

Erratum

Fast simulation of Gaussian random fields

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Abstract. In the paper “Fast simulation of Gaussian random fields”, a typo occurred. Instead of $(p_{k_1 \dots k_d})_i = k_i/l_i$ it should read $(p_{k_1 \dots k_d})_i = (k_i - N_i/2)/l_i$ in Algorithm 3.1, Remark d). For convenience of the reader we reproduce below the complete corrected algorithms.

Keywords. Gaussian random fields, fast Fourier transform, simulation.

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Algorithm 3.1.

Remarks

- The functions FFT and FFT^{-1} include all necessary rescaling depending on the used FFT algorithm and the integers N_i .
- A is a d -dimensional complex-valued array, B is real-valued.
- $x_{k_1 \dots k_d}$ denotes the grid point corresponding to the integers (k_1, \dots, k_d) . The grid points are distributed equidistantly in each direction, i.e., the distance of two arbitrary neighbor grid points in direction e_i is given by a constant Δx_i .
- The points $p_{k_1 \dots k_d}$ in the Fourier domain are given by

$$(p_{k_1 \dots k_d})_i = (k_i - N_i/2)/l_i \quad \text{for } i = 1, \dots, d.$$

Input

- D a d -dimensional rectangular region with l_1, \dots, l_d lengths of the edges,
- N_1, \dots, N_d the numbers of discretization points in each direction, all even,
- $\gamma^{1/2}$ a symmetric, positive function on \mathbb{R}^d ,
- $R(\cdot)$ a function that generates independent $\mathcal{N}(0, |\Delta^N|^{-1})$ -distributed random numbers.

Output

GRF B on D , where the covariance is given by the Fourier transform of γ .

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for  $k_i = 0, \dots, N_i - 1, i = 1, \dots, d$  do
   $B(k_1, \dots, k_d) \leftarrow R()$ ;
end for
 $A \leftarrow \text{FFT } B$ ;
for  $k_i = 0, \dots, N_i - 1, i = 1, \dots, d$  do
   $A(k_1, \dots, k_d) \leftarrow A(k_1, \dots, k_d) \cdot \gamma(p_{k_1 \dots k_d})^{1/2} \cdot |D|^{-1}$ ;
end for
 $B \leftarrow \text{FFT}^{-1} A$ ;

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Algorithm 3.2.

Remarks Same as in Algorithm 3.1.

All calculations have to be done modulo N_i in the i -th direction.

Input Same as in Algorithm 3.1.

Output Same as in Algorithm 3.1.

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for  $k_i = 0, \dots, N_i - 1, i = 1, \dots, d - 1, k_d = 0, \dots, N_d/2$  do
  if  $k_i \in \{0, N_i/2\}$ , for all  $i = 1, \dots, d$  then
     $\text{Re } A(k_1, \dots, k_d) \leftarrow R() \cdot \gamma(p_{k_1 \dots k_d})^{1/2} \cdot |D|^{-1}$ ;
     $\text{Im } A(k_1, \dots, k_d) \leftarrow 0$ ;
  else
     $\text{Re } A(k_1, \dots, k_d) \leftarrow 2^{-1/2} R() \cdot \gamma(p_{k_1 \dots k_d})^{1/2} \cdot |D|^{-1}$ ;
     $\text{Im } A(k_1, \dots, k_d) \leftarrow 2^{-1/2} R() \cdot \gamma(p_{k_1 \dots k_d})^{1/2} \cdot |D|^{-1}$ ;
     $\text{Re } A(N_1 - k_1, \dots, N_d - k_d) \leftarrow \text{Re } A(k_1, \dots, k_d)$ ;
     $\text{Im } A(N_1 - k_1, \dots, N_d - k_d) \leftarrow -\text{Im } A(k_1, \dots, k_d)$ ;
  end if
end for
 $B \leftarrow \text{FFT}^{-1} A$ ;

```

Algorithm 3.3.

Remarks Same as in Algorithm 3.2.

Input Same as in Algorithm 3.2.

Output Same as in Algorithm 3.2.

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function  $\text{create\_random\_field}(D, N_1, \dots, N_d)$ 
 $A \leftarrow \text{complex\_array}(N_1, \dots, N_d)$ ;

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set_array({N1, ..., Nd}, { }, A);
B ← FFT-1 A;
end function

function set_array({N1, ..., Nj}, {kj+1, ..., kd}, A)
for ki = 0, ..., Ni, i = 1, ..., j - 1, kj = 1, ..., Nj/2 - 1 do
  Re A(k1, ..., kd) ← 2-1/2 R() · γ(pk1...kd)1/2 · |D|-1;
  Im A(k1, ..., kd) ← 2-1/2 R() · γ(pk1...kd)1/2 · |D|-1;
  Re A(N1 - k1, ..., Nd - kd) ← Re A(k1, ..., kd);
  Im A(N1 - k1, ..., Nd - kd) ← -Im A(k1, ..., kd);
if (|{N1, ..., Nj}| = 1) then
  Re A(0, ..., kd) ← R() · γ(p0k2...kd)1/2 · |D|-1;
  Im A(0, ..., kd) ← 0;
  Re A(N1/2, ..., kd) ← R() · γ(pN1/2k2...kd)1/2 · |D|-1;
  Im A(N1/2, ..., kd) ← 0;
else
  set_array({N1, ..., Nj-1}, {0, kj+1, ..., kd}, A);
  set_array({N1, ..., Nj-1}, {Nj/2, kj+1, ..., kd}, A);
end if
end for
end function

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