I. INTRODUCTION

Many innovations are inspired by past ideas in a nontrivial way. Tracing these origins and identifying scientific branches is crucial for research inspirations. In this paper, we use citation relations to identify the descendant chart, i.e., the family tree of research papers. Unlike other spanning trees that focus on cost or distance minimization, we make use of the nature of citations and identify the most important parent for each publication, leading to a treelike backbone of the citation network. Measures are introduced to validate the backbone as the descendant chart. We show that citation backbones can well characterize the hierarchical and fractal structure of scientific development, and lead to an accurate classification of fields and subfields.

II. METHODS

To start our analyses, we first denote the references of a paper as its parents, and the articles citing the paper as its offspring. The set of parents and the offspring of a paper \( i \) are denoted by \( \mathcal{P}_i \) and \( \mathcal{O}_i \) with \( p_{i} \) and \( o_{i} \) elements, respectively. Intuitively, the offspring of an important paper should share a similar focus introduced by its influential parent. Less relevant parents will, by contrast, lead to a more heterogeneous descendence. We thus quantify the impact of parent \( \alpha \) on \( i \) by

\[
I_{\alpha \rightarrow i} = \sum_{i' \in \mathcal{O}_\alpha \setminus \{i\}} s_{i'i},
\]

where \( s_{i'i} \) is the similarity between \( i \) and \( i' \). We refer to papers in the set of \( \mathcal{O}_\alpha \setminus \{i\} \) as the peers of \( i \) rooted in \( \alpha \) (see Fig. 1 for an illustration). The higher the overall similarity between \( i \) and the papers in \( \mathcal{O}_\alpha \setminus \{i\} \), the higher the impact of \( \alpha \) on \( i \).

A simple way to measure the similarity between \( i \) and peer \( i' \) is to count the number of their common references, i.e.,

\[
s_{i'i} = |\mathcal{P}_i \cap \mathcal{P}_{i'}|.
\]

However, this similarity measure favors peers with many references, resulting in an impact biased toward parents with a large offspring. This suggests to define a similarity measure based on a random walk from the peers to \( i \). We thus consider a two-step random walk from each peer \( i' \) to \( i \).
FIG. 1. A schematic diagram that shows two peers \( i', i_1, i_2 \) (shaded) of \( i \) rooted from parent \( \alpha \). To compute each \( s_{ii'}^{\text{ant}} \), we consider a random walk from \( i' \) through papers cited by both \( i' \) and \( i \). Specifically, to compute each \( s_{ii'}^{\text{read}} \) we consider a random walk from \( i' \) through papers citing both \( i' \) and \( i \).

that passes through their common references, and we define a contribution to \( s_{ii'} \) as

\[ s_{ii'}^{\text{ant}} = \frac{1}{p_i} \sum_{j \in O_i \cap O_{i'}} \frac{1}{p_j}. \]  

(1)

The superscript represents the authors’ interpretations, as this similarity is measured through the references chosen by the authors of \( i \). A second contribution is instead given by a random walk through articles citing \( i \) and represents the readers’ interpretation of \( i \). In analogy with \( s_{ii'}^{\text{ant}} \), we define

\[ s_{ii'}^{\text{read}} = \frac{1}{o_i} \sum_{j \in O_i \cap O_{i'}} \frac{1}{p_j}. \]  

(2)

As defined in both Eqs. (1) and (2), the higher the random-walk probability from \( i' \) to \( i \), the higher the similarity between \( i' \) and \( i \).

Finally, by combining linearly \( s_{ii'}^{\text{read}} \) and \( s_{ii'}^{\text{ant}} \) and summing over all the peers rooted in \( \alpha \), we obtain the impact of \( \alpha \) on \( i \) as

\[ I_{\alpha \rightarrow i} = \sum_{i' \in O_{\alpha \setminus i}} \left[ f s_{ii'}^{\text{read}} + (1 - f) s_{ii'}^{\text{ant}} \right]. \]  

(3)

with \( f \) to adjust the relative weights on the two contributions. The subsequent analysis is simplified by setting \( f = 0.5 \) unless otherwise specified. We note that citations between peers [16] do not contribute to the above measure, as these citations may complements \( i \) instead of being merely an influential parent of \( i \). The same is true if \( i \) cites many peers rooted from \( \alpha \), which suggests \( \alpha \) is a complement of its peers instead of a mere influential parent.

By keeping only the reference \( \alpha \) with highest \( I_{\alpha \rightarrow i} \), for all \( i \), we obtain a citation backbone denoted as the SIM backbone. Cases of equal scores are extremely rare and do not affect results (in such situations, we arbitrarily choose the latest reference with the highest \( I_{\alpha \rightarrow i} \)). In addition, we examine also the RAN and the LON backbone, which selects, respectively, a random parent and the reference giving rise to the longest path to the root (most likely the latest published parent). Other than serving as a benchmark, the RAN backbone can be informative as the random parent is one of the original references. The LON backbone instead represents a natural choice if progress is always based on recent developments, as one may follow the step-by-step evolution of science represented by the maximum number of steps needed to reach the root.

III. STATISTICAL PROPERTIES OF THE BACKBONE

We will examine the citation network among the journals of the American Physical Society, from the years 1893 to 2009. The dataset is composed of 4.67 \( \times \) 10^6 citations between 4.49 \( \times \) 10^5 publications. In rare cases there are references to contemporary or even posterior published papers. These citations are removed and the network is strictly acyclic.

We note that all papers without reference are potentially the roots of the backbone. As this number is generally greater than 1 and we are limited to the simplest case with one selected ancestor per paper, there may appear multiple roots and hence isolated trees in the backbone. In the subsequent discussion, we will refer to the output of the SIM, RAN, and LON algorithms as backbone, and its isolated components as trees. Since the seemingly isolated roots may be connected by citations other than the APS network, the number of isolated trees would be lower if more comprehensive citation data were used. Nevertheless, such isolated trees may represent a crude classification of papers. Table I summarizes general statistical properties of the group of trees as obtained by SIM, RAN, and LON approaches.

IV. THE STRUCTURE OF THE BACKBONE

In this section, we will discuss and derive measures to validate the citation backbones as representative of descendant charts. Three aspects will be studied. First, we examine the linkage between different generations of papers. Second, we quantify the paper classification as given by the clustering and branching structure in the backbones. Finally, we examine the possible self-similarity in citation backbones.

A. Hierarchy

We first examine the probability of observing an original citation between two papers as a function of their distance in the backbone. If the backbone is meaningful, we expect this quantity to decrease fast as the distance increases. To compute the distance between \( i \) and \( j \), we find the first common ancestor

| TABLE I. Statistical properties of the isolated trees in the SIM, RAN, and LON backbones. Values for RAN are averaged over 100 realizations. We also show the average interval (in years) between the date of publication of a paper and its parent. |
|-----------------|--------|--------|--------|
|                | SIM    | RAN    | LON    |
| Size:           |        |        |        |
| largest tree    | 26 115 | 30 358 | 428 147|
| 2nd largest tree| 25 386 | 15 697 | 592    |
| 3rd largest tree| 21 794 | 11 362 | 471    |
| \( \langle \Delta t \rangle \) parent-offspring | 9.5 y  | 7.4 y  | 2.1 y  |
two papers are at distance \( d \) assuming there is a citation. \( \alpha \)' in the backbone and count the number of steps \( d_{i\alpha'} \) and \( d_{j\alpha'} \) required to go from \( i \) to \( \alpha' \) and from \( j \) to \( \alpha' \). The distance \( d_{ij} \) is then set as \( d_{ij} = d_{i\alpha'} + d_{j\alpha'} \). We consider \( d_{ij} = \infty \) for paper \( i \) and \( j \) in isolated trees. In Fig. 2, we plot \( P(l|d) \) as a function of \( d \) for all SIM, RAN, and LON backbones, where \( l \) denotes the presence of a link, i.e., a citation. As we can see, \( P(l|1) = 1 \) by definition and all \( P(l|d) \) display a power-law decay for small \( d \). The SIM backbone shows a faster decay than other algorithms, suggesting that citations are more localized in the neighborhood of a paper in the SIM backbone. A similar quantity \( P(d|l) \) (see the inset of Fig. 2) also indicates that the SIM backbone is the best representative of the APS network since citations are concentrated at \( d = 2 \) and decay faster as the distance increases.

In addition to \( P(l|d) \), we further consider \( P(l|d_{i\alpha'}, d_{j\alpha'}) \), where \( \alpha' \) is again the first common ancestor of \( i \) and \( j \) in the backbone. This allows us to see whether citations are localized on the specific branch of each paper or spread over different ramifications on the tree. For any pair \( (i,j) \), we take \( i \) as the later published paper such that the only potential citation is \( i \to j \). We show in Figs. 3(a)–3(c) the results of \( P(l|d_{i\alpha'}, d_{j\alpha'}) \) for the three backbones as a function of \( d_{i\alpha'} \) and \( d_{j\alpha'} \). One notes that increasing \( d_{i\alpha'} \) on the line of \( d_{j\alpha'} = 0 \) corresponds to the vertical trace toward the root, while points with \( d_{j\alpha'} \neq 0 \) correspond to the various “ramifications” in the backbone. Both SIM and RAN give a meaningful structure where citations are localized on the descendant chart of the immediate and next immediate ancestor, i.e., the triangle in the bottom left-hand corner. Citations between different ramifications are rare. The LON backbone instead displays a less coherent structure in which citations crossing different lines of research are common. To examine the difference between SIM and RAN, we also show the scaled difference of their \( P(l|d_{i\alpha'}, d_{j\alpha'}) \) as given in the vertical axis of Fig. 3(d). This comparison clearly indicates that SIM gives rise to the

\[
\begin{align*}
\text{FIG. 2. (Color online) Conditional probability } P(l|d) \text{ of observing a citation between two papers at distance } d \text{ on the SIM (red squares), the RAN (black plus), and the LON (green circles) backbone. Results for RAN are averaged over 100 realizations of the backbone (the variance is negligible). Inset: conditional probability } P(d|l) \text{ that two papers are at distance } d \text{ assuming there is a citation.}
\end{align*}
\]

\[
\begin{align*}
\text{FIG. 3. (Color online) Heat maps that show } P(l|d_{i\alpha'}, d_{j\alpha'}) \text{ as a function of } d_{i\alpha'} \text{ and } d_{j\alpha'} \text{ for (a) the SIM backbone, (b) the RAN backbone, and (c) the LON backbone, with citation } i \to j. \text{ Since papers only cite references published before them, the observed dark triangle in LON suggests a rather homogeneous temporal interval between papers and their best LON ancestor, such that citations with } d_{i\alpha'} > d_{j\alpha'} \text{ are highly improbable. Results for RAN are averaged over 100 realizations of the backbone (the variance is negligible). (d) Scaled difference of } P(l|d_{i\alpha'}, d_{j\alpha'}) \text{ between SIM and RAN.}
\end{align*}
\]
most meaningful hierarchy as citations are mainly found on
the descendant chart of the more relevant ancestors instead of
crossing different charts.

B. Clustering

In addition to the crude classification as given by the iso-
lated trees, the branches in a single tree are also informative
to identify research fields and subfields. From the clustering point
of view, the method we have introduced is computationally
efficient [with complexity $O(N)$ as long as connectivity is
not extensive] compared to modularity maximization-based
algorithms [17,18] or hierarchical clustering algorithms [19]
[with complexity at least $O(N^2)$]. Moreover, the clustering
naturally explores the temporal dimension by preserving the
ancestor-descendant relations.

In order to map the backbone into clusters, we consider two
simple approaches that involve only a single parameter. The
first approach makes use of the publication year of papers and
naturally follows the order of publication. We first make a cut
at the year $Y_i$ such that papers printed before $Y_i$ are removed. We
then consider each unconnected component as a different branch,
e.g., a different cluster, in the original backbone, and
as a classification for papers.

The second approach is dependent on the cluster size, which
we consider to be a typical research branch. Starting from the
leaves of the backbone (i.e., papers with no offspring), we
trace toward the root until a branching point is reached. The
branching point is defined as a node of the network from which
at least (i) two ramifications start and (ii) two ramifications are
extended more than $S$ steps. When a branching point satisfies
these requirements, all ramifications originating from it are
considered as different branches, resulting in a classification
of papers. Here we quantify the validity of clustering as a
function of parameter $Y_i$ and $S$.

In order to evaluate the quality of a given clustering, we
use two different measures. The first one—which we call
exclusivity—is a modified modularity measure specific for
directed acyclic graphs. The rationale behind this measure is
to compute the fraction of links of the original network falling
inside the same cluster and compare it with the expected value
for a random directed acyclic graph. We denote the set of
papers assigned to branch $x$ as $X$ and define the exclusivity as

$$E = \left\{ \left. \frac{\sum_{i} p_{i}^{x} - n_{i}^{x}}{n_{i}} \right| i \in X \right\},$$

(4)

where $p_{i}$ is again the number of references of $i$, $p_{i}^{x}$ is the
number of $i$'s references in branch $x$, $n_{i}$ is the number of papers
published before $i$, and $n_{i}^{x}$ is the number of papers published
before $i$ in branch $x$. The term $n_{i}^{x}/n_{i}$ thus corresponds to the
expected fraction of links from $i$ to any element in $X$ in the
random case. To reduce the noise from small clusters, we have
excluded branches with fewer than 10 papers.

The second measure we use is the effective number of PACS—$N_p$—which counts the average number of heterogeneous
PACS in individual branches. Good paper classifications
result in small values of $N_p$. We first denote $r_{p}^{x}$ to be the fraction
of paper in branch $x$, which is labeled by the PACS number $p$,
and note that $\sum_{p} r_{p}^{x} \geq 1$ as papers are always labeled by more
than one PACS number. $N_p$ is then defined as

$$N_p = \frac{1}{\sum_{p} \left( f_{p}^{x} \right)^{2}},$$

(5)

where $f_{p}^{x} = r_{p}^{x} / \sum_{p} r_{p}^{x}$. Therefore, $N_p = 1$ when there is
only one PACS in the branch, which corresponds to the optimal classification of papers. On the other hand, $N_p$ attains
its maximum when all PACS numbers in $X$ have an equal share (i.e., $equal f_{p}^{x}$) and a large $N_p$ thus corresponds to
high heterogeneity inside single clusters. We remark that in
evaluating $N_p$, only the first four digits are used to distinguish
PACS numbers.

In Fig. 4, we plot the $E$ and $N_p$ as a function of the two
parameters $Y_i$ and $S$. Both measures are biased by the
cluster size but in an opposite way. While $N_p$ indicates better
clustering (and thus a lower value) when isolated clusters are
of a smaller size, $E$ indicates better clustering (and thus a
higher value) when clusters are of a larger size. Even with the
compensation by $n_{i}^{x}/n_{i}$ in Eq. (5), we still observe a small
bias of $E$ on cluster size. These biases may influence our
comparison of the identified clusters from the $SIM$, $RAN$, and
$LOV$ backbones, as they have different sizes. Nevertheless, the
combination of the two independent measures clearly indicates
that $SIM$ is the best choice to obtain a meaningful clustering
besides the bias introduced by cluster sizes. Moreover, the
exclusivity of the $SIM$ backbone is higher for any value of the
parameter $S$, which further supports the validity of the
comparison despite the presence of the bias.

C. Self-similarity

Other than the hierarchial and clustering properties,
the backbones may possess self-similarity. Intuitively,
self-similarity may be induced when branches of research succes-
ively generate branches of significant advances. The existence
of fractality in the backbone would provide support for its
relevance with the evolution of science.

To show the self-similarity in networks, one can measure
their fractal dimension by the box-covering method [12,20,21]. In
this approach, the fractal dimension $d$ is defined as the
power-law exponent in

$$N(l_B) \sim l_B^d,$$

(6)

where $N(l_B)$ is the minimum number of boxes, each of radius
$l_B$, required to cover the whole network. To obtain the exact
$N(l_B)$ is difficult, thus we employ the random sequential box-
covering algorithm [21], which gives an approximate $N(l_B)$
with the same scaling. Specifically, we start with all nodes being
“uncovered” and repeat the following procedures until
all nodes become “covered”: (i) randomly pick a seed node,
(ii) find all “uncovered” nodes within a distance of $l_B$ from
the seed, and (iii) increase $N(l_B)$ by 1 if there exists at least one
“uncovered” node and mark all of them as “covered.” Note
that a “covered” node can also be a seed in the subsequent
searches. For the same tree, we show the minimum of $N(l_B)$
among 20 random sequences as our final value for each value of
$l_B$.

We show in Fig. 5 the results of $N(l_B)$ as a function of $l_B$ for
the largest tree in the $SIM$, $RAN$, and $LOV$ backbone. The
results are compared to \( N(I_B) \) of the original citation network. As we can see, \( N(I_B) \) from the LON backbone has the greatest resemblance to power laws, while that of the RAN backbone shows the fastest decay in \( N(I_B) \). The LON tree has a long tail of \( N(I_B) \), as it is longest and largest in size (see Table I). Only the largest tree of a particular realization of the RAN backbone is shown, as similar results are observed in other realizations. Although a long tail is not observed in the SIM tree, it shows a power-law-like behavior up to an intermediate value of \( I_B \).

Similar behaviors are also observed in the other isolated trees of the SIM backbone, as shown in the inset of Fig. 5.

We interpret the results as follows. The observed resemblance to power laws from the SIM and LON backbone may suggest the presence of self-similarity in their descendant chart. While the LON backbone does not possess a meaningful hierarchy or clustering compared to the SIM backbone, its step-by-step structure indeed shows the highest fractality. We note that a rather short power law is also observed in the original network, though characterized by a different exponent from the SIM and LON backbones. On the other hand, such fractality is not observed in the RAN backbone.

V. POTENTIAL APPLICATIONS

In this section, we briefly describe the implications and potential applications of the citation backbone as a descendant chart of research papers. As the backbone is a sketch of the skeleton of scientific development, it can be applied to identify seminal papers. Preliminary results show that a simple measure based on the number of relevant offspring, i.e., followers in the backbone, is sufficient to give a meaningful ranking that is not trivially correlated with the original number of incoming citations (between the two rankings, Kendall’s correlation coefficient is 0.19 and there is an overlap of only seven papers in the top 20 ranks). This serves as a simple yet meaningful definition of the impact of a publication. More refined definitions that take into account the reputation of each relevant offspring and/or the structural role of a given paper in the backbone.
can give an even better selection of fundamental papers. Moreover, our formulation of tunable weight on authors’ and readers’ interpretations in Eq. (3) can be easily incorporated in common ranking algorithms such as Page Rank, where an even repartition of citation importance is assumed instead.

The second application corresponds to the classification of papers. As we have mentioned before, such clustering divides papers into research fields or subfields and offers a basis for a synthetic picture of the state-of-the-art. There are several advantages over conventional classifications, including (i) lower computational complexity, (ii) additional information of subclustering as given by the internal tree structure, and (iii) predictions of future development by considering the rate of growth of subbranches. This last feature is particularly useful to filter the most active directions in the large amount of literature at our disposal.

VI. CONCLUSIONS
We have shown that a simple backbone constructed by the most relevant citations can well characterize the original citation network. Conversely, nontrivial information stored in the citation network can be simply extracted from its backbone. While conventional spanning trees are based on contemporary information, we demonstrated the significance of temporal dimension in citation backbones.
Specifically, we have introduced both a simple approach to identify the most relevant reference for each publication and effective measures to quantify the validity of the resulting backbone. Our results show that the essential features of hierarchy and paper clustering in the original network are well captured by our citation backbone, while this is not the case for other simple approaches. On the other hand, we showed that a resemblance to self-similarity is observed in citation backbones.

In terms of applications, the backbone can be considered as a descendant chart of research papers, which constitutes a useful basis for identifying seminal papers and paper clusters, and in general a synthetic picture of different research fields. In particular, paper classification by means of the backbone is computationally efficient when compared to the conventional clustering approaches, and provides additional information on the cluster structure besides a mere cluster label.

While we only investigated the citation network of the American Physical Society, the same approach can be readily applied to other citation networks. It would also be interesting to examine the potentials of the present approach on other directed acyclic graphs.

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