimal portfolio choice also in a multiperiod environment. Under more general conditions, however, investors will restructure their portfolios in reaction to changes in income, the accumulated wealth and the investment opportunity set. This possibility to adjust the portfolio composition affects the initial investment choice.

From the perspective of investors saving for retirement, the portfolio choice problem changes dramatically during the life cycle. Popular wisdom holds that early in working life, a typical investor should invest almost all of her wealth in stocks as opposed to the risk-free asset. When approaching retirement, the investor should gradually decrease the share of stocks in order to reduce the impact of disastrous losses since these could no longer be offset by increased savings out of labor income.[1]

Life cycle models of portfolio choice are designed to provide a more detailed and more comprehensive analysis of optimal investment policies. Their general set-up can be characterized as follows. An investor has a certain or uncertain life span, usually partitioned into a working life and a retirement phase. Income during working life is most often assumed to be exogenously determined. It is also assumed to be time dependent, stochastic and non-diversifiable, which sets these models apart from other models in the extensive literature on dynamic portfolio choice. Retirement income is either exogenously given or comes from some savings account managed in the interests of the agent. Each year, the agent decides how much to consume and how to allocate savings among various asset classes subject to several constraints. Her objective is to maximize expected discounted life time utility.

The literature on life cycle portfolio choice has evolved in an attempt to find a framework which is rich enough to explain actual household behaviour. The motivation for new models is typically a "puzzle" which is defined as a discrepancy between the predictions of financial theory and the choices investors actually make. Thus, the models reviewed here are designed as *positive* theories. However, as will be discussed later, they can also be interpreted as *normative* models explaining investors how they *should* behave. There is need for such normative approaches, because household data suggest that the investment behaviour of at least a minority of investors is not compatible with welfare maximization.[2]

According to this normative perspective, life cycle models of portfolio choice can also help to improve pension schemes. In most European countries, a 'one-size-fits-all' philosophy still prevails in parts of the mandatory retirement savings system. Thus, welfare gains might be achieved by allowing individualized strategies with enhanced freedom of choice for the employee. Such strategies should be specifically tailored to the individual employee's circumstances. This calls for a flexible analysis, for which life cyle models offer a useful framework.

[1] See e.g. Danthine and Donaldson (2005) p. 76–77.
[2] See Campbell (2006).

During the last five years, research in this field has progressed rapidly, so that a variety of life cycle models of portfolio choice now exists. It is the objective of this state-of-the-art article to survey this stream of literature and to highlight its achievements as well as remaining shortcomings. We will present a basic life cycle model in the next section, before the extensions and modifications of more complex models are discussed in Sect. 3.[3]

2 A basic model of life cycle portfolio decisions

The model presented in this section corresponds to the baseline case analysed by Cocco et al. (2005). We use this setting to illustrate how the investor's objective function, her labor and retirement income as well as her savings account and asset universe are typically modelled.

2.1 The investor's objective function

The investor is assumed to have a finite horizon T and to be concerned about her expected discounted lifetime utility derived from consumption. Taking into account a realistic survival process, expected utility can be written as:

$$E_0 \sum_{t=1}^{T} \beta^t \left(\prod_{j=0}^{t-1} \psi_{j+1|j} \right) u(C_t), \tag{1}$$

where β is the discount factor, $u(\bullet)$ a utility function, C_t periodic consumption and $\psi_{t+1|t}$ the probability to survive up to period $t+1$ conditional on having survived up to period t.[4] In this baseline model and also in some of the more advanced models, $u(\bullet)$ is assumed to be a standard power utility function:

$$u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}. \tag{2}$$

If the investor additionally derives utility from leaving a bequest B_t, the expectation to be maximized becomes:

$$E_0 \sum_{t=1}^{T} \beta^t \Gamma_t \left[\psi_{t|t-1} u(C_t) + \left(1 - \psi_{t|t-1}\right) \varphi(B_t) \right], \tag{3}$$

with

$$\Gamma_t = \begin{cases} \left(\prod_{j=0}^{t-2} \psi_{j+1|j} \right) & \forall \; 1 < t \leq T \\ 1 & \text{for} \quad t = 1 \end{cases}$$

[3] It is important to note that this structure does not correspond to the time of publication of the relevant papers. What we call base model did not necessarily form the starting point for the extensions presented in Sect. 3. It rather serves as a base case for our exposition of more intricate specifications of life cycle models. We chose this structure over one where the reviewed contributions are nested in a general specification because the latter would inflate notation unnecesarily without offering additional insights.

[4] We use E_t to denote expectation conditional on time t information.

and where $\varphi(\bullet)$ is usually also a power utility function, possibly adjusted for a parameter measuring the strength of the bequest motive.

2.2 Labor and retirement income

The investor begins to work at time t_0 and retires at time t_R. During working life, she receives exogenous labor income Y_t with $y_t \equiv \log Y_t$ most commonly modelled as the sum of a permanent component and a transitory shock. The permanent component consists of a deterministic function $f(t, \mathbf{Z}_t)$ of age and individual characteristics $\mathbf{Z}_t$,[5] and of a persistent income component p_t following a random walk:

$$p_t = p_{t-1} + v_t. \tag{4}$$

Thus:

$$y_t = f(t, \mathbf{Z}_t) + p_t + e_t \quad \forall\, t_0 \leq t \leq t_R, \tag{5}$$

where both e_t and v_t are assumed to be iid normally distributed with mean zero. This specification is adopted from the literature on life cycle saving.[6]

While most of the models to be reviewed below agree on (5) as their specification for income during working life, assumptions on retirement income vary. A basic formulation postulates retirement income to be exogenously specified as a constant fraction λ of the permanent component of labor income in the retirement year t_R. Thus,

$$y_t = \log \lambda + \left(f\left(t_R, \mathbf{Z}_{t_R}\right) + p_{t_R}\right) \quad \forall\, t > t_R. \tag{6}$$

More generally, retirement income could be modelled as an arbitrary function of income during working life.

Note that this specification of the investor's income process implies that retirement age (t_R) is exogenous, a simplifying assumption that does not hold in practice. In many real world pension systems retirement benefits can be withdrawn early or late subject to particular constraints, thus making the time of retirement part of the investor's decision problem.[7]

2.3 Saving accounts and asset allocation

In the most basic of all specifications, we can picture the investor to own a savings account to which all savings are automatically credited. The balance can also be drawn down at the investor's discretion.

More specifically, the investor starts any given period t with accumulated financial wealth W_t and realizes labor income Y_t. Together, W_t and Y_t make up all resources available for consumption. These resources are usually denominated *cash-on-hand* (X_t):

$$X_t \equiv W_t + Y_t. \tag{7}$$

[5] The dimension of $\mathbf{Z}_t$ depends on the number of individual characteristics considered.

[6] See e.g. Hubbard et al. (1995), Carroll and Samwick (1997), Gourinchas and Parker (2002).

[7] See e.g. French (2005) for a life-cycle model of savings (but excluding portfolio choice) with endogenous labor supply and retirement.

Next, the investor consumes the desired amount C_t. A fraction of the remaining resources can be invested in a risky asset (α_t), while the remainder $(1 - \alpha_t)$ is invested in a riskless asset. The riskless asset has known, constant gross return R_{rf}. A standard assumption for the risky asset's return is to add to the riskless rate a risk premium κ disturbed by a white noise shock ζ:

$$R_{e,t+1} = R_{rf} + \kappa + \zeta_{t+1}. \tag{8}$$

The investor's asset allocation and asset returns result in a compound portfolio return of

$$\begin{aligned} R_{p,t+1} &\equiv \alpha_t R_{e,t+1} + (1 - \alpha_t) R_{rf} \\ &= \left(\alpha_t (\kappa + \zeta_{t+1}) + R_{rf}\right). \end{aligned} \tag{9}$$

The dynamics of financial wealth can then be summarized as:

$$W_{t+1} = (W_t + Y_t - C_t) R_{p,t+1}. \tag{10}$$

Using (7), this equation can also be re-written in terms of cash-on-hand as:

$$X_{t+1} = (X_t - C_t) R_{p,t+1} + Y_{t+1}. \tag{11}$$

Of course, the investor's asset menu can be broadened to include other assets or loans. Also, liquidity constraints might be considered, which is important if the investor is not allowed to withdraw funds from an account for a fixed period of time.

2.4 Optimization

In summary, in a basic life cycle model of portfolio choice with mortality, the investor controls her consumption $\{C_t\}_{t=t_0}^{T-1}$ as well as her risky asset share $\{\alpha_t\}_{t=t_0}^{T-1}$ throughout lifetime, subject to three sources of uncertainty, the labor income shocks e_t and v_t and the stock return shock ζ_t.[8] The investor's optimization problem can be stated as:

$$\begin{aligned} &\max_{\{\alpha_t\}_{t=t_0}^{T-1}, \{C_t\}_{t=t_0}^{T-1}} \quad (1) \\ &\text{s.t. } (2), (5), (6), (9), (11), 0 \le \alpha_t \le 1, 0 \le C_t \le X_t \text{ and } X_0 \text{ given.} \end{aligned} \tag{12}$$

Notice from (12) that the optimization is subject to standard inequality constraints. Firstly, the fraction of wealth invested in the risky asset is constrained to be between zero and one. This means that the risky asset can neither be sold short nor can the investment in the risky asset be levered up. By the second constraint, consumption is bounded from above by cash-on-hand.

As is well known, a solution to problem (12) is a function (usually called *policy function*) that specifies the optimal time paths for the control variables $\left\{\alpha_t^*\right\}_{t=t_0}^{T-1}$ and $\left\{C_t^*\right\}_{t=t_0}^{T-1}$, depending on the problem's state variables. The state-space of the current basic problem is spanned by three variables: time t, cash-on-hand X_t and the random walk component of labor income p_t. However, the state space can be reduced to the two variables t and X_t by standardizing the entire problem by the permanent component of labor income $f(t, \mathbf{Z}_t) + p_t$.[9]

[8] The variables v_t and ζ_t are often allowed to be mutually correlated.

[9] See Carroll (2002a) for details.

Due to the non-stationary labor income process, the finite horizon and the inequality constraints, the policy function cannot be obtained analytically, but must be approximated numerically. The approximation is based on a recursive representation of the problem by means of the *Bellman* Equation. Denoting by $V_t(\bullet)$ the value function, this equation can be written as:

$$V_t(X_t) = \max_{\substack{0 \le C_t \le X_t, \\ 0 \le \alpha_t \le 1}} \left\{ u(C_t) + \psi_{t+1|t}\, \beta E_t \left[V_{t+1}(X_{t+1}) \right] \right\}$$
$$\text{s.t. (11), (2), (5), (6), (9), } X_0 \text{ given.} \tag{13}$$

In the last period (T) it is optimal to consume all remaining resources which implies that the value function corresponds to the instantaneous utility function: $V_T(X_T) = u(X_T)$. The problem can then be solved by backward induction starting in $T - 1$. Computationally, this is a non-trivial task because the value and policy functions must be computed in every period and at every grid point of X_t by solving a non-linear constrained optimization of an objective function with time-varying state dynamics. Its evaluation requires a three-dimensional numerical integration at peak times (during working life).[10]

Having obtained the policy functions, we know the investor's optimal behavior in every possible state of nature. However, this is of limited interest as many of these states are improbable to arise. We are more interested in typical or average life cycle consumption and asset allocation paths. To this means, it is standard practice in the literature to picture an economy populated by many identical investors ($i = 1, 2, ...N$) for each of whom $\{\zeta_t\}_{i,t=0}^{T}$, $\{e_t\}_{i,t=0}^{t_R}$ and $\{v_t\}_{i,t=0}^{t_R}$ are simulated. Any given triple $\{\zeta_t\}_{i,t=0}^{T}$, $\{e_t\}_{i,t=0}^{t_R}$, $\{v_t\}_{i,t=0}^{t_R}$ implies a specific labor income stream and, together with the policy functions, pins down investor i's optimal life cycle patterns of consumption, portfolio choice and wealth. Having simulated triples for all investors and having determined everyone's optimal life cycle pattern, economy-wide life cycle paths can then be obtained from averaging across the individual investors' optimal choices at every point in time.

As an alternative to the idea of an economy populated by many identical investors, the same simulation could also be interpreted from the point of view of only one investor who is interested in possible paths which might obtain in the future.

2.5 Characteristic results

In Figure 1 we present characteristic results for the basic model. To derive Figure 1, we solved and simulated problem (12) for the parameters reported below the figure.

Panel A shows mean and median forward simulated income as a function of age together with the fifth and ninetyfifth quantiles. The average income profile increases until the investor's late fourties and slightly decreases thereafter. The income drop at retirement age $t_R = 65$ is due to the assumed replacement rate $\lambda = 0.7$ in (6). A visual inspection of the interquantile range shows that the simulation of the labor income process (5) and (4) generates noticeable income heterogeneity throughout life.

[10] During retirement, the required numerical integration is only one-dimensional. We will not further elaborate on the details of this solution here. For further details, see e.g. Carroll (2002a).

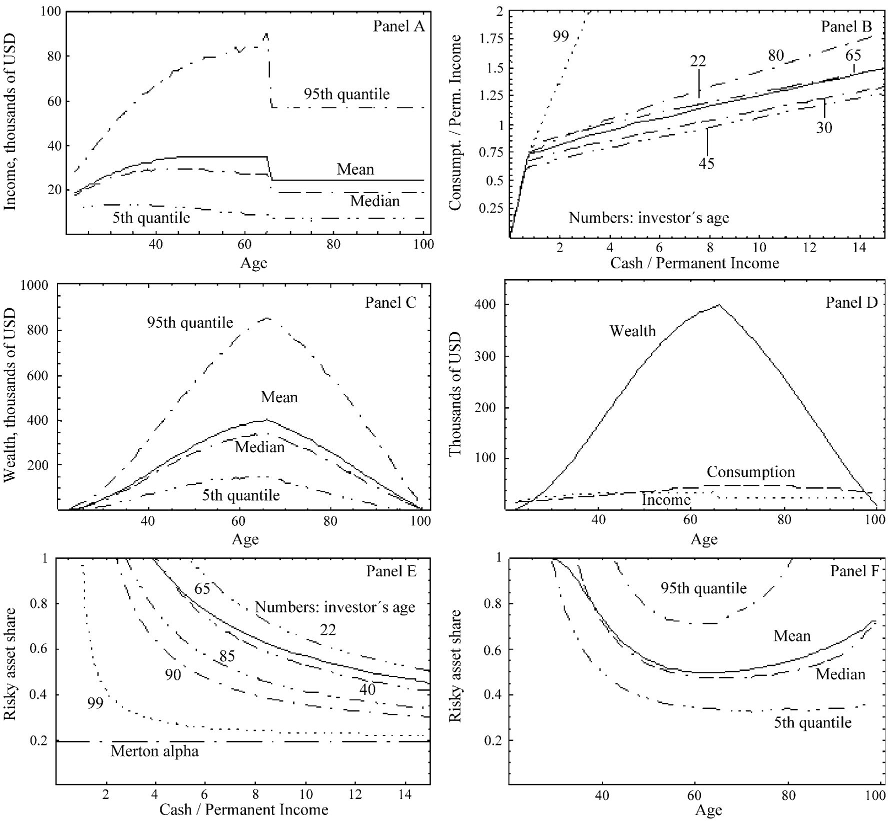

Fig. 1 The figure is based on solving and simulating problem (12) using the following parameters: $t_0 = 22$, $t_R = 65$, $T = 100$, $\lambda = 0.7$, $\gamma = 8$, $\beta = 0.96$, $R_{rf} = 1.02$, $\kappa = 0.04$, $\sigma_\zeta = 0.16$, $\sigma_e = 0.26$, $\sigma_v = 0.10$, $\rho_{v,\zeta} = 0$. The Merton-Alpha implied by this parameter constellation is: 19.53%. The required conditional survival probabilities were obtained from the *National Vital Statistics Report*'s Life Tables for the total population in 2001 for the US; see Arias (2004), p 7. To obtain the deterministic part of labor income we used the third order polynomial estimated by Cocco et al. (2005) for High School graduates in the US; see Cocco et al. (2005), pp 499–500. For this reason, all absolute magnitudes in this figure are in thousands of 1992 USD. The model was solved on a grid with 223 states for the cash-on-hand to permanent income ratio. $\{\zeta_t\}_{i,t=0}^{T}$, $\{e_t\}_{i,t=0}^{t_R}$ and $\{v_t\}_{i,t=0}^{t_R}$ were then simulated 10 000 times. For the simulations we assumed that each investor starts life with zero initial wealth. For each simulation we obtained the corresponding optimal evolution of cash-on-hand, wealth, consumption, and the risky asset share. These were finally used to compute the cross-sectional statistics reported in this figure.

Panel B represents the policy functions for consumption as a fraction of permanent income at different ages. These functions determine the investor's savings at every point in the state space and are thus responsible for wealth accumulation. The consumption policy functions are generally concave, displaying a sharp drop in the marginal propensity to consume at particular cash to permanent income ratios.

Panel C shows the same statistics as panel A, but for forward simulated wealth. In the beginning of working life, the investor is liquidity-constrained and does not save much. But starting in the early thirties, savings become sizable to reach their maximum just before retirement. During the retirement phase, the investor completely dissaves her wealth to be able to consume above income. Heterogeneity is also present in the wealth profiles: 'Rich' households accumulate about nine times as much wealth as 'poor' households at the peak. Notice also that heterogeneity is highest just before retirement.

Panel D shows mean forward simulated income, wealth and consumption as functions of age in one graph. The panel illustrates that the investor is able to smooth consumption out over her life-time using income and wealth accumulation.

Most interesting for our purposes are panels E and F. Panel F shows mean, median, fifth and ninetyfifth quantiles of the investor's forward simulated risky asset share as a function of age. In line with popular wisdom and standard investment advice,[11] the investor has her portfolio fully invested in the risky asset until the early thirties and then gradually reduces this exposure to around 50% at the beginning of retirement. But contrary to standard investment advice, the economic rationale behind this behaviour is not that the young investor has a longer horizon to recoup eventual losses from equity investments. Instead, in the present model the investor's life-time equity profile is the result of a subtle interaction between labor income dynamics, wealth accumulation and age, as is demonstrated in panel E. It plots the investor's policy functions for the risky asset share for various ages as a function of the cash-on-hand to permanent income ratio.

A first important insight conveyed by panel E is that independent of age the policy functions for the risky asset share are convex, downward-sloping curves. This is true because part of the investor's future labor income can be considered an implicit riskless asset holding.[12] Thus, for given future labor income (i.e. at a given age), an investor with low cash-on-hand can afford to invest a high fraction of her financial wealth in the risky asset because she already has a high fraction of her entire wealth (including future labor income) invested in a quasi riskless asset. For an investor with extremely high cash-on-hand, future labor income and thus implicit riskless asset holdings become negligible which is why she invests more conservatively.[13] In fact, for cash-on-hand high enough to completely undermine the existence of risky labor income, the policy functions asymptote towards the constant *Merton*-Alpha, α_M:[14]

$$\alpha_M = \frac{\kappa}{\gamma \sigma_\zeta^2} \tag{14}$$

where σ_ζ^2 denotes the variance of ζ.

A second insight delivered by panel E is that during retirement, the policy functions become more conservative with age, i.e. they shift to the southwest of the

[11] See e.g. Bodie (2003).

[12] This part is smaller the higher the correlation of labor income with the shocks on the risky asset returns.

[13] See also Jagannathan and Kocherlakota (1996).

[14] See Merton (1971). This result was derived assuming continuous-time, perfect markets, a geometric Brownian motion for the risky asset's price, utility of the general HARA form $u(C) = \frac{\gamma}{(1-\gamma)} \left(\frac{\tau C}{\gamma} + \eta \right)^\gamma$ and no labor income. Samuelson (1969) derived the same result in a discrete time setting.

figure. This is due to the fact that the present value of future labor income decreases with age during retirement. For given cash-on-hand this implies a diminishing implicit riskless asset position which must be compensated by a more conservative risky asset share in financial wealth.

To be able to re-trace the forward simulated equity share in panel F, it is important to consider how the policy function for the risky asset share and the cash-on-hand to permanent income ratio interact over the life cycle.[15] In the beginning, the investor's cash-on-hand to permanent income ratio is low and the policy function is aggressive (northeast of panel E) implying a 100% equity investment. Then the cash-on-hand to permanent income ratio gradually increases while the policy functions shift to the southwest. During retirement, the cash-on-hand to permanent income ratio is again reduced while the policy functions still become more conservative. Now, the relative speed of the cash reduction versus the increased portfolio conservatism imposed by the reduction in future labor income determines whether the equity share increases or decreases. If cash is reduced quickly enough, the equity share becomes more aggressive again during retirement, as can be seen in panel F. This finding might appear counter intuitive, but it depends on the exact mortality process used. Mortality is the main determinant of the speed of cash reduction during retirement: in (13), a lower survival rate effectively implies a lower discount factor making the investor more impatient. For example, men usually have lower conditional survival rates than women at advanced ages. As a consequence, in the present model, they would reduce their wealth more quickly and thus invest more aggressively than women during late retirement.

The baseline results turn out to be quite robust. Cocco et al. (2005), e.g., show that riskier labor income, a transitory shock on retirement income, a positive probability (iid) for a low income state during retirement (health shock) and different bequest intensities or equity premia have only minor effects on life cycle asset allocations.[16] However, a higher correlation between permanent labor income shocks and stock returns or a positive probability for labor income to reach a very low state during working life (*disaster state*) generate hump shaped equity profiles peaking at 100% or below between ages 45-50. When the investor is allowed to borrow at the expected equity return, equity holdings are zero until around age 40, then increase to about 100% at age 65 and decrease again during retirement.

2.6 Portfolio puzzles

Basic theoretical models as the one outlined in the previous section generally bear four robust implications: (i) if a fixed cost of stock market participation is included,[17] the participation rate - i.e. the fraction of investors who have paid the fixed cost - in the stock market is predicted to increase rapidly to 100% among young investors and remain constant thereafter. (ii) Young investors are predicted

[15] We do not plot the average cash-on-hand to permanent income ratio separately because it qualitatively resembles the wealth path in panel C of Fig. 1. It reaches a high of about 15 around retirement age.

[16] Of course, this does not necessarily hold in extreme cases. For example, a very high bequest intensity does lead to a less risky equity share decreasing in age.

[17] This could easily be accomplished in the basic model by constraining risky asset holdings using a dummy that equals one if a particular fixed cost has already been paid and zero otherwise.

to be fully invested in stocks and (iii) the risky asset share is expected to decline in age and also in cash-on-hand (iv).

These theoretical results are in stark disagreement with available empirical evidence. Table 1 summarizes several recent empirical studies on household portfolio choice for different countries.[18] These studies all use cross-sectional or panel data from representative household surveys to analyse household portfolio choice in their respective countries. The authors devote particular attention to two central questions: first, what percentage of households owns risky assets (participation or ownership rate) and second, what fraction of their assets do those who own risky assets invest in those assets (conditional risky asset share)? For both items interest is centered on their behavior as a function of the household head's age and particular measures of the household's wealth. To disentangle the ownership and share decisions, most authors employ a sample selection model where the discrete selection equation captures the ownership decision and a continuous equation represents the risky asset share and its determinants. To estimate these models, a variant of Heckman's two-step procedure is usually employed.

Most of the studies find ownership of risky assets to be hump-shaped in age and increasing in wealth. Results on conditional risky asset shares are mixed, but mainly suggest a constant or concave age-pattern and an invariant or increasing behavior with respect to wealth.

Unlike theoretical models, empirical results thus seem to imply anything but complete equity specialization and participation of young investors, or risky asset shares declining in age or in cash-on-hand. Rather, (a) a sizable fraction of households is usually found not to participate in stock markets, (b) conditional asset shares among the young are generally found to be at the same time low and (c) hump-shaped, increasing or constant in age. Also, (d) the risky asset share is mainly found to increase in measures of wealth.[19]

These discrepancies between theory and evidence are sometimes called *portfolio puzzles.*[20] More specifically, the conflicting pair (i versus a) can be termed a *participation puzzle* whereas the pairs (ii versus b), (iii versus c) and (iv versus d) are known as *composition puzzles.* Theorists widely perceive these puzzles as a challenge to build models that yield implications more in line with the data.

3 Modifications and extensions

3.1 Overview

Apart from the motivation to resolve portfolio puzzles, researchers have also extended the basic model of Sect. 2 to gain insight into the restrictiveness of different assumptions. More generally, extended models allow us to study welfare effects of policy proposals and of various institutional settings.

[18] See Ameriks and Zeldes (2001), Bertaut and StarrMcCluer (2002), Banks and Tanner (2002), Eymann and Boersch-Supan (2002), Alessi et al. (2002), Andersson (2001) and Guiso and Jappelli (2002).

[19] See Guiso et al. (2002), Dynan et al. (2003), Carroll (2002b), Hurd (2002), Curcuru (2003), Heaton and Lucas (2000) and Davis, Kubler and Willen (2004) for further empirical evidence on household portfolio choice.

[20] Haliassos and Michaelides (2002) p. 55.

Table 1 Empirical studies on household portfolio choice

Country	US		UK	Germany	Netherlands	Sweden	Italy
Authors and year	Ameriks/Zeldes (2001)	Bertaut/ Starr-McCluer (2002)	Banks/Tanner (2002)	Eymann/ Börsch-Supan (2002)	Alessie et al. (2002)	Andersson (2001)	Guiso/Jappelli (2002)
Data							
Name	Survey of Financial Charact. of Cons., Survey of Consumer Finances (SCF), TIAA-CREF	Federal Reserve Flow of Funds Account, Survey of Consumer Finances (SCF)	Family Expenditure Survey (FES), Financial Research Survey (FRS)	German National Accounts, Deutsche Bundesbank, Income and Expend. Survey (EVS), Spiegel Survey "Soll und Haben", German Socioeconomic Panel (GS OEP)	Dutch National Accounts, Income Panel Survey by Statistics Netherlands, CentER Savings Survey (CSS)	Household Income Distribution (HINK)	Survey of HH Income and Wealth (SHIW)
Type	Cross section (SCF), panel (TIAA-CREF)	Cross section data	Pseudo panel (FES)	Cross section data	Panel data	Rotating panel	Cross section data w/ panel comp.
Period	1962, 1983–1998 (SCF), 1987–1999 (TIAA-CREF)	1983–1998 (both data types)	1987–1988, 1991–1992, 1997–1998 (FRS), 1978–1996 (FES)	1975–1998 (National Acct.), 1990–1997 (Bundesbank), 1978–1993 (EVS), 1980–1997 (GSOEP)	1995/1998 (National Acct.), 1993–1998 (CSS)	1982–1992	1989–1995/1998
Methods	OLS, Probit	Censored model, Sample selection model, Multivariate Probit	Probit, Heckman two-step model	Probit, Heckman two-step model	Random effects Probit	Censored model (Tobit), Heckman two-step model, Fixed eff. w/o select. equ.	Standard sample select. model, Random effects Probit, Conditional Logit
Results							
Risky asset ownership							
as function of age	Hump-shaped	Hump-shaped	Hump-shaped	Essentially flat	Increasing, Probit: u-shaped	Hump-shaped	Hump-shaped
as function of wealth measures	–	Increasing	Increasing	Increasing	Increasing	–	Increasing
Condit. risky asset share							
as function of age	Roughly constant	Hump-shaped	Rather time invariant	Convex	Weakly increasing, Probit: u-shaped	Tobit: hump-shaped, Heckman: no disc. pattern	Fairly constant, concave
as function of wealth measures	–	Increasing	Rather time invariant	Increasing	Weakly increasing	–	Not very responsive

Due to the complexity of the problem and the computation time involved, it is impossible to study all potential determinants of life cycle investing simultaneously. Each model focuses on only a few determinants, as is illustrated in Figure 2. This figure arranges the models included in this review according to four dimensions: the assumptions about the investor's preferences (at the top), the labor income process (at the bottom), and the asset classes available. The distinction between non-financial assets (on the right) and financial assets (on the left) is important, because the retirement income is primarily financed by financial assets and their proceeds. Of course, the distinction is in some way artificial, since stocks and real estate assets cannot always be unambiguously classified.

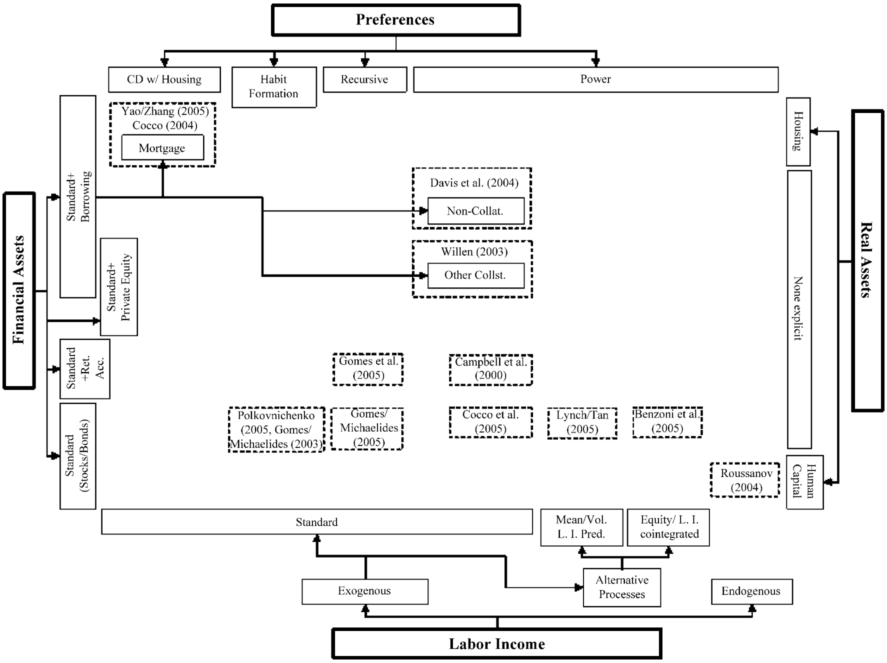

Fig. 2 Classification of reviewed models. The models are surrounded by dotted lines, the classification scheme consists of drawn-through lines

The basic Cocco et al. (2005) model presented in the last section can be seen as a combination of standard assumptions. Our 'routing' through the other contributions is as follows. We will first describe the consequences of including further financial assets apart from stocks and a risk-free savings account. Secondly, we will turn to alternative preference structures, where the focus will be on habit formation utility and recursive preferences. The assets and the labor income processes are kept at their basic formulation. In the third step, we will examine attempts to endogenize assets omitted in previous models, namely human capital and housing. Finally, the implications of more realistic exogenous income processes will be considered. This routing reflects the principal steps taken in the literature to come up with results that are more consistent with empirical observations.

3.2 Retirement savings account

Campbell et al. (2000) extend the basic model of Sect. 2 to analyse welfare implications of alternative retirement savings systems.[21] The investor in Campbell et al. (2000) is assumed to have a mandatory retirement savings account into which she has to pay a fraction of each period's log labor income.

Compared to the basic model above, the retirement account in Campbell et al. (2000) restricts the investor's choices and thus produces a welfare loss. How does the investor react to the new restrictions? Consider first the case where savings in the retirement account are fully invested in the riskfree asset. Compared to the basic model, savings in the retirement account now crowd out liquid wealth. As in the basic model, stockholdings in liquid wealth display a decrease and subsequent increase in midlife, but are generally more aggressive due to the sizable riskfree asset position in the retirement account.[22]

When half of the mandatory account is invested in the risky asset,[23] the decrease and subsequent increase of the equity share in midlife is now more pronounced and pointed. The equity share reaches a low level of about 25% shortly before retirement. The main conclusions, however, are the same as in the basic model.

3.3 Realistic borrowing regimes

A realistic borrowing regime is interesting to study in a life cycle model due to its ambiguous effects on wealth accumulation: On the one hand, the investor might borrow in order to level out transitory income shocks and to smooth consumption. This usually comes at the expense of lower wealth accumulation. On the other hand, provided that the net expected return from levered equity holdings is positive, a higher leverage will increase wealth on average. As Davis et al. (2004) show, findings in this context critically depend on the relative magnitudes of the riskfree rate, the borrowing rate and the expected return on equity: a borrowing rate exceeding the riskfree rate (but not the expected return on equity) is found to decrease average absolute equity holdings substantially and a borrowing rate equal to the expected return on equity (*baseline case*) is found to minimize average absolute equity demand. Also, the participation rate is found to be significantly lower in the *baseline case* than in any of the other borrowing regimes: it is found to increase from 25% at age 20 to full participation not before age 50.

These findings are due to the fact that the relation between the cost of borrowing and equity demand is non-monotonic in the model by Davis et al. (2004): as the borrowing rate increases from the risk free rate towards the equity return,

[21] The model by Campbell and Viceira (2002) is a simplified version of Campbell et al. (2000) used for illustrative purposes in their volume on long term portfolio choice. The only difference to Campbell et al. (2000) is the assumption that all wealth in the illiquid retirement account is invested in the riskfree asset and that there is no fixed cost of stock market participation.

[22] Campbell et al. (2000) set the risk aversion parameter to 5, whereas we used a value of 7. Thus, the set-ups are not perfectly comparable.

[23] In this case, Campbell et al. (2000) reduce the contribution rate from 10% to 6% in order to keep the average replacement rate at the same level as before.

leverage premium and borrowing capacity (present value of lowest possible future labor income discounted at borrowing rate) decrease, which implies declining average equity holdings. When the borrowing rate increases beyond the expected equity return so that the leverage premium becomes negative, investors continue to borrow only to smooth consumption or income shocks and no longer to 'lever up' their equity holdings. The higher the borrowing rate the less inclined they are to borrow. They will thus accumulate more wealth which leads to higher average equity holdings.

Davis et al. (2004) compare predicted debt and equity shares with empirical data from the *Survey of Consumer Finances*. Assuming their *baseline case*, they find a distinctly better fit to the empirical debt data as well as the data on equity holdings and participation rates than in the base model without borrowing. The authors conclude: "In short, a realistic treatment of borrowing also brings the theory closer to the evidence on life-cycle patterns in equity holdings and participation rates".[24]

In a related paper, Willen (2003) shows that a realistic treatment of credit markets can also help to explain the frequently observed dramatic dependence of entrepreneurial households on only one entrepreneurial asset. His model is identical to that of Davis et al. (2004) except for an enriched asset universe. It contains an entrepreneurial asset with an uncertain return, an unsecured loan with limited exposure at a low rate and unlimited exposure at a higher rate. Additionally, there is a loan with a rate between the riskfree rate and the lower unsecured loan rate, that must be collateralized either by equity or by the entrepreneurial asset. Willen (2003) shows that wealth is invested almost exclusively in private equity if the wealth-to-income ratio is low, but that this exposure decreases in wealth-to-income. With rising wealth-to-income ratios over the life-cycle, private equity holdings (as a fraction of all risky investments) will exhibit a decreasing lifetime pattern.

3.4 Alternative preference structures

Preferences used in the basic model are separable across time and states of nature. This means that the marginal utility of consumption in one state of nature and point in time is independent of consumption in other states or points in time. But, as Cochrane (1997) argues, it could well be the case that "the marginal utility of consumption in the sunny state of the world is affected by the level of consumption in the rainy state"[25] and that "yesterday's consumption can have an impact on today's appetite".[26] These arguments call for time and state non-separable preference specifications. Following the literature on the equity premium puzzle,[27] two common non-separabilities have been analysed in life cycle models: habit formation and recursive preferences according to Epstein and Zin (1989).

[24] Davis et al. (2004), p. 20.
[25] Cochrane (1997), p. 19.
[26] Cochrane (1997), p. 20.
[27] Cochrane (1997) and Campbell (2003) review this literature.

3.4.1 Models with habit formation utility

Habit formation preferences formalize the idea that investors get used to their standard of living so that they attempt to make sure that resources will always suffice to consume above habit. Investors with low wealth are exposed to a high risk of consumption to fall short of habit. The young, poor investor would thus be expected to invest more conservatively. This could potentially resolve puzzles ii-b and iii-c.

Gomes and Michaelides (2003) consider habit formation in the model of Sect. 2. The Gomes and Michaelides (2003) investor has objective function (1) only that instantaneous utility is now defined over consumption C_t and habit H_t. They propose two specifications, where utility is either driven by the ratio of consumption to habit (*ratio habit specification*) as in (15):

$$u(C_t, H_t) = \frac{1}{(1-\gamma)} \left(\frac{C_t}{H_t^{\omega_{hs}}} \right)^{(1-\gamma)}, \tag{15}$$

or it is driven by the difference between consumption and habit (*difference habit specification*) as in (16):

$$u(C_t, H_t) = \frac{1}{(1-\gamma)} \left(C_t - \omega_{hs} H_t \right)^{(1-\gamma)}, \tag{16}$$

where ω_{hs} is a parameter that measures habit strength. Habit itself is modelled as a weighted average of past habit and past consumption:

$$H_t = (1 - \omega_{hp}) H_{t-1} + \omega_{hp} C_{t-1}, \tag{17}$$

where ω_{hp} denotes the weight.[28] In a very similar model presented by Polkovnichenko (2006), habit is more simply postulated to be a function of lagged consumption only.

Contrary to the power utility models considered in Sections 2, 3.2 and 3.3, habit formation entails important additional constraints on the evolution of endogenous state variables: First, current consumption must exceed current habit in each period; secondly, current consumption is chosen such that, even for the worst possible realizations of labor income and stock return shocks, resources suffice to consume above habit in the future. Polkovnichenko (2006) derives and illustrates these constraints analytically.[29] Due to their existence, the investment policy strongly depends on the level of wealth.

For intermediate to high wealth, the investment behavior in habit models is similar to expected utility models.[30] However, the closer cash-on-hand is to its minimum feasible level, the more conservative young and old investors must invest to alleviate the magnitudes of the worst case shocks. The effect is more pronounced for young investors because income uncertainty plays a more important role for them.

[28] Subscript *hp* stands for habit persistance.

[29] The constraints maintain that, in a given period and for a given current habit, current cash plus the minimum present value of labor income over a specific horizon must exceed the infimum of the present value of future habits over the same horizon.

[30] See Sect. 2.5 above.

Therefore, unlike the standard CRRA model, one would expect poor, young households to have a conservative equity share in a difference habit model. But contrary to this expectation, neither Gomes and Michaelides (2003) nor Polkovnichenko (2006) find this to be the case in simulations of their baseline models. Gomes and Michaelides (2003), e.g., find an average forward simulated equity share of essentially 100% at all ages. This result proves to be robust in sensitivity analyses. Similarly, Polkovnichenko (2006) finds only minor differences between his baseline simulated equity-age-profile and that implied by an otherwise identical CRRA model.

It is only when Polkovnichenko (2006) calibrates his model with a 0.5% probability of a zero labor income draw in any of the working life years that he finds more conservative stock holdings among young investors in his habit model. Full equity specialization is delayed until the investor's early fourties when it reaches its maximum and starts to decrease again.[31] However, the equity share is still high for young investors: By the age of 30, they are already invested by more than 80% in stocks.[32]

In summary, for the considered ratio and difference habit specifications, equity shares are found to be unrealistically high at close to 100% for all ages. Portfolios are only slightly interior at very young ages. Gomes and Michaelides (2003) conclude: "Contrary to the initial motivation, we find that introducing habit formation preferences in the standard life-cycle asset allocation model actually decreases its ability to match the observed empirical regularities. (...) We conclude that internal habit formation preferences on their own are unlikely to resolve existing portfolio composition puzzles." [33] Only with a disastrous labor income shock in the Polkovnichenko (2006) model do more plausible, hump-shaped and interior age profiles obtain. But, as shown by Cocco et al. (2005), such a shock is able to generate similar profiles in a model with CRRA preferences, too. Nevertheless, the model by Polkovnichenko (2006) is able to resolve one of the puzzles, since it implies - in line with the data - that the risky asset share should increase with cash-on-hand.

3.4.2 Models with recursive preferences and preference heterogeneity

The Epstein and Zin (1989) generalization of the power utility function allows to disentangle the degree of relative risk aversion and the elasticity of intertemporal substitution. Risk aversion affects investment in the risky asset directly as well as indirectly through its effect on precautionary savings. The elasticity of intertemporal substitution mainly accounts for the magnitude of retirement savings. Since total savings are an important determinant of asset allocation behavior, one might expect that appropriately 'tuned' risk aversion and elasticity of intertemporal substitution could resolve one or more of the portfolio puzzles.

[31] These findings remain essentially unchanged for different utility curvature and strenghts of the bequest motive.
[32] With a low income state that obtains with a 1.3% probability, results are qualitatively identical, but young investors' equity share is even higher.
[33] Gomes and Michaelides (2003), p. 5.

The Epstein–Zin investor derives utility from current consumption C_t and from expected, discounted utility next period according to the recursion

$$u_t = \Bigg\{ \left(1 - \beta \psi_{t+1|t}\right) C_t^{1-\frac{1}{\eta}} + \beta E_t \left[\psi_{t+1|t} u_{t+1}^{1-\gamma} + \left(1 - \psi_{t+1|t}\right) v(X_{t+1}) \right]^{\frac{1-1/\eta}{1-\gamma}} \Bigg\}^{\frac{1}{1-1/\eta}}, \tag{18}$$

with terminal condition

$$u_{T+1} = \frac{\iota}{(1-\gamma)} \left(\frac{X_{T+1}}{\iota} \right)^{1-\gamma}, \tag{19}$$

where γ is the coefficient of relative risk aversion and η the elasticity of intertemporal substitution. Considering the second term inside the curly brackets on the right hand side of (18), it becomes apparent that the expected, discounted utility next period is a survival probability weighted function of next period's regular utility, u_{t+1}, realized in case of survival and of next period's utility over bequested wealth, $v(X_{t+1})$, realized in case of death.[34] The weights are given by the conditional probabilities of survival and death next period $\psi_{t+1|t}$ and $\left(1 - \psi_{t+1|t}\right)$, respectively. Unlike standard power utility, preference specification (18) does not require η and γ to be inversely related, but it nests this constellation as a special case: setting $\eta = \frac{1}{\gamma}$ in (18) makes the recursion linear. Solving it forward then yields the standard power utility model.[35]

With $\eta = \frac{1}{\gamma}$, the model by Gomes and Michaelides (2005) is practically identical to the baseline case in Cocco et al. (2005). However, reducing risk aversion (γ) while maintaining a low elasticity of intertemporal substitution (η) produces dramatically lower wealth accumulation at all ages. Highest wealth is now only about 17% of peak wealth in the baseline case. Consequently, with a fixed cost of market participation, full participation in the stock market can be delayed significantly. Yet, due to the lower risk aversion, the equity share of those, who have already payed the fixed cost of participation, is again a flat 100% at all ages. Thus, a recursive preference formulation alone cannot generate both a sensible participation rate and, conditional on participation, a sensible equity profile.

To overcome this problem, Gomes and Michaelides (2005) relax the identical agent assumption. They first solve the optimization problem of an investor with relatively low γ and low η and that of an investor with higher γ and higher η. In the forward simulation[36] they then assume a population composed only of these two types of investors. With this preference heterogeneity the model economy displays a participation rate and an average equity share among participants close to data from the *Survey of Consumer Finances*. A remaining deficiency of this experiment is that it slightly overestimates participation during retirement and more significantly overestimates the equity share of very young investors.

[34] The functional form of $v(\bullet)$ is given by the right-hand side of (19).

[35] See also Campbell and Viceira (2002), p. 42–44.

[36] Actually, Gomes and Michaelides (2005) compute transition distributions, but they also carry out regular Monte Carlo simulations; see their footnote 12.

The model by Gomes et al. (2004) blends elements of Campbell et al. (2000) and Gomes and Michaelides (2005) and extends them to a more realistic tax environment. As in Campbell et al. (2000), the investor owns a liquid savings account and an illiquid retirement account. But while the former is now assumed to be a taxable account, the latter is tax-deferred. Therefore, the investors try to profit from tax arbitrage between the two accounts. As an additional complication, Gomes et al. (2004) model two types of investors, indirect and direct stockholders. The indirect stockholders can hold equity only in their tax-deferred account and are assumed to have low risk aversion (γ) and elasticity of intertemporal substitution (η), whereas direct stockholders are characterized by high γ and η.[37]

In the forward simulation, the age profile of equity as a share of wealth is found to be decreasing in both accounts. Direct stockholders hold mixed portfolios in both accounts which seems to be in conflict with tax arbitrage, but is in line with empirical data. This implication of the model is due to the cross sectional averaging in an economy populated by investors with different lengths of access to the tax-deferred account. The indirect stockholders are less concerned about background risk (due to lower γ) and saving for retirement (due to lower η). Thus, they accumulate less wealth in both accounts and invest nearly the entire tax-deferred wealth in equity.

3.5 Endogenizing human capital

Endogeneity of human capital can be achieved by assuming that the agent derives utility from leisure and receives labor income in relation to her working hours. In this style, Bodie et al. (1992) compare the consumption and portfolio choice problem of an investor who decides on labor supply once and for all to that of an agent who can continuously vary her supply under both a riskless and a risky wage regime. In continuous time the authors are able to derive closed form solutions from which a main lesson can be learnt: Everything else equal, greater labor supply flexibility permits particularly the young investor to hold a higher fraction of financial wealth in risky assets. More recently, Bodie et al. (2004) derive closed forms for the joint consumption-, labor/leisure- and portfolio choice problem of a continuous time life cycle investor (finite horizon, explicit retirement phase) with general habit formation preferences, a risky wage rate and access to a financial market with many assets and stochastic coefficients. With this model, they study the implications of habits and of a deferred retirement date on optimal consumption-, labor/leisure- and portfolio choice.[38]

A different approach to endogenize human capital is suggested by Roussanov (2004). He grants the investor an option to increase her human capital by investing in education. His investor has a standard power utility objective function with

[37] Both direct and indirect shareholders are further subdivided in groups of investors that differ in the number of years since they had first access to a tax-deferred account.

[38] Bodie et al. (1992) and Bodie et al. (2004) are not directly comparable to the models reviewed here: Bodie et al. (1992) is a continuous time model without borrowing restrictions. The wage rate is assumed to be perfectly positively correlated with the return on the risky asset in the risky wage regime and labor income is presumed to be insurable. Similarly, in Bodie et al. (2004) asset holdings are not constrained and borrowing against future labor income is possible for the greater part of the analysis. If borrowing against future labor income is not permitted, they are able to derive explicit solutions for consumption and labor choice, but not for portfolio choice.

a terminal bequest motive. The investor controls her labor income by deciding whether to work or to study in a given period. While working, labor income is an iid random variable which is augmented by the deterministic return to education after having successfully completed the study program. The investor can study only once.[39] While studying, she cannot work, but has to pay education costs. Contrary to previous formulations, working life is not distinguished from retirement through different labor income characteristics. Instead, a high bequest intensity proxies for a retirement phase.

In this setting, Roussanov (2004) finds the equity share to be a non-monotonic function of wealth prior to completion of studies. For low levels of wealth, the investor has a high equity share because riskless investing would not generate sufficient return to afford education. For wealth close to education costs, investing is found to be conservative to ensure sufficiency of funds for education next period. For magnitudes of wealth other than these two extremes, the equity share is found to be a decreasing function of wealth, as in the basic model. In forward simulations, Roussanov (2004) mainly finds interior, hump-shaped age-equity-profiles with a peak of below 80% for all considered levels of initial wealth. Once again, a seemingly unrealistic implication of the Roussanov (2004) model is that poor young investors are fully invested in stocks, but in this setting only as long as they have not yet invested in education.

3.6 Endogenizing housing

A potentially important determinant of savings and investment behavior omitted so far is housing. By considering labor income as net of rental payments, all models analysed so far implicitly assume that housing services can only be rented. Introducing a realistic housing framework is not trivial since housing is at the same time a durable consumption good yielding utility and an asset class of its own.[40]

Cocco (2004) assumes an investor with objective (3), where $\psi_{t+1|t} = 1 \; \forall \, t$ and $u(\bullet)$ is a Cobb–Douglas function with H_t the units of housing consumed and ω_{hp} a parameter measuring relative housing preference:

$$u(C_t, H_t) = \frac{\left(C_t^{1-\omega_{hp}} H_t^{\omega_{hp}}\right)^{1-\gamma}}{(1-\gamma)}. \tag{20}$$

This means the investor is now additionally concerned about consuming units of housing, with a house of bigger size yielding higher utility. House prices are assumed to be a function of the permanent component of labor income. Owning and selling a house involve maintenance and selling costs, respectively. The investor can mortgage against her house at a constant rate up to a given fraction of house value.

In a cross sectional regression of simulated equity share on several explanatory variables, Cocco (2004) finds a positive correlation between financial net worth – defined as the sum of equity, riskless asset holdings net of debt and house value – and the weight of stocks in the portfolio of liquid assets. Also, equity share

[39] The duration of the education is a function of an exogenously given, positive termination probability.

[40] See, e.g., Yao and Zhang (2005).

is found to be positively correlated with mortgage debt because investors with higher future labor income invest more aggressively and are also found to borrow more. Another prediction of the model is that investors with low financial net worth hold leveraged portfolios that are strongly concentrated in real estate. Stock market participation is low among these investors. Stock holdings as a fraction of assets are generally found to increase during working life. It is only after retirement that stock holdings as a fraction of financial and total assets (also including discounted labor income) are lowered, whereas they continue to increase relative to liquid assets. Stock holdings are found to be substantially lower for higher risk aversion, stronger housing preference and higher discount rates. A fixed cost of stock market participation is found to reduce participation significantly. This is due to the lower liquid asset holdings among real estate owners.

Yao and Zhang (2005) study an extended model that allows the agent not only to *own* housing as in Cocco (2004), but also to *rent* housing services. If a renter in a given period, the investor can keep renting or start owning next period. If an owner, she can either keep the house or sell the house and subsequently choose between renting or owning (a house of different size). Yao and Zhang (2005) also assume an exogenous, binomial moving shock which – at times – forces the investor to move and thus – if not a renter – to sell her house. The investor has objective (3) with $u(\bullet)$ given by (20).[41] Housing is considered as an asset class with stochastic return which may be correlated with labor income growth. A mortgage with limited exposure is available to house-owners.

An advantage of the Yao and Zhang (2005) model is that it allows to analyse welfare and portfolio choice implications of the simplifying assumptions 'owning only' (Cocco (2004)) and 'renting only' (all other models in this paper). Both assumptions exclude *combinations* of both possibilities and therefore entail welfare losses: for the assumption 'renting only' these are found to be in the order of 8% of total current wealth, whereas they can reach 25% of total current wealth if only owning is allowed. The simplifying assumptions also induce a bias in portfolio choice: whereas the equity share is slightly upward biased when only renting is allowed, it exhibits a more severe downward bias when only owning is possible.

When stock and housing returns are positively correlated as compared to a zero correlation, owners have a lower equity share, while renters have a higher share throughout life due to the renters' effective short position in housing.

Interesting results also emerge from simulating the model forward for renters aged 20 with zero initial wealth: The fraction of investors owning a house increases rapidly and by age 40, the majority are owners. Interestingly, the average renter aged 20 is predicted to be fully invested in stocks, but to reduce her exposure gradually while accumulating savings to pay down a house. When the renter becomes an owner, the equity share in total assets is reduced (crowding out), whereas it is increased in liquid wealth (diversification benefit). The older the investor, the lower is the equity share in both liquid wealth and net worth.

[41] The bequest motive is different from the one in earlier models in that the investor intends to bequeath an annuity over a given horizon to provide for the beneficiary's consumption and housing. For details, see Yao and Zhang (2005), p. 7.

3.7 Different labor income and stock return dynamics

In all previous models, the vector of innovations to stock returns and labor income $\boldsymbol{\delta}_t \equiv (e_t v_t \zeta_t)'$ was assumed to be independently and identically distributed, where at most a contemporaneous correlation between the shock to stock returns ζ_t and the shock to the permanent component of labor income v_t was allowed. Lynch and Tan (2004) argue for a more realistic treatment of the dynamics of $\boldsymbol{\delta}_t$ that incorporates a macroeconomic perspective. Their idea is to let the conditional distribution of $\boldsymbol{\delta}_t$ depend on the business cycle so that the first two moments of log labor income growth can be explained by a variable that is known to predict the business cycle. Empirically, they find that log labor income growth is negatively and volatility of log labor income growth positively correlated with *lagged* annual dividend yields. Because dividend yields are *counter-cyclical*, this implies that log labor income growth is *pro-cyclical*, whereas volatility of labor income growth is *counter-cyclical*. Since realized stock return is *pro-cyclical*, this further means that realized stock return and future "labor income opportunities" [42] (i.e. labor income growth and volatility of labor income growth) are positively correlated. A positive relation between current returns and future investment opportunities generates a negative hedging demand for stocks by (risk averse) long term investors. Thus, an investor who incorporates predictability of the first two moments of labor income growth into her rationale should have lower equity holdings. This reduction should be stronger for young investors, as labor income plays a particularly prominent role for them.

And indeed, as compared to a standard power utility model without predictability, Lynch and Tan (2004) find that the presence of the two predictability channels (labor income growth and volatility of labor income growth) reduces average stock holdings dramatically for investors with low to medium wealth income ratios in the first month of their horizon. The reduction is of a magnitude of 77.2% for zero wealth to income and still amounts to 68% for wealth to income of 30. An interesting finding also pertains to the average equity share as a function of the wealth to income ratio: with the volatility channel present it becomes sharply increasing for low wealth to income ratios and flat for higher wealth to income ratios. With regard to life cycle effects, Lynch and Tan (2004) find that the two predictability channels lead to a lower leveled hump-shaped age-equity profile peaking around age 50. Finally, adding the two business cycle channels results in elevated non-participation among the youngest investors. For instance, a 22 year old investor with zero wealth is a non-participant in 80% of the cases in the first year. Non-participation is then found to decrease with age, reaching levels below 10% at the early 40's.

Similar to Lynch and Tan (2004), Benzoni et al. (2005) argue that an advanced treatment of labor income and stock return dynamics might produce more realistic life cycle equity holdings. They consider a life cycle model with the standard transitory and permanent idiosyncratic labor income shocks as well as a potentially low contemporaneous correlation between permanent labor income shocks and stock returns. Additionally, as a central element of their model, they account for possible cointegration between labor income and stock returns.

The model by Benzoni et al. (2005) predicts that young agents are not invested in the risky asset approximately until age 30 when stockholdings begin to increase

[42] Lynch and Tan (2004), p. 2.

sub-proportionally to reach a maximum of around 50% shortly before retirement. This finding is due to cointegration: Young investors hold wealth only in the form of future labor income, and the return on their human capital is highly susceptible to stock market risk due to the cointegration effect. Thus, they draw the consequence not to hold any stocks. For older investors with fewer years of working life remaining, return on human capital is less susceptible to market risk because cointegration does no longer affect their future labor income.

4 Conclusions and future perspectives

The main characteristics of all models reviewed in this paper are summarized in Table 2. We have learnt the following lessons from this literature: *standard power utility* life cycle models do not resolve any of the existing portfolio puzzles, they rather contribute to the disaccord between theoretical and empirical results. Introducing a fixed cost of stock market participation to these standard models delays full stock market participation slightly among the very young, but is of no help otherwise. Similarly, *habit formation preferences* do not resolve the puzzles either. Only with a disaster state in the labor income process can participation among the young and conditional equity shares be somewhat reduced. Models with *recursive preferences* give mixed results. On the one hand, endowing the representative investor with low risk aversion and a low elasticity of intertemporal substitution results in a low stock market participation. On the other hand, a low risk aversion predicts high conditional equity shares. Only by forcing preference heterogeneity on the population, both the participation puzzle and the conditional equity share puzzle can be partly resolved.

Endogenizing labor income by granting the investor an opportunity to invest in her own human capital generates interior, hump-shaped age-equity profiles. However, it still implies high equity shares among young investors as these are willing to take significant risk in order to increase the likelihood of earning enough to soon start education.

Models that allow the investor to consume *housing services* and *own real estate* are able to resolve all puzzles at least partially. Some results indicate that it might be important to consider ownership and rental markets at the same time. Yet, such models are cumbersome due to their complex renting/owning dynamics and can only be solved with supercomputers.

Finally, models with *advanced labor income and stock return dynamics* appear to be very successful: they are entirely standard among all model dimensions, yet extend the labor income specification to include either a correlation between current stock returns and future investment opportunities or a cointegration of labor income and stock returns. Both specifications elegantly resolve all puzzles.

In summary, the literature on life cycle portfolio choice has effectively developed a *positive* tradition, according to which portfolio "puzzles" are tackled by the presumption that investors do not generally make "wrong" decisions, but have objective functions or are subject to restrictions not adequately accounted for in the early models. The new models help to resolve "puzzles" by better explaining the prevailing investment behavior. Most importantly, the literature highlights the decisive role played by an investor's labor income in determining optimal lifetime savings and asset allocation.

Table 2 Model overview

Authors	Objectives	Main model features	Main results
Benzoni et al. (2005)	Build model that matches emp. data on equity hold. and stock market partic. by incorpor. cointegration of labor inc. and stock returns	Cont. time model w/ cointegrated lab. inc. and stock returns	Young investors not invested in equities before age 30, then subproport. incr. of equity share to 50% upon ret.
Campbell et al. (2000)	Compare ret. savings systems w/ diff. inv. risk characteristics, under borrowing and stock market partic. constraints	Illiquid ret. savings account under altern. asset allocation regimes, fixed cost of stock market partic.	Welfare gains from equity investm. in ret. acc. if contr. rate is reduced, high partic. and cond. equity shares dur. life
Campbell/Viceira (2002)	Illustrate life cycle models	Same as CCGM except ret. acc. inv. riskfree; no partic. cost	–
Cocco (2004)	Study implications of endogenous housing on portfolio choice in life cycle setting	Housing cons. and ownership possible, mortage avlbl.	Equity share pos. corr. with financial net worth and mortage debt, share incr. during working life
Cocco et al. (2005)	Study effects of heterog. uninsureable labor income and utility costs of suboptimal portfolio rules	Various forms of income shocks, stochast. and non stoch. ret. inc.	Baseline: equity share 100% for ages 25–40, decrease to 50% at 65, slight incr. during ret., substantial welfare costs of suboptimal portf. strat.
Davis et al. (2004)	Study effects of diff. borrowing regimes, build model that matches unsecured debt holdings and equity shares in emp. data	Features borrowing, at rates higher than riskfree rate	Predicts more realistic life cycle profiles for borrowing and equity as fractions of income
Gomes/Michaelides (2003)	Study effects of internal additive and ratio habit formation preferences	Additive and altern. ratio habit formation preferences	Equity share close to 100% throughout life
Gomes/Michaelides (2005)	Match empirical fact of low stock market participation and low cond. equity shares	Epstein Zin preference heterogeneity	With heterog. preferences: matches emp. life cycle partic. rates, slightly overestimates equity share among the young
Gomes et al. (2004)	Study welfare effects of tax deferred ret. accounts (TDA), portfolio choice in presence of TDA‘s, with population of direct and indirect stockholders	Epstein Zin preferences, taxable and tax deferred ret. acc., direct/indirect stockholders	Welfare gains from TDA large for inv. w/ high RA, strong savings incent. of employer matching for high RA-HH, low welfare cost of fixed contrib. rate if it is HH spec.
Lynch/Tan (2004)	Build model that matches emp. data on equity hold. and stock market partic. by incorpor. predictab. of labor inc. growth	Advanced labor inc. dynamics: dividend yield as bussiness cycle predictor, risky ass. ret. and div. yield modelled as VAR	High non-partic. among young, hump-shaped, interior age-equity profiles, low equity holdings among poor young, equity share incr. in wealth/income ratio
Polkovnichenko (2004)	Study effects of internal additive habit formation preferences	Additive habit formation preferences	With disaster state of lab. inc.: hump shaped age-equity profiles, equity share incr. in wealth
Roussanov (2004)	Study effects of endog. illiquid human wealth on dyn. portfolio choice	Investor has option to invest in human capital once	Hump-shaped age-equity profiles, high equity share predicted for poor young investors
Willen (2003)	Build model that predicts low portfolio diversification in entrepreneurial HH as obs. in empirical data	Features an entrepr. asset with stochast. return, secured and unsecured borrowing	Realistic treatment of credit markets leads to real. predict. of portf. of entrepr. HH
Yhao/Zhang (2005)	Study effects of housing ownership and rental market on dyn. portfolio choice, estimate utility cost of simplifying assumption “housing only” or “renting only”	Feature house renting and ownership, with rich dynamics	Interior, hump-shaped equity hold., “renting only”: max welfare cost 8% of current wealth, portf. share upward biased, “owning only” max. welfare cost 25% of curr. wealth, portf. downward biased

However, a shortcoming of the "fitting" ambition prevailing in the literature is that it tends to lose *normative* ground because it does not provide constructive life-cycle investment advice for private investors. Campbell (2006) shows that a minority of households, particularly those with less education, actually make investment choices which almost surely do not maximize their welfare. To get these investors to change their behaviour, it seems important to offer insight from models thoroughly adapted to the specific household situation. These models should adopt a normative approach, which means that the *assumptions* should fit household characteristics, but model parameters should *not* be "fitted" to the current portfolio structures. The latter kind of "fitting", which is characteristic for the *positive* approach, should only be carried out if there is reason to believe that actual and welfare-maximizing behaviour of sample households coincide. In future research, it seems important to find ways to separate these different types of investors and to adopt a positive or normative approach accordingly.

Another shortcoming of the literature is that all models are set in partial equilibrium, i.e. asset returns and earnings processes or wage rates are not determined endogenously. However, partial equilibrium might not suffice when evaluating welfare consequences of policy proposals in applied work, e.g. in the field of retirement provision.

We thus see the following future perspectives for the branch of research adressed in this paper: first, there is a tendency to integrate life-cyle models into large scale models with aggregate demand and supply features or into a general equilibrium analysis. For example, Cerny et al. (2005) analyse the impact of demographic changes and various pension reform proposals on the demand for housing and financial assets in the UK. In their model, the investor solves a life-cycle problem similar to those presented here, but embedded in an aggregate OLG model.

Second, several recent models can explain some or even all of the stylized empirical findings on household portfolio choice.[43] As the models are partially competing, this could create an increased interest in methods to identify them empirically as well as in ways to assess and compare their 'goodness of fit'. In the consumption literature, the *method of simulated moments* is often used in related exercises. For this method to be applicable to the estimation of problems similar to those reviewed here, however, two conditions need to be fulfilled: on the one hand, improved and richer micro data sets are required and on the other hand, methods are needed to overcome the computational burden associated with simulated method of moments estimators in situations where it is prohibitively time-consuming to generate data from the economic model, as is the case with elaborate dynamic programming problems like those reviewed here.

Third, we see scope for a move of the literature to normative reasoning and to providing life-cycle investment advice for private investors. The results obtained so far call for a more accurate and detailed analysis of individual and household labor income dynamics. Several questions are immediately apparent, such as: Which features of labor income differ across investors? How do these differences affect life cycle portfolio choice? Can we catalogue stylized features of labor income processes together with the implied optimal portfolio behaviour? To establish such a taxonomy of labor income features, it will be necessary to have a closer empirical look at the labor income process. Analyses of the correlation patterns between

[43] See Yao and Zhang (2005), Lynch and Tan (2004) and Benzoni et al. (2005).

labor income and various assets' return processes as well as potential cointegration structures (in the spirit of Lynch and Tan (2004) and Benzoni et al. (2005)) are of eminent interest. This points to the importance of empirical labor economics for life-cycle portfolio choice and suggests a closer collaboration between researchers in the fields of financial and labor economics in the future. Ultimately, this research could then enable us to better understand how investors *should* behave, why their behavior might look puzzling and where they might actually be wrong.

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