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Some New Results on Industrial Sector Mode-Locking and Business Cycle Formation

Bernd Süssmuth[*] Ulrich Woitek[†]

[*]Department of Business and Economics, Munich University of Technology, Germany, bernd.suessmuth@wi.tum.de

[†]Institute for Empirical Research in Economics, University of Zurich, Switzerland, u.woitek@iew.unizh.ch

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Bernd Süssmuth and Ulrich Woitek

Abstract

Business cycles in different industries have a tendency to synchronize with one another in what appears to be a national business cycle. Using simulation and time series techniques in the time and frequency domain, we offer econometric support for the industrial sector mode-locking hypothesis, extending recent work by Selover, Jensen and Kroll (2003). In addition, we propose an economic motivation of the underlying nonlinear model.

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1. Introduction

Selover, Jensen, and Kroll (2003) motivate their recent study on industrial sector mode-locking by the fact that business cycles in different industries have a tendency to synchronize with one another in what appears to be a national business cycle. Extending their seminal work, we analyze how more narrowly defined industrial sectors are synchronized and how this phenomenon can be modelled. There is a body of theoretical literature relating to their "less contrived and more economically intuitive" mode-locking model based on the concept of nonlinear van der Pol oscillators. Similarily, Hillinger and Weser (1988) combine second order differential equations to model synchronization through the mode-locking mechanism. Incorporating information externalities (Caplin and Leahy 1993, Zeira 1994, Gale 1996a, and Molina 2003), it is straightforward to economically motivate a sectoral version of the nonlinear model by Hillinger and Weser (Süssmuth 2002a, 2003).[1] The empirical part of our study presents econometric support for sectoral mode-locking on a relatively high level of disaggregation.

The remainder of the paper is structured as follows: Section 2 briefly outlines our nonlinear model of endogenous investment cycles incorporating information externalities. Section 3 investigates a necessary condition of nonlinear sectoral mode-locking models: the failure of linear models to explain synchronization. In the second part of Section 3, we present empirical evidence of mode-locking between 450 different US manufacturing sectors based on spectral analytic techniques. Section 4 contains a thorough simulation study of the model, and Section 5 concludes. Appendices A to E complete our study.

2. The Model

2.1 The basic mode-locking model of investment

Industry i selects a quadratic cost-minimizing path of production capital, taking the trajectory of desired capital as given:

$$\min_{k_i(t)} \int_{t_0}^{\infty} \left[\alpha_i k_i^2 + \beta_i \left(k_i^{'} \right)^2 + \gamma_i \left(k_i^{''} \right)^2 \right] e^{\rho(t_0 - t)} dt, \tag{2.1}$$

where ρ represents the discount rate. All costs are formulated in terms of deviations k_i of actual capital K_i from its desired value K_i^*; i.e., $k_i = K_i^* - K_i$, $k_i^{'} = \frac{d}{dt}(K_i^* - K_i)$, etc. As demonstrated by Hillinger, Reiter and Weser (1992), this optimization leads to the following second order differential equation (Appendix A):

$$I_i^{'} = K_i^{''} = \frac{\alpha_i}{\gamma_i \rho^2} (K_i^* - K_i) - \frac{\beta_i}{\gamma_i \rho} K_i^{'} = a_i (K_i^* - K_i) - b_i K_i^{'}, \tag{2.2}$$

$$\text{where } a_i = \frac{\alpha_i}{\gamma_i \rho^2} \text{ and } b_i = \frac{\beta_i}{\gamma_i \rho}.$$

[1] Related empirical studies include Entorf (1991), Hornstein and Praschnik (1997), Forni and Reichlin (1998), Christiano and Fitzgerald (1998), Hornstein (2000), and Süssmuth (2002b).

The second order accelerator (SOA) equation (2.2) reflects the inertia of the investment process due to institutional frictions.[2] The parameter a_i is mainly responsible for the rate at which industrial investment is adjusted, i.e. for the period length of the cyclic series described by (2.2). Parameter b_i determines the rate of damping. As shown in Süssmuth (2002a; 2003), a straightforward way to introduce interactions among sectors that are consistent with endogenously-timed herding scenarios (Gul and Lundholm, 1995, and Gale, 1996b) is to incorporate a nonlinear feedback from the aggregate variable:

$$I_i^{'} = K_i^{''} = a_i \left\{ 1 + \chi_i \cdot \Psi \left[(K_i^* - K_i) \sum_j I_j \right] \right\} (K_i^* - K_i) - b_i K_i^{'}. \tag{2.3}$$

A simple example for the transform function Ψ is

$$\Psi[x] = \begin{cases} +1 & \text{for } x > 0 \\ 0 & \text{for } x = 0 \\ -1 & \text{for } x < 0 \end{cases} . \tag{2.4}$$

The parameter χ_i can be interpreted as the strength of individual interaction with the aggregate behavior (j denotes all $j \neq i$ industrial sectors).[3] Some intuition, motivation, and justification for equations (2.3) and (2.4) is given in Appendix C.

2.2 A generalization of the synchronization mechanism

Equation (2.3) is of the so called Hill's class of equations, since it assigns the second temporal derivative of a variable $K_i^{''}$ to a function including a temporally variable coefficient $a_i(t) = f\left(K_i(t); \sum_j I_j(t)\right)$ (Arnol'd 1983). The coupled van der Pol oscillators of Selover *et al.* (2003) are also in this class of equations. In the limit case of (relatively) homogeneously fluctuating sectoral investments, (2.3) represents a (quasi-)Mathieu-type equation, i.e. an equation with constant coefficients in the leading term ($K_i^{'}$) and oscillating coefficients in the lower-order terms (K_i). Mathieu-type equations have the potential to resonantly stimulate phases and to periodically entrain them (Arnol'd 1983, Hillinger and Weser 1988). This effect depends on the forcing amplitude or depth of modulation, corresponding to the strength of interaction with the aggregate χ_i restricted to $|\chi_i| < 1$. In addition, a certain constellation of system-frequency to frequency of perturbation or driving signal (*SFDF*-ratio) is required. If many such constellations for different values of χ_i exist, these zones of periodic entrainment are visualized in Arnol'd tongues (Arnol'd 1983, Mosekilde *et al.* 1992, Sterman and Mosekilde 1994). For systems of Mathieu-type, the by far largest zone of periodic entrainment for a depth of modulation $|\chi_i| \in (0, 1]$ is given by an *SFDF*-ratio of approximately 0.5, which is intrinsic to (2.3).

[2] The analogy of equation (2.2) to Euler equations of investment in the standard Ramsey model with time-to-build lags is shown in Appendix B.

[3] Due to the atomistic structure of the units (for the simulation study of Section 4 $i = 1, ..., N$ sectors, where $N >> 10$) $\sum_j I_j$ can accurately be approximated by $\sum_i I_i$.

Consider the case of weakly heterogeneous quasi-cyclic investing behavior on the sectoral level. Investment I_i "idealistically fluctuates" (as in the following indicated by a single line arrow) according to a sinusoidal function depending on microeconomic characteristics, i.e. the SOA (2.2) as a solution to (2.1). For the sake of simplicity, this solution takes the form $\cos(a_i t)$, abstracting from any damping: $K_i^{''} = I_i^{'} = f\left[(K_i^* - K_i)\right] \to \cos(a_i t)$.

Now suppose that aggregate investment is a combination of many quasi-cycles with roughly equal periodicities. Under these conditions, the sum of its first-order term approximately follows the same cyclical dynamics as $I_i^{'}$, i.e. $\cos(a_i t)$. This implies that $\sum_j I_j \approx \int_t^T I_i^{'} dt = \int_t^T f\left[(K_i^* - K_i)\right] dt \to \sin(a_i t)$. The inner term $\left[(K_i^* - K_i)\sum_j I_j\right] \to \cos(a_i t)\sin(a_i t) = \frac{1}{2}\sin(2a_i t)$ shows the doubled frequency of the outer oscillation $(K_i^* - K_i) \to \cos(a_i t)$ of (2.3), and the *SFDF*-ratio equals 0.5.

Proposition 1 *Whenever two distinct endogenous cycles with certain frequencies* $\theta_x \wedge \theta_y$*:* $x = \sin(\theta_x t) \wedge y = \sin(\theta_y t)$ *are coupled, the stylized model of coupled cycles can be transformed into an acceleration form similar to equation (2.3). A* $1/2$*-relationship of system-frequency to frequency of driving signal (SFDF-ratio* $= 0.5$*) is intrinsic to these models.*

Proof. See Appendix D.

Given the generality of the argument, the above sketched model can serve as an explanatory base for mode-locking. Stylized formulations like the one of Selover *et al.* (2003) can be enriched by a more meaningful economic argumentation based on economic objectives.

3. Empirical Evidence

3.1 Data

We analyze variable new real capital spending (base year 1987) taken from the NBER US Manufacturing Productivity Database. The dataset consists of 450 SIC 4-digit industrial series, covering annual observations for the period 1958 to 1992. For detail see Bartelsman and Gray (1996).

3.2 The failure of linear models

First, we test the detrended disaggregate investment series for significant autoregressive (AR) dynamics as well as for an impact of aggregate investment activity on the industrial level. We consider the following models

$$y_{i,t} = \phi_{i,1}\sum_{j\neq i}^{\widetilde{N}} y_{j,t} + \phi_{i,2}\sum_{j\neq i}^{\widetilde{N}} y_{j,t-1} + \phi_{i,3}\; y_{i,t-1} + \phi_{i,4}\; y_{i,t-2}; \tag{3.1}$$

$$y_{i,t} = \phi_{i,1}\sum_{i}^{N} y_{i,t} + \phi_{i,2}\sum_{i}^{N} y_{i,t-1} + \phi_{i,3}\; y_{i,t-1} + \phi_{i,4}\; y_{i,t-2}, \tag{3.2}$$

i.e. (3.1) excludes series i in the construction of the aggregate, while it is included in (3.2) ($N = 450$, $\widetilde{N} = 449$). We set the AR order to $p = 2$. The null hypotheses are

$$H_0: \ \phi_{i,1} = \phi_{i,2} = 0 \tag{3.3}$$

$$H_0: \ \phi_{i,3} = \phi_{i,4} = 0. \tag{3.4}$$

We test both restricted models against the unrestricted (3.1) and (3.2). Table 1 reports the shares of series for which (3.3) and (3.4) could be rejected for three levels of significance and filtering methods. Up to about half of the sectoral series show significant AR dynamics and an impact from the actual and lagged aggregate.
The majority of contemporaneous aggregate coefficient estimates $\widehat{\phi}_{i,1}$ shows a positive sign, suggesting a positive feedback (Table 2). The impact from the aggregate lagged by one period is negligible.

Table 1:AR dynamics and aggregate feedback: Rejecection shares (F-test)

	significance level: 1%			significance level: 5%			significance level: 10%		
		(3.3)	(3.4)		(3.3)	(3.4)		(3.3)	(3.4)
a.	(3.1)	21.3%	17.8%	(3.1)	39.1%	38.4%	(3.1)	48.7%	51.6%
	(3.2)	21.6%	17.6%	(3.2)	39.8%	38.2%	(3.2)	50.0%	51.6%
b.	(3.1)	18.9%	15.8%	(3.1)	31.3%	32.9%	(3.1)	42.7%	45.1%
	(3.2)	19.3%	15.8%	(3.2)	32.0%	32.9%	(3.2)	43.8%	45.1%
c.	(3.1)	21.1%	15.8%	(3.1)	39.1%	28.7%	(3.1)	50.2%	40.7%
	(3.2)	21.8%	15.3%	(3.2)	39.6%	28.7%	(3.2)	50.9%	40.7%

Note: a. First log differences, b. BKM filter; see Baxter and King (1999) and A'Hearn and Woitek (2002), c. HP(100)-filter; see Hodrick and Prescott (1997).

Table 2: Sizes and signs of estimated aggregate feedback coefficients

	$\frac{1}{N}\sum_i \widehat{\phi}_{i,1}$	$\frac{1}{N}\sum_i \widehat{\phi}_{i,2}$	$\frac{1}{N}\sum_i \lvert\widehat{\phi}_{i,1}\rvert$	$\frac{1}{N}\sum_i \lvert\widehat{\phi}_{i,2}\rvert$	$\widehat{\phi}_{i,1} < 0$	$\widehat{\phi}_{i,2} < 0$
	0.002	0.001	0.002	0.001	42	138
a.	(47.5%)*	(16.6%)*				
	0.002	0.002	0.002	0.001	36	138
	(49.3%)*	(16.6%)*				
	0.002	0.000	0.002	0.001	67	230
b.	(40.0%)*	(16.2%)*				
	0.002	-0.001	0.002	0.001	64	230
	(39.7%)*	(16.2%)*				
	0.002	0.000	0.002	0.001	80	265
c.	(41.3%)*	(20.2%)*				
	0.002	0.000	0.002	0.001	77	267
	(43.7%)*	(20.0%)*				

Note: a., b., and c. as given in Table 1 above (values rounded on third digit); $\star$: share of significant estimates at 10% level

In the next step, the ability of the linear model to reproduce basic business cycle stylized facts on the industry and aggregate level is analyzed. We consider three cases: linear models with (i) a purely idiosyncratic, (ii) a sectorally interdependent, and (iii) a composed idiosyncratic and common shock structure. The cyclical component of investment in industry s is denoted by y_t^s. The elements of the vector z_t are aggregate variables exogenous to the industrial sector. As first order VAR process, z_t^s can be modeled as a vector of sector specific exogenous variables. These are uncorrelated across sectors and also follow a first order VAR process. In addition, the profit function of sector s can be approximated by a quadratic function in the lags of y_t^s, in z_t^s and z_t. The result is a linear policy function (abstracting from the irrelevant constant) of the form

$$y_t^s = \sum_{i=1}^{p} \alpha_{1i} y_{t-i}^s + \alpha_2 z_t + \alpha_3 z_t^s + e_t, \tag{3.5}$$

where e_t denotes the error term. Since the exogenous variables z^s and z are AR processes, they have (infinite) MA-representations in terms of their driving shocks ε^s. Therefore, (3.5) can be rewritten as

$$A^s(L)\, y_t^s = B^s(L)\, \varepsilon_t^s, \tag{3.6}$$

where L is the lag operator. The estimated parameters of

$$y_t^s = \sum_{i=1}^{p} \beta_i y_{t-i}^s + \varepsilon_t^s, \tag{3.7}$$

are used to recursively simulate the time series $y_t, \ldots, y_T$ (Lütkepohl 1991, Appendix D).[4]

Two models are generated: The first *MI*, i.e. a multiple time series model with VAR generation process as described above, and the second *MII*, where $u_t^s \sim N\left(0, \sigma_{\widehat{\varepsilon}_t^s}\right)$, i.e. a linear model with independent sectoral shocks. The number of simulations is 1000. In each step, the standard deviation of the resulting aggregate is calculated to compare it with the standard deviation of the actual aggregate[5] The results are reported in the first two rows of Table 3. There are no remarkable differences between *MI* and *MII*. This can be due to the fact that the average correlation coefficient between estimated sectoral shocks is very low (7%). On average, the empirical standard deviation is 3.2 times higher than the standard deviations of the simulated aggregate series. This suggests that a common shock structure, e.g. in the form of feedback from the contemporaneous aggregate, is needed to create the volatility of the aggregate

[4] The series in Section 3.2 are detrended using the BKM filter. Results remain qualitatively unchanged for other detrending methods. The first two values of the detrended actual series are treated as presample values in the VAR model. This strategy ensures the starting phase constellations of the empirical micro-cycles. As the underlying orignal series, the simulated time series have 29 observations.

[5] $\sigma_{y^a} = 5057.79$.

series.

The models *MIII* ($\widehat{\Sigma}_\varepsilon$ incorporated) and *MIV* ($\widehat{\Sigma}_\varepsilon$ not incorporated) are modifications of *MI* and *MII*, including the aggregate as additional regressor in the estimations and employing the contemporaneous simulated aggregate in a simultaneous VAR. Results are displayed in the third and fourth row of Table 3. Again, there are no differences between *MIII* and *MIV*. The average correlation coefficient between estimated sectoral shocks is now 1.4%. The ratio of empirical to simulated standard deviations has switched to roughly 1 to 2.7.

Table 3: Standard deviations of MC-simulated aggregate cycles

Model	Mean	Median	Maximum	Minimum	Standard deviation
MI	1608.97	1599.19	2575.62	963.79	252.40
MII	1580.50	1568.88	2575.50	935.05	250.34
MIII	13856.89	13810.34	20447.70	8154.37	2118.24
MIV	13800.45	13729.84	21147.56	8260.93	2101.51

MI: Model (3.5) with covariance matrix $\widehat{\Sigma}_\varepsilon$; *MII*: Model (3.5) with independent sectoral shocks; *MIII*: Model (3.5) with covariance matrix $\widehat{\Sigma}_\varepsilon$ and including the aggregate; *MIV*: Model (3.5) with independent sectoral shocks and including the aggregate.

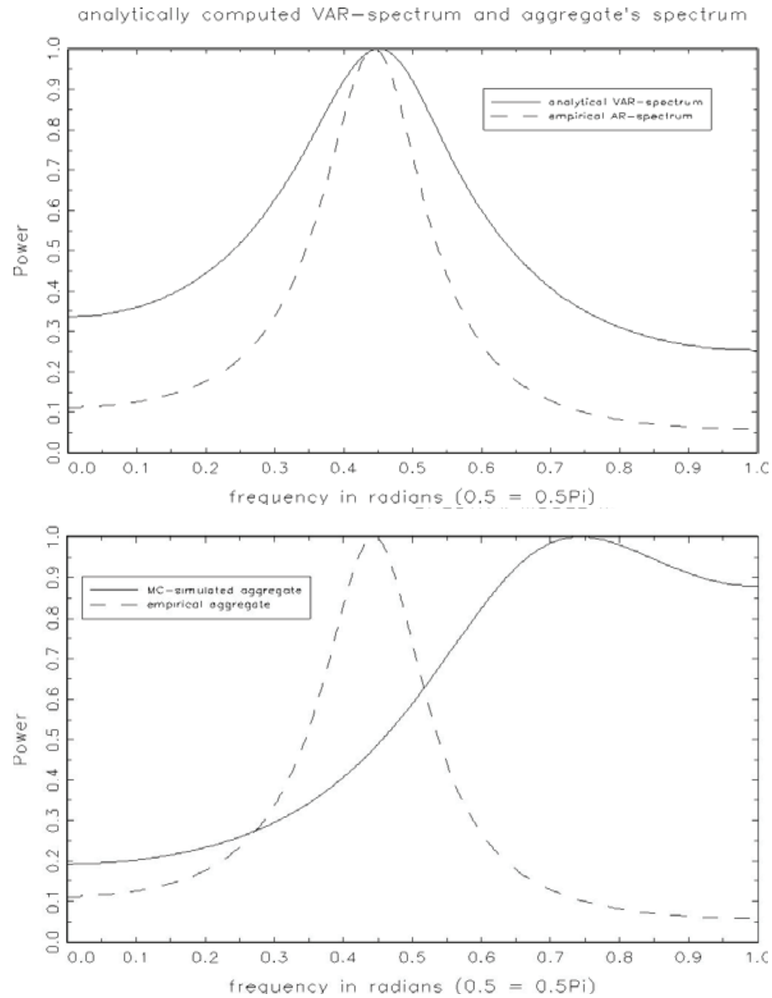
As there are no major differences between *MI* and *MII* as well as *MIII* and *MIV*, the comparison of the spectral characteristics of the implied and/or simulated aggregate and the empirical aggregate is based on Models *MI* and *MIII*. We calculate the analytical spectrum for the aggregate series of *MI*. Writing *MI* as a VAR(2)-model, the aggregate spectrum can be derived from a linear combination of the underlying processes (Koopmans, 1995). The resulting spectrum plotted against the spectrum of the empirical aggregate is displayed in the first window of Figure 1.[6] The spectrum implied by the VAR(2)-model *MI* (solid line) is peaking nearly at the same frequency as the spectrum of the empirical series (dashed line). But there is a difference concerning the shape: the VAR(2)-model's aggregate spectrum is relatively flat, suggesting that the aggregate series is modeled too smooth (Table 3). In addition, it is too noise- rather than signal-driven in contrast to its empirical analogue. The second window of Figure 1 displays the spectrum for the MC-simulated aggregate series (solid line) for which the adjusted AR(2)-coefficients were the nearest neighbors of the median coefficient vector of AR(2)-models adjusted to the 1000 simulated aggregate series of *MIII*. The spectrum is shifted towards higher frequencies compared with the empirical aggregate cycle.

The above tests of nested models suggest that the cycle in aggregate investment plays a role in determining industrial investment cycles. Nevertheless, employing the aggregate variable in the framework of VAR simulations as a common shock variable (besides idiosyncratic and weakly correlated sectoral shocks) does not lead to an improvement of explanatory power. In summary, these results suggest

[6] The simulated aggregate (solid line) together with the actual manufacturing investment aggregate (dashed line) are displayed in Figure 2.

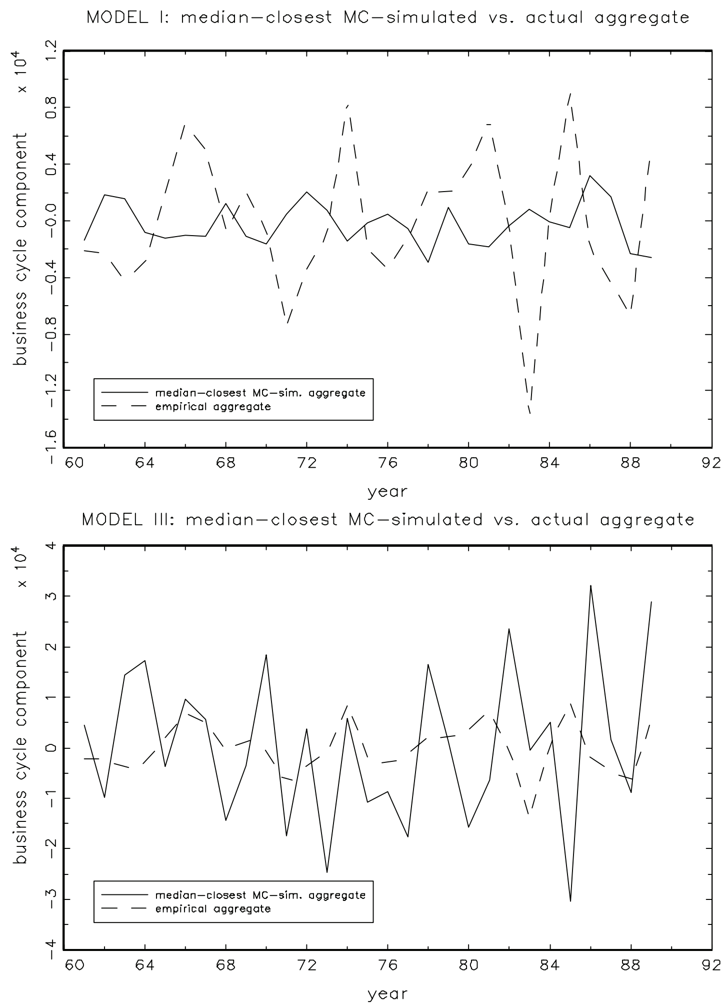
that the aggregation process might be more adequately described and modeled by a nonlinear process that incorporates the aggregate variable.

Figure 1: Spectrum of modeled vs. actual aggregate cycle



Note: First window: theoretical spectrum;
Second window: spectrum based on MC-simulations of *MIII*.

Figure 2: MC-simulated vs. actual aggregate cycle



Note: First (second) window: based on MC-simulations of *MI* (*MIII*).

3.3 Sectoral comovement and synchronization over time

Recent contributions on industrial business cycle comovement measure the outcome of a synchronization process, rather than the process itself and its development over time.[7] Periodic entrainment or mode-locking on the industrial level implies that there is a drawing together of different sectoral phases and periodicities. The crucial point is that this process does not happen instantaneously according to a potentially underlying phase- or mode-locking regime, as would be the case in a common shock scenario, but rather develops over time until synchronization reaches its full impact or a temporary state of desynchronization. An adequate technique to investigate this phenomenon is parametric (multivariate) spectral analysis that allows for time dependent measures by applying the Kalman filter.[8] Based on the time dependent (V)AR models we calculate the following measures for each point in time:

(a) Share of Total Variance

Integrating the univariate spectrum in the interval $[-\pi, \pi]$, we obtain the overall variance of the series. Thus, it is possible to calculate the proportion of variance in a certain frequency band $[\omega_1, \omega_2]$. This measure helps us to assess the relative importance of the cyclical components in a frequency band of 3-7 years, i.e. the prominent cycle length of the US investment series.[9]

(b) Explained Variance

Using the decomposition of the squared coherency into an in-phase and an out-of-phase component, we present another measure of synchronization.[10] The share of total variance in a frequency band can be decomposed into a part which is explained by the variance of another series in the same frequency band, and an unexplained part. The explained variance can be further decomposed into an in-phase and an out-of-phase component: The higher the in-phase component, the higher the comovement between the two series in the frequency band of interest. We analyze the similarity of the cyclical structure by calculating the Euclidean distance between the spectral shape of the aggregate and the spectral shape of the disaggregated industries. The more similar the spectra, the lower the distance measure on the ordinate. As can be seen from Figure 3, the results point uniformly towards an increasing similarity.[11] From period 1 corresponding to the year 1958 (in the BKM-filter case 1960) to period 32 corresponding to the year 1992 (1990) the median of the distance measure obviously falls. We calculate the

[7]This includes Selover *et al.* (2003) who, on the one hand, argue that the different oscillators or sectors should tend toward the same compromise frequency if they are really mode-locking. On the other hand, however, they investigate a realized common frequency (as if sectors are and always have been mode-locked) rather then the tendency to mode-lock, synchronize or desynchronize over time.

[8]The detailed estimation method is described in Appendix E.

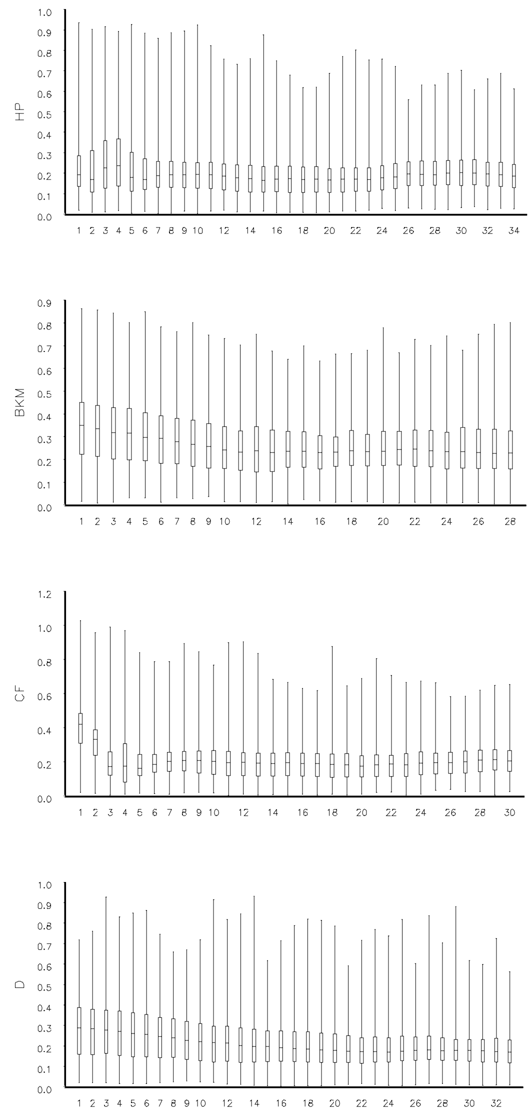
[9]The average cycle length length is about 4.5 to 5 years, both for the industrial investment series and in the MFI aggregate (see Süssmuth, 2002b). Corresponding results for the overall spectral shape as well as for different cycle ranges are available on request from the authors.

[10]This decomposition is based on the dynamic correlation proposed by Croux, Forni, and Reichlin (2001).

[11]The only exception, i.e. a less clear cut increase in similarity, is given by the HP-filtered series.

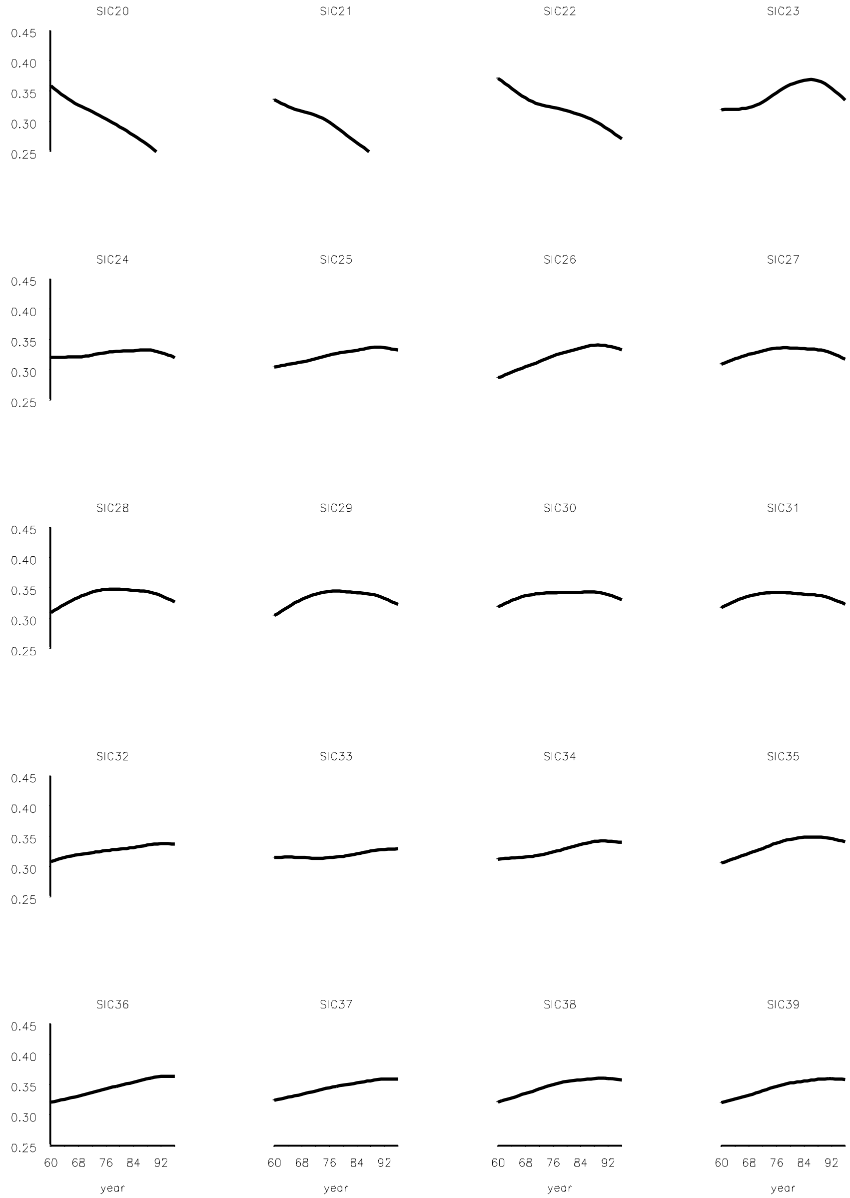
change in the in-phase explained variance as discussed above. We focus on the 5-7 and 3-5 years range, and group the results on the 2-digit level (Figure 4 and 5). Apart from food, tobacco products, textile mill products, and apparel and other textiles, the comovement increases over time. As expected, however, the synchronization effect is more pronounced in the 3-5 years range, and relatively less so in the 5-7 years range. This evidence of a weak relationship between these industrial cycles and the main economy confirms Hornstein (2000, p. 30) who notes that "these are industries which are subject to shocks exogenous to the aggregate economy, like weather ... and whose contribution to the aggregate economy is limited." Contrary to the studies of Christiano and Fitzgerald (1998) and Hornstein (2000), which are based on business cycle components of US sectoral employment series, we find that the petroleum and coal products industry does increasingly comove with the aggregate, at least in the 3-5 years range.

Figure 3: Changes in the Similarity of the Spectral Shape, 3-7 Years Range



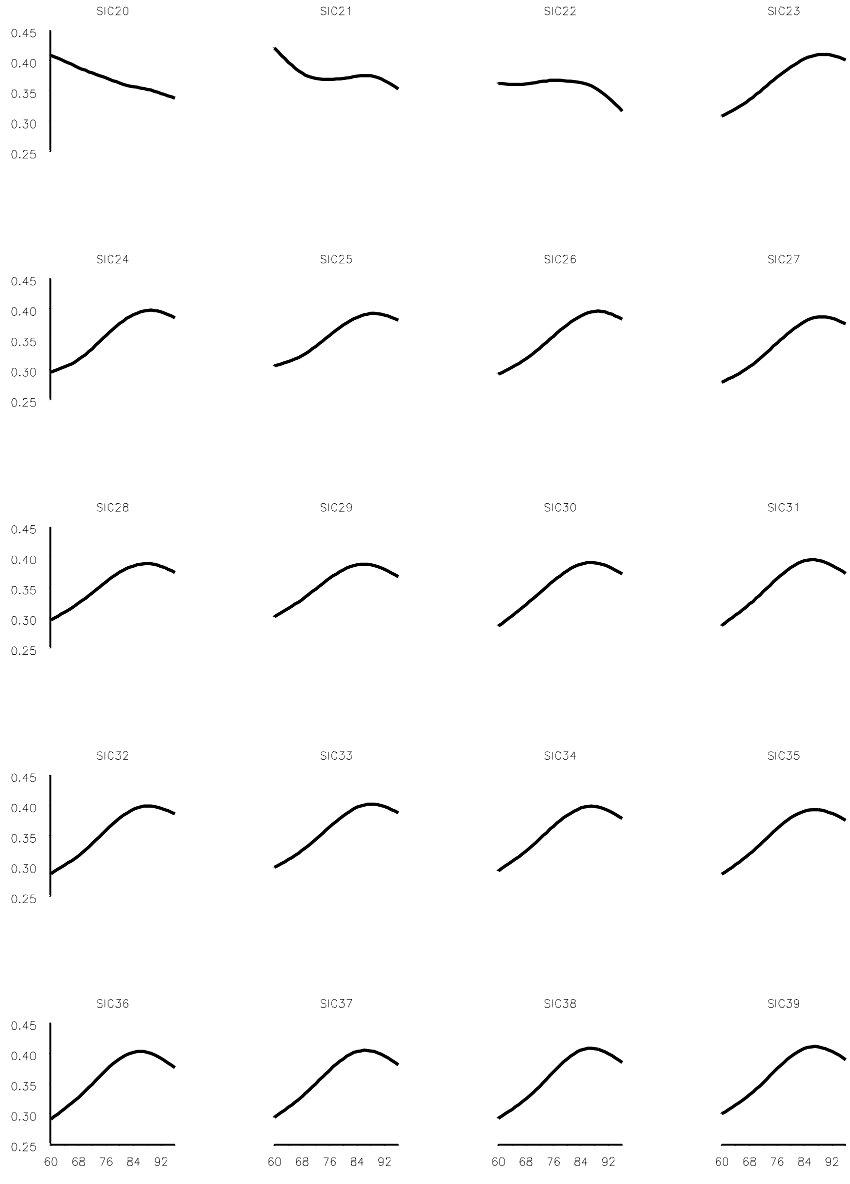
Notes
HP: Hodrick-Prescott Filter (smoothing weight: $\lambda = 100$); BKM: modified Baxter-King Filter; CF: Christiano-Fitzgerald Filter; D: Difference Filter.
The bottom of the box corresponds to the 25th percentile, the line drawn through the box marks the 50th percentile (median) and the top of the box the 75th percentile of the distribution. The two whiskers represent the minimia and maxima for each year.

Figure 4: Changes in the Comovement with the Aggregate, 2-Digit Level, 5-7 Years Range, BKM Filter



Notes:
SIC20: Food and kindred products, SIC21: Tobacco products, SIC22: Textile mill products, SIC23: Apparel and other textile, SIC24: Lumber and wood, SIC25: Furniture and fixtures, SIC26: Paper and allied, SIC27: Printing and publishing, SIC28: Chemicals and allied, SIC29: Petroleum and coal products, SIC30: Rubber and misc. plastics products, SIC31: Leather and leather products, SIC32: Stone, clay and glass, SIC33: Primary metals, SIC34: Fabricated metals, SIC35: Industrial machinery and equipment, SIC36: Electronic and electric equipment, SIC37: Transportation equipment, SIC38: Instruments and related products, SIC39: Miscellaneous industries

Figure 5: Changes in the Comovement with the Aggregate, 2-Digit Level, 3-5 Years Range, BKM Filter



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4. Simulations

We calibrate the model sketched in Section 2 on the base of spectral estimations of the discrete analogues of the underlying SOA equations and empirical initial values. The complementary parts of (2.2) imply the following roots:

$$\lambda_{i,\ 1,2} = \frac{1}{2}\left(-b_i \pm \sqrt{b_i^2 - 4a_i}\right).$$

For $a_i > 0.25b_i$ the discriminant is negative, the Eigenvalues are conjugate complex and the solution takes the form of a damped cycle:

$$I_i\left(t\right) = Ae^{\rho_i t}\cos\left(\omega_i t - \phi_i\right).$$

The damping rate ρ, frequency ω and period P are given by $\rho_i = -b_i/2$, $\omega_i = 0.5\sqrt{4a_i - b_i}$, and $P_i = 2\pi/\omega_i$, respectively. In reality, we do not observe investment behavior as described by equations (2.2) or (2.3) and (2.4) in continuous time. To find empirically consistent parameter values a_i and b_i, we need to rely on the discrete analogues of these equations. These are given by a set of AR(2) equations. AR spectral analysis enables us to obtain estimates $\widehat{P}_i$ along with moduli mod $_i$ of the complex root of the respectively adjusted AR(2) model for the $i = 1, ..., 450$ sectoral cycle components. This information allows to compute estimates of b_i from

$$\widehat{b_i} = 2\widehat{\rho_i} = 2\ln\left(\text{mod } i\right).$$

Using these estimates and the above definitions of ω_i and P_i, we obtain empirical measures for a_i:

$$\widehat{a_i} = \frac{1}{4}\left[\left(4\pi/\widehat{P_i}\right)^2 + \widehat{b_i}\right].$$

Given estimates of $\widehat{P}_i$ and mod $_i$, initial values for sectoral stocks and flows of capital, i.e. $K_i\left(0\right)$ and $I_i\left(0\right)$ in detrended form, and a law of motion for K_i^*, we simulate equations (2.2) and the equation system (2.3) and (2.4) for the case $\chi_i = 0$.

Estimates based on the NBER data provide information on $\widehat{P}_i$ and mod $_i$. After detrending (BKM filter), the initial period 0 corresponds to the year 1961.[12] The values of desired capital $K_i^*\left(t\right)$ should be expressed in terms of observable quantities, typically current sales and some measures of the cost of capital. While for current sales data are available on the SIC 4-digit level, cost of capital are not available and in general difficult to measure. Furthermore, as equipment is intended to serve as a means of production for many years, relating desired capital to current sales only is unsatisfactory: desired capital should not be dominated by short-run fluctuations, but rather by long-run expectations. It is, therefore, straightforward to assume that desired capital reacts only to permanent changes

[12] Following the specifiaction of Baxter and King (1999), the BKM bandpass filter shortens an annual series by three data points at the beginning and end of the observation period.

in sales. In our model, the permanent component of sales is adequately captured by the trend, and therefore the desired capital stock follows the trend path only. Since (2.3) expresses variables as deviations from long-term trend, desired capital stocks are throughout set to zero in the course of the simulation study.

We will focus on the top 135 sectors in terms of volatility due to the fact that these contribute more than four fifth to the aggregate cycle's standard deviation (Süssmuth 2003). In addition, sectors are divided into a part of sectors that interact with the aggregate (type I industries) and a non-interacting part (type II industries). The classification is based on estimates of bivariate squared coherency (sc) of aggregate and disaggregate cycles (equation E.5, Appendix E). Coherency is a prerequisite or necessary, but not sufficient condition for synchronization. If a sectoral series shows weak or no coherency with the aggregate over the whole observation period, it will be modeled by a non-interacting SOA (2.2). Accordingly, we treat the 110 sectoral series showing an sc-value ≥ 0.45 with the aggregate MFI cycle as type I, the remaining 25 sectors as type II industries.

In a first step, we set $\chi_i = \overline{\chi}$ for all $i = 1, ..., 110$ type I industries and minimize the difference between resulting simulated aggregate cycle and empirical aggregate cycle (based on the standard prognostic measures mean squared error, mean absolute error, and root mean squared error). A local minimum[13] is found for $\overline{\chi} \approx 0.6$ (Figure 7).

We assume the following linear relationship dependent on the bivariate sc estimates:

$$\chi_i - \overline{\chi} = \gamma \left(sc_i - \overline{sc} \right), \tag{4.1}$$

where $\overline{sc}$ denotes the mean of estimated sc-values, i.e. $\frac{1}{110} \sum_{i=1}^{110} sc_i$, and γ the factor of proportionality. For a single trial value $\widetilde{\gamma}$ system (2.3), (2.4) can now be simulated and optimized with respect to γ. This parameter reflects the strength of interaction with the aggregate activity or, in other words, the weight the individual industry assigns to the observation of aggregate investment activity relative to its own information.

A further extension in comparison to earlier versions of the model and to Selover *et al.* (2003) is the inclusion of exogenous shocks. The modification of equation (2.3) is

$$I_i^{'}(t) = -a_i \left[1 + \chi_i \cdot \Psi \left(-K_i(t) \sum_{i,j} I_{i,j}(t) \right) \right] K_i(t) - b_i K_i^{'}(t) + \xi_i(t), \tag{4.2}$$

where Ψ is defined as in (2.4). Subscript i now denotes type I, and subscript j type II industries. The idiosyncratic shock terms are $\xi_i(t) \sim N\left(\mu_i^{AR}, \sigma_i^{AR}\right)$, where μ_i^{AR} is the mean of the error vector of the respective AR model underlying the ith sectoral spectral estimation model, and σ_i^{AR} is the corresponding standard deviation of the ith error vector.

To control for the relative weight of these idiosyncratic sectoral shocks, the

[13] Global in the range $\overline{\chi} \in (0.001, 1.500)$.

parameter ϑ is introduced, with

$$\xi_i(t) = \vartheta\epsilon_i(t), \tag{4.3}$$

where $\epsilon_i(t) \sim N\left(\mu_i^{AR}, \sigma_i^{AR}\right)$ for all $i = 1, ..., 110$ type I sectors. The non-interacting type II industries behave according to

$$I_j^{'}(t) = -a_j K_j(t) - b_j K_j^{'}(t) + \xi_j(t), \text{ where} \tag{4.4}$$

$\xi_j(t) = \vartheta\epsilon_j(t)$ and $\epsilon_j(t) \sim N\left(\mu_j^{AR}, \sigma_j^{AR}\right)$ with $j = 1, ..., 25$.

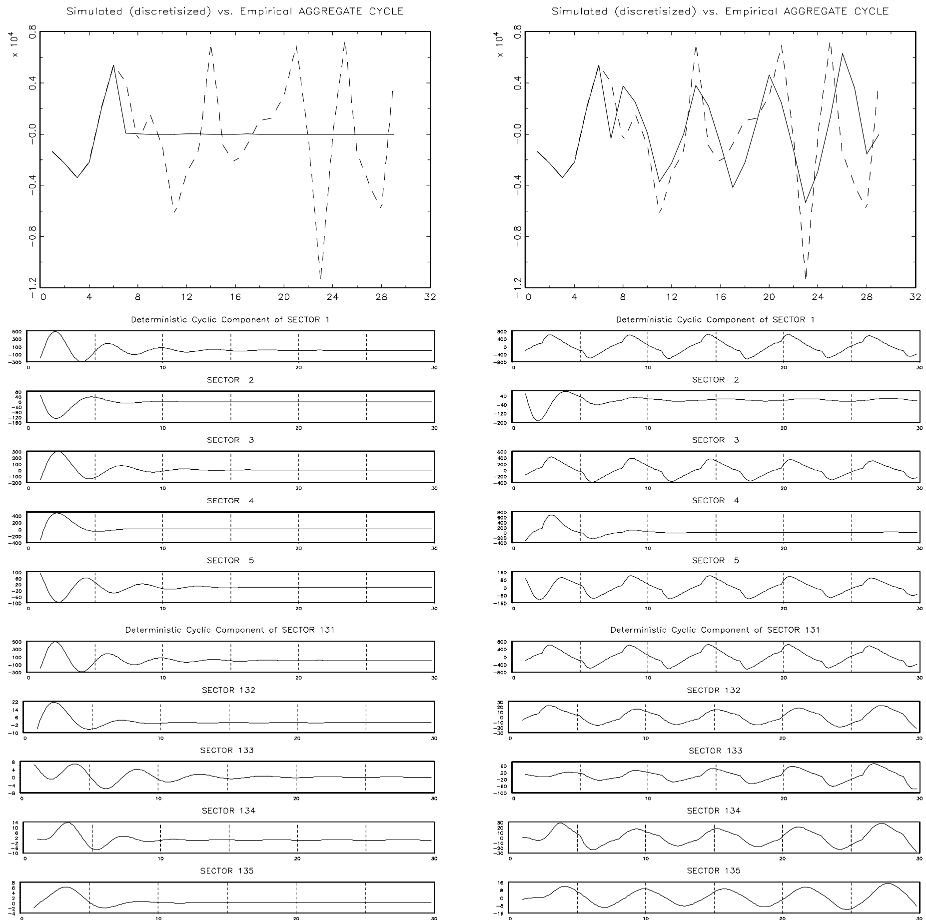
Simulation results demonstrate that changing the ϑ-values has in general less impact on the results than changing the interaction-parameter γ (Süssmuth 2003). The best performance in terms of matching a chosen set of spectral and volatility measures as well as of maximizing a standard prediction measure is achieved for parameter constellations with γ in between 0.2 and 0.4, and ϑ with a value of about 1. An exemplary simulation run is displayed in Figure 8.

5. Conclusion

Our study extends the analysis by Selover *et al.* (2003) in three ways: (i) a more meaningful economic model, explicitly based on an optimization objective, (ii) a further disaggregation of analyzed industries (450 as opposed to 23 sectors), and (iii) a time dependent measure of synchronization, allowing to quantitatively assess the process rather than the outcome of industrial sector mode-locking. With regard to (iii), we find that the comovement of industrial sectors with the aggregate cycle increased over the last four decades, apart from few industries which are predominantly subject to exogenous shocks. This result is confirmed by simulations of the economic model. The results support the hypothesis of a time varying profile of US sectoral business cycle comovement and mode-locking.

Figure 7: Simulations of the sectoral mode-locking model of investment

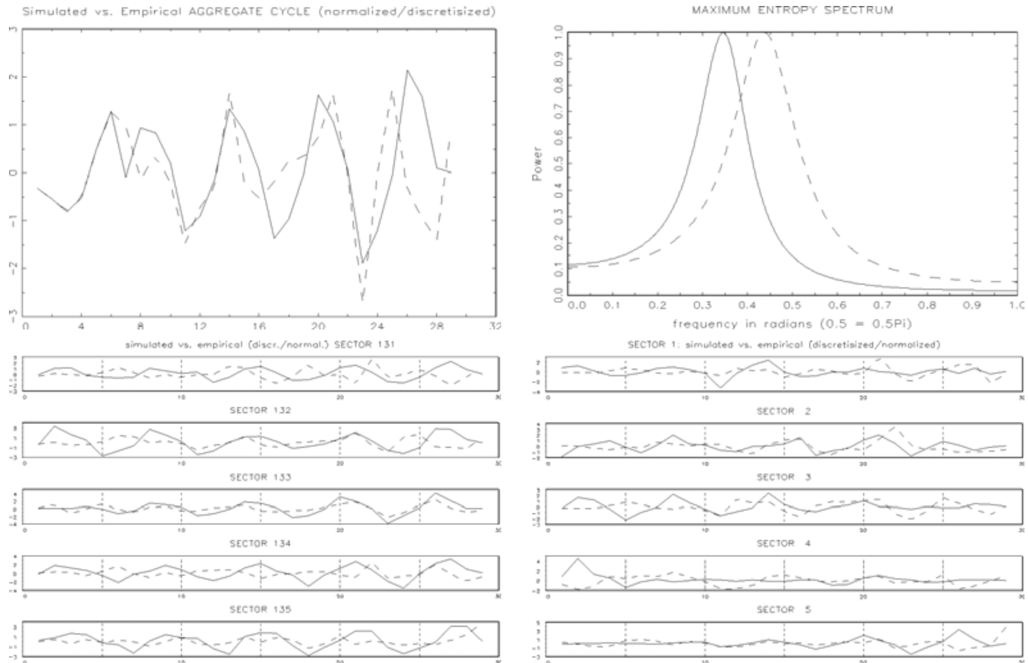
$\chi_i = \chi = 0$ $\qquad$ $\chi_i = \chi = 0.67$



Note: Dashed: Empirical series; below aggregate windows: First/last five of top135 most volatile sectors:

Sector 1	motor vehicles & car bodies (SIC 3711)	Sector 131	frozen fruits & veg. (SIC 2037)
Sector 2	petroleum refining (SIC 2911)	Sector 132	machine tool acc. (SIC 3545)
Sector 3	motor vehicle parts & acc. (SIC 3714)	Sector 133	gypsum products (SIC 3275)
Sector 4	blast furnaces & steel mills (SIC 3312)	Sector 134	canned specialities (SIC 2032)
Sector 5	paper mills (SIC 2621)	Sector 135	cereal breakfast food (SIC 2043)

Figure 8: (Dis-)Aggregate dynamics for sample simulation run: $\gamma = 0.4$, $\vartheta = 0.8$



Note: Solid (dashed) line: simulated (empirical) series; first (second) window: aggregate series (spectra);

performance measures for aggregate series: *period length*: 5.99 (actual: 4.55), *mod*: 0.83 (0.75), *RMSE*: 0.51, *absolute discrepancy in standard deviation terms*: 8.5 (= 0.002%);

below aggregate windows: First/last five of top135 most volatile sectors:

Sector 1	motor vehicles & car bodies (SIC 3711)	Sector 131	frozen fruits & veg. (SIC 2037)
Sector 2	petroleum refining (SIC 2911)	Sector 132	machine tool acc. (SIC 3545)
Sector 3	motor vehicle parts & acc. (SIC 3714)	Sector 133	gypsum products (SIC 3275)
Sector 4	blast furnaces & steel mills (SIC 3312)	Sector 134	canned specialities (SIC 2032)
Sector 5	paper mills (SIC 2621)	Sector 135	cereal breakfast food (SIC 2043)

Appendix A. Motivation and Derivation of Equation (2.2)

All costs are expressed in terms of deviations k_i of actual capital K_i from its desired value K_i^*; i.e., $k_i = K_i^* - K_i$, $k_i^{'} = \frac{d}{dt}(K_i^* - K_i)$, etc. They are assumed to be given by a sum of contributions quadratic in k_i, $k_i^{'} = i_i$ and $k_i^{''} = i_i^{'}$, where $i_i = k_i^{'}$ denotes the individual investment deficit. The motivation is an intertemporal optimization calculus in the presence of three cost components outlined in more detail in the three assumptions below:

Assumption 1. Costs due to excess capital
$k_i > 0$: *excess capital of an industry leading to inflexibility and enhanced depreciation causing costs.*
$k_i < 0$: *underequipment with capital, i.e., an industrial capital deficit. This leads to missing production possibilities and excessive capacity utilization.*

Assumption 2. Costs due to changes of the capital stock:
$k_i^{'} > 0$ *and* $k_i^{'} < 0$ *mark situations where imperfect substitutability with other production factors leads to costs or, in the case of close to perfect substitutability, leads to a subefficient use of the other input factors and thereby cause costs.*

Assumption 3. Costs due to changes of the investment strategy:
$k_i^{''} = i_i^{'} > 0$ *and* $k_i^{''} = i_i^{'} < 0$ *reflect changes in the investment strategy. Changes of contractual commitments and supplier's arrangements are costbearing consequences of this behavior.*

Based on these three components, it is possible to formulate the following cost function:

$$C_i\left(k_i, k_i^{'}, k_i^{''}\right) = \alpha_i k_i^2 + \beta_i \left(k_i^{'}\right)^2 + \gamma_i \left(k_i^{''}\right)^2. \tag{A.1}$$

Every investing unit determines the time paths of production capital in such a way that the present value of all cost components, i.e. (A.1) discounted with an appropriate discount rate ρ, is minimized:

$$\min_{k_i(t)} \int_{t_0}^{\infty} \left[\alpha_i k_i^2 + \beta_i \left(k_i^{'}\right)^2 + \gamma_i \left(k_i^{''}\right)^2\right] e^{\rho(t_0 - t)} dt. \tag{A.2}$$

Given initial values $K_i(t_0)$ and $I_i(t_0)$, the relevant transversality conditions are:

$$\begin{aligned} \lim\nolimits_{t\to\infty} i_i e^{\rho(t_0-t)} &= k_i^{'} e^{\rho(t_0-t)} = 0, \\ \lim\nolimits_{t\to\infty} i_i^{'} e^{\rho(t_0-t)} &= k_i^{''} e^{\rho(t_0-t)} = 0, \\ \lim\nolimits_{t\to\infty} i_i^{''} e^{\rho(t_0-t)} &= k_i^{'''} e^{\rho(t_0-t)} = 0. \end{aligned} \tag{A.3}$$

¿From (A.2) and (A.3), it is possible to derive the following Euler equation for $k_i(t)$ by means of standard techniques of variational analysis:

$$\alpha_i k_i + \beta_i \left(\rho - \frac{d}{dt}\right) k_i^{'} + \gamma_i \left(\rho - \frac{d}{dt}\right)^2 k_i^{''} = 0. \tag{A.4}$$

The characteristic polynomial of this fourth order differential equation in k_i is:

$$P(x) = \alpha_i + \beta_i (\rho - x) x + \gamma_i (\rho - x)^2 x^2. \tag{A.5}$$

$P(x)$ is quadratic in $y = (\rho - x) x$. The two solutions in terms of y are

$$y_{1,2} = \frac{1}{2\gamma_i} \left(-\beta_i \pm \sqrt{\beta_i^2 - 4\alpha_i\gamma_i} \right). \tag{A.6}$$

Solving $y = (\rho - x) x = \rho x - x^2$ for x and substituting (A.6), leads to the following potential solutions of (A.5) that fulfill the transversality conditions:

$$\begin{aligned} x_{1,2} =& \frac{\rho}{2} - \sqrt{\frac{\rho^2}{4} + \frac{1}{2\gamma_i} \left(\beta_i \pm \sqrt{\beta_i^2 - 4\alpha_i\gamma_i} \right)} \\ =& \frac{\rho}{2} \left[1 - \sqrt{1 + \frac{2}{\gamma_i^2\rho^2} \left(\beta_i \pm \sqrt{\beta_i^2 - 4\alpha_i\gamma_i} \right)} \right] \end{aligned} \tag{A.7}$$

The solutions are oscillatory for $\beta_i^2 < 4\alpha_i\gamma_i$, i.e., sufficiently large values of the cost parameters associated with discrepancies in the individual capital stock and changes in the investment strategy. Equation (A.7) can be simplified to[14]

$$\begin{aligned} x_{1,2} \approx& \frac{\rho}{2} \left[1 - 1 + \frac{1}{\gamma_i^2\rho^2} \left(\beta_i \pm \sqrt{\beta_i^2 - 4\alpha_i\gamma_i} \right) \right] \\ \approx& \frac{1}{2\gamma_i\rho} \left(\beta_i \pm \sqrt{\beta_i^2 - 4\alpha_i\gamma_i} \right). \end{aligned} \tag{A.8}$$

The values of $x_{1,2}$ given by (A.8) are the roots of the polynomial

$$\alpha_i + \beta_i \rho x + \gamma_i \rho^2 x^2, \tag{A.9}$$

which is the characteristic polynomial of the differential equation

$$\alpha_i k_i + \beta_i \rho k_i' + \gamma_i \rho^2 k_i'' = 0. \tag{A.10}$$

Finally, substitution of the relationship $k_i = K_i^* - K_i$ and assuming $(K_i^*)' = (K_i^*)'' = 0$ leads to

$$I_i' = K_i'' = \frac{\alpha_i}{\gamma_i\rho^2} (K_i^* - K_i) - \frac{\beta_i}{\gamma_i\rho} K_i' = a_i (K_i^* - K_i) - b_i K_i', \tag{A.11}$$

$$\text{where } a_i = \frac{\alpha_i}{\gamma_i\rho^2} \text{ and } b_i = \frac{\beta_i}{\gamma_i\rho}.$$

[14] We apply $\sqrt{1+x} \approx 1 + \frac{x}{2}$, assuming sufficiently small values of x.

Appendix B.

The following paragraphs demonstrate the analogy of the second order dynamics (2.2) to the Euler equation of a standard Ramsey model with time-to-build (TTB) lags[15] (Kydland and Prescott, 1982). For the sake of convenience, the TTB Euler equation of investment is derived as an analogue to the second order differential equation (2.2) in a discrete time framework. The derivation follows Oliner, Rudebusch and Sichel (1999), under standard assumptions from the literature.

The firm's production function is Cobb-Douglas with constant returns to scale:

$$Y_t = F\left(K_{t-1}, L_t\right) = AK_{t-1}^{\theta} L_t^{1-\theta}, \tag{B.1}$$

where Y_t and L_t are output and employment during period t, and K_{t-1} is the capital stock at the end of period $t-1$. The marginal product of capital is

$$F_{K_t} \equiv \partial Y_{t+1} / \partial K_t = \theta Y_{t+1} / K_t. \tag{B.2}$$

Capital is a quasi-fixed factor subject to the usual quadratic adjustment costs, while employment is assumed to be variable. Let I_t denote gross investment during period t. The adjustment cost function, abstracting from interactions between capital and labor, is

$$C\left(I_t, K_{t-1}\right) = \left[\alpha_0 \left(I_t / K_{t-1}\right) + \left(\alpha_1 / 2\right) \left(I_t / K_{t-1}\right)^2\right] K_{t-1}. \tag{B.4}$$

The partial derivatives of $C\left(I_t, K_{t-1}\right)$ are

$$C_{I_t} = \alpha_0 + \alpha_1 \widetilde{I}_t \text{ and } C_{K_{t-1}} = -\left(\alpha_1 / 2\right) \widetilde{I}_t^2, \tag{B.4}$$

where $\widetilde{I}_t \equiv I_t / K_{t-1}$. For the firm's investment decision to be well defined, C_I must be increasing with the level of investment, i.e., $\partial C_t / \partial I_t = \alpha_1 / K_{t-1}$ must be > 0, implying that $\alpha_1 > 0$.

All markets are perfectly competitive. Both input prices are normalized by the price of output (p_t). The resulting real price of capital and the real wage are denoted by p_t^I and w_t, respectively.

In addition, we assume that investment projects are subject to TTB lags: Let S_t denote the value of projects started in period t. All projects take τ periods to complete, so that additions to the capital stock in period t equal project starts in period $t-\tau$. Then

$$K_t = (1-\delta) K_{t-1} + S_{t-\tau}. \tag{B.5}$$

Let ϕ_i denote the proportion of a project's total value that is put in place i periods after its start, with $\phi_0, ..., \phi_\tau \geq 0$ and $\sum_{i=0}^{\tau} \phi_i = 1$. Thus, investment at

[15] Under the assumptions of (i) short foresight, (ii) constant marginal installation costs of new capital, and (iii) no depreciation.

t is given by

$$I_t = \sum_{i=0}^{\tau} \phi_i S_{t-i}. \tag{B.6}$$

Firms maximize the expected value of their real future profits

$$V_t = E_t \left(\sum_{s=0}^{\infty} \beta_{t,s}^{*} \pi_s \right),$$

where $\beta_{t,s}^{*} = \Pi_{j=t+1}^{s} \beta_j$ denotes the discount factor from period s back to t and

$$\pi_s = F\left(K_{s-1}, L_s\right) - C\left(\sum_{i=0}^{\tau} \phi_i S_{s-i}, K_{s-1}\right) - w_s L_s - p_s^I \left(\sum_{i=0}^{\tau} \phi_i S_{s-i}\right)$$

subject to equation (B.5).

The relevant two first-order conditions are

$$\sum_{i=0}^{\tau} \phi_i E_t \left[\beta_{t,t+i}^{*} \left(p_{t+i}^I + C_{I_{t+i}}\right)\right] = E_t\left(\beta_{t,t+\tau}^{*} \lambda_{t+\tau}\right) \tag{B.7}$$

$$\begin{aligned} & E_t\left[\beta_{t,t+\tau}^{*}\left(F_{K_{t+\tau}} - C_{K_{t+\tau}}\right)\right] = \\ & E_t\left[\beta_{t,t+\tau}^{*} \lambda_{t+\tau} - (1-\delta)\,\beta_{t,t+\tau+1}^{*} \lambda_{t+\tau+1}\right]. \end{aligned} \tag{B.8}$$

Eliminating the terms in λ combining (B.7) and (B.8) and comprising expectational error ϵ and (scaled) output terms by Λ_t, we obtain after some rearrangement

$$\begin{aligned} & \sum_{i=0}^{\tau} \phi_i \Delta p_{t+i+1}^I + \sum_{i=0}^{\tau} \alpha_0 \phi_i \Delta \beta_{t,t+i+1}^{*} + \\ & \quad + \frac{1}{2} \alpha_1 \beta_{t,t+\tau+1}^{*} \widetilde{I}_t^2 + \sum_{i=0}^{\tau} \alpha_1 \phi_i \Delta \widetilde{I}_t = \Lambda_t, \end{aligned} \tag{B.9}$$

where

$$\begin{aligned} \Delta p_{t+i+1}^I &\equiv (1-\delta)\,\beta_{t,t+i+1}^{*} p_{t+i+1}^I - \beta_{t,t+i}^{*} p_{t+i}^I \\ \Delta \beta_{t,t+i+1}^{*} &\equiv (1-\delta)\,\beta_{t,t+i+1}^{*} - \beta_{t,t+i}^{*} \\ \Delta \widetilde{I}_t &\equiv (1-\delta)\,\beta_{t,t+i+1}^{*} \widetilde{I}_{t+i+1} - \beta_{t,t+i}^{*} \widetilde{I}_{t+i}. \end{aligned}$$

If we now assume (i) no depreciation, i.e. $\delta = 0$, (ii) constant marginal costs of installing new capital, i.e. $p_t^I = p_{t+1}^I = ...$, and (iii) short foresight in the sense that $\beta^*_{t,t<t+\tau+1} >> \beta^*_{t,t+\tau+1}$ for all t,

$$\left(\alpha_1\phi_1\beta^*_{t,t+2} - \alpha_1\phi_2\beta^*_{t,t+2}\right)\widetilde{I}_{t+2} - \left(\alpha_1\phi_1\beta^*_{t,t+1} - \alpha_1\phi_0\beta^*_{t,t+1}\right)\widetilde{I}_{t+1} - \\ \alpha_1\phi_0\beta^*_{t,t}\widetilde{I}_t = \Lambda_t$$

is obtained for $\tau = 2$ and

$$\left(\alpha_1\phi_0\beta^*_{t,t+1} - \alpha_1\phi_1\beta^*_{t,t+1}\right)\widetilde{I}_{t+1} - \alpha_1\phi_0\beta^*_{t,t}\widetilde{I}_t = \Lambda_t$$

for $\tau = 1$, where $\Lambda_t = f\left(\beta^*, \theta, Y, \epsilon, \tau\right)$.

Appendix C.

For an individual capital deficit (excess) $K_i^* - K_i > 0$ ($K_i^* - K_i < 0$) and aggregate disinvestment activity $\sum_j I_j < 0$ model (2.3) implies $\partial I_i^{'}/\partial \sum_j I_j < 0$ ($\partial I_i^{'}/\partial \sum_j I_j > 0$), i.e. a slowing down (speeding up) of the pace of the investment flow. This behavior can be justified on the grounds of an endogenously-timed herding scenario.

Consider two sectors i and j. The investors in these industries face a myopic two-sided investment decision problem: whether and if so when to run a certain investment project of a continuous flow of investment opportunities additional to its medium-term stock smoothing objective (2.1). Each of these projects has a specific value equaling the state of the world ω.[16] The state is not directly observed, instead there is a signal, μ, for i and j at $t = 1$, respectively. Let μ_t^i be the signal of an investor in sector i at time t. Signals μ^i and μ^j are assumed independent and identically drawn from a uniform distribution with range $[-1, 1]$, i.e. $\mu^{i,j} \sim U[-1, 1]$. These signals do not change over time, and ω is set equal to the sum of signals, $\omega = \mu^i + \mu^j$. Actions are defined as: $x^i = 1 \Longleftrightarrow$ "invest"; and $x^i = 0 \Longleftrightarrow$ "do not invest". In each period actions are made simultaneously. Agents in i and j cannot observe each others' actions at a myopic point in time. However, in period 2, the investor of sector i will know the action performed by the investor of industry j in period 1, and through the observed choice of activity some information about the nature of the j-signal is revealed. By assumption there is no pre-play communication, i.e. the possibility to meet and clear signals. Payoffs $\pi_t^{i,j}$ equal the state of the world discounted by ρ:

$$\pi_t^{i,j} = \begin{cases} \rho^{t-1}\omega & \text{if} \quad x^{i,j} = 1 \\ 0 & \text{if} \quad x^{i,j} = 0 \end{cases} . \tag{C.1}$$

To solve the short-term decision problem, we consider the problem faced by an investor of sector i. Let us establish the following simple rules: (i) invest (i.e., $x^i = 1$) if and only if $E\left[\pi_t^i\right] > 0$; (iia) if an investment is to be made, then

[16] We assume a fixed ω at the beginning of time and a discrete time framework. It can be shown that results remain qualitatively the same in the case of ω following a Markow process and/or a continuous time world.

make it at $t = 1$ if and only if $E\left[\pi_1^i\right] > E\left[\pi_2^i\right]$, otherwise wait. In these rules the profit function explicitly includes discounting the short run time horizon. This might seem a sensible rule to adopt, but while we are capturing a notion of the cost of delay since we consider an implicit $\rho < 1$ in the second period payoff, we are not capturing the benefit of delay, i.e. the option value of waiting. This value comes about because of the possibility that for some reason a unit of sector i may have invested at time 1 even though doing so was foolish given the information available at time 2. Next, define a symmetric signal value $\bar{\mu}$ such that $\mu^i > \bar{\mu} > 0 \iff x^i = 1$. We have not yet said anything about what to do at $t = 2$, but we have already defined an alternative decision rule for $t = 1$: (iib) invest at $t = 1$ (i.e. set $x_1^i = 1$) if and only if $\mu^i > \bar{\mu} > 0$. The threshold signal $\bar{\mu}$ is symmetric and can be derived as follows.

Consider the cost of delay that can be seen intuitively as $(1-\rho)\,\mu^i$, i.e. the expected payoff at the myopic point in time 1 minus the expected payoff at 2. Next, consider the benefit in delay. Here we need to take into account the possibility of regret, where an investment made at $t = 1$ actually seems less sensible when information made available at $t = 2$ is revealed. Such information is obtained if it is observed that an investor of industry j did not invest at $t = 1$, therefore revealing that $\mu^j < \bar{\mu}$ which provides some evidence that the state of the world is less likely to merit investment. This can be avoided if the investing unit in i waits and provides the option value of waiting which occurs with probability $\Pr\left[\mu^j < \bar{\mu}\right]$. The option value can therefore be defined as the expected loss avoided in i by not investing at $t = 1$ in the event that in sector j nobody does invest at $t = 1$:

$$-\rho \Pr\left[\mu^j < \bar{\mu}\right]\left\{\mu^i + E\left[\mu^j \mid \mu^j < \bar{\mu}\right]\right\}. \tag{C.2}$$

Consider the condition which leaves the marginal decision-maker indifferent when deciding to invest at $t = 1$. Indifference occurs when the option value exactly offsets the costs of delay. This is the standard value matching condition for a dynamic planning problem. This condition implicitly defines the value $\bar{\mu}$ using the properties of the uniform distribution:

$$\begin{aligned}(1-\rho)\,\bar{\mu} &= -\,\rho \Pr\left[\mu^j < \bar{\mu}\right]\left\{\mu^i + E\left[\mu^j \mid \mu^j < \bar{\mu}\right]\right\}\\ \bar{\mu} &= \frac{-(4-2\rho) \pm \left[(4-2\rho)^2 + 12\rho^2\right]^{\frac{1}{2}}}{6\rho}.\end{aligned} \tag{C.3}$$

For $\rho \in (0,1)$ and $\bar{\mu} \in [-1,1]$ we can rule out one of this two results, eliminating

$$\bar{\mu} = \frac{1}{6}\rho^{-1}\left\{-(4-2\rho) - \left[(4-2\rho)^2 + 12\rho^2\right]^{\frac{1}{2}}\right\} \notin [-1,1] \text{ for } \rho \in (0,1). \tag{C.4}$$

This leaves the value of $\bar{\mu}$ uniquely given by

$$\bar{\mu} = \frac{1}{3} + \frac{2}{3\rho}\left\{\left[\rho^2 - \rho + 1\right]^{\frac{1}{2}} - 1\right\}. \tag{C.5}$$

Equation (C.5) is well defined for $\rho \in (0,1)$ and gives a range of values for $\bar{\mu}$ of $\bar{\mu} \in \left(0, \frac{1}{3}\right]$ that can be roughly approximated by the linear function $\bar{\mu} = \frac{1}{3}\rho$. It has been shown that there exists a unique value $\bar{\mu}$ such that if $\mu^i > \bar{\mu}$ the cost of delay is strictly offset by the option value of waiting. This is due to the fact that the cost of delay is rising in μ^i (and falling in ρ) which therefore defines the optimal decision rule for an investor of sector i at $t = 1$. The assumption of a positive option value to delay immediately implies that $\bar{\mu} > 0$.

An additional delay (speed up) effect on $I_i^{'}$ as described by (2.3) in combination with (2.4) and sketched in the first paragraph above will be triggered in case the sign of the capital deficit of an investor in i, i.e. the private signal, deviates from (matches with) the action performed in j, i.e. the aggregate activity in (2.3).

To illustrate this, consider no investment or disinvestment at the myopic point in time $t = 1$ in sector i, implying $\mu^i < \bar{\mu}$. Investment will benefit the investor in i if $E\left[\pi_2^i\right] >> 0$. Thereby two rationales at $t = 1$ are possible and will be considered in turn:

In the first case $\mu^i \in (-1,0]$, implying $E\left[\pi_2^i\right] < 0$, it would be rational to decide to invest only if new information suggested a rise in $E\left[\pi_2^i\right]$. An investor in i must have observed one of two possible histories: $x_1^j = 1$ or $x_1^j = 0$. Only if $x_1^j = 1$ is observed, expectation of π_2^i would be raised as follows:

$$E\left[\pi_2^i \mid x_1^j = 1\right] = \mu^i + E\left[\mu^j \mid \mu^j > \bar{\mu}\right] = \mu^i + \frac{1 + \bar{\mu}}{2} >> \mu^i = E\left[\pi_1^i\right] \qquad \text{(C.6)}$$

$$E\left[\pi_2^i \mid x_1^j = 0\right] = \mu^i + E\left[\mu^j \mid \mu^j < \bar{\mu}\right] = \mu^i - \frac{1 - \bar{\mu}}{2} < \mu^i = E\left[\pi_1^i\right]. \qquad \text{(C.7)}$$

Since this is a symmetric problem, the same holds true for an investor in j if $\mu^j \in (-1,0]$. Therefore, without positive investment in j at $t = 1$ expectations would only be raised if $x_1^i = 1$. Without positive investment in any of the two sectors, expectations will not increase in $t = 2$, which leads to additional disinvestment in $t = 2$, and hence no rise in expectation occurs at $t = 3$, and so on. As additional disinvestment is time consuming there is an incentive to speed up. The same, of course, holds for additional positive investments and a positive private signal.

In the second case, $\mu^i \in \left(0, \bar{\mu}\right)$ and $E\left[\pi_2^i\right] > 0$, an investor in i was delaying despite expecting positive profit because of the positive option value to delay. This option value has however been spent. If $x_1^j = 1$ the agent in sector i would have been better off investing at $t = 1$ and would have done so had she realized that an agent in j would definitely invest. There will be investment in i at $t = 2$ since there will be no further revelations from j. Now if $x_1^j = 0$, an investor of industry i will lower her payoff expectation as will an agent in j. Therefore if it was optimal for them to delay at $t = 1$ it is optimal to delay at $t = 2$ *a fortiori* and so it will be optimal not to invest at $t = 2, 3, 4, \ldots$ etc. The flow of starting projects is interrupted, i.e. investment accelerations will slow down.

The transform function $\Psi(x)$ in equation (2.4) in combination with (2.3) captures the described scenarios. Translating the $x = (K_i^* - K_i)\sum_j I_j$ product to a $\{-1, 0, 1\}$ index is our $\Psi(x)$-form chosen for practical reasons with regard to a simulation-based calibration of parameter γ and its comparison to ϑ. Other less tractable specifications like $\Psi(x) = x$ yield similar results.

Appendix D. Proof of Proposition 1

Suppose the following basic laws of motion based on the general principle of superposition:

$$x(t) = a_{x,1}\sin(\rho_x t) + a_{x,2}\cos(\rho_x t), \tag{D.1}$$

$$y(t) = a_{y,1}\sin(\rho_y t) + a_{y,2}\cos(\rho_y t), \tag{D.2}$$

where $a_{i,2} = A_i\cos(B) \wedge a_{i,1} = A_i\sin(B)$ for $i = x, y$, respectively.
As $x = A_x[\sin(\rho_x t)\sin(B) + \cos(\rho_x t)\cos(B)] = A_x[\cos(\rho_x t) - B]$, we may re-write

$$x(t) = A_x\cos(\rho_x t - B_x), \tag{D.1'}$$

$$y(t) = A_y\cos(\rho_y t - B_y). \tag{D.2'}$$

Let us call these base modes "outer" modes of the system and denote them out_x, out_y, respectively: $out_x = f(\rho_x)$, $out_y = f(\rho_y)$. Now we cast the basic system in an "accelerator-representation" by constructing the second temporal derivatives of (D.1') and (D.2'), assuming a time dependent frequency and using $\frac{d\rho_i}{dt} = (\rho_i)' = \omega_i^u$, where ω_i^u, $i = x, y$, denotes the respective "uncoupled" frequency:

$$x'(t) = -A_x\sin(\rho_x t)\,\omega_x^u \Rightarrow x''(t) = -A_x\cos(\rho_x t)\,(\omega_x^u)^2,$$

$$y'(t) = -A_y\sin(\rho_y t)\,\omega_y^u \Rightarrow y''(t) = -A_y\cos(\rho_y t)\left(\omega_y^u\right)^2.$$

According to (D.1') and (D.2'), we get:

$$x''(t) = -x(t)\,(\omega_x^u)^2, \tag{D.3}$$

$$y''(t) = -y(t)\left(\omega_y^u\right)^2. \tag{D.4}$$

To develop the mode-lock system, we introduce coupling by replacing ω_x^u, ω_y^u with ω_x^c, ω_y^c, where superscript "c" denotes "coupled," and $\omega_x^c = \omega_x + k_x\sin(\rho_x t - \rho_y t)$, $\omega_y^c = \omega_y + k_y\sin(\rho_y t - \rho_x t)$, i.e. we add to the respective base frequency $k_x\sin(\rho_x t - \rho_y t)$ and $k_y\sin(\rho_y t - \rho_x t)$. In quadratic terms, see (D.3) and (D.4), this implies analogously adding $k_x\left[\frac{1}{2} - \frac{1}{2}\cos(2\rho_x t - 2\rho_y t)\right]$,

$k_y\left[\frac{1}{2}-\frac{1}{2}\cos\left(2\rho_y t-2\rho_x t\right)\right]$, respectively:

$$x''\left(t\right)=\left\{\omega_x^2+k_x\left[\frac{1}{2}-\frac{1}{2}\cos\left(2\rho_x t-2\rho_y t\right)\right]\right\}x\left(t\right), \tag{D.5}$$

$$y''\left(t\right)=\left\{\omega_y^2+k_y\left[\frac{1}{2}-\frac{1}{2}\cos\left(2\rho_y t-2\rho_x t\right)\right]\right\}y\left(t\right). \tag{D.6}$$

As can be seen from these equations, the inner modes in_x, in_y are functions of the doubled values of ρ_i, $i=x,\,y$, i.e. the frequencies of the outer modes: $in_x=f\left(2\rho_x\right)$, $in_y=f\left(2\rho_y\right)$. The 1/2-relationship is intrinsic. Recently the existence of a critical coupling k_c was proven for frequencies of cyclic behaving entities, that show a Gaussian distribution with standard deviation σ, determined by $k_c=\sqrt{2/\pi^3}\sigma$ (Néda et al. 2000, p. 6988). Accordingly, for $k>k_c$ synchronization of the cyclic behaviors of the entities is possible. Unfortunately, this is only true for the special case of a) a population of globally coupled, i.e. $k_x=k_y$, non-identical cyclic behaving entities and b) in the limit equilibrium dynamics, i.e. for the number of entities $N\rightarrow\infty$ and $t\rightarrow\infty$. Furthermore, for the economically more meaningful case of (weakly) heterogeneous cyclical behaviors that are damped and disturbed by stochastic shocks, numerical simulations seem to be more adequate to determine interaction or coupling parameters like χ_i in (2.3).

Appendix E. Decomposed and Time Dependent Spectra

Consider a univariate AR model of order p, with residual variance σ^2. Its spectrum is given by

$$f\left(\omega\right)=\frac{1}{2\pi}\frac{\sigma^2}{\left|1-\sum_{j=1}^{p}a_j e^{-i\omega j}\right|^2};\ \omega\in\left[-\pi,\pi\right]. \tag{E.1}$$

With a VAR model of order p, the spectral density matrix is given by

$$\mathbf{F}\left(\omega\right)=\frac{1}{2\pi}\mathbf{A}\left(\omega\right)^{-1}\sum\mathbf{A}\left(\omega\right)^{-\star};\ \omega\in\left[-\pi,\pi\right]. \tag{E.2}$$

$\sum$ is the error variance-covariance matrix of the model, and $\mathbf{A}\left(\omega\right)$ is the Fourier transform of the matrix lag polynomial $\mathbf{A}\left(L\right)=I-A_1L-\ldots A_1L^p$.[17]

The total area under the spectrum in (E.1) equals the variance:

$$\gamma(0)=\int_{-\pi}^{\pi}f(\omega)d\omega. \tag{E.3}$$

In other words, we can look at it as the plot of a decomposition of the variance against frequencies in the interval $[0,\pi]$. After normalizing the spectrum using $\gamma(0)$, the area under the curve from ω_1 to ω_2 in Figure 3 is (half) the share of

[17]The superscript '$\star$' denotes the complex conjugate transpose.

total variance which can be attributed to waves in this frequency range. In the univariate case, the measure period length referred to in Section 3.3 is the cycle length associated with the peak in the spectrum $f_{max(\omega)}$.

To analyze the lead-lag relationship between the aggregate and the individual industries, we calculate the explained variance and the phase shift. These measures are based on the elements of the spectral density matrix, $\mathbf{F}(\omega)$. The off-diagonal elements of $\mathbf{F}(\omega)$, $f_{xy}(\omega)$, are the cross-spectra. The cross spectrum at frequency ω is a complex number and given by

$$f_{xy}(\omega) = c_{xy}(\omega) - iq_{xy}(\omega)\,;\ \omega \in [-\pi, \pi]\,, \tag{E.4}$$

where $c_{yx}(\omega)$ is the cospectrum and $q_{yx}(\omega)$ the quadrature spectrum. The explained variance is based on the squared coherency $sc(\omega)$:

$$sc(\omega) = \frac{|f_{xy}(\omega)|^2}{f_x(\omega) f_y(\omega)};\ 0 \le sc(\omega) \le 1. \tag{E.5}$$

This measure assesses the degree of linear relationship between two series, frequency by frequency. If we are interested in the extent to which the variance of cyclical components of the series X_t in the frequency band $[\omega_1, \omega_2]$ can be attributed to corresponding cyclical components in series Y_t, we can use $sc(\omega)$ to decompose the fraction of overall variance in this interval into an explained and an unexplained part:

$$\begin{aligned}
\int_{\omega_1}^{\omega_2} f_x(\omega) &= \int_{\omega_1}^{\omega_2} sc(\omega) f_x(\omega)\, d\omega + \int_{\omega_1}^{\omega_2} f_u(\omega)\, d\omega, \text{ where} \\
\int_{\omega_1}^{\omega_2} sc(\omega) f_x(\omega)\, d\omega &\equiv \text{“explained” variance}, \\
\int_{\omega_1}^{\omega_2} f_u(\omega)\, d\omega &\equiv \text{“unexplained” variance}.
\end{aligned} \tag{E.6}$$

We will use this decomposition, to compare the degree of linear relationship between cycles in different series for different cycle ranges (Figure 3).

Another measure which can be derived from the cross-spectrum is the phase spectrum. It measures the phase shift between two cycles at frequency ω, and allows to judge the lead-lag relationship between the two series frequency by frequency:

$$\phi_{xy}(\omega) = -\arctan \frac{q_{jk}(\omega)}{c_{jk}(\omega)};\ \omega \in [-\pi, \pi]\,. \tag{E.7}$$

The phase spectrum measures the phase lead of the series X over the series Y at a frequency ω. We will present the phase shift for the frequency where the univariate spectra reach there maximum. As pointed out by Croux *et al.* (2001), a measure like the (isolated) squared coherency presented above is not suited for analyzing the comovement of time series, inasmuch it does not contain information about possible phase shifts between cycles in the series X_t and Y_t.

In this sense, the correlation coefficient in time domain is more informative, since it is calculated lag by lag, providing both information on the lead-lag structure and the degree of linear relationship between the two series. We can overcome this problem by also presenting the phase spectrum. However, the phase spectrum is difficult to interpret, since it is only defined mod2π, and cannot easily be summarized over a frequency band like in the case of the explained variance. Croux *et al.* (2001) propose an alternative measure, the so-called dynamic correlation $\rho(\omega)$, which measures the correlation between the "in-phase" components of the two series at frequency ω:

$$\rho(\omega) = \frac{c_{xy}(\omega)}{\sqrt{f_x(\omega) f_y(\omega)}}; \; -1 \leq \rho(\omega) \leq 1. \tag{E.8}$$

Using

$$sc(\omega) = \frac{|f_{xy}(\omega)|^2}{f_x(\omega) f_y(\omega)} = \frac{c_{xy}(\omega)^2 + q_{xy}(\omega)^2}{f_x(\omega) f_y(\omega)}, \tag{E.5'}$$

we are able to further decompose the expression in equation (E.6):

$$\begin{aligned}
\int_{\omega_1}^{\omega_2} f_x(\omega) &= \int_{\omega_1}^{\omega_2} sc(\omega) f_x(\omega)\, d\omega + \int_{\omega_1}^{\omega_2} f_u(\omega)\, d\omega \\
&= \int_{\omega_1}^{\omega_2} \frac{c_{xy}(\omega)^2 + q_{xy}(\omega)^2}{f_x(\omega) f_y(\omega)} f_x(\omega)\, d\omega \\
&\quad + \int_{\omega_1}^{\omega_2} f_u(\omega)\, d\omega \\
&= \int_{\omega_1}^{\omega_2} \frac{c_{xy}(\omega)^2}{f_x(\omega) f_y(\omega)} f_x(\omega)\, d\omega \\
&\quad + \int_{\omega_1}^{\omega_2} \frac{q_{xy}(\omega)^2}{f_x(\omega) f_y(\omega)} f_x(\omega)\, d\omega \\
&\quad + \int_{\omega_1}^{\omega_2} f_u(\omega)\, d\omega,
\end{aligned} \tag{E.6'}$$

where

$$\begin{aligned}
\int_{\omega_1}^{\omega_2} \frac{c_{xy}(\omega)^2}{f_x(\omega) f_y(\omega)} f_x(\omega)\, d\omega &\equiv \text{"explained" variance: in-phase} \\
\int_{\omega_1}^{\omega_2} \frac{q_{xy}(\omega)^2}{f_x(\omega) f_y(\omega)} f_x(\omega)\, d\omega &\equiv \text{"explained" variance: out-of-phase} \\
\int_{\omega_1}^{\omega_2} f_u(\omega)\, d\omega &\equiv \text{"unexplained" variance}
\end{aligned}$$

We obtain a time dependent spectrum by transforming an AR model of order p

into state space form, treating the parameters as unobservable state vector:

$$\begin{aligned} x_t =& \begin{pmatrix} x_{t-1} & x_{t-1} & \dots & x_{t-p} \end{pmatrix} \mathbf{a}_t + \epsilon_t, \text{ where} \\ \mathbf{a}_t =& \begin{pmatrix} a_{1,t} & a_{2,t} & \dots & a_{p,t} \end{pmatrix}', \\ \mathbf{a}_t =& \mathbf{a} + \mathbf{T}\mathbf{a}_{t-1} + \zeta_t. \end{aligned} \tag{E.9}$$

The errors ϵ_t and ζ_t are assumed to be serially uncorrelated with variances σ^2 and Q_t, respectively. The transition matrix $\mathbf{T}_t$ is assumed to be a diagonal matrix. The value of the elements on the diagonal is 0.9. Thus, the parameters follow a stationary AR(1) process. The parameters of the model are estimated using the Kalman filter.[18]

In the case of the VAR model, we start with the following equation:

$$\begin{aligned} \mathbf{x}_t =& \mathbf{c} + \sum_{j=1}^{p} \mathbf{A}_j \mathbf{x}_{t-j} + \mathbf{u}_t = \\ =& \underbrace{\begin{pmatrix} \mathbf{c} & \mathbf{A}_1 & \dots & \mathbf{A}_p \end{pmatrix}}_{\mathbf{A}} \underbrace{\begin{pmatrix} 1 & \mathbf{x}_{t-1}^T & \dots & \mathbf{x}_{t-p}^T \end{pmatrix}^T}_{\mathbf{Z}_{t-1}} + \mathbf{u}_t = \\ =& \mathbf{A}\mathbf{Z}_{t-1} + \mathbf{u}_t;\ \mathbf{u}_t \sim iid\left(\mathbf{0}, \mathbf{H}\right). \end{aligned} \tag{E.10}$$

Vectorizing equation (E.10), and allowing the parameters of the VAR to be time dependent, gives

$$\mathbf{X}_t = \left(\mathbf{Z}_{t-1}' \otimes \mathbf{I}\right) \underbrace{\mathrm{vec}\mathbf{A}_{t-1}}_{\boldsymbol{\alpha}_{t-1}} + \mathbf{u}_t; \tag{E.11}$$

which is the measurement equation in our state space version of equation (E.10). The transition equation for the VAR parameters is given by

$$\alpha_t = \mathbf{T}\alpha_{t-1} + \eta_t;\ \eta_t \sim iid\left(0, \mathbf{Q}\right). \tag{E.12}$$

Again, we assume the matrix $\mathbf{T}$ to be a diagonal matrix with elements $\rho = 0.9$ on the diagonal, forcing the time path for the parameters to be a damped AR(1) process. The elements in the covariance matrices $\mathbf{H}$ and $\mathbf{Q}$ are treated as hyperparameters, and the likelihood function derived based on the cumulated prediction errors is maximized with respect to these parameters. The solution of this estimation procedure implies a time path for α_t, Thus allowing the spectral density matrix in equation (E.2) to be time dependent.[19]

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[18] See Harvey (1992) for details.

[19] To estimate the VARs, we use Geoffrey Shuetrim's Kalman filter code (http://ideas.repec.org/c/apr/aprsft/cd0002.html).

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