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A Distributional Approach to Multiple Stochastic Integrals and Transformations of the Poisson Measure

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Abstract We study a class of 'nonpoissonian' transformations of the configuration space and the corresponding transformations of the Poisson measure. For some class of Poisson measures we find conditions which are sufficient for the transformed measure (which in general is nonpoissonian) to be absolutely continuous with respect to the initial Poisson measure and get the expression for the corresponding Radon–Nikodym derivative. To solve this problem we use a distributional approach to Poisson multiple stochastic integrals.

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Key words Poisson random measures · multiple stochastic integrals · absolute continuity · Radon–Nikodym derivative.

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1 Introduction

Let G be a metric space, $\mathcal{B}$ its Borel σ-algebra, $\mathcal{B}_0$ the ring of bounded Borel subsets of G. Let Π be a σ-finite measure on G. Suppose that $\Pi(V) < \infty$ for every $V \subset \mathcal{B}_0$.

We denote by $\mathcal{X} = \mathcal{X}(G)$ the space of (locally finite) configurations on G. By definition

$$\mathcal{X}(G) = \{X \subset G : |X \cap V| < \infty \text{ for all V} \subset \mathcal{B}_0\},$$

where $|A|$ denotes the cardinality of the set A. We equip $\mathcal{X}$ with the vague topology $\mathcal{O}(\mathcal{X})$, i.e., the weakest topology such that all functions $\mathcal{X} \to \mathbb{R}$

$$X \mapsto \sum_{x \in X} f(x)$$

are continuous for all continuous functions f: $G \to \mathbb{R}$ with bounded support. The Borel σ-algebra corresponding to $\mathcal{O}(\mathcal{X})$ will be denoted by $\mathcal{B}(\mathcal{X})$. This is the smallest σ-algebra for which the mapping

$$X \mapsto |X \cap V|$$

is measurable for any $V \subset \mathcal{B}_0$.

We say that a probability measure P on $(\mathcal{X}, \mathcal{B}(\mathcal{X}))$ is the Poisson measure with intensity measure Π, if for every $V \in \mathcal{B}_0$

$$P(X : |X \cap V| = k) = e^{-\Pi(V)} \frac{\Pi(V)^k}{k!}.$$

For more details, see e.g. [1, 6, 7]

The interesting case of course is the one where $\Pi(G) = \infty$. Only in this case the typical configurations are infinite. Thus from now on we suppose $\Pi(G) = \infty$.

In what follows we suppose that $G = S \times [0, \infty)$, where S is a complete separable metric space, and the measure Π is of the form $\Pi(\mathrm{d}\theta, \mathrm{d}x) = \pi(\mathrm{d}\theta)p(x)\mathrm{d}x$, $\theta \in S$, $x \in [0, \infty)$, $\mathrm{d}x$ is a Lebesgue measure on $[0, \infty)$, π is a finite measure on S.

We suppose that for every $\varepsilon > 0$ the function p is strictly positive, bounded, continuous on the set $[\varepsilon, \infty)$.

Suppose that the measure Π satisfies the condition

$$\int_G \min(x, 1)\mathrm{d}\Pi < \infty \tag{1}$$

It follows from (1) that with probability 1 the projection of the configuration X into $[0, \infty)$ has the single limit point 0 and, for any $\varepsilon > 0$, $|X \cap G_\varepsilon| < \infty$, where $G_\varepsilon = S \times [\varepsilon, \infty)$.

Let F be a measurable mapping F: $\mathcal{X} \to \mathcal{X}$, and PF^{-1} be the corresponding transformation of the measure P. By definition, for any $A \in \mathcal{B}(\mathcal{X})$

$$PF^{-1}(A) = P(F^{-1}(A)). \tag{2}$$

In the present paper we find conditions such the measure PF^{-1} is absolutely continuous with respect to the measure P and get then the expression for the corresponding Radon–Nikodym derivative $\frac{\mathrm{d}PF^{-1}}{\mathrm{d}P}$.

For the special case of the mapping

$$X = \{(\theta, x)\}_{(\theta,x)\in X} \mapsto F(X) = \big\{\big(\theta,\ f(\theta, x)\big)\big\}_{(\theta,x)\in X} \tag{3}$$

where for every $\theta \in S$ $f(\theta, \cdot) : [0, \infty) \to [0, \infty)$ is a diffeomorphism, the transformation of the Poisson measure is nothing but a change of intensity measure. The intensity measure of the transformed Poisson measure is equal to $\pi(\mathrm{d}\theta)\frac{p(f^{-1}(x))}{|f'(f^{-1}(x))|}\mathrm{d}x$. For this 'Poissonian' transformation the absolute continuity conditions were first obtained by Skorokhod [15] (see also [1, 4, 8, 18]). We remark that for mappings of this type the 'new' position of each point depends on the 'old' position only and does not depend on the other points of the configuration.

In the present paper we consider 'nonpoissonian' transformations of the measure P. This means that the 'new' position of each point depends not only on the 'old' position but also on other points of the configuration. Nonpoissonian transformations of the Poisson processes have been studied by Privault and coworkers [9–14]. Privault's results correspond to the techniques developed by Ramer, Kusuoka and Zakai for the Wiener case. For the Poisson process, Privault used a probability space of the form of a countable product of the exponential distribution and the product of countably many copies of Lebesgue measure on [0, 1]. In our work we develop our analysis on the space of configurations. It is known that the space of configurations can be considered as an infinite dimensional manifold [1, 17]. It can also be considered as a suitable probability space for interesting random processes (see e.g.[2, 16]).

We should note that our work is not based on the L_2-theory, because some of the interesting applications (for example nongaussian stable measures) do not have finite second moments. Instead of the L_2-theory on which much work on Poisson infinite dimensional analysis are based we use another approach based on the theory of generalized functions [5]. We consider the Poisson measure P as a generalized function and under the condition (1) construct its regularization. For this we propose a new construction of Poisson multiple stochastic integrals. This is based on the point of view of generalized functions and is different from the theory of iterated integrals with respect to compensated Poisson measure developed in a paper by Surgailis [17].

The paper is organized as follows. In the first section we construct the regularization formula for the Poisson measure P. In the second section we consider the problems connected with definition of the mapping $\mathcal{X} \to \mathcal{X}$. In the third section we get a sufficient condition for the absolute continuity of the transformed measure. In the fourth section we consider some examples.

2 The Multiple Stochastic Integrals

By $\mathcal{X}_0$ we denote the space of all finite configurations. For this space we have a representation of the form

$$\mathcal{X}_0 = \bigcup_{k=0}^{\infty} G^k/\pi_k, \tag{4}$$

where G^k/π_k is the factor space of G^k with respect to the permutation group π_k. In this representation, the term with $k = 0$ corresponds to the empty configuration, the term with $k = 1$ corresponds to single-point configurations, the next term corresponds to double-point configurations, and so on.

We consider $S \times \{0\}$ as an absorbing state in $S \times [0, \infty)$ i.e. for every $\theta \in S$ the point $(\theta, 0)$ disappears from the configuration. This means that at first we equip $\mathcal{X}$ with the topology of disjoint unions and that for every $k \in \mathbb{N},\ \theta \in S$ we then identify the following configurations:

$$\{(\theta_1, x_1), \dots, (\theta_k, x_k), (\theta_{k+1}, 0)\} = \{(\theta_1, x_1), \dots, (\theta_k, x_k)\}. \tag{5}$$

In other words for every k we 'attach' $G^k \times (S \times \{0\})$ to G^k.

Let f: $\mathcal{X}_0 \to \mathbb{R}$ be a function on $\mathcal{X}_0$. In accordance with (4) f has a representation of the form

$$f = (f_0, f_1, f_2, \dots),$$

where f_k is the restriction of f to G^k/π_k. By definition f_k is a symmetric function on G^k. Using (5) it follows that for every $k \in \mathbb{N},\ \theta \in S$

$$f_{k+1}((\theta_1, x_1), \dots, (\theta_k, x_k), (\theta_{k+1}, 0)) = f_k((\theta_1, x_1), \dots, (\theta_k, x_k)). \tag{6}$$

Let us define the sequence of difference operators Δ^k, $k \in \mathbb{N}$. For every $k \in \mathbb{N}$ the domain of the operator Δ^k is taken to be set of all functions f from G^k into $\mathbb{R}$. For f: $G \to \mathbb{R}$ we have

$$\Delta f(\theta, x) = \Delta^1 f(\theta, x) = f(\theta, x) - f(\theta, 0). \tag{7}$$

Similarly, for a function f: $G^k \to \mathbb{R}$ we put

$$\Delta^k f = \Delta_{x_1} \Delta_{x_2} \dots \Delta_{x_k} f. \tag{8}$$

By definition $\Delta^k f$ is the increment of f on the rectangle $[0, x] \subset [0, \infty)^k$, $x = (x_1, \dots, x_k)$ (the parameter $\theta \in S^k$ is fixed).

Now we define the multiple stochastic integral. At first we define it as a function on $\mathcal{X}_0$.

Definition 1 Let $g = g(\overline{x}_1, \dots, \overline{x}_k)$, $\overline{x}_i = (\theta_i, x_i)$ be a symmetric function on G^k. We define the k-multiple stochastic integral $I_k(g)$ of g in the following way

$$I_k(g)(X) = \sum_{\{\overline{x}_1, \dots, \overline{x}_k\} \subset X} \Delta^k g(\overline{x}_1, \dots, \overline{x}_k), \tag{9}$$

where $X \in \mathcal{X}_0$. The summation in the latter expression is extended over all subsets of X which consist of k points (namely $\{\overline{x}_1, \dots, \overline{x}_k\}$).

If $k = 0$ then $g = g_0 = const$ and by definition we put $I_0(g) = g_0$.

Lemma 1 For every symmetric function g: $G^k \to \mathbb{R}$ we have

$$|I_k(g)| \le I_k(|\Delta^k g|). \tag{10}$$

Proof We have

$$\begin{aligned}|I_k(g)(X)| &= \Big|\sum_{\{\overline{x}_1,\ldots,\overline{x}_k\}\subset X} \Delta^k g(\overline{x}_1,\ldots,\overline{x}_k)\Big| \\ &\le \sum_{\{\overline{x}_1,\ldots,\overline{x}_k\}\subset X} |\Delta^k g(\overline{x}_1,\ldots,\overline{x}_k)| \\ &= \sum_{\{\overline{x}_1,\ldots,\overline{x}_k\}\subset X} \Delta^k |\Delta^k g(\overline{x}_1,\ldots,\overline{x}_k)| \\ &= I_k(|\Delta^k g|).\end{aligned}$$

□

Theorem 1 *For every* $f\colon \mathcal{X}_0 \to \mathbb{R}$ *we have*

$$f = \sum_{k=0}^{\infty} I_k(f_k).$$

Proof Let us define boundary operators δ_i, $i = 1, \ldots, k$, $k \in \mathbb{N}$. By definition, the function $\delta_i f_k$ is obtained from f_k by taking $x_i = 0$. Obviously, $\delta_i = 1 - \Delta_i$, where by Δ_i we denote the above defined operation Δ with respect to the variable x_i.

For every finite set I of natural numbers, put

$$\delta_I = \prod_{i\in I} \delta_i, \quad \Delta_I = \prod_{i\in I} \Delta_i.$$

For every fixed n, let CI denote the set $\{1, \ldots, n\} - I$. We note that for any f the function $\delta_I f_n$ depends only the variables (θ_i, x_i), $i \in CI$.

Let $|X| = n$, $X = \{\overline{x}_1, \ldots, \overline{x}_n\}$, $\overline{x}_i = (\theta_i, x_i)$. Using the identity

$$1 = \prod_{i=1}^{n} (\delta_i + \Delta_i) = \sum_I \Delta_I \delta_{CI} \tag{10a}$$

we get

$$\begin{aligned} f(X) = f_n(\overline{x}_1, \ldots, \overline{x}_n) &= \sum_{I\subset\{1,\ldots,n\}} \Delta_I \delta_{CI} f_n \\ &= \sum_{k=0}^{n} \sum_{I\subset\{1,\ldots,n\}, |I|=k} \Delta_I \delta_{CI} f_n \\ &= \sum_{k=0}^{n} \sum_{\substack{\{j_1,\ldots,j_k\}\subset \\ \subset\{1,\ldots,n\}}} \Delta^k f_k(\overline{x}_{j_1}, \ldots, \overline{x}_{j_k}) \\ &= \sum_{k=0}^{n} I_k(f_k) = \sum_{k=0}^{\infty} I_k(f_k).\end{aligned}$$

This completes the proof. □

Our next aim is to extend the stochastic integrals I_k to $\mathcal{X}$. Obviously it is possible to extend them directly and pointwise using (8) only if the support of the function g is a compact set in G^k. Otherwise we can extend I_k only P-a.s.

Suppose $f = f(\overline{x}_1, \dots, \overline{x}_k)$, $\overline{x}_i = (\theta_i, x_i)$ is a symmetric function on the domain G^k, and let $I = \{i_1, \dots, i_j\}$ be an arbitrary subset of $\{1, \dots, k\}$. We shall use the short notation

$$\partial_I = \frac{\partial^j f}{\partial x_{i_1} \dots \partial x_{i_j}}.$$

Let $\delta > 0$ be a positive number, by $\| f\|_{k,\delta}$ we denote the norm

$$\| f\|_{k,\delta} = \sum_{i=0}^{k} \binom{k}{i} \sup_{G(i,\delta)} |\partial_{\{1,\dots,i\}} f| (c_1(\delta))^i (2c_2(\delta))^{k-i}, \tag{11}$$

where

$$G(i, \delta) = (G \setminus G_\delta)^i \times (G_\delta)^{k-i},$$

$$c_1(\delta) = \int_{G\setminus G_\delta} x \mathrm{d}\Pi, \ c_2(\delta) = \int_{G_\delta} \mathrm{d}\Pi. \tag{12}$$

Lemma 2 If f is a symmetric function on G^k then for any $\delta > 0$

$$\left| \int_{G^k} \Delta^k f \mathrm{d}\Pi^k \right| \leq \| f\|_{k,\delta} \tag{13}$$

Proof Denote by B_i the set of all $\overline{x} \in G^k$, $\overline{x} = ((\theta_1, x_1), \dots, (\theta_k, x_k))$ such that cardinality of the set $\{j : x_j \leq \delta\}$ is equal to i. It is clear that the B_i are disjoint and

$$\bigcup_{i=0}^{k} B_i = G^k.$$

Using the symmetry of the function f we get

$$\left| \int_{B_i} \Delta^k f \mathrm{d}\Pi^k \right| \leq \binom{k}{i} \sup_{G(i,\delta)} |\partial_{\{1,\dots,i\}} f| (c_1(\delta))^i (2c_2(\delta))^{k-i}.$$

This completes the proof. □

For every function f: $\mathcal{X}_0 \to \mathbb{R}$, $f = (f_k)_{k\in\mathbb{N}_0}$, $\mathbb{N}_0 = \{0\} \cup \mathbb{N}$ and any arbitrary number $\varepsilon > 0$ we define the function f_ε: $\mathcal{X} \to \mathbb{R}$ by the formula

$$f_\varepsilon(X) = f(X \cap G_\varepsilon). \tag{14}$$

f_ε can be looked upon as an approximation of f.

We say that a measurable function $f\colon \mathcal{X}_0 \to \mathbb{R},\ f = \{f_k\}_{k=0}^{\infty}$ belongs to the space $FA(\delta)$ if the following norm is finite

$$\|f\|_{FA(\delta)} = \sum_{k=0}^{\infty} \frac{\|f_k\|_{k,\delta}}{k!}. \tag{15}$$

We write $\mathbb{E}\, f_\varepsilon$ for $\int f_\varepsilon \mathrm{d}P$.

Theorem 2 *If $f \in FA(\delta)$ then for every $\varepsilon > 0$*

$$\mathbb{E}\, f_\varepsilon = \sum_{k=0}^{\infty} \frac{1}{k!} \int_{G_\varepsilon^k} \Delta^k f_k \mathrm{d}\Pi^k. \tag{16}$$

Proof We have

$$\int f_\varepsilon \mathrm{d}P = e^{-\Pi(G_\varepsilon)} \sum_{k=0}^{\infty} \frac{1}{k!} \int_{G_\varepsilon^k} f_k \mathrm{d}\Pi^k. \tag{16a}$$

Using the identity (10a) we get

$$\begin{aligned}\int_{G_\varepsilon^k} f_k \mathrm{d}\Pi^k &= \sum_I \Pi(G_\varepsilon)^{|I|} \int_{G_\varepsilon^{|CI|}} \Delta_{CI}\delta_I f_k \prod_{i\in CI} \Pi(\mathrm{d}\bar{x}_i)\\ &= \sum_{j=0}^{k} \Pi(G_\varepsilon)^j \binom{k}{j} \int_{G_\varepsilon^{k-j}} \Delta^{k-j} f_{k-j} \mathrm{d}\Pi^{k-j}.\end{aligned}$$

Substituting this expression into (16a) we obtain

$$\begin{aligned}\int f_\varepsilon \mathrm{d}P &= e^{-\Pi(G_\varepsilon)} \sum_{k=0}^{\infty} \frac{1}{k!} \sum_{j=0}^{k} (\Pi(G_\varepsilon))^{k-l} \binom{k}{k-l} \int_{G_\varepsilon^l} \Delta^l f_l \mathrm{d}\Pi^l\\ &= e^{-\Pi(G_\varepsilon)} \sum_{l=0}^{\infty} \frac{1}{l!} \sum_{k=l}^{\infty} \frac{\Pi(G_\varepsilon))^{k-l}}{(k-l)!} \int_{G_\varepsilon^l} \Delta^l f_l \mathrm{d}\Pi^l\\ &= \sum_{l=0}^{\infty} \int_{G_\varepsilon^l} \Delta^l f_l \mathrm{d}\Pi^l.\end{aligned}$$

This completes the proof. □

Corollary We have

$$\mathbb{E}(I_k(g))_\varepsilon = \frac{1}{k!} \int_{G_\varepsilon^k} \Delta^k g \mathrm{d}\Pi^k. \tag{17}$$

Proof This follows from Theorem 2 and (9). □

Later we prove that, for every $f \in FA(\delta)$ the approximating function f_ε (14) converges in L_1 as $\varepsilon \downarrow 0$ to some extended function $f^{\sim}$. At first we consider this problem for the stochastic integrals.

Theorem 3 *Let $g\colon G^k \to \mathbb{R}$ be a measurable symmetric function and assume $\|g\|_{k,\delta} < \infty$. Then*

$$\lim_{\varepsilon\to 0}(I_k(g))_\varepsilon$$

exists in the L_1 sense.

Remark We call this limit an extended stochastic integral and use for it the same notation $I_k(g)$ as in (9).

Proof For every $\varepsilon > 0$ let g_ε denote the function

$$g_\varepsilon((\theta_1, x_1), \dots, (\theta_k, x_k)) = g((\theta_1, x_1\mathbf{1}_{[\varepsilon,\infty)}(x_1)), \dots, (\theta_k, x_k\mathbf{1}_{[\varepsilon,\infty)}(x_k))),$$

where $\mathbf{1}_{[\varepsilon,\infty)}$ is the indicator function of the set $[\varepsilon, \infty)$.

It is easy to show that

$$\Delta^k g_\varepsilon = \Delta^k g \cdot \mathbf{1}_{G^k_\varepsilon} \tag{18}$$

and

$$(I_k(g))_\varepsilon = I_k(g_\varepsilon).$$

Let $0 < \varepsilon_1 < \varepsilon_2$. We have

$$\begin{aligned}\mathbb{E}|(I_k(g))_{\varepsilon_1} &- (I_k(g))_{\varepsilon_2}| \\ &= \mathbb{E}|(I_k(g) - I_k(g_{\varepsilon_2}))_{\varepsilon_1}| \\ &= \mathbb{E}|(I_k(g - g_{\varepsilon_2}))_{\varepsilon_1}|\end{aligned}$$

Using (10), (13), (17) and (18) we get

$$\begin{aligned}\mathbb{E}|(I_k(g - g_{\varepsilon_2}))_{\varepsilon_1}| &\le \mathbb{E}(I_k(|\Delta^k(g - g_{\varepsilon_2})|))_{\varepsilon_1} \\ &= \frac{1}{k!}\int_{G^k_{\varepsilon_1}} \Delta^k(|\Delta^k(g - g_{\varepsilon_2})|)\mathrm{d}\Pi^k \\ &= \frac{1}{k!}\int_{G^k_{\varepsilon_1}} |\Delta^k(g - g_{\varepsilon_2})|\mathrm{d}\Pi^k \\ &= \frac{1}{k!}\int_{G^k_{\varepsilon_1}\setminus G^k_{\varepsilon_2}} |\Delta^k g|\mathrm{d}\Pi^k \underset{\varepsilon_1,\varepsilon_2\to 0}{\longrightarrow} 0.\end{aligned}$$

Using Cauchy's criterion we get the statement of the theorem. □

Now we construct the extended function associated with an arbitrary function $f \in FA(\delta)$.

Theorem 4 *For every function $f = \{f_k\}_{k=0}^\infty \in FA(\delta)$ the L_1-limit of f_ε as $\varepsilon \downarrow 0$ exists. Calling it $f^\sim$ we have*

$$f^\sim = \sum_{k=0}^\infty I_k(f_k) \tag{19}$$

(note that in (19) we use the extended stochastic integral $I_k(f_k)$ given by Theorem 3).

Proof Using Theorem 1 and (14) we get

$$f_\varepsilon = \sum_{k=0}^{\infty} (I_k(f_k))_\varepsilon.$$

Every term of this sum converges to $I_k(f_k)$ as $\varepsilon \to 0$. Using (10), (17) and (5) we get

$$\mathbb{E}|(I_k(f_k))_\varepsilon| \le \mathbb{E}(I_k(|\Delta^k f_k|))_\varepsilon = \frac{1}{k!}\int_{G_\varepsilon^k} |\Delta^k f_k| \mathrm{d}\Pi^k$$
$$\le \frac{1}{k!}\int_{G^k} |\Delta^k f_k| \mathrm{d}\Pi^k \le \frac{1}{k!}\| f_k\|_{k,\delta}.$$

This completes the proof. □

Corollary For $F \in FA(\delta)$ we have

$$\mathbb{E} f^{\sim} = \int f^{\sim} \mathrm{d}P = \sum_{k=0}^{\infty} \frac{1}{k!}\int_{G^k} \Delta^k f_k \mathrm{d}\Pi^k. \tag{20}$$

Proof This follows easily by taking $\varepsilon \to 0$ in (16). □

Remark Let us make a few comments on formula (20). Consider the measure P as a linear functional $f \mapsto (P, f) = \mathbb{E} f$. If the function f is finite in the sense that it only depends on the configurations in a finite volume, (so that $f(X) = f(X \bigcap G_\varepsilon)$ for some $\varepsilon > 0$) then we can represent our linear functional in the following form

$$(P, f) = \mathbb{E} f = e^{-\Pi(G_\varepsilon)} \sum_{k=0}^{\infty} \frac{1}{k!}\int_{G^k} f_k \mathrm{d}\Pi^k.$$

But this formula works only if f is finite. For arbitrary f we need some regularization [5]. One can consider the formula (20) as a regularization of this linear functional. It is not surprising that this regularization is in the form of a sum where each term $\int_{G^k} \Delta^k f_k \mathrm{d}\Pi^k$ is a standard regularization [5] of the linear functional $(\Pi^k, f_k) = \int_{G^k} f_k \mathrm{d}\Pi^k$. For the first time this regularization formula appears in [16]. Note that in [3] a formula for the regularization of the linear functional (P, f) was already proved in the case where the measure Π has a stronger singularity at the point 0 (so that $\int_G \min(x, 1)\mathrm{d}\Pi = \infty$ and $\int_G \min(x^2, 1)\mathrm{d}\Pi < \infty$). In this case the regularization formula for the linear functional $(P, f) = \mathbb{E} f$ has the following form

$$(P, f) = \mathbb{E} f = \sum_{k=0}^{\infty} \frac{1}{k!}\int_{G^k} \widetilde{\Delta}^k f_k \mathrm{d}\Pi^k,$$

where $\int_{G^k} \widetilde{\Delta}^k f_k \mathrm{d}\Pi^k$ is the standard regularization of the linear functional $(\Pi^k, f_k) = \int_{G^k} f_k \mathrm{d}\Pi^k$, so that for $k = 1$ we have

$$\widetilde{\Delta} f(\theta, x) = f(\theta, x) - f(\theta, 0) - x f'(\theta, x) \cdot \mathbf{1}_{[0,1]}(x).$$

3 The Extension of the Mapping from $\mathcal{X}_0$ to $\mathcal{X}$

Initially we define our mapping as a mapping from $\mathcal{X}_0$ to $\mathcal{X}_0$ and then, using results of the second section, construct an extended mapping. Note that we always consider mappings that change only the real coordinate of each point of the configuration (so that $X = \{(\theta_1, x_1), \ldots, (\theta_k, x_k)\} \mapsto Y = \{(\theta_1, y_1), \ldots, (\theta_k, y_k)\}$).

Let $F_0 = F_0(\overline{x}, X)$ be a continuous function of the arguments $\overline{x} = (\theta, x) \in G,\ X \in \mathcal{X}_0$ and taking values in $[0, \infty)$. Suppose that

a) F_0 is differentiable with respect to $x \in [0, \infty)$ with $\frac{\partial}{\partial x} F_0 > 0$,
b) For every $L > 0$

$$\sup_{\overline{x} \in S \times [0, L]} \| F_0(\overline{x}, \cdot) - F_{0,\varepsilon}(\overline{x}, \cdot) \|_{FA(\delta)} \underset{\varepsilon \to 0}{\longrightarrow} 0. \tag{21}$$

Let us define the mapping F: $\mathcal{X}_0 \to \mathcal{X}_0$ by the rule

$$X = \{\overline{x}_1, \ldots, \overline{x}_k\} = \{(\theta_1, x_1), \ldots, (\theta_k, x_k)\} \mapsto$$

$$Y = \{(\theta_1, F_0(\overline{x}_1, X)), \ldots, (\theta_k, F_0(\overline{x}_k, X))\} \tag{22}$$

(by definition we put $\{\emptyset\} \mapsto \{\emptyset\}$).

By F_k we denote the restriction of F to G^k so that $F = \{F_k\}_{k=0}^{\infty}$. Using Theorem 4 and (21) for every $\overline{x} \in G$ we extend (in the L_1-sense) the function $F_0(\overline{x}, \cdot)$ to $\mathcal{X}$. Moreover, for every $L > 0$, we shall extend this function uniformly with respect to $\overline{x} \in S \times [0, L]$.

Let $\{L_n\},\ L_n > 0,\ L_n \to \infty$ be an increasing sequence of positive numbers. Using standard arguments we prove that for P -a.s. $X \in \mathcal{X}$ the function $F_0((\theta, x), X)$ is continuous with respect to $x \in [0, \infty)$. We use for this extended function F: $\mathcal{X} \to \mathcal{X}$ the same notation F.

4 The Absolute Continuity Conditions

Let $F = \{F_k\}_{k=0}^{\infty}$ be a mapping $\mathcal{X}_0 \to \mathcal{X}_0$ defined as in the third section, and let us denote by the same letter F also the extended (in the sense explained in the third section mapping.

We suppose that for every $k \in \mathbb{N}$ and for every $\theta_1, \ldots, \theta_k \in S$ the mapping $(x_1, \ldots, x_k) \mapsto (y_1, \ldots, y_k)$, where

$$((\theta_1, y_1), \ldots, (\theta_k, y_k)) = F_k((\theta_1, x_1), \ldots, (\theta_k, x_k)), \tag{23}$$

is a diffeomorphism. By J_k we denote the corresponding Jacobi matrix.

By Φ we denote the class of all bounded, continuous and finite function on $\mathcal{X}$ (where finite is understood in the sense of the remark at the end of the second section).

It is obvious that if we find a function $u \in L_1(\mathcal{X}, P)$ such that for every $\varphi \in \Phi$

$$\int \varphi \circ F \mathrm{d}P = \int \varphi u \mathrm{d}P \tag{24}$$

then we get immediately that the measure PF^{-1} is absolutely continuous with respect to P and

$$\frac{\mathrm{d}PF^{-1}}{\mathrm{d}P} = u.$$

In what follows we at first define the function u as a function on $\mathcal{X}_0$ and then, using the methods of the second section extend this function from $\mathcal{X}_0$ to $\mathcal{X}$.

Using (20) we get

$$\int \varphi \circ F \mathrm{d}P = \sum_{k=0}^{\infty} \frac{1}{k!} \int_{G^k} \Delta^k (\varphi_k \circ F_k) \mathrm{d}\Pi^k.$$

After a change of variables in every term of the latter sum we obtain

$$\int \varphi \circ F \mathrm{d}P = \sum_{k=0}^{\infty} \frac{1}{k!} \int_{G^k} \Delta^k \varphi_k \psi_k \mathrm{d}\Pi^k, \tag{25}$$

where $\psi_k = \frac{\mathrm{d}\Pi^k F_k^{-1}}{\mathrm{d}\Pi^k}$, so that for $\overline{x} = (\overline{x}_1, \ldots, \overline{x}_k)$, $\overline{x}_j = (\theta_j, x_j)$, $p_k(\overline{x}) = p(x_1)p(x_2)\ldots p(x_k)$

$$\psi_k(\overline{x}) = \frac{p_k(F_k^{-1}(\overline{x}))}{p_k(\overline{x})|J_k(F_k^{-1}(\overline{x}))|}, \tag{26}$$

(where we assumed that p_k and J_k do not vanish). Now we transform the R.H.S. of (24). Using (20) we get

$$\int \varphi u \mathrm{d}P = \sum_{k=0}^{\infty} \frac{1}{k!} \int_{G^k} \Delta^k (\varphi_k u_k) \mathrm{d}\Pi^k. \tag{27}$$

Let us define boundary operators δ_i, $i = 1, \ldots, k$, $k \in \mathbb{N}$. By definition, the function $\delta_i f_k$ is obtained from f_k by taking $x_i = 0$. Obviously, $\delta_i + \Delta_i$ is the identity operator, where by Δ_i we denote the operation Δ with respect to the variable x_i.

For every finite set I of natural numbers, we put

$$\delta_I = \prod_{i \in I} \delta_i, \quad \Delta_I = \prod_{i \in I} \Delta_i.$$

By CI we denote the set $\{1, \ldots, k\} \setminus I$.

Lemma 3 Let f, g be symmetric functions on G^k. We have

$$\Delta^k(fg) = \sum_{I \subset \{1,\ldots,k\}} \Delta_I \delta_{CI} f \Delta_{CI} g, \tag{28}$$

where the summation in the latter expression is extended over all subsets of $\{1, \ldots, k\}$.

Proof We have

$$\begin{aligned}\Delta^k(fg) &= \Delta^k\Big(\big(\prod_{i=1}^{k}(\Delta_i+\delta_i)f\big)g\Big)\\ &= \Delta^k\Big(\sum_{I\subset\{1,\ldots,k\}}(\Delta_I\delta_{CI}f)g\Big)\\ &= \sum_{I\subset\{1,\ldots,k\}}\Delta^k((\Delta_I\delta_{CI}f)g).\end{aligned}$$

To conclude the proof, it remains to observe that

$$\Delta^k((\Delta_I\delta_{CI}f)g) = \Delta_I\delta_{CI}f\Delta_{CI}g.$$

□

If we combine (27) and (6) with Lemma 3 we get (l being equal to the cardinality of the set I)

$$\begin{aligned}\int\varphi u\mathrm{d}P &= \sum_{k=0}^{\infty}\frac{1}{k!}\sum_{l=0}^{k}\binom{k}{l}\int_{G^k}\Delta^l\varphi_l\Delta^{k-l}u_k\mathrm{d}\Pi^k\\ &= \sum_{k=0}^{\infty}\sum_{l=0}^{k}\frac{1}{l!(k-l)!}\int_{G^k}\Delta^l\varphi_l\Delta^{k-l}u_k\mathrm{d}\Pi^k\\ &= \sum_{l=0}^{\infty}\frac{1}{l!}\sum_{k=l}^{\infty}\frac{1}{(k-l)!}\int_{G^k}\Delta^l\varphi_l\Delta^{k-l}u_k\mathrm{d}\Pi^k\\ &= \sum_{l=0}^{\infty}\frac{1}{l!}\sum_{k=0}^{\infty}\frac{1}{k!}\int_{G^{k+l}}\Delta^l\varphi_l\Delta^k u_{l+k}\mathrm{d}\Pi^{l+k}\\ &= \sum_{l=0}^{\infty}\frac{1}{l!}\int_{G^l}\Delta^l\varphi_l\Big(\sum_{k=0}^{\infty}\frac{1}{k!}\int_{G^k}\Delta^k u_{l+k}\mathrm{d}\Pi^k\Big)\mathrm{d}\Pi^l. \qquad (29)\end{aligned}$$

Now we define an operator A. The domain of the operator A is taken to be the set of measurable functions $\{f:\mathcal{X}_0\to\mathbb{R},\ f=\{f_k\}_{k=0}^{\infty}\}$ for which $\sum_{k=0}^{\infty}\frac{1}{k!}\int_{G^k}|\Delta^k f_{l+k}|\,\mathrm{d}\Pi^k$ is finite for any $l\geq 0$. By definition we put

$$(Af)_l = \sum_{k=0}^{\infty}\frac{1}{k!}\int_{G^k}\Delta^k f_{l+k}\mathrm{d}\Pi^k. \qquad (30)$$

Substituting (30) in (29) we get

$$\int\varphi u\mathrm{d}P = \sum_{l=0}^{\infty}\frac{1}{l!}\int_{G^l}\Delta^l\varphi_l(Au)_l\mathrm{d}\Pi^l = \sum_{k=0}^{\infty}\frac{1}{k!}\int_{G^k}\Delta^k\varphi_k(Au)_k\mathrm{d}\Pi^k. \qquad (31)$$

Comparing (24), (25) and (31) we get the equation

$$Au=\psi. \qquad (32)$$

Our next aim is to construct the inverse operator A^{-1}. For this we define another operator T.

For the symmetric function g: $G^k \to \mathbb{R}$ we define the function $Tf : G^{k-1} \to \mathbb{R}$ by the formula

$$Tg(\overline{x}_1, \ldots, \overline{x}_{k-1}) = \int_G \Delta_k g(\overline{x}_1, \ldots, \overline{x}_{k-1}, \overline{x}_k) \mathrm{d}\Pi(\overline{x}_k) \tag{33}$$

(in this expression by Δ_k we denote the operator Δ with respect to the variable x_k). Further, for the function $f \in FA(\delta)$, $f = \{f_k\}_{k=0}^{\infty}$ by definition we put

$$Tf = \{Tf_{k+1}\}_{k=0}^{\infty}.$$

Lemma 4 For any function $f \in FA(\delta)$ we have $Af = (\exp T)(f)$ with

$$(\exp T)(f) = \sum_{k=0}^{\infty} \frac{T^k f}{k!} \tag{34}$$

Proof First let us consider the action of the operator A on the stochastic integrals. It is easy to prove that if $f = I_k(g)$ then

$$Af = \sum_{j=0}^{k} \frac{1}{j!} I_{k-j}(T^j g).$$

Now, for any function $f \in FA(\delta)$ using theorem 1 we have

$$\begin{aligned} Af &= A\left(\sum_{k=0}^{\infty} I_k(f_k)\right) = \sum_{k=0}^{\infty} A(I_k(f_k)) \\ &= \sum_{k=0}^{\infty}\sum_{j=0}^{k} \frac{1}{j!} I_{k-j}(T^j f_k) = \sum_{j=0}^{\infty} \frac{1}{j!} \sum_{k=j}^{\infty} I_{k-j}(T^j f_k) \\ &= \sum_{j=0}^{\infty} \frac{1}{j!}\left(\sum_{k=0}^{\infty} I_k(T^j f_{k+j})\right) = \sum_{j=0}^{\infty} \frac{1}{j!} T^j f = (\exp T)(f). \end{aligned}$$

□

Now it is clear that

$$A^{-1} f = \exp(-T)(f) = \sum_{k=0}^{\infty} (-1)^k \frac{T^k f}{k!}$$

or

$$(A^{-1} f)_l = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \int_{G^k} \Delta^k f_{l+k} \mathrm{d}\Pi^k. \tag{35}$$

Further, from (32) and (35) it follows that on $\mathcal{X}_0$ the function u is equal to $\exp(-T)\psi$. Now, to extend this function to $\mathcal{X}$ we must prove that $\exp(-T)\psi$ belongs to $FA(\delta)$.

Lemma 5 For every function $f \in FA(\delta)$ we have

$$\| \exp(-T) f \|_{FA(\delta)} \le \sum_{k=0}^{\infty} \frac{2^k}{k!} \| f_k \|_{k,\delta}. \tag{36}$$

Proof For simplicity we consider only the case $G = [0, \infty)$. Using (11), (35) and (13) we get

$$\begin{aligned}
\|(A^{-1} f)_l\|_l &\le \sum_{i=0}^{l} \binom{l}{i} \sup_{[0.\delta]^i \times [\delta,\infty)^{l-i}} \left| \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \int_{G^k} \Delta^k \partial_{\{1,\dots,i\}} f_{k+l} \mathrm{d}\Pi^k \right| \cdot (c_1(\delta))^i (2c_2(\delta))^{l-i} \\
&\le \sum_{i=0}^{l} \binom{l}{i} \sup_{[0.\delta]^i \times [\delta,\infty)^{l-i}} \sum_{k=0}^{\infty} \frac{1}{k!} \int_{G^k} |\Delta^k \partial_{\{1,\dots,i\}} f_{k+l}| \mathrm{d}\Pi^k \cdot (c_1(\delta))^i (2c_2(\delta))^{l-i} \\
&\le \sum_{i=0}^{l} \binom{l}{i} \sup_{[0.\delta]^i \times [\delta,\infty)^{l-i}} \sum_{k=0}^{\infty} \frac{1}{k!} \sum_{j=0}^{k} \binom{k}{j} \sup_{[0,\delta]^j \times [\delta,\infty)^{k-j}} |\partial_{\{1,\dots,i+j\}} f_{k+l}| \\
&\quad \times (c_1(\delta))^j (2c_2(\delta))^{k-j} (c_1(\delta))^i (2c_2(\delta))^{l-i} \\
&= \sum_{k=0}^{\infty} \frac{1}{k!} \sum_{i=0}^{l} \sum_{j=0}^{k} \binom{l}{i} \binom{k}{j} \sup_{[0.\delta]^{i+j} \times [\delta,\infty)^{k+l-(i+j)}} |\partial_{\{1,\dots,i+j\}} f_{k+l}| \\
&\quad \times (c_1(\delta))^{i+j} (2c_2(\delta))^{k+l-(i+j)} \\
&= \sum_{k=0}^{\infty} \frac{1}{k!} \sum_{m=0}^{k+l} \sup_{[0.\delta]^m \times [\delta,\infty)^{k+l-m}} |\partial_{\{1,\dots,m\}} f_{k+l}| \\
&\quad \times \sum_{i=0}^{\min(m,l)} \binom{l}{i} \binom{k}{m-i} (c_1(\delta))^m (2c_2(\delta))^{k+l-m} \\
&\le \sum_{k=0}^{\infty} \frac{1}{k!} \sum_{m=0}^{k+l} \sup_{[0.\delta]^m \times [\delta,\infty)^{k+l-m}} |\partial_{\{1,\dots,m\}} f_{k+l}| \\
&\quad \times \binom{l+k}{m} (c_1(\delta))^m (2c_2(\delta))^{k+l-m} = \sum_{k=0}^{\infty} \frac{1}{k!} \| f_{k+l} \|_{k+l,\delta}.
\end{aligned}$$

Summing w.r.t. l we get

$$\begin{aligned}
\| A^{-1} f \|_{FA(\delta)} &= \sum_{l=0}^{\infty} \frac{1}{l!} \| (A^{-1})_l \|_{l,\delta} \le \sum_{l=0}^{\infty} \frac{1}{l!} \sum_{k=0}^{\infty} \frac{1}{k!} \| f_{k+l} \|_{l,\delta} \\
&= \sum_{l=0}^{\infty} \frac{1}{l!} \sum_{j=l}^{\infty} \frac{\| f_j \|_j}{(j-l)!} = \sum_{j=0}^{\infty} \frac{1}{j!} \| f_j \|_j \left(\sum_{l=0}^{j} \frac{j!}{l!(j-l)!} \right) \\
&= \sum_{j=0}^{\infty} \frac{1}{j!} \| f_j \|_j 2^j.
\end{aligned}$$

This completes the proof of the Lemma 5. □

Now, taking into account (24), (25), (31), (32) and (36) we obtain immediately the following result.

Theorem 5 *Suppose that the function* $F_0(\overline{x}, X)$, *where* $\overline{x} \in G$, $X \in \mathcal{X}_0$, *satisfies* (21) *and the function* $F\colon \mathcal{X}_0 \to \mathcal{X}_0$ *defined by* (22) *satisfies* (23). *We also suppose that the functions*

$$\psi_k = \frac{\mathrm{d}\Pi^k F_k^{-1}}{\mathrm{d}\Pi^k}$$

satisfy (6) *and for some* $\delta > 0$

$$\sum_{k=0}^{\infty} \frac{2^k}{k!} \|\psi_k\|_{k,\delta} < \infty.$$

Then the measure PF^{-1} *is absolutely continuous with respect to* P *and*

$$\frac{\mathrm{d}PF^{-1}}{\mathrm{d}P} = (e^{-T}\psi)^{\sim}.$$

5 Some Examples

5.1 'Poissonian' Transformations

At first we consider the case where the function F_0 is of the form $F_0(\overline{x}, X) = f(\overline{x}) = f(\theta, x)$, $\theta \in S$, $x \in [0, \infty)$. We suppose that for every $\theta \in S$ $f(\theta, \cdot)$ is a diffeomorphism on $[0, \infty)$ and $f(\theta, 0) = 0$. By $\varphi(\overline{x}) = \varphi(\theta, x)$ we denote the inverse function $f^{-1}(\theta, \cdot)$ and by $\varphi'(\overline{x})$ we denote its derivative w.r.t. x.

In this case we have

$$\psi_k(\overline{x}_1, \ldots, \overline{x}_k) = \prod_{i=1}^{k} \psi(\overline{x}_i),$$

where

$$\psi(\overline{x}) = \frac{p(\varphi(\overline{x}))\varphi'(\overline{x})}{p(x)}.$$

We remark that the function $\psi = \{\psi_k\}_{k=0}^{\infty}$ satisfies $T\psi = \lambda\psi$, with $\lambda = \int_G \Delta\psi \, d\Pi$. Therefore $e^{-T}\psi = e^{-\lambda}\psi$. Now from the Theorem 5 it follows that if $\psi(\theta, 0) = 1$ and $\|\psi\|_{1,\delta} < \infty$ then PF^{-1} is absolutely continuous with respect to P and

$$\frac{\mathrm{d}PF^{-1}}{\mathrm{d}P}(X) = (e^{-\lambda}\psi)^{\sim}(X) = e^{-\lambda} \lim_{\varepsilon \to 0} \prod_{\overline{x} \in X \cap G_\varepsilon} \psi(\overline{x}) = e^{-\lambda} \prod_{\overline{x} \in X} \psi(\overline{x}).$$

5.2 'Nonpoissonian' Transformations

At first for every $k > 0$ we define an inverse mapping $\Phi_k = F_k^{-1}$. We define the function $\Phi_0(\overline{x}, X)$ by the formula

$$\Phi_0(\overline{x}, X) = xe^{\varphi(\overline{x})H(\sum_{\overline{y}\in X}\gamma(\overline{y}))} \tag{40}$$

and then, by definition, we put

$$\begin{aligned}\Phi_k(\overline{x}_1, \dots, \overline{x}_k) &= \Phi_k((\theta_1, x_1), \dots, (\theta_k, x_k))\\ &= \big((\theta_1, \Phi_0(\overline{x}_1, X)), (\theta_2, \Phi_0(\overline{x}_2, X)), \dots, (\theta_k, \Phi_0(\overline{x}_k, X))\big)\end{aligned}$$

where $X = \{\overline{x}_1, \dots, \overline{x}_k\}$.

We remark that the functions φ and γ in (40) are functions of the arguments $\theta \in S$, $x \in [0, \infty)$. By φ' (resp. γ') we denote the derivative of f (resp. γ) with respect to the real argument. We suppose that the functions $\varphi, \gamma,\ H$ are measurable and satisfy the following conditions

1. a) $\varphi(\theta, \cdot) \in C_b^{(2)}([0, \infty))$ for every $\theta \in S$
   b) $\varphi(\theta, 0) = 0$
   c) $\varphi' \geq 0$
   d) The functions $x\varphi'(\overline{x})$ and $x\varphi''(\overline{x})$ are bounded (uniformly by $\theta \in S$).
2. a) $\gamma(\theta, \cdot) \in C_b^{(1)}([0, \infty))$ for every $\theta \in S$,
   b) The function $x\gamma'(\overline{x})$ is bounded (uniformly by $\theta \in S$),
   c) For some $\delta > 0$ $\gamma(\theta, x) = 0$ for every $x < \delta$, $\theta \in S$.
3. a) $H \in C_b^{(1)}[0, \infty)$
   b) $H \geq 0$ and $H' \geq 0$.

We also suppose that the intensity measure Π is of the following form

$$\Pi(\mathrm{d}\theta, \mathrm{d}x) = \pi(\mathrm{d}\theta)\frac{\mathrm{d}x}{x^{1+\alpha}}, \quad \alpha \in (0, 1). \tag{41}$$

We cannot check directly the conditions of the third section, but later we prove that under the conditions 1–3 the mapping Φ_k for every k has a strictly positive Jacobi matrix and the determinant of this matrix is larger than 1. This means that Φ_k is an injective mapping. Also it is easy to prove that $\Phi_k(G^k) = G^k$ (so, Φ_k^{-1} is uniquely defined) and that the function $F = \{F_k\}_{k=0}^{\infty} = \{\Phi_k^{-1}\}_{k=0}^{\infty}$ can be extended to $\mathcal{X}$.

Our next aim is to calculate the functions ψ_k. At first we calculate the Jacobian of the mapping Φ_k. We recall that all differential operations take place with respect to the real variables, and $\theta \in S$ is a parameter.

The Jacobi matrix of the mapping ψ_k is equal to

$$\begin{pmatrix} e^{H\varphi(\overline{x}_1)} & 0 & 0 & \cdots & 0 \\ 0 & e^{H\varphi(\overline{x}_2)} & 0 & \cdots & 0 \\ 0 & 0 & e^{H\varphi(\overline{x}_3)} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & e^{H\varphi(\overline{x}_k)} \end{pmatrix}$$

$$\times \begin{pmatrix} \begin{matrix} 1+x_1\varphi'(\overline{x}_1)H+ \\ +x_1\varphi(\overline{x}_1)H'\gamma'(\overline{x}_1) \end{matrix} & x_1\varphi(\overline{x}_1)H'\gamma'(\overline{x}_2) & \cdots & x_1\varphi(\overline{x}_1)H'\gamma'(\overline{x}_k) \\ x_2\varphi(\overline{x}_2)H'\gamma'(\overline{x}_1) & \begin{matrix} 1+x_2\varphi'(\overline{x}_2)H+ \\ +x_2\varphi(\overline{x}_2)H'\gamma'(\overline{x}_2) \end{matrix} & \cdots & x_2\varphi(\overline{x}_2)H'\gamma'(\overline{x}_k) \\ x_3\varphi(\overline{x}_3)H'\gamma'(\overline{x}_1) & x_3\varphi(\overline{x}_3)H'\gamma'(\overline{x}_2) & \cdots & x_3\varphi(\overline{x}_3)H'\gamma'(\overline{x}_k) \\ \vdots & \vdots & \ddots & \vdots \\ x_k\varphi(\overline{x}_k)H'\gamma'(\overline{x}_1) & x_k\varphi(\overline{x}_k)H'\gamma'(\overline{x}_2) & \cdots & \begin{matrix} 1+x_k\varphi'(\overline{x}_k)H+ \\ +x_k\varphi(\overline{x}_k)H'\gamma'(\overline{x}_k) \end{matrix} \end{pmatrix}$$

(where $H = H(\sum_{i=1}^k \gamma(\overline{x}_i)),\ H' = H'(\sum_{i=1}^k \gamma(\overline{x}_i))$).

To calculate the determinant of the latter matrix we multiply the second row of this matrix by $x_1\varphi(\overline{x}_1)$ and after this subtract the first row multiplied by $x_2\varphi(\overline{x}_2)$. Then we multiply the third row by $x_1\varphi(\overline{x}_1)$ and after this subtract the first row multiplied by $x_3\varphi(\overline{x}_3)$ and so on. After these transformations we get a matrix with nonzero elements on the first row, first column and on the main diagonal. It can easily be checked that

$$J_k = e^{H\sum_{i=1}^k \varphi(\overline{x}_i)}\Bigg(\prod_{j=1}^k (1 + x_j\varphi'(\overline{x}_j)H) \\ + \sum_{m=1}^k \Big(x_m\varphi(\overline{x}_m)\gamma'(\overline{x}_m)H' \prod_{j\neq m}(1 + x_j\varphi'(\overline{x}_j)H)\Big)\Bigg). \tag{42}$$

Thus we have

$$\psi_k(\overline{x}_1,\dots,\overline{x}_k) = e^{-\alpha H\cdot\sum_{i=1}^k \varphi(\overline{x}_i)}\cdot\Bigg(\prod_{j=1}^k (1 + x_j\varphi'(\overline{x}_j)H) \\ + \sum_{m=0}^k \Big(x_m\varphi(\overline{x}_m)\gamma'(\overline{x}_m)H' \prod_{j\neq m}(1 + x_j\varphi'(\overline{x}_j)H)\Big)\Bigg),$$

(where $H = H(\sum_{i=1}^k \gamma(\overline{x}_i)),\ \ H' = H'(\sum_{i=1}^k \gamma(\overline{x}_i))$).

Our next aim is to estimate $\|\psi_k\|_{k,\delta}$. For this we calculate the derivatives of the function ψ_k. In accordance with (11) it is enough to only compute $\partial_{\{1,\dots,i\}}\psi_k$ on the set $(G\setminus G_\delta)^i \times G_\delta^{k-i}$ where $\gamma(\overline{x}_1) = \cdots = \gamma(\overline{x}_i) = 0$.

We have

$$\partial_{\{1,\dots,i\}}\ \psi_k\ (\overline{x}_1,\dots,\overline{x}_k) = e^{-\alpha H(\sum_{i=1}^k \varphi(\overline{x}_i))}$$
$$\times \Bigg((-\alpha H)\sum_{j=1}^{i}\varphi'(\overline{x}_j) + H^i \prod_{j=1}^{i}\big(\varphi'(\overline{x}_j)+x_j\varphi''(\overline{x}_j)\big)$$
$$\times \prod_{j=i+1}^{k}\big(1+x_j\varphi'(\overline{x}_j)H\big) + H^i \sum_{m=i+1}^{k}\Big(x_m\varphi(\overline{x}_m)\gamma'(\overline{x}_m)H'\prod_{j=1}^{i}\big(\varphi'(\overline{x}_j)+x_j\varphi''(\overline{x}_j)\big)$$
$$\times \prod_{\substack{j=i+1\\ j\neq m}}^{k}\ (1+x_j\varphi'(\overline{x}_j)H)\Big)\Bigg).$$

We see that

$$\sup_{(G\setminus G_\delta)^i\times G_\delta^{k-i}} |\partial_{\{1,\dots,i\}}\psi_k| \le M^k,$$

where the constant M does not depend on k.

Thus we get

$$\|\psi_k\|_{k,\delta} \le M^k(c_1(\delta)+2c_2(\delta))^k$$

and

$$\sum_{k=0}^{\infty}\frac{2^k}{k!}\|\psi_k\|_{k,\delta} < \infty.$$

So, from Theorem 5 we have the absolute continuity of PF^{-1} with respect to P and

$$\frac{\mathrm{d}PF^{-1}}{\mathrm{d}P} = (e^{-T}\psi)^{\sim}.$$

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