PARTS, COUNTERPARTS, AND MODAL OCCURRENTS

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A modal occurring is an individual entity that stretches across possible worlds. It is a trans-world individual. It consists of parts that are located in different worlds just as ordinary objects consist of parts that are located at different places, or processes and other events (so-called temporal occurrences) consist of parts that are located at different times. If we do not think that quantification over possible worlds is intelligible, then the notion of a modal occurring will be unintelligible to us. (Some philosophers feel that way about the notion of a temporal occurring.) If we think that quantification over possible worlds is intelligible but that mereological fusions should be restricted to entities existing in the same world, then to us the notion of a modal occurring will be intelligible but empty. (Three-dimensionalists feel that way about temporal occurrences, at least insofar as these are meant to include objects besides events.) However, if we think that quantification over possible worlds is intelligible and that mereological fusions should not be so restricted, then we are well off. For then we can make sense of modal talk even if we cannot make sense of trans-world identity.

The purpose of this paper is to illustrate how this can be done. First I will lay out the basic apparatus of the theory of modal occurrences. Next I will show how to interpret modal talk extensionally within the apparatus regardless of whether trans-world identity is allowed, i.e., regardless of whether distinct worlds can overlap. Then I will show that ruling out trans-world identity by requiring possible worlds to be pairwise discrete is all that is needed to reconstruct the bulk of counterpart theory (i.e., to define the concept of a counterpart and to derive the standard postulates governing that concept). This will throw some new light on
counterpart theory itself. Finally, I will argue that the account allows us to see the indeterminacy of our modal intuitions as being part and parcel with the indeterminacy of our criteria for individuating modal occurrences, and that this indeterminacy is best explained in terms of semantic (as opposed to theoretical or ontic) vagueness.

Mereological preliminaries

I will assume a standard mereological background constructed around the binary relational predicate ‘x is part of y’, written ‘Pxy’\(^1\). I take this predicate to be true when \(x\) is any sort of part of \(y\), including an improper part, so that \(Pxy\) will be consistent with \(x\) and \(y\) being the same. The predicate for proper parthood, along with other familiar mereological predicates, can be introduced by definition in the usual way:

1. \(PPxy =_d f Pxy \land \neg x = y\)
x is a proper part of \(y\) iff \(x\) is a part of \(y\) but is distinct from \(y\).

2. \(Oxy =_d f \exists z (Pzx \land Pzy)\)
x overlaps \(y\) iff something is part of both.

3. \(POxy =_d f Oxy \land \neg x = y\)
x properly overlaps \(y\) iff \(x\) overlaps \(y\) but is distinct from \(y\).

As axioms I will assume those of standard extensional mereology, except for the principle of unrestricted fusion. Thus, I will assume parthood to be a reflexive, antisymmetric, and transitive relation—a partial ordering—subject to a strong supplementation principle:

4. \(\forall x Pxx\)
Everything is part (viz., an improper part) of itself.

\(^1\) For an overview of mereology see Simons 1987.
(5) $\forall x \forall y (P_{xy} \land P_{yx} \rightarrow x=y)$  
No two things are part of each other.

(6) $\forall x \forall y (P_{xy} \land P_{yz} \rightarrow P_{xz})$  
The parts of a thing’s parts are themselves part of that thing.

(7) $\forall x \forall y (\forall z (P_{zx} \rightarrow O_{zy}) \rightarrow P_{xy})$  
If every single part of $x$ overlaps $y$, then $x$ itself is part of $y$.

I shall not assume the axiom of unrestricted fusion but I shall not impose any special conditions pro or against any kind of mereological fusion, either. In the present context I will leave the issue open. If, given a condition $\phi$, there is a mereological fusion of all the $\phi$ers, then it is unique by (7) and we can refer to it by a definite description\textsuperscript{2}:

(8) $\sigma x \phi =_{df} \exists z \forall y (O_{zy} \leftrightarrow \exists x (\phi \land O_{yx}))$  
The fusion of all $x$ such that $\phi$ (if it exists) is that thing which overlaps all and only those things which overlap some $x$ such that $\phi$.

The notation for finitary sums is introduced accordingly:

(9) $x + y =_{df} \sigma z (z=x \lor z=y)$  
The sum of $x$ and $y$ is the fusion of $x$ and $y$ (if it exists).

**Modal occurrents**

The theory of modal occurrents can be built on the basis of mereology by adding two more primitives, namely, a one-place predicate ‘$Wx$’ (read: ‘$x$ is a possible world’) and an individual constant $\alpha$ (for ‘the actual world’). These primitives are governed by the following axioms:

\textsuperscript{2} I will assume descriptions to be handled in standard, Russellian fashion.
(10) \( \mathcal{W} \alpha \)
\( \alpha \) is a world—the actual world.

(11) \( \forall x \exists y (W_y \land Oxy) \)
Everything overlaps some world.

(12) \( \forall x (W_x \land \exists y PPxy) \)
Every world includes something.

The first axiom is to fix the intended role of ‘\( \alpha \)’ and the second axiom is to fix the intended meaning of ‘\( \mathcal{W} \)’ on the understanding that a world is the mereological fusion of everything in it. The third axiom is only needed to go along with the standard assumption that there are no «empty worlds», an assumption which generalizes—modally—the assumption that the domain of discourse is non-empty. Strictly speaking this assumption is not essential and one could allow for empty worlds just as one can allow for the empty domain; but this would introduce complications into the logical machinery which would only becloud the basic picture\(^3\). On the other hand, there is no need to require that all worlds overlap, or that no two worlds overlap, for such requirements do not belong to a general formal theory of worlds. For the same reason we need not require that worlds be maximal individuals, i.e., that no world is part of another.

With this apparatus in hand, a modal occurrence is defined, quite simply, as any object in the domain of quantification whose mereological make up does not include any worlds:

(13) \( M_x =_{df} \forall y (W_y \to \neg Pyx) \)
\( x \) is a modal occurrence iff no world is part of \( x \).

Typically, a modal occurrence is a trans-world individual—i.e., it has fragments that are too large to be mereologically included in a single world. This is how the concept has been originally introduced in the literature (though a preferred idiom is David Lewis’s ‘modal continuant’\(^4\)). However, our definition includes world-

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3 On these matters I refer to Garson 1984.
4 The terminology is introduced in Lewis 1983: 41. I prefer my idiom because it parallels the distinction between occurcents and continuants familiar from the literature on
bound individuals as a limit case and is therefore to be understood very broadly. We can always introduce the limit case by definition:

\[(14) \quad PMx =_{df} \forall y(Wy \rightarrow \neg Pxy)\]
\[x \text{ is a proper modal occurrent iff } x \text{ is (a modal occurrent which is) not part of any world } y.\]

Given our modest assumptions concerning possible worlds, it is also possible for a trans-world modal occurrent to be world-bound, provided there is a world large enough to contain it all. But whether or not a modal occurrent is world-bound, it certainly has world-bound parts, the largest of which may be thought of as the «actualizations» (or stages) of that occurrent in the relevant worlds. We can make this more precise through the following auxiliary definitions:

\[(15) \quad WBxy =_{df} Mx \land Wy \land Pxy\]
\[x \text{ is world-bound in } y \text{ iff } x \text{ is a modal occurrent and } y \text{ is a world wholly containing } x.\]

\[(16) \quad WFxyz =_{df} Pxy \land WBxz\]
\[x \text{ is a world fragment of } y \text{ in } z \text{ iff } x \text{ is a part of } y \text{ which is world-bound in } z.\]

\[(17) \quad WSxyz =_{df} WFxyz \land \forall u(WFuyz \rightarrow \neg PPzu)\]
\[x \text{ is a world stage of } y \text{ in } z \text{ iff } x \text{ is a maximal world fragment of } y \text{ in } z.\]

Some immediate corollaries of these definitions:

\[(18) \quad \forall x \forall y(WBxy \rightarrow WSxxxy)\]
Every world-bound occurrent is a world stage of itself (in the relevant world).

\[\text{temporal persistence: a temporal occurrent is an entity that has temporal parts whereas a temporal continuant is an entity which is present in its entirety at any time at which it exists. By analogy, then, I would use ‘modal continuant’ to refer to those items which are present in their entirety in any worlds in which they exist—exactly the opposite of a (proper) modal occurrent.}\]
(19) \( \forall x \forall y \forall z (WSxyz \land WByz \rightarrow x = y) \)

Every world-bound occurrence coincides with its own world stage (in the relevant world).

(20) \( \forall x \forall y \forall z \forall u (WSxyz \land WSuyz \rightarrow x = u) \)

Every occurrence has at most one world stage in any world.

Given (20), we can introduce a definite description to talk about world stages (where they exist):

(21) \( x^y =_{df} \exists z WSzxy \)

The unique stage (if it exists) of occurrence \( x \) in world \( y \).

Modalities

To interpret modal discourse against the background of the formal theory of modal occurrences, we interpret names and predicates as usual, i.e., as objects and relations defined on a universe of discourse. In particular, ‘\( \alpha \)’ will pick out an object in the extension of ‘W’, i.e., a world. However, the intuition is that ordinary names such as ‘Pavarotti’ correspond to proper modal occurrences and ordinary predicates such as ‘is a tenor’ correspond to sets of world stages. (Ordinary relational predicates such as ‘is a better tenor than’ will correspond to relations among world stages.) A statement such as

(22) Pavarotti is a tenor

will then count as true iff the actual world stage of Pavarotti is a tenor. And a statement such as

(23) Pavarotti might have been a ballerina

will count as true iff some world stage of Pavarotti is a ballerina\(^5\). This intuition is familiar from the literature on four-dimensionalism, according to which a ordinary proper name such as

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\(^5\) This is the intuition put forward in Lewis 1983: 40–42. See also Lewis 1986: 213–217.
‘Pavarotti’ denotes a temporally extended individual and a statement of the form

(24) Pavarotti will be a ballerina

is true iff there is a future temporal part of Pavarotti which is a ballerina. In effect, if we construed possible worlds intuitively as temporal stages of the actual world, then the theory of modal occurrences outlined here would yield a theory of temporal occurrences which is compatible with much literature on the topic. (This would be especially evident if we assumed all worlds to be pairwise discrete, for typically a four-dimensional ontology has no room for continuants, i.e., things wholly existing at different times.)

Keeping with this basic framework, the intuition can be generalized as follows. Let L be the given modal (first-order) language and let \( L^* \) be the non-modal language that results from L by replacing the modal operators with the primitive vocabulary of mereology and the theory of modal occurrences. For every formula \( \phi \) of L we define a corresponding \( L^* \)-formula \( \phi' \) («\( \phi \) holds at \( y \)») by recursion,

\[
(25) \quad \text{if } \phi = Rt_1 \ldots t_n, \text{ then } \phi' = Rt_1' \ldots t_n'
\]

\[
\text{if } \phi = \neg \psi, \text{ then } \phi' = \neg \psi'
\]

\[
\text{if } \phi = \psi \cdot \chi, \text{ then } \phi' = \psi' \cdot \chi'
\]

\[
\text{if } \phi = \forall x \psi, \text{ then } \phi' = \forall x(\exists! x' \rightarrow \psi')
\]

\[
\text{if } \phi = \exists x \psi, \text{ then } \phi' = \exists x(\exists! x' \land \psi')
\]

\[
\text{if } \phi = \Box \psi, \text{ then } \phi' = \forall x(Wx \rightarrow \psi')
\]

\[
\text{if } \phi = \Diamond \psi, \text{ then } \phi' = \exists x(Wx \land \psi')
\]

where ‘\( \cdot \)’ is any binary connective and ‘\( \exists! \)’ is the existence predicate defined by:

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6 Lewis himself is a proponent of a four-dimensionalist ontology in which tensed statements are interpreted according to this schema. See e.g. Lewis 1986: 204.

7 The first conjunct in the clause for ‘\( \exists! \)’ is actually redundant if descriptions are understood à la Russell. We leave it in to ensure immediate duality between ‘\( \exists! \)’ and ‘\( \forall \)’.
(26) \( \exists x = \text{df} \exists y (y = x) \).

Then the translation of any given L-formula \( \phi \) into \( L^* \) is given by the function \( \tau \) defined by setting:

(27) \( \tau(\phi) = \phi^\alpha \).

Here are some illustrative examples:

(28) \( \tau(Fb) = Fb^\alpha \)

'b is F' means 'The actual world stage of b is F'. (Thus, if b has no world fragments in the actual world, then the statement is false.)

(29) \( \tau(\forall x Fx) = \forall x (E!x^\alpha \rightarrow Fx^\alpha) \)

'Everything is F' means 'Every actual world stage is F'.

(30) \( \tau(\square Fb) = \forall x (Wx \rightarrow Fb^\alpha) \)

'Necessarily b is F' means 'In every world, b's world stage is F'. (Thus, if there is a world in which b has no world fragments, then the statement is false.)

(31) \( \tau(\Diamond \exists x Fx) = \exists y (Wy \land \exists x (E!x^\alpha \land Fx)) \)

'Possibly, something is F' means 'There is a world in which something has a stage that is F'.

(32) \( \tau(\exists x \Diamond Fx) = \exists x (E!x^\alpha \land \exists y (Wy \land Fx)) \)

'Something is possibly F' means 'Something which has an actual world stage has a possible world stage that is F'.

(33) \( \tau(\forall x (Fx \rightarrow \square Gx)) = \forall x (E!x^\alpha \rightarrow (Fx^\alpha \rightarrow \forall z (Wz \rightarrow Gx^\alpha))) \)

'Every F is necessarily G' means 'Everything that has an actual world stage that is F has in every world a stage that is G'. (Thus, if something x has an actual world stage that is F, then the statement is false if there are worlds in which x lacks a world stage altogether even if every possible world stage of x is indeed G.)

It is worth observing that our definition of 'modal occurrent' (13) makes it impossible to use this machinery to express modal
discourse that involves counterfactualizing about entire worlds. One might want to say, for example, that this world of ours could have been better than it is, or that it might have been something else than a world, or that something which is not a world (e.g., Switzerland) might have been a world. These are not statements that can be expressed in L* unless we relax our definitions so as to allow for modal occurrents which include worlds among their parts. As a matter of fact, there would be nothing wrong with such a revision (except that ‘M’ would then have to be assumed as a primitive predicate). However, the definition given above reflects a limitation that is common to all familiar ways of expressing the semantics of modal discourse, so I will stick to it for comparative convenience.

**Counterparts**

At this point it is natural to establish a link between all the things that are world stages of the same occurrent. And the natural way of doing so is to think of such things as *counterparts* of one another, so that our translation scheme would effectively amount to a scheme for representing counterfactual statements about world stages in terms of factual statements about their counterparts. For example, on this understanding a statement such as

(23) Pavarotti might have been a ballerina

would be true just in case there is at least one counterpart of the actual world stage of Pavarotti which is a ballerina. More generally, let us define:

(34) $Ax =_{df} PPx \alpha$

$x$ is actual iff $x$ is part of the actual world.

(35) $Ixy =_{df} PPxy \land Wy$

$x$ is in $y$ iff $y$ is a world of which $x$ is a proper part.
(36) \( \text{Cxy} \equiv_{df} \exists z \exists u \exists v (\text{Mz} \land \text{WS}xz u \land \text{WSy}z v) \)

x is a counterpart of y iff x and y are world stages of a common occurrence z. (Here the fact that we are not assuming the unrestricted fusion axiom is relevant, for otherwise anything x in any world u would count as a counterpart of anything y in any world v for the simple reason that x and y would count as world stages of their sum \( x + y \). Without that axiom, however, we are free to think of the occurrences that exist as being counterpart-interrelated, in fact maximally counterpart-interrelated.\(^8\).)

Together with ‘W’, these three predicates form the primitive vocabulary of Lewis’s counterpart theory.\(^9\) And the appropriateness of these definitions is shown by the fact that the bulk of counterpart theory can now be derived from our three axioms as long as we make one extra assumption—viz., that there are no trans-world identities:

(37) \( \forall x \forall y (\text{W}x \land \text{W}y \rightarrow \neg \text{PO}xy) \)

No two worlds overlap.

Given this additional assumption, all the postulates of counterpart theory can be proved as theorems:

(38) \( \forall x \forall y (\text{Ix}y \rightarrow \text{W}y) \)

Nothing is in anything except a world. [This follows directly from definition (35)]

(39) \( \forall x \forall y \forall z (\text{Ix}y \land \text{Ixz} \rightarrow y = z) \)

Nothing is in two worlds. [From axiom (37) via definitions (1), (2), (3), and (35)]

(40) \( \forall x \forall y (\text{C}xy \rightarrow \exists z \text{Ixz}) \)

Whatever is a counterpart is in a world. [Directly from definition (36), second conjunct, via definitions (15), (16), (17), and (35)]

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8 Such occurrences correspond to what Lewis (1983: 41) calls ‘modal continuants’ and Lewis 1986: 214 calls ‘*-possible individuals’.

9 See Lewis 1968.
(41) \( \forall x \forall y (Cxy \rightarrow \exists z lyz) \)
Whatever has a counterpart is in a world. [Directly from
definition (36), third conjunct, via definitions (15), (16),
(17), and (35)]

(42) \( \forall x \forall x \forall z (lxy \land lzy \land Cxz \rightarrow x=z) \)
Nothing is a counterpart of anything else in its world.
[From corollary (20) via definitions (17), (35), and (36)]

(43) \( \forall x \forall y (lxy \rightarrow Cxx) \)
Anything in a world is a counterpart of itself. [From cor-
ollary (18) via definitions (15), (35), and (36)]

(44) \( \exists x (Wx \land \forall y (lxy \leftrightarrow Ay)) \)
Some world contains all and only actual things. [From
axiom (10) via definitions (34) and (35)]

(45) \( \exists x Ay \)
Something is actual. [From axioms (10) and (12) via de-
finition (34)]

There is but one notable difference between this reconstruction
and Lewis's original formulation, namely, our axioms have the
following consequence which is not provable in Lewis's
counterpart theory:

(46) \( \forall x \forall y (Cxy \leftrightarrow Cyx) \)
Counterpathood is symmetric. [From definition (36)]

To rule this out we would have to introduce C as a primitive rela-
tion and replace definition (36) by an axiom which preserves only
the implication from left to right:

(47) \( \forall x \forall y (Cxy \rightarrow \exists z \exists u \exists y (Mz \land WSxz \land WSyuv)) \)
A thing and its counterparts are world stages of a com-
mon occurrent.

Then every postulate of counterpart theory would still be provable
except for (43), which would have to be assumed as an
independent axiom.
Comparisons

We have derived counterpart theory from the theory of modal occurrences, and this may be viewed as an exercise in theory formulation. However, there is a difference between the way we would now express modal discourse in counterpart theory and the way modal discourse is expressed in Lewis’s original version. The difference concerns the way ordinary names are interpreted, and consequently the definition of the translation function from \( L \) to \( L^* \). For us, as we have seen, a name such as ‘Pavarotti’ denotes a modal occurrent and a statement such as

(23) Pavarotti might have been a ballerina

is true iff there is some world stage of Pavarotti which is a ballerina. For Lewis ‘Pavarotti’ denotes only the actual world stage of a counterpart-interrelated modal occurrent, and (23) is true iff there is some counterpart of Pavarotti, i.e., some world stage of that occurrent, which is a ballerina. Formally, this means that for Lewis the translation function \( \tau \) is based on a different recursive definition of \( \phi^v \), namely:

(25') if \( \phi = R_{t_1 \ldots t_n} \), then \( \phi^v = \phi \)
if \( \phi = \neg \psi \), then \( \phi^v = \neg \psi^v \)
if \( \phi = \psi \cdot \chi \), then \( \phi^v = \psi^v \cdot \chi^v \)
if \( \phi = \forall x \psi \), then \( \phi^v = \forall x (Ixy \rightarrow \psi^v) \)
if \( \phi = \exists x \psi \), then \( \phi^v = \exists x (Ixy \land \psi^v) \)
if \( \phi = \square \psi x_1 \ldots x_n \), then \( \phi^v = \forall z \forall z_1 \ldots \forall z_n (W_{z} \land I_{z} \psi \land C_{z} x_{j} \land \ldots \land I_{z} z \land C_{z} x_{n} \rightarrow \psi^{z} z_{j} \ldots z_{n}) \)
if \( \phi = \Diamond \psi x_1 \ldots x_n \), then \( \phi^v = \exists z \exists z_1 \ldots \exists z_n (W_{z} \land I_{z} \psi \land C_{z} x_{j} \land \ldots \land I_{z} z \land C_{z} x_{n} \land \psi^{z} z_{j} \ldots z_{n}) \)

With reference to our earlier examples, this yields the following alternative translations:

(28') \( \tau(Fb) = Fb \)
‘\( b \) is \( F \)’ means ‘\( b \) is \( F \)’.

(29') \( \tau(\forall x Fx) = \forall x (Ix \alpha \rightarrow Fx) \)
‘Everything is \( F \)’ means ‘Everything actual is \( F \)’.
(30') \( \tau(\Box Fb) = \forall x \forall y (Wy \land Ixy \land Cxb \to Fx) \)

‘Necessarily \( b \) is \( F \)’ means ‘Every counterpart of \( b \), in any world, is \( F \)’.

(31') \( \tau(\Diamond \exists x Fx) = \exists y (Wy \land \exists x (Ixy \land Fx)) \)

‘Possibly, something is \( F \)’ means ‘There is a world in which something is \( F \)’.

(32') \( \tau(\exists x \Diamond Fx) = \exists x (Ix\alpha \land \exists y \exists z (Wz \land Iyz \land Cyx \land Fy)) \)

‘Something is possibly \( F \)’ means ‘Something actual has a counterpart that is \( F \)’.

(33') \( \tau(\forall x (Fx \to \Box Gy)) = \forall x (Ix\alpha \to (Fx \to \forall y \forall z (Wz \land Iyz \land Cyx \to Gy))) \)

‘Every \( F \) is necessarily \( G \)’ means ‘If something actual is \( F \), then its counterparts, in any world, are \( G \)’.

There are several differences between these translations and our earlier translations in (28)–(33). How important are they?

One difference, for example, is that the truth of the translation in (30) depends on whether \( b \) has a world stage in every world, while the translation in (30') does not depend on that—that is, it does not depend on the corresponding question of whether \( b \) has a counterpart in every world. A similar point could be made about (33) and (33'); the former, but not the latter, depends on whether \( x \) has a world stage (respectively: a counterpart) in every world. However these are differences that reflect only a minor disagreement concerning the scope of modal statements: the translation function defined in (25) takes a modal statement to range over all possible worlds whereas Lewis’s translation function (25') takes a modal statement to range exclusively over those worlds in which the relevant counterparts exist. This is a minor difference because it can easily be accommodated by changing the relevant clauses in one translation function or the other. For example, we could replace the last two clauses of (25) with the following clauses and the difference would disappear\(^\text{10}\):

\(^\text{10}\) The amendment of the clause for ‘\( \exists \)’ is actually redundant if descriptions are understood \( \textit{a la} \) Russell. Again, we include it here for the sake of duality.
(25'') if $\phi = \square \psi x_1, \ldots, x_n$, then $\phi^v = \forall z(W_z \land E!x_1^z \land \ldots \land E!x_n^z \\
\rightarrow (\psi x_1^z, \ldots, x_n^z)^z)$

if $\phi = \Diamond \psi x_1, \ldots, x_n$, then $\phi^v = \exists z(W_z \land E!x_1^z \land \ldots \land E!x_n^z \\
\land (\psi x_1^z, \ldots, x_n^z)^z)$

Another difference is that Lewis’s translation function (25’) exploits the possibility that an object (a world-bound individual) has more than one counterpart in some worlds, whereas our translation function (25) ignores that possibility altogether. More precisely, definition (36) allows an object to have more than one counterpart per world, but (25) as well as its revised version (25'') say that the truth-value of a modal statement about the actual world stages of certain occurrents $x_1, \ldots, x_n$ depends exclusively on the properties of the possible world stages of these occurrents—and each occurrent has at most one world stage in every world. So, for example, according to both (25) and (25'') a statement such as

(23) Pavarotti might have been a ballerina

is true iff Pavarotti’s world stages comprise a ballerina. These world stages are counterparts of Pavarotti$^\alpha$ (i.e., of that actual thing that Lewis calls ‘Pavarotti’). They are those counterparts of Pavarotti$^\alpha$ which fall under the same name: ‘Pavarotti’. But Pavarotti$^\alpha$ may have other counterparts besides those. He will have as many counterparts as there are modal occurrents that share the same actual world stage as the occurrent Pavarotti. And those are not taken into account by our clauses for expressing modal statements in terms of modal occurrents (whereas they are taken into account by Lewis’s clauses).

If desired, however, this difference between the two translation functions can also be eliminated. All we have to do is to emend our clauses so that a statement such as (23) will be true iff at least one counterpart of Pavarotti$^\alpha$ is a ballerina, regardless of whether that counterpart is a world stage of Pavarotti or of some other modal occurrent whose actual world stage is Pavarotti$^\alpha$. In general, this amounts to revising (25) or (25'') along the following lines:
(25′′′) if $\phi = \Box \psi x_1, \ldots, x_n$, then $\phi^y = \forall z_1 \forall z_2 \ldots \forall z_n (Wz \land Cz_1 z_1^y \land \ldots \land Cz_n z_n^y \rightarrow (\psi z_1 \ldots z_n)^y)$

if $\phi = \Diamond \psi x_1, \ldots, x_n$, then $\phi^y = \exists z_1 \exists z_2 \ldots \exists z_n (Wz \land Cz_1 z_1^y \land \ldots \land Cz_n z_n^y \land (\psi z_1 \ldots z_n)^y)$

These conditions are now pretty close to those of (25′).

So it does not look as though the choice between taking C as a primitive relation, as in Lewis’s original formulation of counterpart theory, or defining it in terms of modal occurrents, as in our formulation, will make much difference at all. As Lewis would say, the two theories appear to be verbal variants of one and the same technique for expressing modal talk extensionally. New terminology is not a new theory\(^{11}\).

However that is not true. One important difference between the two theories concerns the way they handle indeterminacy – the sort of indeterminacy that gets into counterfactuals about named individuals\(^ {12}\). And this is an important difference insofar as this sort of indeterminacy – has sometimes been claimed to cause troubles for counterpart theory.

Suppose that we have doubts about (23). Suppose, that is, that there is indeterminacy about whether or not Pavarotti might have been a ballerina. Assuming that there is no indeterminacy as to who Pavarotti is in this world, this means that there is indeterminacy as to what Pavarotti’s counterparts are. What sort of indeterminacy is this? In the original formulation of counterpart theory it certainly need not be a form of ontological indeterminacy. If the counterpart relation is understood as any old relation of comparative overall similarity, then the indeterminacy is merely semantic (or pragmatic): the predicate ‘C’ can suffer from the vagueness of our stipulations, and such vagueness can be dealt with in semantic (or pragmatic) terms. It can be differently resolved in different contexts. This is how Lewis himself sees the matter, and he insists that vagueness is therefore not a problem. Still, C is the primitive relation of counterpart theory. So to say that C is vague (albeit only semantically or pragmatically vague)

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11 This is claimed both in Lewis 1983 and in Lewis 1986.
12 The point is made informally in Bennett 1988: 63–64, though with reference to a slightly different way of understanding the theory of modal occurrents.
is to concede that the theory is vague—it is a theory built around a vague primitive predicate. And to some people this is enough to say that the theory itself—and the reductive account of modality that the theory is supposed to provide—is seriously defective.

Consider now the theory of modal occurrences. In fact, let us distinguish two cases depending on whether we go along with the initial formulation (25) (or its revision based on (25\(^{11}\)) or with the later formulation based on (25\(^{111}\)). If we go along with the latter, then the indeterminacy is not at all semantic (or pragmatic). The indeterminacy exhibited by a statement such as (23) is truly ontological. This is because C itself is construed as an ontological relation, in terms of P (which is an ontological relation). If it is indetermined whether there is a counterpart of the actual Pavarotti which is a ballerina, then it must be indetermined whether there exists a modal occurring \(x\) such that \(x^\alpha = \text{Pavarotti}^\alpha\) and \(x\) is a ballerina for some world \(y^{13}\). So it is indetermined whether certain occurrences exist. So there is indeterminacy as to what there is. The result is that now counterpart theory is no longer a vague theory, but it calls for a vague ontology.

By contrast, if we go along with the initial formulation (25) (or its revision based on (25\(^{11}\)), taking ordinary names to denote modal occurrences, then the indeterminacy of a statement such as (23) need not be a source of worry: it does not make the theory vague and it does not call for a vague ontology. Simply, the statement is *prima facie* indeterminate because it is indeterminate which modal occurring is picked out by the name ‘Pavarotti’, hence which world stages count as relevant counterparts of Pavarotti’s actual stage. And this *prima facie* indeterminacy is real if some of the admissible candidates contain a world stage that is a ballerina while other candidates do not. In fact, a supervenational semantics suggests itself naturally here. A term is indeterminate insofar as there appear to be many ways of assigning it a referent, all of them compatible with the way the term is used. Hence the truth-value of a statement containing indeterminate terms is naturally construed as a function of its truth-values under the various admissible ways of assigning a unique referent to those indeterminate terms. If the statement is true under all such assignments,

\[13\] Let us assume that the predicate ‘is a ballerina’ involves no vagueness.
then we may take it to be true *simpliciter*; the unmade semantic stipulations do not matter because what the statement says is true regardless (or *super-true*, as Kit Fine has it). This is the case of a non-modal statement such as (22), for example:

(22) Pavarotti is a tenor.

Likewise, if the statement comes out false under every admissible way of assigning a unique referent to its indeterminate terms, then we may regard it as false (or *super-false*) in spite of the indeterminacy. The negation of (22) would be a case in point. It is only when the statement is true under some assignments and false under others that there is trouble. In such cases, nothing will settle the question for us and the statement will fall into a truth-value gap. This seems to be precisely the situation exhibited by a statement such as (23).

To sum up, then, we have three different accounts of the indeterminacy that gets into counterfactuals, depending on which theory we work with. On Lewis’s original formulation of counterpart theory, it is the *theory* itself that is vague. On its closest mereological reconstruction in terms of modal occurents, it is the *ontology* that is vague. And on our initial formulation of the theory of modal occurents, it is just the names that we use that are vague: the indeterminacy exhibited by a statement such as (23) is just a case of *semantic* vagueness on a par with many others, and can be handled like any other case. In this sense, ‘Pavarotti’ is modally vague just as ‘Everest’ is spatially vague or ‘the industrial revolution’ is temporally (and spatially) vague. I think that these differences between the three accounts are important. And I think that they speak in favor of the theory of modal occurents insofar as ontological vagueness and theoretical vagueness can be objected to on independent grounds.

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14 See Fine (1975), where a supervaluational semantics for vague predicates is developed. The notion of a supvaluation goes back to van Fraassen (1966), who used it to provide a semantics for free logic, i.e., a logic in which singular terms may lack a reference. Our situation here is perfectly dual.

15 Heller (1990) has a similar account of the indeterminacy involved in our intuitions about persistence and identity through time.

16 I summarize my views on ontological vagueness in Varzi 2001.
A coda on existence and possibility

What are the consequences of this way of putting things for the prospects of modal extensionalism, i.e., the view that one can provide an extensional reductive analysis of modal discourse? Clearly, one important consequence is that by placing the vague-ness of our modal discourse in the semantics of the names we use, rather than in the underlying ontology or in the semantics of the primitive notions around which the analysis is formulated, the account is not directly affected by the phenomenon of vague-ness—neither more nor less than a physicalist account of the mind-body problem, for example, is affected by the vagueness of ordinary proper names. (Is the candy that Pavarotti is presently chewing part of his body? Will it be part of Pavarotti’s body only after he has swallowed it? Only after he has digested it? On closer look, our earlier assumption concerning the determinacy of the actual Pavarotti will have to be discharged too.) There is, however, also a seemingly negative consequence of our way of explaining modality in terms of modal occurrents. For the appeal to modal occurrents may be said to yield a paradox.

Briefly put, the paradox is that on the proposed reductive analysis of modality one is forced to say that all possible things exist (modal realism), but not all existing things are possible. This follows from the fact that possibility is explained in terms of what goes on in different worlds, hence what goes on across worlds is not possible. The possible individuals are the world-bound stages, so the trans-world individuals (the proper modal occurrents) are impossible. They exist, but they cannot possibly exist. And what goes for existence goes for truth. Let Pavarotti₀ be one of the various modal occurrents that can plausibly be associated with the vague name ‘Pavarotti’. Then the statement

(48) Pavarotti₀ exists

is a necessary falsehood. It is true that there is such a thing as Pavarotti₀, but it cannot be true.

This puzzle arises in the theory of modal occurrents but not in the original theory of counterparts in which names designate what we have called world stages, i.e., things that bear the relation I
(in) to the things which are W (possible worlds). At least, the puzzle does not arises in the original theory as long as we do not formulate it in mereological terms. Accordingly, we would have another difference between counterpart theory and the theory of modal occurents—and a remarkable difference to say the least.\footnote{As a matter of fact Lewis (1983) does interpret I as P and he does accept unrestricted composition, so the puzzle does affect his understanding of the theory; but it need not.}

Now, one could fix things by changing the relevant notion of existence. To exist in a world is not to be part of that world (i.e., to be in that world, as per definition (36)). Rather, to exist in a world is to have parts in that world. If we accept this, then Pavarotti\(_0\) would exist and the paradox would dissolve. Not only, but the difference between such terms as ‘the winged horse’ and ‘the round square’ (say) would be restored too: the former, not the latter, would denote an object that has parts in some possible worlds, so the former, not the latter, would denote (perhaps vaguely) a possibly existing thing. And what goes for existence goes for truth. The proposition

(49) The-winged-horse exists

would be possibly true, but

(50) The-round-square exists

would be necessarily false.\footnote{The point is made in Hudson 1999.}

If this way out is accepted, then the initial paradox dissolves, and so does the relevant difference between the theory of modal occurents and the theory of counterparts. Moreover, modal realism is vindicated and modality explained away: to be possible is just to exist in some world, i.e., to have parts that exist in some world.

On the other hand, one could object that a change in the notion of existence is a change in the strength of the reductive analysis. For the reductive analysis presupposes what Lewis calls the principle of Plenitude: every way that a world could be is a way that some world is.\footnote{See Lewis 1986: 86–92.} Roughly speaking, this means that anything can
coexist with anything, and anything can fail to coexist with anything. *Strictly* speaking, the no-overlap principle (37) prevents this from being true in counterpart theory, so what the principle really says is this: any number of possible individuals has duplicates which coexist in some world as well as duplicates that do not coexist—where two things are duplicates iff they have exactly the same natural properties and their parts can be mapped onto one another in such a way that corresponding parts have the same natural properties too. (Never mind the details of defining «natural property».) The principle of Plenitude is crucially important if the account is to deliver a complete analysis of the notion of possibility in extensional terms, for it is this principle that guarantees that the analysis leaves «no gaps in logical space». But now there is trouble. For there is no way that modal occurrences can have duplicates which coexist. There is no way, that is, that modal occurrences can «recombine» because such a recombination would violate the no-overlap axiom. If there is a world that contains the whole of Pavarotti₀, then that world will also include among its parts the actual world stage of Pavarotti, contrary to the no-overlap principle²⁰. So if Pavarotti₀ is a possible thing, then it cannot recombine with other occurrences. And if it can recombine then it is not a trans-world occurrence.

The only way out, it seems to me, is to give up the no-overlap principle itself—or else we must go back to a vague theory of counterparts²¹.

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20 This point was brought to my attention by Borghini 2000: §3.2.1.
21 Thanks to Andrea Borghini, Berit Brogaard, and Chris Partridge for helpful comments on an early draft of this paper.
References


