Parentheticals as Conventional Implicatures*

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1 Introduction

In his Logic and conversation, Grice (1989, chap. 2) proposed a much commented distinction between what is said and what is implied.¹ This distinction is of particular relevance for constituents that are not the complement or the modifier of another constituent within a sentence, like heureusement ‘fortunately’, je pense ‘I think’ or d’après Marie ‘according to Mary’. In current usage, such expressions are called parentheticals. This is actually misleading. Because many of these expressions are often (see evaluative adverbs such as heureusement) or always (see je pense ‘I think’, paraît-il ‘I hear’) prosodically incident, the class of parentheticals tends to be equated with that of expressions which are or can be incidentals. However, the semantico-pragmatic property is clearly distinct from the prosodic property, as shown by Bonami et al., chapter 11: parenthetical adverbs such as heureusement, for instance, may occur either as incidentals or with an integrated prosody, just like modal adverbs, which we show are not parentheticals. In this paper, we are primarily interested in adverbials that qualify an assertion either by modalizing it (modal adverbs) or by signalling that its truth is warranted by a particular source (belief and report expressions). They are particularly puzzling since, although most of them (excluding modals,

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¹Grice also uses ‘suggested’ and ‘meant’.
as we will see) are not a part of ‘what is said’, they interact with the assertive force of the sentence in which they occur. For instance, sentences with *paraît-il* ‘I hear’ are certainly less authoritative than the same sentences without the adverbial. We propose that parentheticals fall into the general category of conventional implicatures, that is, constraints on interpretation which are lexically triggered but do not contribute to delineate the referential content of a sentence (the type of situation which the sentence purports to describe). Formally, we capture the distinction between parentheticals and non-parentheticals by distinguishing between two different kinds of updates in the dynamic multimodal multiagent framework of Gerbrandy (1998). The paper is organized as follows: in section 2, we review the different tests and show that they support the distinction we propose. In section 3, we discuss the theoretical status of the distinction. In 3.1, we show that parentheticals must be conventional implicatures, if anything. In 3.2, we characterize the status of conventional implicatures in dynamic semantics. Finally, in 3.3 and 3.4 we present our formal treatment for parentheticals.

## 2 Tests for the distinction

How do we determine that a constituent is part of what is said? We will consider six tests, some of them being mentioned in the literature (see for instance Borillo (1976), Molinier and Lévrier (2000) for French; Wilson (1975), Infantidou (1994), Rouchota (1998) for English).

### 2.1 The *oui/non* test

With *Oui* ‘yes’ answers, it seems that the speaker can refer to the whole host sentence, including the adverbial. For instance, in (1a,b,c), B’s answer can be interpreted as expressing agreement on the choice of the modality.

(1) a. A — Jean a probablement changé de voiture  
‘John probably got a new car’
B — Oui, c’est même plus que probable  
‘Yes, it’s even more than probable’

b. A — Jean a malheureusement eu un accident  
‘Unfortunately, John had an accident’
B — Oui, c’est très ennuyeux  
‘Yes, that’s a real problem’

c. A — Jean a eu un accident, paraît-il  
‘John had an accident, I hear’
B — Oui, je l’ai aussi entendu dire  
‘Yes, I also heard of that’
However, *non* 'no' is not symmetric to *oui* in all examples.²

(2) a. A — Jean a probablement changé de voiture
   'John probably got a new car'
   B — Non, c'est improbable
   'No, it's improbable'

   b. A — Jean a malheureusement eu un accident
   'Unfortunately, John had an accident'
   B1 — Non, ??c'est bien fait pour lui. Il conduit trop vite
   'No, he deserves it. He drives too fast'
   B2 — Non, ??tu es bien content, avoues-le; tu le détestes
   'No, be frank, you are glad of that; you hate him'

   c. A — Jean a eu un accident, paraît-il
   'John had an accident, I hear'
   B1 — Non, personne n'a dit ça
   'No, nobody said that'
   B2 — Non, ??personne n'était au courant
   'No, nobody knew'

In examples (2b), B1's and B2's answers cannot refer to modalities such as 'It is unfortunate that' or 'The speaker A considers that it is unfortunate that'. The case of (2c) is more complex and we return to it in the next section.

2.2 The *vrai/faux* test

In this test, one tries to imagine an answer whereby B echoes or opposes directly A's assertion by asserting that what A said is true/false. We illustrate the *C'est faux* ('It's false') case.

(3) a. A — Jean a probablement changé de voiture
   'John probably got a new car'
   B — C'est faux, c'est improbable
   'It's false, it's improbable'

   b. A — Jean a malheureusement eu un accident
   'Unfortunately, John had an accident'
   B1 — ??C'est faux, c'est bien comme ça
   'It's false, that is OK'
   B2 — ??C'est faux, tu es bien content, avoues-le; tu le détestes
   'It's false, be frank, you are glad of that; you hate him'

²Actually, *oui* has the same behavior as *non*. For instance, if, in (1c), B answers by *Moi aussi* 'Me too', it can only mean 'I had an accident too', not 'I heard it too'. 
c. A – Jean a eu un accident, paraît-il
   ‘John had an accident, I hear’
B1 – C’est faux, personne n’a dit ça
   ‘It’s false, nobody said that’
B2 – ??C’est faux, personne n’était au courant
   ‘It’s false, nobody knew’

In (3a), B takes the modality into account. In contrast, (3b) replicates
the observation (2b). B fails to deny that it is unfortunate (for the
speaker A) that John had an accident. (3c) exhibits the same contrast
as (2c) between a denial based on ‘personne n’a dit ça’ and ‘personne
n’était au courant’. B2’s answers in (2c) and (3c) are unnatural because
they fail to refer to the modality. To interpret these answers, we have to
find a discourse relation which might connect Non or C’est faux with
these propositions. Relations like Narration, Elaboration, Contrast,
are not good candidates. The most reasonable choices are Justification
or Convergence. A Justification relation between α and β can be glossed
by ‘α since β’. A Convergence relation obtains when two propositions
point in the same direction (through entailment or implicature). Justi-
fication can be signalled by puisque and Convergence by d’ailleurs
(roughly equivalent to ‘also’ in this context) or de plus (‘moreover’).
(4) illustrates the differences. B1’s and B2’s answers show that Justifi-
cation is not possible with the first proposition while Convergence is.
B4’s answer shows that neither relation is possible with personne n’était
au courant. This is to be expected since this sentence presupposes that
John had an accident, a proposition which is explicitly denied by the
first sentence of the answer (Non and C’est faux). Note that, for Justi-
fication to be appropriate with B1, the answer would have to pick up
the reportedness modality and produce a meaning like ‘It cannot be
the case that you heard that since nobody said that’.

(4) A – Jean a eu un accident, paraît-il
   ‘John had an accident, I hear’
B1 – ??Non/C’est faux puisque personne n’a dit ça
   ‘No/It’s false since nobody said anything like that’
B2 – Non/C’est faux, d’ailleurs personne n’a dit ça
   ‘No/It’s false, also, nobody said anything like that’
B3 – ??Non/C’est faux puisque personne n’était au courant
   ‘No/It’s false since nobody knew’
B4 – ??Non/C’est faux, d’ailleurs personne n’était au courant
   ‘No/It’s false, also nobody knew’

3See Mann and Thompson (1988), Sanders et al. (1992), Lascarides and Asher
(1993) for standard repertoires of discourse relations.
2.3 The conditional test

Assuming that conditional sentences can have an implicative interpretation of the form \( \phi \Rightarrow \psi \), if the modality is integrated into what is said in \( \phi \), it may have effects on the truth or relevance of the conclusion \( \psi \). For instance, the adjunct *on Tuesday* in *John was in Germany on Tuesday* is a part of what is said because it plays an essential role in the implicative connection of sentences like *If John was in Germany on Tuesday, he was not in San Francisco*. This test is consistent with the idea that the *probably* modality is a part of what is said but the other two cannot be.

(5) a. Si Jean a probablement changé de voiture, il a probablement aussi acheté une voiture d’occasion
   ‘If John probably got a new car, he also probably bought a second-hand one’

b. Si Jean a probablement changé de voiture, ??il a aussi acheté une voiture d’occasion
   ‘If John probably got a new car, he also bought a second-hand one’

c. Si Jean a malheureusement démissionné, ??alors il est également malheureux que son bras droit ait démissionné
   ‘If John unfortunately resigned, then it is also unfortunate that his assistant resigned’.

In (5a), the *si*–clause is preferably interpreted as echoing some previous judgment. For instance, the speaker echoes what another speaker said or implied. The other possible interpretation, under which the speaker herself introduces the modal judgment, is less natural since it would correspond to a reading like ‘If I believe that \( \phi \); then \( \psi \)’, where the speaker doubts the existence of her own mental states. (5b) sounds strange because if it is only probable that John got a new car, asserting that it is a second-hand one is too strong. Under the interpretation that the first judgment (it is unfortunate that John has resigned) entails the second (it is unfortunate that his assistant resigned), (5c) is out, because the first modality cannot be integrated into the antecedent and escapes the entailment relation, which is necessary for *aussi* to be justified. The sentence does not mean ‘If it is unfortunate that …, then …’.

*Parait-il* raises an additional problem. This modality is not compatible with a *si*–clause because it is speaker-centered and means something like ‘I heard that’. So, saying *Si Jean a, parait-il, démissionné* (‘If John resigned, I hear’) would, at best, amount to saying ‘If I heard that John resigned’, an improbable case of doubting the existence of one’s
own perceptions or mental states. To circumvent this problem, one can use another possible interpretation of conditional sentences where the sentence points to a contrast between two propositions. (6a) illustrates the interpretation and (6b) shows that paraît-il is compatible with the si-clause in this case. (6d) shows that paraît-il cannot be a part of what is said and that its interpretation cannot be equated with (6c).

(6) a. Si Marie est grande, Jean est petit  
     'If Mary is tall, John is short'

     b. Si, du moins paraît-il, les impôts augmentent, en revanche, le chômage baisse  
     'If taxes are increasing, at least according to what I hear, in contrast, unemployment is decreasing'

     c. Si j’ai entendu dire que les impôts augmentaient, Jean a entendu dire le contraire. Comment savoir?  
     'If I heard that taxes are increasing, John heard the contrary. How could we know?'

     d. Si, du moins paraît-il, les impôts augmentent, ??Jean a entendu dire le contraire.  
     'If taxes are increasing, at least according to what I hear, John heard the contrary'

2.4 Discourse attachment

The aim of this section is to clarify the illocutionary status of parentheticals. We saw in section 2.2 that attachment problems through discourse relations are responsible for certain differences in acceptability. Asher (2000) proposes that parentheticals are attached to the clause they modify by discourse relations like Comment, Evidence, etc. In Asher’s Segmented DRT (SDRT) (Asher, 1993; Lascarides and Asher, 1993), attaching a discourse segment, or constituent $\beta$ to another constituent $\alpha$ is only possible in the following two cases, where $\gamma$ denotes the last constituent in the sequential order of discourse:

1. $\alpha = \gamma$.

2. $\gamma$ is subordinated to $\alpha$ via a subordination discourse relation.

Attachment can hold between constituents which do not correspond to speech acts in the usual sense (Searle, 1969). For instance, in Asher and Lascarides (1998), the DRSs corresponding to presuppositions can be attached to other constituents.\(^4\)

The attachment properties of parentheticals show that they are not genuine speech acts. Let us consider the triple (7). (7a) connects two

\(^4\)We assume here that presuppositions are not speech acts. For a different view, see Ducrot (1972).
assertions by a Justification relation. The second assertion is presented
as a reason to believe that the proposition expressed by the first as-
sertion is true. (7b) and (7c) contain the additional judgment that the
situation associated with the first sentence is a good thing. This is the
result of inserting an independent assertion in (7b), and a parenthetical
adverb in (7c). (7b) is much better if $\beta$ is connected by Explanation
to $\gamma$, not to $\alpha$. In other words, the preferred interpretation of the
discourse is that the fact that the basketball group voted for Mary is a
justification of the assertion that it is a good thing. This suggests that
the Comment relation between $\alpha$ and $\gamma$ is not a subordination relation
but a Coordination relation, which, in SDRT, is predicted to block the
attachment of $\beta$ to $\alpha$.

Two observations are in order for (7c). First, the parenthetical is not
integrated into what is said since the first sentence cannot be para-
phrased by ‘It is a good thing that Mary will be elected to head the
club’. Assume the contrary; the Justification connection would then be
unclear: how could the decision of the basketball group possibly affect
the felicity of Mary being elected? Second, the parenthetical is not ad-
ded in a separate speech act, unlike the parallel judgment in (7b), since
we do not observe the same effect in (7c) as in (7b) with respect to
attachment. We conclude that, in SDRT, fortunately is neither a part
of the asserted content nor a separate speech act–based constituent.

(7) a. Mary will be elected to head the club, since the basketball
group decided to vote for her

b. Mary will be elected to head the club. ($\neg \alpha$) This is a good
thing. ($\neg \gamma$) $\#$Since the basketball group decided to vote for
her ($\neg \beta$)

c. Mary will, fortunately, be elected to head the club, since the
basketball group decided to vote for her

2.5 Interrogatives

If modal adverbs are integrated into what is said, why are they odd in
yes–no questions (8a)? One would expect that they combine with the
interrogative modality to produce readings like ‘Is it probable / likely
/ etc. that $\phi$?’. Two points should be noted in this respect. First, the
combination of parentheticals with the interrogative modality is not
uniform. For instance, heureusement is out in yes–no questions while
malheureusement ‘unfortunately’ is acceptable.5 Second, the behavior
of modal adverbs might be explained by particular scope properties.

5See Est-ce que Jean $\alpha$, heureusement / malheureusement, découvert la ré-
ponse? ‘Did John, fortunately / unfortunately find the answer?’.
Molinier and Lévrier (2000) note that four modal French adverbs (*forcément, fatalement ‘of necessity’, obligatoirement ‘obligatorily’ and nécessairement ‘necessarily’) can occur after the negation marker pas, in contrast with other modals (9). They can also occur in questions (8).

(8) a. Est-ce que Jean a probablement démissionné?
   ‘Did John probably resign?’

   b. Est-ce que Jean a nécessairement / forcément etc. démissionné
   ‘Did John necessarily resign?’

(9) a. Jean n’a (*nécessairement / *forcément etc.) pas (nécessairement
   / forcément etc.) démissionné
   ‘John did (necessarily) not (necessarily) resign’

   b. Jean n’a (probablement) pas (*probablement) démissionné
   ‘John did (probably) not (probably) resign’

(9) indicates that the four mentioned modals can occur in the scope of the main sentential operator, i.e. the negation. If we assume that, in interrogatives, the main operator is a question operator, we can account for the parallelism between (8b) and (9a) in terms of scope. Certain modal adverbs (e.g. *probablement) must take wide scope, whereas others (e.g., *forcément) don’t follow this rule. This predicts that (8a) means something like ‘It is probable that (did John resign?)’, hence its oddity. If this conjecture is right, the question test pertains to the scope problem, not to the ‘said’ vs. ‘implied’ distinction. The reader is referred to Ferrari (1995) for a more systematic treatment of similar scope problems.

2.6 Declarative verbs

Following Bach (1994), an expression is part of what is said if it can occur in the complement clause of a declarative verb, and conversely, is not part of what is said if it cannot. The test is based on the behavior of speech act adverbs such as frankly, which are uncontroversially parentheticals, see (10a). However, the test is not convincing. First, as shown in (10b), the French equivalent is acceptable for many speakers. Second, it conflicts with the other tests that we have discussed, since evaluative adverbs or reportive incidentals are perfectly acceptable, as shown by (10c). Speakers who reject (10b) seem to restrict the relevance of speech act adverbs to the actual speech act, to the exclusion of a reported speech act.

(10) a. Mary said that *frankly John is incompetent

   b. Marie a dit que, (??)franchement, Jean était incompétent
c. Marie a dit que Jean était, malheureusement / paraît-il, mal-
ade
lit.: Mary said that John was, unfortunately / she hears, ill

Summarizing, we see that (i) two tests (declarative clause embedding and interro-
gatives) are not significant and (ii) the other four support the hypothesis of a difference between modals and parentheticals. Specifically, modals are interpreted as a part of the assertion while parentheticals are not. So the question naturally arises of the exact nature of their contribution. Rossari (2002) has independently provided a convergent analysis for causal parentheticals and non-parentheticals.

3 The contribution of parentheticals

3.1 Parentheticals convey conventional implicatures

If parentheticals are not part of the assertion, the semantic options left to us are: (i) they introduce presuppositions, (ii) they introduce implicatures. The first possibility is unlikely. Parentheticals like *paraît-il* or *I hear* do not pass the standard tests that detect presuppositions (Soames, 1989; Geurts, 1999). Some of them, e.g. *heureusement* ‘fortunately’ are considered as ‘factive’ (Bartsch, 1975). The term may be misleading since it suggests an analogy with factive verbs (see Bonami et al. (chapter 11, this volume) for a discussion). However, factivity-preserving environments for factive verbs do not have the same effect on the adverbs mentioned.

(11) a. Est-ce que tu sais que Marie a réussi son examen?
   ‘Do you know that Mary passed her exam?’

   b. Est-ce que, malheureusement, Marie aurait raté son examen?
   ‘Would Mary have –unfortunately– failed her exam?’

While (11a) still carries the presupposition that Mary passed, this is not the case for (11b). The adverb only applies to possible events (of Mary failing). As to *paraît-il*, it is not compatible with questions.

Certain parentheticals correspond to detachable lexical material. One can suppress *fortunately* in *Fortunately, John was elected* without changing the truth-conditions of the sentence. This is less clear for *I hear. John was elected, I hear* is more cautious than *John was elected*. However, as shown in section 2, the judgments of truth and falsity ignore the parenthetical, a fact which suggests that its contribution to the truth of the sentence is only indirect. A parenthetical cannot be ‘cancelled’. For instance, *If John has a son, his son is certainly proud of his father* suspends the presupposition that John has a son, that is, cancels the default effect of ‘his son’. In contrast, *If it is really a
good thing that John was elected, then, fortunately, he was elected is hardly interpretable. According to Grice (1989), the two properties of detachability and non-cancellability are the hallmark of conventional implicatures, and we may assume as a starting point that parentheticals trigger such implicatures.  

3.2 The status of conventional implicatures
What is the Gricean status of implicatures? Grice proposes that discourse markers like therefore convey the implicature that there is a consequence relation between two propositions. Similarly, one might say that paraît–il conveys the implicature that the speaker heard that \( \phi \), where \( \phi \) is the asserted content.

(12) Marie a, paraît–il, réussi son examen
‘Mary passed her exam, I hear’:
assertion: ‘Mary passed her exam’
implicature: ‘I heard that Mary passed her exam’

The problem with Grice’s approach is that implicatures are mostly described in a negative way (as ‘non-assertions’). In this respect, their contribution to the sentence meaning remains somewhat obscure. How is it, for instance, that one cannot deny implicatures? After all, if they simply had a different content from assertions, one could deny this content.  

Grice was actually aware of the problem (Grice, 1989, chap. 5). He proposes that conventional implicatures are associated with non-central speech acts which rely on the execution of other, more central ones. For instance, the act of adding (e.g. associated with ‘moreover’) only makes sense if there are two assertions (‘A moreover B’ supposes ‘A’ and ‘B’). Grice notes that the dependence of the non-central speech act \( X \) upon the central one(s) should be described in a way that accounts for the impossibility of using \( X \) for ‘saying’ something. One must also keep in mind that the central ingredient in Grice’s analysis of linguistic meaning is intention. Roughly speaking, by asserting that \( \phi \) the speaker \( a \) intends to make the hearer believe that \( \phi \) and believe that \( a \) believes \( \phi \) through the identification of this very intention. We ignore the type of circularity involved in this definition (see Barwise and Moss (1996) on this topic), but we retain the idea of an intentional pro-

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6 Generally speaking, recent literature on presuppositions (Beaver, 2001; Geurts, 1999) shows that attempts to put presuppositions and implicatures on a par (Gazdar, 1979; Karttunen and Peters, 1979) are misguided.

7 Rouchota’s 1998 and Asher’s 2000 skepticism as to the standard Gricean approach stems partly from the fact that Grice did not really provide an account of conventional implicatures.
cess. Together, intentionality and non–centrality suggest the following picture. The information communicated by a speaker $a$ is partitioned into:

1. what is said (= asserted), that is, what the speaker intends to be added to the common ground, and

2. what is conventionally implied, that is, what the speaker intends to be added to the hearers’ beliefs with respect to what the speaker believes.

Note that, in the second case, the speaker certainly intends in certain cases to convince the hearers that the implicature is true. But this would be achieved in an indirect way, through the fact that the hearers espouse the speaker’s belief because they trust her. To paraphrase what Stalnaker (1973) observed for presuppositions in such cases, the speaker ‘may want to communicate a proposition indirectly’. We then distinguish between the following two kinds of effect for an assertive speech act.

**Definition 1** Let $A$ be an assertive speech act whereby $a$ asserts that $\phi$ and conventionally implicates that $\psi$ in the presence of $b$, then the effect of $A$ on $b$ includes at least the two following updates:

1. If $b$ trusts $a$ on $\phi$, she updates her belief state with $\phi$ and with the proposition that $a$ believes $\phi$,
2. if $b$ trusts $a$ on $\psi$, she updates her belief state with the proposition that $a$ believes $\psi$.

The update in (1) is the intended effect of the speech act $A$.

How can we account for the behavior of denials like *C’est faux* ‘It’s false’? Adjectives like *true* ‘true’ and *faux* ‘false’ select for propositional entities. So, in themselves, they cannot tell apart asserted and implied propositions. This suggests that it is the demonstrative pronoun *c’* ‘this, that’ that selects the asserted proposition. This is confirmed by the fact that, with other adjectives, the same effect obtains.

(13) $A$ = Jean a raté son examen, il paraît
‘John failed his exam, I hear’

$B$ = C’est malheureux / étonnant
‘It’s unfortunate / surprising’

= ‘It is unfortunate / surprising that John failed his exam’

More generally, it seems that anaphors on non–asserted material are impossible or marginal. This is evidenced by anaphoric pronouns and
by the linking law of Ducrot (1972),\textsuperscript{8} which says that discourse markers cannot exploit presupposed material.

(14) a. Jean a raté son examen, il paraît. Je m’y attendais
   ‘John failed his exam, I hear. I expected that’
   = ‘I expected that he would fail his exam’
   ≠ ‘I expected that I would hear that he failed his exam’

   b. Jean a cessé de fumer. ??Pourtant, il connaissait les risques
   ‘John stopped smoking. Yet he was aware of the risks’
   ≠ ‘John was smoking, yet he was aware of the risks’

In (14a), the clitic pronoun \textit{y} cannot refer to the reportive modality. In (14b), the opposite discourse marker cannot refer to the presupposition that John has been smoking for some time.

Summarizing, our proposal amounts to keeping the truth-conditional and the epistemic status of implicatures separate. Being propositions, implicatures can correspond (or not) to the facts. Then, they are truth-conditional, and we agree with Asher (2000) on this point. Moreover, implicatures are ‘dynamic’, that is, they can be added to the belief states of the discourse participants. In these two respects, implied propositions do not differ from asserted propositions. However, in contrast to asserted propositions, implied propositions are not added to the common ground. So, although they are dynamic, their epistemic locus is different, as evidenced by the impossibility of referring to them through anaphoric markers (pronouns, discourse markers).

3.3 Problems with standard dynamic semantics

Following Stalnaker (1978) and Veltman (1996) in particular, we model assertions as information updates. Given a set of epistemic alternatives for an agent \textit{a}, an assertion that \(\phi\) may lead \textit{a} to eliminate the alternatives that are not consistent with \(\phi\). Such approaches are not entirely appropriate to our goals for two reasons. First, they are not concerned with embedded belief, making it difficult to represent what agents believe about others’ beliefs. We will take this aspect into account by using a multiagent representation system.

Second, they do not make room for modal updates. Consider Veltman’s approach. An agent believes that \(\phi\) iff \(\phi\) is true in every epistemic alternative available to the agent. In contrast to ‘ordinary’ propositions, which give rise to eliminative updates, modal propositions such as \textit{Might} \(\phi\) are static. At a given information state, they are simply

\textsuperscript{8}Loi d’enchaînement in French. We assume here that presuppositions are not asserted and that apparent evidence to the contrary can be disposed of along the lines of Von Fintel (2001).
true or false. \( \text{Might } \phi \) is true at \( S \) iff \( \phi \) is true in at least one \( s \in S \). The update of \( S \) with \( \text{Might } \phi \) succeeds if \( \text{Might } \phi \) is true at \( S \). Otherwise, it ‘fails’, that is, it produces the absurd information state \( \emptyset \). However, updates triggered by modal sentences are intuitively perceived as adding information, and are thereby not reducible to formula testing. For instance, in (15), a possible interpretation is that the speaker, having learned that John’s decision has not been approved by the committee, updates her information state with the proposition that John will probably resign.

(15) La décision de Jean n’a pas été approuvée par le comité? Alors, il va probablement démissionner

‘John’s decision has not been approved by the committee? Then, he is probably going to resign’

We noted in the introduction that parentheticals qualify assertions. In other terms, they somehow affect the content with which the discourse is updated. For instance, in (16b), the update concerns what is said, i.e. the proposition that John has resigned; however, there is a strong feeling that what is said in (16b) is, in some sense, weaker than what is said in (16a), where there is no qualification by \( \text{paraît-il} \).

(16) La décision de Jean n’a pas été approuvée,

‘John’s decision has not been approved,’

a. donc il a démissionné

‘so he resigned’

b. donc il a, paraît-il, démissionné

‘so he resigned, I hear’

3.4 Extending the standard semantics

We extend update-based approaches in two directions. First, we consider sets of agents communicating their belief states to each other. This can be done in multiagent dynamic epistemic logic, e.g. Gerbrandy (1998) or Van Ditmarsch (2002). Second, to cope with modal updates, we admit partiality in the semantics. In ordinary possible worlds, every proposition is either true or false. We let partiality in through undetermined propositions. For space reasons, we will consider only a simplified version of Gerbrandy (1998) approach, based on finite possibilities.

Definition 2 Possibilities

Let \( \mathcal{P} \) be a set of propositions, \( \mathcal{A} \) a finite set of agents (\( a, b, \) etc.) and \( \mathcal{M} \) a finite set of unary modal operators. A possibility based on \( \mathcal{P} \) and \( \mathcal{A} \) is a function \( \pi \) which assigns to each proposition of \( \mathcal{P} \) one of the values 0, 1, or ? and to each pair \( \langle x, M_i \rangle \), with \( x \in \mathcal{A} \) and \( M_i \in \mathcal{M} \), a
set of possibilities, called an information state or i.s. \((s, s', \text{ etc.})\). We define \(\nu\) to be the following special (‘undefined’) possibility: \(\nu(p) = \?) for every \(p \in \mathcal{P}\), \(\nu(\langle x, M_i \rangle) = \{\nu\}\) for every \(\langle x, M_i \rangle\). \(\pi \uparrow \mathcal{P}\) is the root of \(\pi\).

\(\nu\) is used to stop possibility expansion. A ‘preterminal’\(^9\) possibility \(\pi\) returns \(\nu\) for any \(\langle x, M_i \rangle\) argument. Preterminal possibilities ‘measure’ the introspective power of agents. The undefined possibility makes every formula undefined (\(\nu \not\models \phi\) for every \(\phi\)). To simplify the semantic definition of truth and the definition of updates, we consider only normal possibilities, that is, possibilities where no i.s. contains \(\nu\), except, possibly, for the ‘terminal’ i.s. \(\{\nu\}\).

**Definition 3** A possibility \(\pi\) is normal iff it contains no subpossibility \(\pi’\) such that \(\pi’(\langle x, M_i \rangle) = s, \nu \in s\) and \(s \neq \{\nu\}\), for some \(\langle x, M_i \rangle\).

A possibility \(\pi\) is limited if it ends in \(\nu\) ‘everywhere’. A limited possibility is pictured in figure 1 below.

![Figure 1](image)

**Figure 1** A limited normal possibility

More technically, consider the possibility \(\pi\); all its branches are streams of the form \(\langle r, \pi_1, \pi_2, \ldots \rangle\), where \(r\) is the root of \(\pi\), \(\pi_1\) is one of the possibilities of \(\pi(\langle x, M_i \rangle)\), for some \(x \in \mathcal{A}\) and some \(M_i \in \mathcal{M}\), \(\pi_2\) is one of the possibilities in \(\pi_1(\langle y, M_j \rangle)\) for some \(y\) and some \(M_j\), etc. In figure 1, the grey square (the root) and circles determine a branch. A possibility is limited when all its branches are of the form \(\langle r, \pi_1, \ldots, \pi_n, \nu, \nu, \ldots \rangle\), that is when every branch has a finite head before the infinite subbranch \(\langle \nu, \nu, \ldots \rangle\). Labelled branches are the streams of the form:

\(\langle r, \langle x_1, M_1 \rangle, \pi_1, \langle x_2, M_2 \rangle, \pi_2, \ldots, \langle x_n, M_n \rangle, \nu, \langle x_{n+1}, M_{n+1} \rangle, \nu, \ldots \rangle\).

They correspond to all the paths in the possibility with agent–operator pairs as labels. In figure 1, the grey square and circles connected by the arrows correspond to the labelled branch \(\langle r, \langle a, B \rangle, \pi_1, \langle b, B \rangle, \pi_2, \langle a, B \rangle, \pi_3 \rangle\).

\(^9\)‘Preterminal’ and ‘terminal’ are used metaphorically. Actually, the ‘terminal’ \(\nu\) cycles infinitely into itself.
\( \langle c, B \rangle, \varnothing, \ldots \). In order to be able to use standard recursion instead of corecursion (Barwise and Moss, 1996), we consider only limited normal possibilities. To define updates, we need the notion of truth at a possibility. We assume the standard definitions of truth for partial modal logic (Jaspars and Thijsse, 1996).

**Definition 4 Semantics for possibilities**

Let \( \phi \) be a formula; \( \varnothing \models \phi \); \( \phi \) is true, false or undefined at \( \pi \neq \varnothing \), in symbols \( \pi \models \phi \), \( \pi \models \phi \), \( \pi \models \phi \) iff:

1. The main connective/operator of \( \phi \) is non-modal and the truth-values of the subformulas in \( \phi \) given by \( \pi \) obey the standard definition for partial logic.\(^{10}\)

2. \( \pi \models ( \models ) \Box a \psi \) iff \( \pi' \models \psi \) for every \( \pi' \in \pi(\langle \Box, a \rangle) \) \( (\pi' \models \psi \) for some \( \pi' \in \pi(\langle \Box, a \rangle)). \)

3. \( \pi \models ( \models ) \Diamond a \psi \) iff \( \pi' \models \psi \) for some \( \pi' \in \pi(\langle \Diamond, a \rangle) \) \( \pi' \models \psi \) for every \( \pi' \in \pi(\langle \Diamond, a \rangle)). \)

Our next task is to define updates. Since possibilities admit of indetermination (\( \mathbb{I} \)), adding the information that \( \phi \) may suppress some indetermination but is not deterministic in the general case; hence the following definition for possibilities.

**Definition 5 Updates**

A. If \( \pi \models \phi \), \( \pi \models \phi = \{ \pi \} \). If \( \pi \models \phi \), \( \pi \models \phi \) is the set such that \( \pi' \in \pi + \phi \) iff:

1. if \( \pi \models \psi \), \( \pi' \models \psi \) for any \( \psi \),
2. if \( \pi \models \psi \), \( \pi' \models \psi \) for any \( \psi \),
3. if \( \phi = \psi \land \chi \), \( \pi' \in (\pi + \psi) + \chi \),
4. if \( \phi = \psi \lor \chi \), \( \pi' \in \pi + \psi \lor \pi' \in \pi + \chi \), or \( \pi' \in \pi + \chi \), or \( \pi' \in (\pi + \psi) + \chi \),
5. if \( \phi = \neg \psi \), \( \pi' \in \pi + \psi \), where \( \psi \) is the result of pushing \( \neg \) one step inward (i.e. \( (\psi_1 \land \psi_2) \land = \neg \psi_1 \lor \neg \psi_2 \), etc.),
6. if \( \phi = \Box a \psi \), \( \forall \pi'' \in \pi(\langle \Box, a \rangle)(\pi'((\Box, a)) \in \pi'' + \psi) \),
7. if \( \phi = \Diamond a \psi \), \( \exists \pi'' \in \pi(\langle \Diamond, a \rangle)(\pi'((\Diamond, a)) \in \pi'' + \psi) \),
8. \( \pi' \) does not differ from \( \pi \) except as a consequence of applying 3, 4, 5, 6, 7.

B. If \( \Pi \) is a set of possibilities, \( \Pi \models \phi \) is \( \{ \pi' \mid \pi' \in \pi + \phi \) for some \( \pi \in \Pi \} \).

For instance, if every \( \pi'' \) in \( \pi(\langle \Diamond, a \rangle) \models \phi \), every \( \pi' \) is such that some possibility in \( \pi'((\Diamond, a)) \) is a member of \( \pi'' + \phi \) for some \( \pi'' \). We are

\(^{10}\)E.g., \( \pi \models A \lor B \) iff \( \pi \models A \) and \( \pi \models B \), etc. As usual, \( \pi \models \phi \) iff \( \pi \models \phi \) and \( \pi \models \phi \). This extends to modal formulas.
specifically interested in belief updates, where the intended effect of asserting $\phi$ is that every agent believes $\phi$, or equivalently that the i.s. at $\pi((x, B))$ satisfies $\phi$ for any $x \in A$. It is also common knowledge that every agent believes that every other agent believes that $\phi$. Therefore, for any $x$ and $y$ in the set of agents, any $\langle x, B \rangle$ link from a possibility in $\pi((y, B))$ leads to a state where $\phi$ holds. We ignore updates that go beyond the introspective power of agents. In practice, this means that we update the initial possibility with every expression of the form $B_{x_1}B_{x_2}\ldots B_{x_n}\phi$ that does not force us to update the $\langle u, u_1, \ldots \rangle$ sub-branches. For instance, in figure 1, we will not update with $B_aB_bB_aB\phi$ because this would force us to update $\langle u \rangle$. Analogously, we will not update with $B_a(W_aW_bB_b)$, where $\phi$ is the modal expression $W_aW_bB_b$. This shows that, if $\phi$ is sufficiently complex, any update will be impossible, unless we increase the introspective power of agents.

**Definition 6** Belief updates.

The multiagent assertive belief update of $\pi$ with $\phi$, in symbols $\pi \boxplus \phi$ is the set $((\pi + \beta_1) + \ldots) + \beta_k$ where the $\beta_i$'s are all the expressions of the form $B_{x_1}\ldots B_{x_m}\phi$ such that updating $\pi$ with them does not force us to update $\langle u \rangle$.

Since $A$ and $M$ are finite, the set of $\beta_i$'s is finite too. Common belief updates correspond to assertions. Note that, in (6), we have disregarded the possibility that an agent may be insincere and does not update her own information states with $\phi$. Should this be taken into account, we would modify definition (6) by excluding all branches $B_a\phi$, $B_aB_a\phi$, that is $(B_a)^n\phi$ from the update procedure.

For implicatures, we need updates that do not apply to the hearers. For instance, for two agents $a$ and $b$, if $a$ implies that $\phi$, the only directly intended effect is that $b$ believes that $a$ believes $\phi$, not that $b$ himself believes $\phi$. We use a definition parallel to (6), except for the fact that the belief expressions all end with $B_a\phi$. The non-sincerity of $a$ may be mimicked by excluding $(B_a)^n\phi$ branches, as in the previous case.

**Definition 7** The multiagent $a$-centered implicative belief update of $\pi$ with $\phi$, in symbols $\pi \boxplus_a \phi$ is the set $((\pi + \beta_1) + \ldots) + \beta_k$ where the $\beta_i$'s are all the expressions of the form $B_{x_1}\ldots B_{x_m}B_a\phi$ such that updating with them does not force us to update $\langle u \rangle$.

Modal adverbs such as *probablement* 'probable' give rise to assertive modal updates of the form $\pi \vdash Prob \phi$. Parentheticals behave differently. They give rise to two updates. The implicature they convey enters a speaker-centered implicative belief update (def. 7). The assertion they qualify enters an assertive belief update (def. 6). However, in
contrast to non-qualified assertions, the asserted content is modalized in a way that reflects the hedging profile of the parenthetical. For instance, $\phi$, parait-il ‘I hear’ gives rise to an implicative update with the proposition that the speaker heard that $\phi$ and to an assertive update with the (modal) proposition that is true only in these worlds where what the speaker heard about $\phi$ is true. More generally,

**Definition 8** If $a$ reports that $\phi$ from the source $\sigma$, the modal formula $AGR_{\sigma} \phi$ (‘$\phi$ if one agrees with $\sigma$’) is true at $\pi$ iff $\phi$ is true everywhere at $\pi((\sigma, AGR))$, which corresponds to the worlds where what $\sigma$ says about $\phi$ is true.

When the speaker uses parait-il or d’après X ‘according to X’, she triggers an assertive update with $AGR_{\sigma} \phi$ or $AGR_{X} \phi$, $\sigma$ being an unknown source of information.

## 4 Conclusion

In this chapter, we have considered the status of parentheticals that qualify assertions, contrasting them with modal adverbs. We have argued that they are not part of what is said and that they trigger conventional implicatures à la Grice. We have proposed to represent such implicatures as updates of the mutual information that concerns the hearer’s beliefs and have shown how this can be done in a finitist version of Gerbrandy’s theory of possibilities. However, we have also taken into account the fact that, as qualifiers of assertions, such parenthetics contribute to the update of the common ground in a specific way. The distinction between the propositional content of an epistemic update and its locus allows us to make room for different dimensions of update and to solve the Gricean problem of non-central speech acts.

## References


