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Does the Fornax dwarf spheroidal have a central cusp or core?

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ABSTRACT

The dark matter dominated Fornax dwarf spheroidal has five globular clusters orbiting at $\sim$1 kpc from its centre. In a cuspy cold dark matter halo the globulars would sink to the centre from their current positions within a few Gyr, presenting a puzzle as to why they survive undigested at the present epoch. We show that a solution to this timing problem is to adopt a cored dark matter halo. We use numerical simulations and analytic calculations to show that, under these conditions, the sinking time becomes many Hubble times; the globulars effectively stall at the dark matter core radius. We conclude that the Fornax dwarf spheroidal has a shallow inner density profile with a core radius constrained by the observed positions of its globular clusters. If the phase space density of the core is primordial then it implies a warm dark matter particle and gives an upper limit to its mass of $\sim$0.5 keV, consistent with that required to significantly alleviate the substructure problem.

Key words: methods: *N*-body simulations – galaxies: dwarf – galaxies: individual: Fornax – galaxies: star clusters.

1 INTRODUCTION

The Fornax dwarf spheroidal is a dark matter dominated satellite orbiting the Milky Way. It has five globular clusters that are at a projected distance from the centre of 1.60, 1.05, 0.43, 0.24 and 1.43 kpc (Mackey & Gilmore 2003) as well as further substructure at a projected distance of 0.67 kpc (Coleman et al. 2005). These star clusters move within a dense background of dark matter and should therefore be affected by dynamical friction, causing them to lose energy and spiral to the centre of the galaxy. We will show later that, if Fornax has a cosmologically consistent density distribution of dark matter, the orbital decay time-scale of these objects from their current positions is $\lesssim$5 Gyr. This is much shorter than the age of the host galaxy, presenting us with the puzzle of why these five globulars have not merged together at the centre forming a single nucleus (Tremaine, Ostriker & Spitzer 1975; Tremaine 1976).

Several groups have studied the origin of nuclei in galaxies, e.g. Lotz et al. (2001) carried out Monte Carlo simulations, which show that some, but not all, of the nuclei of dwarf elliptical galaxies could indeed have formed through coalescence of their globular clusters. Additionally, they observed several dE galaxies and found out that within the inner few scalelengths, their sample appeared to be depleted of bright clusters. Oh & Lin (2000) used numerical simulations to show that in dwarf galaxies with relatively weak external tidal perturbations, dynamical friction can lead to significant orbital decay of globular clusters and the formation of compact nuclei within a Hubble time-scale.

Oh, Lin & Richer (2000) gave two possible models for the observed spatial distribution of Fornax globulars. One possibility they proposed is that the dark matter consists of massive black holes which transfer energy to the globulars, preventing them from sinking to the centre of the galaxy. Another possibility they investigated was to postulate a strong tidal interaction between the Milky Way and Fornax which also could inject energy into their orbits and the central core of the dSph. This latter idea is probably ruled out due to the proper motion observations of Fornax (Dinescu et al. 2004) which suggest it is already at closest approach on an extended orbit which never takes it close to the Milky Way.

Here, we investigate another possibility for the lack of a nucleus in Fornax, namely that the central dark matter distribution has a very shallow cusp or core which dramatically increases the dynamical friction sinking time-scale (Hernandez & Gilmore 1998). This would be inconsistent with dark haloes that form within the cold dark matter (CDM) cosmology which have cusps steeper than -1 on all mass scales from 10^{-6} to $10^{15}\,\mathrm{M}_{\odot}$ (Dubinski & Carlberg 1991; Diemand, Moore & Stadel 2005).

Controversial evidence for cored mass distributions in dwarf spiral galaxies has been debated for over a decade (Moore 1994). The inner structure of spheroidal galaxies is harder to determine, however Kleyna et al. (2003) claimed that the second peak in the stellar number density in the nearby Ursa Minor dwarf spheroidal (UMi dSph), is incompatible with cusped CDM haloes. With their observations, they show that this substructure has a cold kinematical signature and that its radial velocity with respect to its host galaxy

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is very small. Such a cold configuration could only survive intact if the stars orbited within a cored mass distribution where the orbital frequencies are all identical (harmonic potential) and phase mixing does not occur.

The stellar kinematical data for Fornax suggest that it is dark matter dominated with a mass to light ratio of the order of 20 within its optical extent. Due to the uncertainty on the orbital anisotropy, the mass distribution can only be weakly constrained – the data is consistent with either cusped or cored density distributions (Łokas 2002). However, the normalization (or mass within the central 1 kpc) is better constrained. In the inner ∼1 kpc of a cored halo, the mean density is approximately six times lower than that in a cusped halo. Furthermore, the velocity distribution function of the background particles is hotter than a cusped halo. These facts conspire to significantly increase the dynamical friction time-scale in a cored mass distribution.

In this paper, we construct cored and cuspy dark matter potentials and calculate orbital decay and sinking times using high-resolution numerical simulations together with analytic calculations (Chandrasekhar 1943). The haloes are consistent with the kinematical data for Fornax. We follow circular and eccentric orbits of single and multiple globular clusters. Although many dynamical friction studies have been carried out before (White 1983; Hernquist & Weinberg 1989; Capuzzo-Dolcetta & Vicari 2005), we are not aware of any studies within constant density cores at the resolution used in this paper, although the recent study explored the effects of sinking objects on various cusp structures (Merritt et al. 2004). In Section 2, we present the numerical methods we used, the analytical computation of the sinking times which are compared to our high-resolution numerical simulations. In Section 3, we discuss our results and draw our conclusions.

2 RESULTS

We carry out a series of self-consistent simulations to examine the orbital behaviour of massive particles moving within a dark matter or stellar background. We use the parallel multistepping *N*-body tree-code, PKDGRAV2, developed by Joachim Stadel (Stadel 2001). We construct stable particle haloes using the techniques developed by Kazantzidis, Magorrian & Moore (2004). These models have density distributions that are described by the α, β, γ law (Hernquist 1990):

$$\rho(r) = \frac{\rho_0}{(r/r_s)^\gamma \left[1 + (r/r_s)^\alpha\right]^{(\beta-\gamma)/\alpha}}. \tag{1}$$

For our simulations we used 'NFW-like' haloes (Navarro, Frenk & White 1996; Moore et al. 1999b) with $\alpha = 0.5$–1.5, $\beta = 3.0$ and $\gamma = 0.5$–1.5, or cored haloes with $\alpha = 0.5$–1.5, $\beta = 3.0$ and $\gamma = 0.0$. In the former case we have $\rho_0 = 0.0058\,\mathrm{M}_\odot\,\mathrm{pc}^{-3}$ and $r_s = 2.4$ kpc. This cuspy halo has a virial mass of $2.0 \times 10^9\,\mathrm{M}_\odot$. The concentration parameter is 15 but our results would not change with a lower concentration, since in either case we are within the asymptotic cusp part of the density profile.

We use a three-shell model (Zemp et al., in preparation); 10^5 particles for the innermost sphere with 100 pc radius, 10^5 particles for the shell between 100 and 500 pc and 10^5 particles for the rest of the halo. The softening lengths of the particles in these shells are 1, 10 and 100 pc, respectively. The results were found not to be sensitive to these values. The particle masses are 58, 569 and $3.2 \times 10^4\,\mathrm{M}_\odot$. These models are stable in isolation but allow us to achieve very high resolution at the halo centre where we wish to follow the dynamical friction.

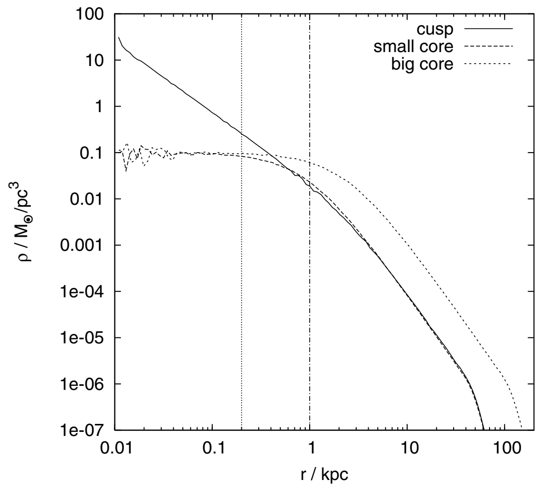

Figure 1. The initial radial density profiles for the three different haloes used in the simulations. The vertical lines indicate the size of the core of the cored haloes.

For a small cored halo we have $\rho_0 = 0.10\,\mathrm{M}_\odot\,\mathrm{pc}^{-3}$ and $r_s = 0.91$ kpc (NB: the radius at which the slope of the density profile is shallower than -0.1 is approximately 200 pc which defines the constant density region in this model). This halo has a virial mass of $2.0 \times 10^9\,\mathrm{M}_\odot$ and the concentration parameter is 40. Again, we use a three-shell model that has 10^5 particles for the innermost sphere with 300 pc radius, 10^5 particles for the shell between 0.3 and 1.1 kpc and 3×10^5 particles for the rest of the halo. The softening lengths of the particles in these shells are 3, 30 and 300 pc, respectively. The particle masses are 89, 1640 and $7572\,\mathrm{M}_\odot$. For a big cored dark matter halo we have basically the same parameters as for the halo with the small core, except for the scalelength $r_s = 2.2$ kpc (here the constant density region is approximately 1 kpc), the virial mass $M_{\mathrm{vir}} = 3.0 \times 10^{10}\,\mathrm{M}_\odot$ and the particle masses, which are in this case 106, 3625 and $1.2 \times 10^5\,\mathrm{M}_\odot$. The density profiles of these three haloes are shown in Fig. 1.

The density profiles agree fairly well with the constraints made by observations of Fornax (Łokas 2002; Walker et al. 2006). We did actually perform the same simulations and treatment as described in the following with the very haloes proposed by Łokas (2002) with the same results. We also repeated several of our simulations with haloes modelled with 10 times as many particles than described above. These high-resolution runs show exactly the same features as the low-resolution runs, but with less noise.

Where available, we will present the high-resolution graphs in this paper. The globular clusters are modelled as single particles of mass $M_{\mathrm{GC}} = 2 \times 10^5\,\mathrm{M}_\odot$ with a softening of 10 pc. We do not expect our conclusions to change if we used a particle model for each globular since they are stable against tidal disruption within Fornax. We start the globulars outside the core, mostly on circular orbits and let them orbit, expecting them to spiral in to the centre of their respective host haloes due to dynamical friction. The distance from the centre of the host halo as a function of time, $r(t)$, can be computed using Chandrasekhar's dynamical friction formula (Binney & Tremaine 1987), which is given by

$$F = -\frac{4\pi \ln\Lambda(r)\rho(r) G M_{\mathrm{GC}}^2}{v_c^2(r)} \times \left\{ \mathrm{erf}\left[\frac{v_c(r)}{\sqrt{2}\sigma(r)}\right] - \frac{2 v_c(r)}{\sqrt{2\pi}\sigma(r)} \mathrm{e}^{-v_c^2(r)/2\sigma^2(r)} \right\}, \tag{2}$$

which gives the force acting on the massive particle crossing the halo. The density profile $\rho(r)$ is given by our equation (1), and we assume that the velocity distribution is isotropic and Maxwellian at all radii. Of course this assumption does not hold, but is good enough for our purposes (Kazantzidis et al. 2004). We can then easily calculate the velocity dispersion using the Jeans equation:

$$\sigma^2(r) = \frac{1}{\rho(r)} \int_r^\infty \frac{M(r')\rho(r')}{r'^2}\, \mathrm{d}r'. \tag{3}$$

We find similar sinking times for eccentric orbits, therefore for brevity we show only the circular orbits in this paper and leave the detailed parameter space study for a future paper which explores the technical aspects of dynamical friction in structures with different density profiles. Additionally, we assume that $M_{\rm GC} \gg m_{\rm par}$. This is a little problematic in the case of the cuspy and the big cored potential because the particles in the outermost shell have $m_{\rm par} = 3 \times 10^4\,\mathrm{M}_\odot$ and $m_{\rm par} = 1.2 \times 10^5\,\mathrm{M}_\odot$, respectively. However, these particles rarely penetrate the innermost 0.5 kpc of the halo. In equation (2), $\ln \Lambda\ (r)$ is the Coulomb logarithm:

$$\ln \Lambda(r) = \frac{b_{\rm max}\sigma^2(r)}{G M_{\rm GC}}. \tag{4}$$

In this definition, $b_{\rm max}$ is the largest impact parameter to be considered. This parameter is defined by one of the assumptions Chandrasekhar made while deriving the above dynamical friction formula: the intruder must be moving through a medium with constant density, therefore $b_{\rm max}$ is the greatest distance for which this is still valid. We keep $b_{\rm max}$ as a free parameter when fitting our analytic formulae to the simulations. We find for the cuspy haloes $b_{\rm max} = 0.25$ kpc and for the cored ones $b_{\rm max} = 1.0$ kpc. In this equation, $v_{\rm c}(r)$ is the circular velocity at radius r. The force exerted by dynamical friction on the perturber is tangential with respect to its movement and thus causes the cluster to lose angular momentum per unit mass at a rate

$$\frac{\mathrm{d}L}{\mathrm{d}t} = \frac{Fr}{M_{\rm GC}} = -\frac{4\pi \ln \Lambda(r)\rho(r) G M_{\rm GC} r}{v_{\rm c}^2(r)} \times \left\{ \mathrm{erf}\left[\frac{v_{\rm c}(r)}{\sqrt{2}\sigma(r)}\right] - \frac{2 v_{\rm c}(r)}{\sqrt{2\pi}\sigma(r)} \mathrm{e}^{-v_{\rm c}^2(r)/2\sigma^2(r)} \right\}. \tag{5}$$

Since the cluster continues to orbit at a speed $v_{\rm c}(r)$ as it spirals to the centre, its angular momentum per unit mass at radius r is at all times $L = r v_{\rm c}(r)$. Substituting the time derivative of this expression into equation (5) we obtain

$$\frac{\mathrm{d}r}{\mathrm{d}t} = -\frac{4\pi \ln \Lambda(r)\rho(r) G M_{\rm GC} r}{v_{\rm c}^2(r) \mathrm{d}[r v_{\rm c}(r)]/\mathrm{d}r} \times \left\{ \mathrm{erf}\left[\frac{v_{\rm c}(r)}{\sqrt{2}\sigma(r)}\right] - \frac{2 v_{\rm c}(r)}{\sqrt{2\pi}\sigma(r)} \mathrm{e}^{-v_{\rm c}^2(r)/2\sigma^2(r)} \right\}. \tag{6}$$

Substituting values for the initial radii we obtain the analytical curves drawn in Fig. 2 plotted on top of the results from the numerical simulations.

For the cuspy potential the analytic calculation agrees very well with the numerical simulation. Haloes with a core give a poorer agreement. After an initial sinking rate that agrees well with the analytic expectation, the globulars sink faster as they approach twice the core radius, and then stop sinking at the core radius. The analytic formula predicts a continued, but slow infall to the centre. This resonance/scattering effect will be investigated in a more detailed paper (Read et al., in preparation). We note, however, that it is not trivially due to the fact that the globular is of comparable mass to that enclosed by its orbit; the radius at which $M(r) = M_{\rm GC}$ is approximately three times smaller than the core radius (see Fig. 4). The stalling results are apparent in both of the cored halo simulations (small core and big core). We conclude that the presence of a central density core leads to the infall of the clusters stopping at the core radius; the problem is in this sense scalable.

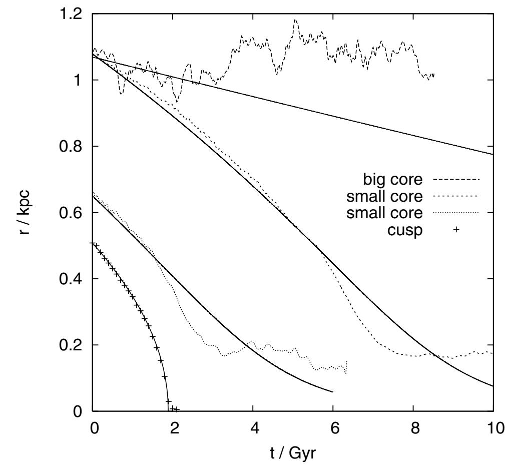

Figure 2. Radial distance of the single globular cluster from the centre of its host halo as a function of time. We start the calculations with the globular at different initial radii for clarity. Solid curves are the analytic estimates, dashed curves are from the numerical simulations.

2.1 Particle noise and halo centring

It is difficult to define the centre of the constant density core. The potential has a negative minimum, but $\sqrt{N}$ noise perturbations can have deeper potentials than the core itself. Thus, defining the centre using the most bound particle gives a large error in determining the centre. We found that a shrinking spheres method can work, so long as one takes the centre of mass of a sphere containing most of the core. If you continue to shrink the sphere based on a small number ($<10^4$) particles, then this also picks out the largest Poisson fluctuation in the core. The centre defined using $\sim 10^5$ particles gives a robust estimate.

In our standard-resolution simulations, we use 10^5 particles in the high-resolution region each of mass $89\,\mathrm{M}_\odot$. Simulating the entire halo at this resolution would require $\sim 4 \times 10^7$ particles. However, as discussed above, $\sqrt{N}$ noise can be substantial, even at this resolution. This may introduce spurious heating, preventing the globulars from sinking. We can investigate this by examining the orbit of the globular cluster. For a perfectly smooth spherical potential the orbit of the globular cluster would always remain in its initial orbital plane. Fluctuations from particle representation will cause deviations from this plane. Once the globular reaches the high density centre in the cases of the low-resolution runs fluctuations in the orbital plane become very large (fluctuations in $L_z/L \sim 0.5$ per cent) – the relaxation time is very short and acts to counter dynamical friction. We therefore carried out simulations with ten times as many particles. At this resolution the fluctuations are greatly reduced (fluctuations in $L_z/L \sim 0.05$ per cent).

2.2 Multiple globular clusters

Finally, we reran these simulations using five globulars to study the effect of having multiple sinking objects. Perhaps interactions

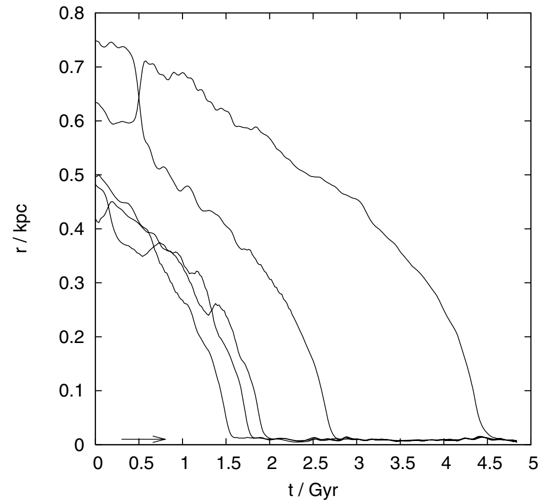

Figure 3. Radial distance of the five globular clusters from the centre of their host halo as function of time, as they orbit within a cusped density distribution. The arrow indicates the radius at which $M_{\mathrm{GC}} = M(r)$.

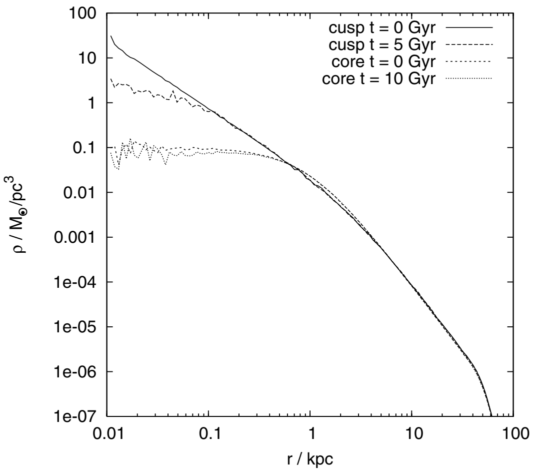

Figure 5. Radial density profile of the dark matter haloes. Only the dark matter is shown, the mass of the globular clusters is neglected. The initial conditions are compared with the final state of the system. An isolated halo, evolved for the same time, has exactly the same density profile as the initial conditions.

between the globulars themselves may prevent them from sinking to the central cusp and merging. We distribute the globulars randomly, what position and plane of the orbit concerns, with distances to the centre between 0.2 and 0.8 kpc. The clusters are again placed on circular orbits around the centre of their host halo and are evolved with PKDGRAV2.

Interestingly, the clusters do not prevent one another from falling to the centre, but instead create an interesting prediction. Fig. 3 shows the infall as a function of time for the five globulars in the cuspy dark matter halo. Notice that all of the clusters fall to the centre. However, clusters which start out at very similar radii arrive ∼1 Gyr apart. At any given time, even for very similar initial conditions, the clusters occupy a range of radii. By contrast, for the case with a central dark matter core (see Fig. 4), although the globulars still arrive at different times due to interactions, they stall at the core radius; they do not sink to the centre even within 20 Gyr.[1] Thus, if Fornax does have a central constant density core we should expect the clusters to stall at some minimum radius. No globular cluster could possibly get any closer to the centre of the halo than the core radius. The lower limit of the core size is constrained by the smallest observed projected cluster distances to be 0.24 kpc.

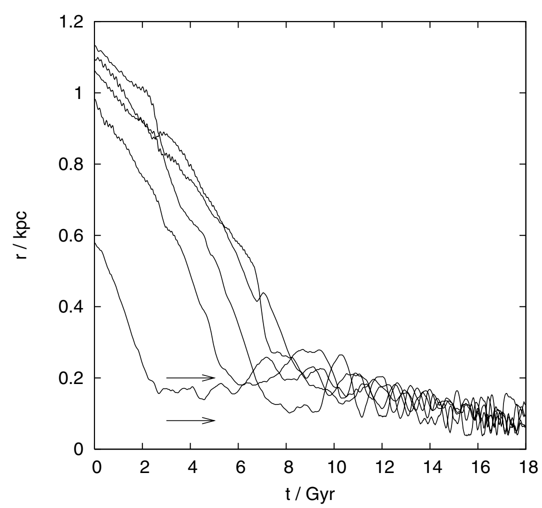

Figure 4. Radial distance of the five globular clusters from the centre of the host halo as function of time within the cored potential. The upper arrow indicates the size of the core and the lower arrow indicates the radius at which $M_{\mathrm{GC}} = M(r)$.

Finally, one can see in Fig. 5 that for the case where there is a cusp and the globular clusters do spiral in, they displace the dark matter from the centre. Dark matter particles move out of the nucleus. The density of the dark matter in the centre drops by more than an order of magnitude, an effect that does not happen in cored haloes. We note that several authors have recently studied this process in detail. For example, El-Zant, Shlosman & Hoffman (2001) and Merritt et al. (2004) study the change in an initial cuspy density profile due to the frictional effects of sinking objects. This process allows initially cusped density profiles to be transformed into nearly harmonic cores as the globulars fall in. For the case of the five globulars in the Fornax halo, the maximum initial central density slope for which this can occur is approximately 0.5. Thus in order for these data to be consistent with a CDM halo, the central cusp must have been modified and flattened to a slope shallower than this value. Exotic scenarios which might achieve this include infall and subsequent blowout of massive gas clouds or star clusters (Read & Gilmore 2005).

3 CONCLUSIONS

The Fornax dwarf spheroidal has five globular clusters orbiting at a projected radius of ∼1 kpc from its centre. Using a cuspy CDM potential with central slope steeper than 0.5 and normalized to match the inferred properties from the kinematical data, we find that these globulars would all sink to the halo centre within 5 Gyr.

By contrast, we showed that if Fornax has a constant density central core then the dynamical friction time becomes arbitrarily long – the globulars stop sinking at the edge of the core, thus the

[1]Recall that this stalling behaviour was also present in the run with a larger core size and so is not peculiar to the small core density profile.

present position of the *innermost* globular gives a lower limit of the core radius of the dark matter distribution. Since CDM uniquely predicts that all haloes are cusped, this suggests that the dark matter distribution within Fornax is not cold, but may be warm dark matter or some other candidate. Alternatively, the mass distribution has been modified through some exotic dynamical phenomenon such as rapid mass-loss (Read & Gilmore 2005). If the phase space density of the core, measured as $Q \sim 10^{-5}\,\mathrm{M}_{\odot}\,\mathrm{pc}^{-3}\,(\mathrm{km\,s}^{-1})^{-3}$ in our model, is primordial then it implies a warm dark matter particle of mass $\sim$0.5 keV (Hogan & Dalcanton 2000; Dalcanton & Hogan 2001). This is consistent with that required to largely solve the substructure problem (Moore et al. 1999a).

Although the dwarf spheroidals around the Milky Way do not contain prominent nuclei, about 30 per cent of dwarf spheroidals (dEs) in clusters are nucleated. If these nuclei form by the merging of star clusters then we must conclude that these galaxies have cuspy mass distributions. This could be due to the fact that transformation to dE via galaxy harassment gives an exponential distribution of stars which usually dominate the central mass distribution. Therefore, globulars could sink via friction against the stellar background. Most nucleated dEs are near the cluster centre where harassment is important which supports this idea.

Our numerical simulations show that the standard Chandreskhar estimate of sinking time-scales works well for cuspy cores but fails completely for cored mass distributions. In addition we showed that over 10^6 particles are required within the core region to suppress heating from particle noise (Weinberg 1998). This is particularly important in a cored mass distribution where the potential minimum is quite shallow. The fact that cored mass distributions do not give rise to dynamical friction is an important result. We believe that this is due to orbit-resonant scattering. This will be the subject of a forthcoming paper (Read et al. in preparation).

We have shown that a natural solution to the timing problems for Fornax's globular clusters is a central dark matter core. A prediction of this model is that the clusters have a well-defined minimum radius. Fornax is not alone in showing indirect evidence for such a core. The UMi dSph galaxy also has substructure which appears to have survived longer than is possible within a cuspy halo. Understanding the origin of these constant density cores could be one of the most exciting challenges facing astronomy in the next few years.

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