Introduction

2013 was the 20th anniversary of the first multiple-point statistics (MPS) simulation method, which was published in the seminal paper of Guardiano and Srivastava (1993), under the impulsion of Andre Journel at Stanford University, Stanford, California, who fathered the MPS concept and nurtured its development (Journel 1993; Journel and Zhang 2006). This work led to a new research field that addressed geological realism in stochastic simulations of the underground. At that time in 1993, computational limitations made the proposed method impossible to apply in real situations. Nevertheless a pathway was open for a new way of thinking about geostatistics. Specifically, three radical innovations were brought forward into a global framework: (1) the concept of MPS instead of two point statistics, such as covariances or variograms to model complex patterns, (2) the necessity of using training images to derive the MPS and (3) the use of non-parametric statistics. It was only in 2002 that these ideas were developed to the point where they became fully applicable. An efficient algorithm (snepim), based on the single normal equation, was proposed by Strebelle (2002), again a student of Andre Journel. This paper was rapidly recognized by the community to be a major breakthrough. It received the prize of the best paper in Mathematical Geology for 2002. Currently, it is the second most cited paper of Mathematical Geosciences with 247 citations, just after a paper that is 10 years older by Noel Cressie.

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Today, MPS has grown into a very active field of research. Numerous new methods and algorithms have been developed, with applications ranging from geology to remote sensing or climate simulation. Despite this profusion, MPS is still in its infancy. Fundamental questions remain to be answered from a theoretical point of view. The limitations of the multiple-point approach need to be clearly defined, and strategies have to be implemented to make these methods more applicable.

This special issue aims at providing a combination of papers that address the theoretical foundation of the multiple-point approach, describe new algorithms and demonstrate how the methods can be used in practice. It is divided into two parts: the first one (this issue) is focused on theory; the second (published later this year) will be devoted to applications of MPS methods.

The present volume starts with a very fundamental issue. Xavier Emery and Christian Lantuejoul question if a training image can be a substitute for a random field model. In particular, they investigate the impact of the finite size of the training images. They show clearly that a training image cannot be considered as a substitute to a random field model in most practical situations. When the TI is not large enough, the statistics of the MPS simulation are different from the statistics of the underlying random field model that was used to generate the training image.

Given the number of different simulation algorithms that have been developed in recent years, the necessity has emerged to develop a framework to compare these algorithms. This is what Xiaojin Tan, Pejman Tahmasebi and Jef Caers propose in the second paper of this special issue, with a general methodology that allows comparing realizations coming from different simulation algorithms, in terms of their variability and across a range of spatial scales.

The next paper is a contribution by Sebastien Strebelle and Claude Cavelius, who revisit the well-known snesim algorithm to solve some of its computational issues, namely, memory consumption and large CPU cost. This is accomplished by introducing the concept of sub-grids and by an optimal choice of the template size.

The special issue then continues on the theme of algorithmic improvements of MPS simulation algorithms, with a paper by Julien Straubhaar and Duccio Maliverni demonstrating how they have solved the long-standing problem of relocating hard conditioning data in the presence of multiple grids.

The paper by Odd Koldbjoernsen, Marita Stien, Heidi Kjonsberg, Bjørn Fjellvoll and Petter Abrahamsen is focused on Markov Mesh models which constitute an alternative method for the construction of spatial random fields from a training image. As opposed to the traditional MPS methods, the Markov Mesh models are formulated as a generalized linear model. In this paper, the authors show the latest advances in this field: how multiple grids can be used in this framework.

The two last papers deal with a key issue in all MPS applications: how to get a three-dimensional training image? Lin Hu, Yongshe Liu, Claude Scheepens, Al Schulz and Ron Thompson consider the situation when an initial facies model of a reservoir is used as training image for simulating the same reservoir and to impair variability to the initial model. This goes against the usual vision that the training image should be a conceptual model (non-local and stationary) of the type of heterogeneities.
Last but not least, Alessandro Comunian, Sanjeev K. Jha, Beatrice M.S. Giambastiani, Gregoire Mariethoz and Bryce F.J. Kelly discuss how process-imitating methods of fluvial deposits can be used in conjunction with MPS to generate training images. They illustrate their methodology using a beautiful example in Eastern Australia: the Lower Namoi aquifer.

We hope that the papers presented in this volume will inspire new debates, research, and developments in the broad field of MPS and training image-based geostatistics. We also hope that it will motivate the potential users to test those methods and challenge their developers to improve them.

References