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On direction of dependence

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Abstract Under the assumption of the existence of linear relationship between two random variables, new formulas are introduced to express the coefficient of correlation. One of these formulas, the fourth power of the correlation coefficient is used to determine the direction of dependency between two random variables. Also an interpretation of the correlation coefficient as an asymmetric function of kurtosis coefficient and skewness coefficient of dependent variable and independent variable is provided. In the absent of the intercept in linear regression, the correlation coefficient is also expressed as a ratio of coefficients of variation between independent and dependent variables.

Keywords Asymmetric interpretation of the correlation coefficient · Causality · Correlation coefficient · Kurtosis coefficient · Linear regression · Response variable · Coefficient of variation · Skewness coefficient

1 Introduction

The Galton–Pearson correlation coefficient is probably the most frequently used statistical tool in applied sciences, and up to now different interpretations for it have been provided. Rodgers and Nicewander (1988) provided 13 interpretations for it. Rovine and von Eye (1997) and Falk and Well (1997) show a collection of algebraic

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and geometric interpretations of the correlation coefficient. An elegant properly of the correlation coefficient similar to that of given random variable which is defined by its mean and variance can be found in Nelsen (1998). Nelsen (1998) shows that the correlation coefficient is equal to the ratio of a difference and sum of two moments of inertia about certain line in the plane. Dodge and Rousson (2000) provided four new asymmetric interpretations in case of symmetrical error in the linear relationship of two variables including the cub of the correlation coefficient. Using the relationship found in their paper, and assuming the existence of linear relation between two random variables, they determined the direction of dependence in the linear regression model. That is, they provided a model on the basis of which one can make a distinction between dependent and independent variables in a linear regression. The direction dependence between two variables, when they follow the Laplace distributions, were provided by Dodge and Wittaker (2000) using graphical model approach. Muddapur (2003) arrives at the same relationship and found yet another formula between the correlation coefficient and the ratio of two coefficients of kurtosis. However, the author does not indicate how it could be used in determining the direction of dependence between two variables in simple linear regression.

In Sect. 2, we provide new formulas for the correlation coefficient, and in Sect. 3, the concept of the directional dependency between two variables is presented and a new procedure is given for determining the direction of dependency between response and explanatory variables in the linear regression. Finally in Sect. 3 of this article we provide some elements for testing statistical hypothesis in determining the directional dependency in linear regression.

2 Asymmetric interpretations of the correlation coefficient

Let X be a random variable and n be a natural number. Then, the nth central moment about the mean, $\mu_X = E(X)$, is defined as

$$E(X - \mu_X)^n. \tag{1}$$

The classical notion of the skewness and the kurtosis are moment-based, given in the univariate case by the standardized third and fourth central moment. The coefficient of skewness of X is

$$\gamma_X = E\left(\frac{X - \mu_X}{\sigma_X}\right)^3, \tag{2}$$

in which σ_X is the standard deviation of X, and the coefficient of excess kurtosis is defined by

$$\kappa_X = E\left(\frac{X - \mu_X}{\sigma_X}\right)^4 - 3. \tag{3}$$

The coefficient of variation of random variable X, denoted by CV_X, is given by

$$\mathrm{CV}_X = \frac{\sigma_X}{\mu_X} \tag{4}$$

The correlation coefficient between two random variables X and Y is defined as follows

$$\rho = \frac{\mathrm{Cov}(X, Y)}{\sigma_X \sigma_Y} \tag{5}$$

where $\mathrm{Cov}(X, Y)$ is covariance between X and Y, σ_X^2 and σ_Y^2 are variances of X and Y, respectively.

Dodge and Rousson (2000, 2001), showed that the cube of the correlation coefficient can be expressed as the ratio of the skewness of two correlated variables and used this expression for determining the directional dependency between two variables. In this article we show that the fourth power of the correlation coefficient can be expressed as the ratio of the excess kurtosis of the response to the excess kurtosis of the explanatory variable. The same relationship is given for the fifth power of the correlation coefficient.

Proposition 1 *Let us consider two random variables X and Y that are related by*

$$Y = \alpha + \beta X + \varepsilon \tag{6}$$

where, the skewness and the excess kurtosis coefficients of the random variables X and Y are not zero, α is the intercept and β is the slop parameter, and ε is an error variable that is independent of X and has normal distribution with zero mean and fixed variance. In this model we have

$$\rho^3 = \frac{\gamma_Y}{\gamma_X},$$
$$\rho^4 = \frac{\kappa_Y}{\kappa_X},$$
$$\rho^5 = \frac{E\left(\frac{Y-\mu_Y}{\sigma_Y}\right)^5 - C_3^5 \gamma_Y}{E\left(\frac{X-\mu_X}{\sigma_X}\right)^5 - C_3^5 \gamma_X}.$$

where $C_n^m = \frac{m!}{n!(m-n)!}$.

Proof Under the linear model (6) we have

$$\rho = \beta \frac{\sigma_X}{\sigma_Y}. \tag{7}$$

Since X is independent of ε, starting with (6) we can write

$$\mathrm{Var}(Y) = \mathrm{Var}(\alpha + \beta X + \varepsilon)$$

if and only if

$$\sigma_Y^2 = \beta^2\sigma_X^2 + \sigma_\varepsilon^2.$$

Dividing both sides of this equation by σ_Y^2 we have

$$1 = \left(\beta\frac{\sigma_X}{\sigma_Y}\right)^2 + \left(\frac{\sigma_\varepsilon}{\sigma_Y}\right)^2.$$

and use (7) we have

$$1 - \rho^2 = \left(\frac{\sigma_\varepsilon}{\sigma_Y}\right)^2. \tag{8}$$

From (6) we can write

$$\begin{aligned}(Y - \mu_Y) &= (\alpha + \beta X + \varepsilon - E\,(\alpha + \beta X + \varepsilon))\\ &= (\alpha + \beta X + \varepsilon - [\alpha + \beta\mu_X + \mu_\varepsilon])\\ &= \beta\ (X - \mu_X) + (\varepsilon - \mu_\varepsilon)\,.\end{aligned} \tag{9}$$

Dividing both sides of (9) by σ_Y and using (7) we obtain

$$\left(\frac{Y - \mu_Y}{\sigma_Y}\right) = \rho\left(\frac{X - \mu_X}{\sigma_X}\right) + \frac{\sigma_\varepsilon}{\sigma_Y}\left(\frac{\varepsilon - \mu_\varepsilon}{\sigma_\varepsilon}\right). \tag{10}$$

From 4th power of both sides of (10) and under expectation we have

$$\begin{aligned}E\left(\frac{Y - \mu_Y}{\sigma_Y}\right)^4 &= \rho^4 E\left(\frac{X - \mu_X}{\sigma_X}\right)^4\\ &\quad + C_2^4\,\rho^2 E\left(\frac{X-\mu_X}{\sigma_X}\right)^2\left(\frac{\sigma_\varepsilon}{\sigma_Y}\right)^2 E\left(\frac{\varepsilon-\mu_\varepsilon}{\sigma_\varepsilon}\right)^2 + \left(\frac{\sigma_\varepsilon}{\sigma_Y}\right)^4 E\left(\frac{\varepsilon-\mu_\varepsilon}{\sigma_\varepsilon}\right)^4\end{aligned}$$

Therefore we have:

$$\begin{aligned}E\left(\frac{Y - \mu_Y}{\sigma_Y}\right)^4 - 3 = \rho^4&\left[E\left(\frac{X - \mu_X}{\sigma_X}\right)^4 - 3 + 3\right]\\ &+ C_2^4\,\rho^2 E\left(\frac{X - \mu_X}{\sigma_X}\right)^2\left(\frac{\sigma_\varepsilon}{\sigma_Y}\right)^2 E\left(\frac{\varepsilon - \mu_\varepsilon}{\sigma_\varepsilon}\right)^2\\ &+ \left(\frac{\sigma_\varepsilon}{\sigma_Y}\right)^4\left[E\left(\frac{\varepsilon - \mu_\varepsilon}{\sigma_\varepsilon}\right)^4 - 3 + 3\right] - 3\end{aligned}$$

and after simplification and using (8) we have

$$\kappa_Y = \rho^4\kappa_X + (1 - \rho^2)^2\kappa_\varepsilon.$$

If $\kappa_\varepsilon = 0$, we have

$$\kappa_Y = \rho^4 \kappa_X \tag{11}$$

and (as long as $\kappa_X \neq 0$)

$$\rho^4 = \frac{\kappa_Y}{\kappa_X}. \tag{12}$$

The expression (12) represents the correlation coefficient between two random variables, which shows that the fourth power of the correlation coefficient is the ratio of the excess kurtosis of the response variable and the excess kurtosis of the explanatory variable. This has a natural interpretation: add a symmetric error to a explanatory variable and you get a response variable with less kurtosis. Also, the fourth power of the correlation may be described as the percentage of kurtosis which is preserved by a linear model. □

Dodge and Rousson (2000) proved that under assumption of symmetry of the error variable and under model (6), the cube of the correlation coefficient is equal to the ratio of the skewness of the response variable and the skewness of the explanatory variable,

$$\rho^3 = \frac{\gamma_Y}{\gamma_X} \tag{13}$$

where γ_X and γ_Y are skewness coefficients of X and Y respectively (as long as $\gamma_X \neq 0$). We can derive (13) in the same way that we obtained (12). From third power of both sides of (10) and under expectation we have

$$E\left(\frac{Y-\mu_Y}{\sigma_Y}\right)^3 = \rho^3 E\left(\frac{X-\mu_X}{\sigma_X}\right)^3 + \left(\frac{\sigma_\varepsilon}{\sigma_Y}\right)^3 E\left(\frac{\varepsilon-\mu_\varepsilon}{\sigma_\varepsilon}\right)^3$$

and by using (8) we have

$$\gamma_Y = \rho^3 \gamma_X + (1-\rho^2)^{\frac{3}{2}} \gamma_\varepsilon$$

where γ_ε is the skewness coefficient of the error variable. If the error variable is symmetric, then $\gamma_\varepsilon = 0$ and

$$\gamma_Y = \rho^3 \gamma_X. \tag{14}$$

If we assume that X and Y are asymmetric, from fifth power of both sides of (10) and under expectation we can obtain

$$E\left(\frac{Y-\mu_Y}{\sigma_Y}\right)^5 = \rho^5 E\left(\frac{X-\mu_X}{\sigma_X}\right)^5 + C_3^5\left(\rho^3\gamma_X\left(\frac{\sigma_\varepsilon}{\sigma_Y}\right)^2 + \rho^2\left(\frac{\sigma_\varepsilon}{\sigma_Y}\right)^3\gamma_\varepsilon\right) + \left(\frac{\sigma_\varepsilon}{\sigma_Y}\right)^5 E\left(\frac{\varepsilon-\mu_\varepsilon}{\sigma_\varepsilon}\right)^5 \quad (15)$$

If we assume that $E(\frac{\varepsilon-\mu_\varepsilon}{\sigma_\varepsilon})^3 = E(\frac{\varepsilon-\mu_\varepsilon}{\sigma_\varepsilon})^5 = 0$, then from (14) and (15) we have

$$\left(E\left(\frac{Y-\mu_Y}{\sigma_Y}\right)^5 - C_3^5\gamma_Y\right) = \rho^5\left(E\left(\frac{X-\mu_X}{\sigma_X}\right)^5 - C_3^5\gamma_X\right) \quad (16)$$

Hence, we obtain a new expression for the correlation coefficient as follows

$$\rho^5 = \frac{E\left(\frac{Y-\mu_Y}{\sigma_Y}\right)^5 - C_3^5\gamma_Y}{E\left(\frac{X-\mu_X}{\sigma_X}\right)^5 - C_3^5\gamma_X} \quad (17)$$

This formulae represents another asymmetric face of the correlation coefficient.

In the what follows, we present some new faces of the correlation coefficient. Later we use some of these formulas for determining the direction of dependency between two variables.

2.1 The ratio of excess kurtosis to skewness

By dividing Eq. (11) to Eq. (14) we obtain

$$\frac{\kappa_Y}{\gamma_Y} = \rho\frac{\kappa_X}{\gamma_X}. \quad (18)$$

From this equation (as long as $\gamma_X \neq 0$, $\gamma_Y \neq 0$ and $\kappa_X \neq 0$), we obtain yet another expression for the correlation coefficient as

$$\rho = \frac{\left(\frac{\kappa_Y}{\gamma_Y}\right)}{\left(\frac{\kappa_X}{\gamma_X}\right)} = \frac{f(Y)}{f(X)} = \frac{g(\kappa_Y, \gamma_Y)}{g(\kappa_X, \gamma_X)}. \quad (19)$$

This Eq. (19) signifies that we can expressed the correlation coefficient as a ratio of a function of Y to the same function of X. This ratio is an asymmetric function of the excess kurtosis and the skewness coefficients of dependent and independent random variables.

From (11), (14), and (18), we can obtain the following inequality between response variable and explanatory random variable

$$\left|\frac{\kappa_Y}{\gamma_Y}\right| \leq \left|\frac{\kappa_X}{\gamma_X}\right|, \quad (20)$$

and

$$\gamma_Y^2 + \kappa_Y^2 < \gamma_X^2 + \kappa_X^2. \tag{21}$$

2.2 The ratio of two coefficients of variation

The correlation coefficient can also be expressed as the ratio of two coefficients of variation of random variables related by a linear function of X and Y in the following form

$$Y = \beta X + \varepsilon. \tag{22}$$

Proposition 2 *Let us consider two random variables X and Y that are related by regression model*

$$Y = \beta X + \varepsilon, \tag{23}$$

where ε is a normal error variable with zero mean and fixed variance that is independent of X and $\beta \in R$ is a constant. In this model,

$$\rho = \frac{\mathrm{CV}_X}{\mathrm{CV}_Y} \tag{24}$$

and

$$\left(\frac{\mathrm{CV}_X}{\mathrm{CV}_Y}\right)^3 = \frac{\gamma_Y}{\gamma_X} \tag{25}$$

Proof Based on the (23) we have

$$\mu_Y = \beta \mu_X \tag{26}$$

then, from (7) and (26) we have

$$\rho = \beta \frac{\sigma_X}{\sigma_Y} = \frac{\beta \sigma_X / \beta \mu_X}{\sigma_Y / \mu_Y} = \frac{\mathrm{CV}_X}{\mathrm{CV}_Y}$$

For the second part, we have

$$E\left(\frac{Y - \mu_Y}{\mu_Y}\right)^3 = E\left(\frac{\beta(X - \mu_X) + \varepsilon}{\beta \mu_X}\right)^3$$
$$= E\left(\frac{X - \mu_X}{\mu_X}\right)^3$$

Then

$$E\left(\frac{Y-\mu_Y}{\sigma_Y}\right)^3\left(\frac{\sigma_Y}{\mu_Y}\right)^3 = E\left(\frac{X-\mu_X}{\sigma_X}\right)^3\left(\frac{\sigma_X}{\mu_X}\right)^3$$

Therefore

$$\rho^3 = \left(\frac{\mathrm{CV}_X}{\mathrm{CV}_Y}\right)^3 = \frac{\gamma_Y}{\gamma_X}$$

(as long as $\gamma_X \neq 0$) which was obtained by Dodge and Rousson (2000). □

3 Direction of dependency in linear regression

Let $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$ be a random sample of n observations from random variables X and Y. Consider the situation that a linear relationship exists between two random variables X and Y in the following form

$$Y = \alpha + \beta X + \varepsilon. \tag{27}$$

In (27) the random variable Y is a function of the random variable X, and X is assumed to be independent of the error variable ε. In this situation we say that the response variable Y depends on the explanatory variable X, and the direction of dependency is from X to Y. Equation (27) can also be thought as a causal relationship between explanatory variable (cause) and response variable (effect). If X causes Y, then we select the model (27). On the other hand, if Y causes X, then we select the model

$$X = \alpha' + \beta' Y + \varepsilon'. \tag{28}$$

In (28) the error variable ε' is independent of the explanatory variable Y. In both models (27) and (28) we assume that, the error variable has a normal distribution with zero mean and fixed variance.

If we wish to investigate the direction of dependency, we may hesitate between model (27) and model (28). To answer such a question, Dodge and Rousson (2000) proposed a method for determining the direction of dependency in a linear regression based on the assumption that the skewness coefficient of the error variable is zero. In situations where such a condition may not exist, this article presents new procedures for determining the direction of dependency. We propose to exploit the asymmetric properties of the correlation coefficient given in Sect. 2 to distinguish the response from the explanatory variable.

When Y is the response variable and X is the explanatory variable, we have

$$\rho^4 = \frac{\kappa_Y}{\kappa_X}.$$

Since $0 < \rho^4 < 1$, we have

$$\rho^4 = \frac{\kappa_Y}{\kappa_X} < 1$$

which implies

$$\kappa_Y < \kappa_X \tag{29}$$

as long as $0 < |\rho| < 1$.
This shows that the kurtosis of the response variable is always smaller than the kurtosis of the explanatory variable. In this case we choose model (27) over model (28). We can apply similar argument and based on (28) and we can obtain the following equation

$$\kappa_X < \kappa_Y. \tag{30}$$

We may hence prefer that model for which the appropriate inequalities (29) or (30) are satisfied in the sample. This strategy has important advantage that no assumption is made about skewness of the response and the explanatory variables.

The process of determining direction dependency can be stated in the framework of testing statistical hypothesis.

$$\begin{aligned} H_0 &: \kappa_Y < \kappa_X \\ H_1 &: \kappa_Y > \kappa_X \end{aligned} \tag{31}$$

If the null hypothesis is rejected in favor of the alternative hypothesis, one could choose model (28) rather than model (27).

Now consider the following equation from the definition of the excess kurtosis coefficient

$$\begin{aligned} \kappa_X &= E\left(\frac{X-\mu_X}{\sigma_X}\right)^4 - 3 \\ &= \operatorname{Var}\left\{\left(\frac{X-\mu_X}{\sigma_X}\right)^2\right\} + \left\{E\left(\frac{X-\mu_X}{\sigma_X}\right)^2\right\}^2 - 3 \\ &= \operatorname{Var}\left\{\left(\frac{X-\mu_X}{\sigma_X}\right)^2\right\} - 2 \end{aligned}$$

then

$$\kappa_Y < \kappa_X \quad \text{if and only if} \quad \operatorname{Var}\left\{\left(\frac{Y-\mu_Y}{\sigma_Y}\right)^2\right\} < \operatorname{Var}\left\{\left(\frac{X-\mu_X}{\sigma_X}\right)^2\right\},$$

and

$$\kappa_Y > \kappa_X \quad \text{if and only if} \quad \operatorname{Var}\left\{\left(\frac{Y-\mu_Y}{\sigma_Y}\right)^2\right\} > \operatorname{Var}\left\{\left(\frac{X-\mu_X}{\sigma_X}\right)^2\right\}.$$

Therefore, we can compare variances of two dependent variables in replace of the (31), which is equivalent to

$$\begin{aligned} H_0 &: \operatorname{Var}\left\{\left(\frac{Y-\mu_Y}{\sigma_Y}\right)^2\right\} < \operatorname{Var}\left\{\left(\frac{X-\mu_X}{\sigma_X}\right)^2\right\} \\ H_1 &: \operatorname{Var}\left\{\left(\frac{Y-\mu_Y}{\sigma_Y}\right)^2\right\} > \operatorname{Var}\left\{\left(\frac{X-\mu_X}{\sigma_X}\right)^2\right\} \end{aligned} \tag{32}$$

Comparing the variances of random variables arises in a variety of situations such as psychology (e.g., Levy 1976; Lord and Novick 1968), and medicine (Choi and Wette 1972). Comparing variances can also be important when measuring effect size (e.g., Doksum 1977; Doksum and Sievers 1976). In meta-analysis, because equal variances are frequently assumed (e.g., Hedges and Olkin 1985), it is important to test this hypothesis.

An early attempt for comparing the variances of two random variables was made by Morgan (1939) and Pitman (1939) under the assumption of bivariate normality. Morgan (1939) and Pitman (1939) noted that the covariance between $X-Y$ and $X+Y$ is equal to $\sigma_X^2 - \sigma_Y^2$. Therefore equality of variances can be tested simply by testing whether the correlation between $X-Y$ and $X+Y$ is zero. Cohen (1986) proposed two procedures based on a nonparametric bootstrap. McCulloch (1987) suggested a modification of the Morgan–Pitman test. Wilcox (1989, 1990) discussed several procedures for comparing variances of dependent groups.

3.1 Special case 1

Let us consider the two random variables, X and Y, where a linear relationship exists between them in the following form

$$Y = X + \varepsilon \tag{33}$$

or

$$X = Y + \varepsilon'. \tag{34}$$

Under model (33) we have $\rho^2 = \frac{\sigma_X^2}{\sigma_Y^2}$ (obtained from (7) when $\beta = 1$) and then $\sigma_Y^2 > \sigma_X^2$, and under model (34) we can obtain that $\sigma_Y^2 < \sigma_X^2$. Then, the procedure of determining the direction of dependency is the procedure of comparing the variances of two dependent random variables X and Y as follows

$$\begin{aligned} H_0 &: \sigma_Y^2 > \sigma_X^2 \\ H_1 &: \sigma_Y^2 < \sigma_X^2 \end{aligned} \tag{35}$$

3.2 Special case 2

Now consider the situation that a linear relationship exists between two random variables X and Y in the following form

$$Y = \beta X + \varepsilon. \tag{36}$$

In (36) the random variable Y is a function of the random variable X, and X is assumed to be independent of the error variable ε. In this situation we say that the response variable Y depends on the variable X, and the *direction of dependency* is from X to Y. Equation (36) can also be thought as a causal relationship between explanatory variable (cause) and response variable (effect). If X causes Y, then we select the model (36). In the other hand, if Y causes X, then we select the model

$$X = \beta' Y + \varepsilon'. \tag{37}$$

In (37) the error variable ε' is independent of the explanatory variable Y. In both models (36) and (37) we assume that, the error variable has a normal distribution with zero mean and fixed variance.

Under assumptions of the model (36) and from (24), we can conclude that the coefficient of variation of the response variable is larger than the coefficient of variation of the explanatory variable. Then, we choose model (36) over model (37) if

$$\mathrm{CV}_Y > \mathrm{CV}_X. \tag{38}$$

Similarly we choose model (37) over model (36), if

$$\mathrm{CV}_Y < \mathrm{CV}_X. \tag{39}$$

The process of determining of the direction of dependency is equivalent to the following hypothesis testing

$$\begin{aligned} H_0 &: \mathrm{CV}_Y > \mathrm{CV}_X \\ H_1 &: \mathrm{CV}_Y < \mathrm{CV}_X \end{aligned} \tag{40}$$

If the null hypothesis is rejected in favor of the alternative hypothesis, one could choose model (37) rather than model (36).

Verrill and Johnson (2007) obtain confidence bounds for a ratio of two coefficients of variation, provide a test for the comparison of two coefficients of variation.

4 Concluding remark

When a linear relationship exists between two random variables, Dodge and Rousson (2000) provided formulae for determining the direction of dependency between them, when their distribution error is symmetric or the skewness of error variable is zero.

Cox (1992) discusses similar problem in the linear regression in the study of dependence of one or more response variables to one or more explanatory variables and relates this problem to cause and effect. In this article we provided new formulas for the case when the distribution of error variable has a zero excess kurtosis. Also we give five new asymmetric interpretations of the correlation coefficient. One of this formulas is representing the correlation coefficient as the ratio of coefficient of variation of the independent variable over the dependent variable. A first step toward testing the hypothesis of direction dependency between two variables has been taken by showing the equivalency of hypothesis testing of two kurtosis coefficients to testing the variances of two dependent random variables in a linear regression model. Further work in this direction is required to arrive at classical statistical testing for direction of dependency.

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