Few-Body Syst (2011) 50: 367–369
DOI 10.1007/s00601-010-0200-y

**Ingo Sick**

# Troubles with the Proton rms-Radius

Received: 1 November 2010 / Accepted: 15 December 2010 / Published online: 14 January 2011


**Abstract** Due to the peculiar shape of the proton charge density $\rho(r)$ the value of the $rms$-radius $r_{rms}$ determined from electron scattering data depends strongly on the density $\rho(r)$ at large radii, which is not fixed by scattering data. Supplementing the data with the large-$r$ shape of $\rho(r)$ resulting from the Fock components $(\mathrm{n}+\pi, \ldots)$ dominating the large-$r$ behavior produces a more reliable value. This radius agrees with the one we previously extracted, but disagrees with the one recently obtained from muonic Hydrogen. The origin of the discrepancy is not understood.

The proton charge $rms$-radii determined from individual experiments on elastic electron–proton scattering cover the range 0.82–0.92 fm. A reanalysis of the *world* e-p cross section and polarization transfer data [1], which fixed several shortcomings, yielded $r_{rms} = 0.894 \pm 0.018$ fm. It is worrisome, though, that other recent determinations [2,3] using the same data base find values between 0.877 and 0.912 fm.

Further study reveals, that the unusual shape of $\rho$—roughly an exponential one—is responsible. The upper limit of the integral that determines $r_{rms}$ for $R = \infty$, $[\int_0^R \rho(r) r^4 dr]^{1/2}$, has to be extended to 2.8 times the $rms$-radius in order to get to 98% of the true radius, see Fig. 1. For the data used to get the radius, typically at momentum transfers $0.5 < q < 1.5$ fm$^{-1}$, the effect of the tail density responsible for the remaining 2% amounts to < 0.2%, which is not measurable with available technology.

As a consequence, the value of $r_{rms}$ depends on assumptions on the large-r behavior implicit in the parameterization of the form-factor $G_e(q)$ used to fit the data. From the e-p data alone there thus is little hope to determine $r_{rms}$ with a model uncertainty as low as one percent.

In order to overcome this uncertainty we have analyzed the e-p data by adding a constraint on the *shape* of the density at large $r$, where $\rho(r) < 0.01\, \rho(0)$. At large $r$, one expects the least-bound Fock component—for the proton the $\mathrm{n} + \pi^+$-component—to entirely dominate the density. The asymptotic shape of the $\pi$ wave function, a Whittaker function $W_{-\eta,3/2}(2\kappa r)/r$, is easily calculated from the pion mass and removal energy. Folding of the corresponding density with the intrinsic pion charge distribution, known from $e - \pi$ scattering, yields the shape of the proton charge density $\rho(r)$ for very large $r$. Accounting for other, more strongly bound, Fock-components of the proton involving the $\Delta$ [4] changes little, other that allowing to explain with the same parameters the tail densities for proton *and* neutron, see Fig. 2.

We have analyzed the *world* e-p data, both cross sections and analyzing powers, up to a momentum transfer of 12 fm$^{-1}$, including the 2-photon exchange corrections [2] and including the tail density for $r > 1.3$ fm (corrected for relativistic effects [7–10]). The data were fitted with $G_e(q)$ and $G_m(q)$ parameterized in terms of SOG [11].

Presented at the 21st European Conference on Few-Body Problems in Physics, Salamanca, Spain, 30.8–3.9.

I. Sick (✉)
Department of Physics, University of Basel, Basel, Switzerland
E-mail: ingo.sick@unibas.ch

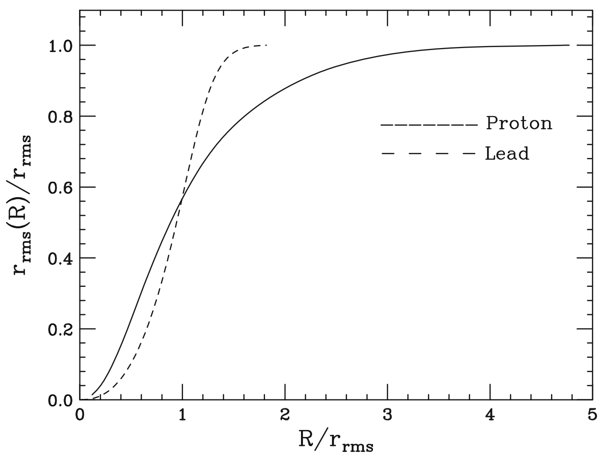


**Fig. 1** $[\int_0^R \rho(r)\, r^4 dr / \int_0^\infty \rho(r)\, r^4 dr]^{1/2}$ as a function of the upper integration limit $R$

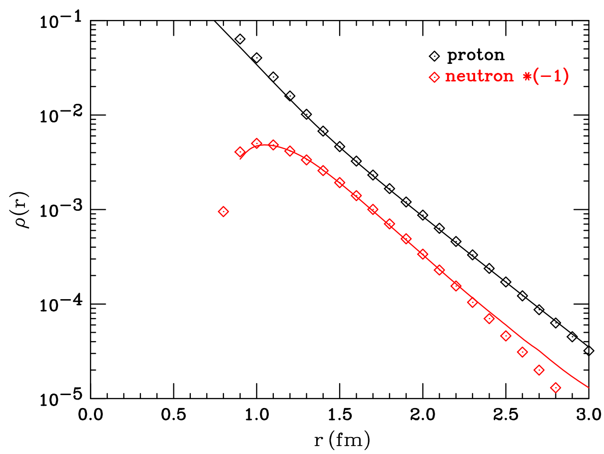


**Fig. 2** Tail densities for proton and neutron. *Diamond* density from published phenomenological fits of the e-p data [1,5,6], *line* asymptotic shape from model described in text

For 605 data points, the $\chi^2$ of the fit is 518 (812) when floating (or not) the normalization of the individual cross section data sets. The resulting *rms* charge radius is $0.894 \pm 0.008$ fm, the error bar covering both the random and the systematic errors. The radius agrees with the one previously extracted [1], but is substantially more accurate.

Contrary to fits *without* tail constraint, where the floating of the data enhances the problem of the model dependence and changes the radius by $> 0.02$ fm, the change with constraint is negligible, $< 0.002$ fm. The fixed shape of the tail density suppresses unphysical behavior of the fit at very low $q$ where $G_e(q)$ is not constrained by the data.

This radius can be compared to the one extracted from transition energies in atomic Hydrogen, $0.877 \pm 0.007$ fm, see [12,13].

Recently, the radius has been determined via the Lamb shift in muonic Hydrogen [14]. Pohl et al. find $r_{rms} = 0.8418 \pm 0.0007$ fm, in stark contrast to the values determined from both e-p scattering and electronic Hydrogen.

We list below a number of potential sources for this disagreement, without being able to identify a clear “suspect”.

Are there missing QED-terms in the analysis of the $\mu$-X ray data? The same type of calculations led to good agreement between radii from $\mu - X$ and (e,e) for $A \geq 12$.

Are the polarization/two photon exchange corrections in doubt? The calculated ones both for $\mu$-X and (e,e) are very small; more than order-of-magnitude increases would be needed.

Are there common problems to all (e,e) data such as radiative corrections? At the low $q$ relevant here, these would show up basically as a $q$-independent factor, and would be compensated when floating the normalization of the data.

Does the present set of (e,e) data have some common defect? A new MAMI experiment [15] gives a radius in agreement with the one from the previous *world* data when analyzed the same way, *i.e.* by floating the data and *without* tail constraint.

Is there a problem with the Zemach moment [16] as claimed by de Rujula who proposes a huge $\langle r^3 \rangle_{(2)} = 36.6\ \text{fm}^3$? The third Zemach moment has been determined [17] from a high-quality Pade fit of the *world* e-p data [1] $\langle r^3 \rangle_{(2)} = 2.71(13)\ \text{fm}^3$.

Could there be uncertainties in the solution of the (relativistic) 2-body problem? There is excellent agreement between the respective radii for the heavier nuclei, so at least for the case of small recoil effects this is not a solution.

So, for the time being we are faced with a serious discrepancy that calls for an explanation.

## References


1. Sick, I.: On the rms-radius of the proton. Phys. Lett. B **576**, 62 (2003)
2. Arrington, J., Melnitschouk, W., Tjon, J.A.: Global analysis of elastic form factor data with two-photon exchange corrections. Phys. Rev. C **76**, 035205 (2007)
3. Borisyuk, D.: Proton charge and magnetic rms radii from elastic ep scattering data. Nucl. Phys. A **843**, 59 (2010)
4. Dziembowski, Z., Holtmann, H., Szcsurek, A., Speth, J.: Pionic corrections to nucleon electromagnetic properties in a light-cone framework. Ann. Phys. **258**, 2 (1997)
5. Galster, S., et al.: Elastic electron–deuteron scattering and the electric neutron form factor. Nucl. Phys. B **32**, 221 (1971)
6. Kelly, J.J.: Simple parameterizations of nucleon form factors. Phys. Rev. C **70**, 068202 (2004)
7. Mitra, A.N., Kumari, I.: Relativistic form factor for clusters with nonrelativistic wave functions. Phys. Rev. D **15**, 261 (1977)
8. Holzwarth, G.: Baryons as solitons in effective chiral field theories. Prog. Part. Nucl. Phys. **36**, 161 (1996)
9. Ji, X.: A relativistic skyrmion and its form factors. Phys. Lett. B **254**, 456 (1991)
10. Licht, A.L., Pagnamenta, A.: Wave functions and form factors for relativistic composite particles. Phys. Rev. D **2**, 1156 (1970)
11. Sick, I.: Model-independent nuclear charge densities from elastic electron scattering. Phys. Lett. B **44**, 62 (1972)
12. Udem, T., et al.: Phase-coherent measurement of the hydrogen 1s–2s transition frequency with an optical frequency interval divider chain. Phys. Rev. Lett. **79**, 2646 (1997)
13. Melnikov, K., van Ritbergen, T.: Three-loop slope of the Dirac form factor and the 1S Lambshift in hydrogen. Phys. Rev. Lett. **84**, 1673 (2000)
14. Pohl, R., et al.: The size of the proton. Nature **466**, 213 (2010)
15. Bernauer, J.C., et al.: High-precision determination of the electric and magnetic form factors of the proton. Phys. Rev. Lett. **105**, 242001 (2010)
16. de Rujula, A.: QED is not endangered by the proton size. Phys. Lett. B **693**, 555 (2010)
17. Friar, J.L., Sick, I.: Muonic hydrogen and the third Zemach moment. Phys. Rev. A **72**, 040502 (2005)